PHI 312 : pset 5

1. Suppose that X and Y are sets, and that $f : X \to Y$ is an injection. Show that if X is non-empty (i.e. has at least one element), then there is a function $g : Y \to X$ such that $gf = 1_X$.

2. Show that the function g in the previous problem is a surjection.

3. Let $\Sigma = \{q_0, q_1, \dots\}$, and let T be the theory in Σ with axioms $q_i \to \neg q_j$, for all i, j such that $i \neq j$. Let $\Sigma' = \{p_0, p_1, \dots\}$, and let T' be the theory with axioms $p_0 \to p_i$, for $i = 0, 1, 2, \dots$.

   (a) How many models does T have? How many models does T' have?

   (b) Show that there is no essentially surjective translation $f : T \to T'$.