

Summary of facts established in Chapter 1.

1. It is false that: if an arrow is both epi and mono then it is iso.
2. If  $f, g$  are monic then  $gf$  is monic. If  $f, g$  are epic then  $gf$  is epic.
3. If  $gf$  is monic then  $f$  is monic. If  $gf$  is epic then  $g$  is epic.
4. In **Mon**, **Rng** and **Top**, there are epis that are not surjective.
5. In **Grp**, an arrow is monic iff it is injective, and epic iff it is surjective. (Ex. 5)
6. An arrow with a left inverse is monic. The converse does not hold (e.g.  $\mathbb{Z}_3 \twoheadrightarrow S_3$  in **Grp**).
7. An arrow with a right inverse is epic. The converse does not hold.
8. If  $f$  is monic and split epic then  $f$  is iso. By duality, also for epic and split monic.
9. An arrow is regular iff it has a left or right inverse. (Ex. 7)
10. In **Set**, epis are split (axiom of choice).
11. In **Set**, monos from a nonempty set are split.
12. If  $f$  is a split mono (split epi), and  $F$  is a functor, then  $Ff$  is a split mono (split epi).
13. If  $f$  is iso then  $Ff$  is iso.
14. It is false that: a functor  $F$  must preserve monos and epis.
15. Any functor from **Set** preserves epis as well as monos from a nonempty set.
16. Faithful functors reflect monics. i.e. if  $F : C \rightarrow B$  is faithful and  $Ff$  is monic, then  $f$  is monic.