

To show: $(p \wedge q) \rightarrow r \Rightarrow p \rightarrow (q \rightarrow r)$

Fitch:

1		$(p \wedge q) \rightarrow r$
2		p
3		q
4		$p \wedge q$
5		r
6		$q \rightarrow r$
7		$p \rightarrow (q \rightarrow r)$

Lemmon:

1	(1)	$(p \wedge q) \rightarrow r$	A
2	(2)	p	A
3	(3)	q	A
2,3	(4)	$p \wedge q$	2,3 $\wedge$ in
1,2,3	(5)	r	1,4 $\rightarrow$ el
1,2	(6)	$q \rightarrow r$	3,5 $\rightarrow$ in
1	(7)	$p \rightarrow (q \rightarrow r)$	2,6 $\rightarrow$ in

Sequent Calculus:

1.	$(p \wedge q) \rightarrow r \Rightarrow (p \wedge q) \rightarrow r$	A
2.	$p \Rightarrow p$	A
3.	$q \Rightarrow q$	A
4.	$p, q \Rightarrow p \wedge q$	2,3 $\wedge$ in
5.	$(p \wedge q) \rightarrow r, p, q \Rightarrow r$	1,4 $\rightarrow$ el
6.	$(p \wedge q) \rightarrow r, p \Rightarrow q \rightarrow r$	5 $\rightarrow$ in
7.	$(p \wedge q) \rightarrow r \Rightarrow p \rightarrow (q \rightarrow r)$	6 $\rightarrow$ in

To show: $\exists x(Px \wedge Qx) \Rightarrow \exists xPx$

Fitch:

1		$\exists x(Px \wedge Qx)$
2		$\overline{a} \quad Pa \wedge Qa$
3		$\overline{\quad} \quad Pa$
4		$\exists xPx$
5		$\exists xPx$

Lemmon:

1	(1)	$\exists x(Px \wedge Qx)$	A
2	(2)	$Pa \wedge Qa$	A
2	(3)	Pa	2 $\wedge$ el
2	(4)	$\exists xPx$	3 $\exists$ in
1	(5)	$\exists xPx$	1,2,4 $\exists$ el

Sequent Calculus:

1.	$\exists x(Px \wedge Qx) \Rightarrow \exists x(Px \wedge Qx)$	A
2.	$Pa \wedge Qa \Rightarrow Pa \wedge Qa$	A
3.	$Pa \wedge Qa \Rightarrow Pa$	2 $\wedge$ el
4.	$Pa \wedge Qa \Rightarrow \exists xPx$	3 $\exists$ in
5.	$\exists x(Px \wedge Qx) \Rightarrow \exists xPx$	1,4 $\exists$ el

To show: $\forall x(Nx \rightarrow Ox), \neg\exists yOy \Rightarrow \neg\exists xNx$

Fitch:

1		$\forall x(Nx \rightarrow Ox)$
2		$\neg\exists yOy$
3		$\exists xNx$
4		^a Na
5		$Na \rightarrow Oa$
6		Oa
7		$\exists yOy$
8		$\exists yOy \wedge \neg\exists yOy$
9		$\exists yOy \wedge \neg\exists yOy$
10		$\neg\exists xNx$

Lemmon:

1	(1)	$\forall x(Nx \rightarrow Ox)$	A
2	(2)	$\neg\exists yOy$	A
3	(3)	$\exists xNx$	A
4	(4)	Na	A
1	(5)	$Na \rightarrow Oa$	1 $\forall$ elim
1,4	(6)	Oa	4,5 $\rightarrow$ elim
1,4	(7)	$\exists yOy$	6 $\exists$ intro
1,2,4	(8)	$\exists yOy \wedge \neg\exists yOy$	2,7 $\wedge$ intro
1,2,3	(9)	$\exists yOy \wedge \neg\exists yOy$	3,4,8 $\exists$ elim
1,2	(10)	$\neg\exists xNx$	3,9 $\neg$ intro

Sequent Calculus:

1.	$\forall x(Nx \rightarrow Ox) \Rightarrow \forall x(Nx \rightarrow Ox)$	A
2.	$\neg\exists yOy \Rightarrow \neg\exists yOy$	A
3.	$\exists xNx \Rightarrow \exists xNx$	A
4.	$Na \Rightarrow Na$	A
5.	$\forall x(Nx \rightarrow Ox) \Rightarrow Na \rightarrow Oa$	1 $\forall$ elim
6.	$\forall x(Nx \rightarrow Ox), Na \Rightarrow Oa$	4,5 $\rightarrow$ elim
7.	$\forall x(Nx \rightarrow Ox), Na \Rightarrow \exists yOy$	6 $\exists$ intro
8.	$\forall x(Nx \rightarrow Ox), \neg\exists yOy, Na \Rightarrow \exists yOy \wedge \neg\exists yOy$	2,7 $\wedge$ intro
9.	$\forall x(Nx \rightarrow Ox), \neg\exists yOy, \exists xNx \Rightarrow \exists yOy \wedge \neg\exists yOy$	3,8 $\exists$ elim
10.	$\forall x(Nx \rightarrow Ox), \neg\exists yOy \Rightarrow \neg\exists xNx$	9 $\neg$ intro