PHI 312 : pset 5

1. Let A and B be Boolean algebras. Let $f : A \to B$ be a function such that $f(a \vee b) = f(a) \vee f(b)$, and $f(\neg a) = \neg f(a)$. Show that f is a homomorphism.

2. An element $a \in B$ is said to be an **atom** if $a \neq 0$, and for all $x \in B$, if $x \neq 0$ and $x \leq a$, then $x = a$. If a is an atom, define $f : B \to \{0, 1\}$ by $f(x) = 1$ iff $a \leq x$. Show that f is a homomorphism.

3. Please solve **one** of the following two problems:

   (a) Show that there is no Boolean algebra with exactly three elements.

   (b) Show that if A and B are Boolean algebras with exactly four elements, then there are exactly two isomorphisms $f : A \to B$ and $g : A \to B$.