

CPL practice

Updated: September 19, 2007

1. Strengthening the Antecedent

$$P \rightarrow Q \vdash (P \wedge R) \rightarrow Q$$

$$(P \vee Q) \rightarrow R \vdash P \rightarrow R$$

2. Weakening the Consequent

$$P \rightarrow Q \vdash P \rightarrow (Q \vee R)$$

$$P \rightarrow (Q \wedge R) \vdash P \rightarrow Q$$

3. Associativity of $\vee$

$$P \vee (Q \vee R) \dashv\vdash (P \vee Q) \vee R$$

4. Commutativity of $\vee$

$$P \vee Q \dashv\vdash Q \vee P$$

5. [No name]

$$A \rightarrow C, B \rightarrow C \vdash (A \vee B) \rightarrow C$$

6. Transitivity

$$P \rightarrow Q, Q \rightarrow R \vdash P \rightarrow R$$

7. Prefixing

$$P \rightarrow Q \vdash (R \rightarrow P) \rightarrow (R \rightarrow Q)$$

8. Suffixing

$$P \rightarrow Q \vdash (Q \rightarrow R) \rightarrow (P \rightarrow R)$$

9. Contraction

$$P \rightarrow (P \rightarrow Q) \vdash P \rightarrow Q$$

10. Expansion

$$P \rightarrow Q \vdash P \rightarrow (P \rightarrow Q)$$

11. Permutation

$$P \rightarrow (Q \rightarrow R) \vdash Q \rightarrow (P \rightarrow R)$$

12. Fusion

$$P \rightarrow (Q \rightarrow R) \dashv\vdash (P \wedge Q) \rightarrow R$$

13. Ex Falso Quodlibet

$$P \wedge \neg P \vdash Q$$

14. [No name]

$$P \rightarrow (Q \rightarrow R) \vdash (P \rightarrow Q) \rightarrow (P \rightarrow R)$$

15. DeMorgan's Laws

$$\neg P \wedge \neg Q \dashv\vdash \neg(P \vee Q)$$

$$\neg P \vee \neg Q \dashv\vdash \neg(P \wedge Q)$$

16. Excluded Middle

$$\vdash P \vee \neg P$$

17. Distribution

$$(P \wedge Q) \vee (P \wedge R) \dashv\vdash P \wedge (Q \vee R)$$

$$(P \vee Q) \wedge (P \vee R) \dashv\vdash P \vee (Q \wedge R)$$

18. [No name]

$$P \rightarrow Q, R \rightarrow S \vdash (P \vee R) \rightarrow (Q \vee S)$$

19. Positive Paradox

$$Q \vdash P \rightarrow Q$$

20. Negative Paradox

$$\neg P \vdash P \rightarrow Q$$

21. Chain Order

$$\vdash (P \rightarrow Q) \vee (Q \rightarrow P)$$

22. Material Implication

$$P \rightarrow Q \dashv\vdash \neg P \vee Q$$

$$\neg(P \rightarrow Q) \dashv\vdash P \wedge \neg Q$$

23. Peirce's Law

$$\vdash ((P \rightarrow Q) \rightarrow P) \rightarrow P$$

24. Glivenko Sentence

$$\vdash (P \leftrightarrow Q) \vee (P \leftrightarrow R) \vee (Q \leftrightarrow R)$$

25. Weak Reductio

$$\vdash (P \rightarrow \neg P) \rightarrow \neg P$$

26. Modus Tollens

$$P \rightarrow Q, \neg Q \vdash \neg P$$