Curie's Principle and spontaneous symmetry breaking

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Abstract In 1894 Pierre Curie announced what has come to be known as Curie's Principle: the asymmetry of effects must be found in their causes. In the same publication Curie discussed a key feature of what later came to be known as spontaneous symmetry breaking: the phenomena generally do not exhibit the symmetries of the laws that govern them. Philosophers have long been interested in the meaning and status of Curie's Principle. Only comparatively recently have they begun to delve into the mysteries of spontaneous symmetry breaking. The present paper aims to advance the discussion of both of these twin topics by tracing their interaction in classical physics, ordinary quantum mechanics, and quantum field theory. The features of spontaneous symmetry that are peculiar to quantum field theory have received scant attention in the philosophical literature. These features are highlighted here, along with a explanation of why Curie's Principle, though valid in quantum field theory, is nearly vacuous in that context.

1. Introduction

The statement of what is now called Curie's Principle was announced in 1894 by Pierre Curie:

> (CP) When certain effects show a certain asymmetry, this asymmetry must be found in the causes which gave rise to it (Curie 1894, p. 401).[1]

This principle is vague enough to allow some commentators to see in it profound truth while others see only falsity (compare Chalmers 1970; Radicati 1987; van Fraassen 1991, pp. 23-24; and Ismael 1997). I will give a formulation of (CP) that makes it virtually analytic. Nevertheless, I claim, the principle so understood is useful in guiding the search for explanations of observed asymmetries.

Some commentators credit Curie (1894) with recognizing the importance of the phenomena that lie at the the roots of the concept of spontaneous symmetry breaking (see, for example, Cao 1997, p. 281). If this attribution is correct it would seem to create an irony for other commentators who take spontaneous symmetry breaking to undermine Curie's Principle (see Radicati 1987). Part of the difficulty here is due to the facts that there is no canonical definition of spontaneous symmetry breaking and that this notion is used in different ways in different contexts. At least three different strands to the discussion of this concept can be distinguished.

The first and, perhaps, most fundamental strand leads to cases where there is a symmetry of the equations of motion or field equations of a system that is not a symmetry of a state of the system that is of special physical significance.[2] Without the qualifier "of special physical significance,"[3] cases of spontaneous symmetry breaking would be all too easy to find. That a law of motion/field equation obeys a symmetry principle–say, time reversal invariance or invariance under spatial rotations–does not imply that a particular solution, or a particular state belonging to the solution, exhibits the symmetry at issue. Indeed, the solutions states exhibiting the symmetry at issue may be the exception rather than the rule. For example, Einstein's gravitational field equations are time reversal invariant, but the set of Friedman-Walker-Robertson cosmological solutions that are time symmetric about some time slice are of "measure zero" (see Castagnino et al. 2003). And one would guess that a similar "measure zero" result would hold for rotationally symmetric states in a classical or quantum theory with laws of motion that are invariant under spatial rotations. I take it that this is the kind of point Curie was making when he wrote that "it is asymmetry that creates phenomena" (Curie 1894, p. 400).[4] But if it is true that the symmetries so beloved by physicists are typically broken by the phenomena we actually observe, one can wonder about the warrant for setting such store by these symmetries (see Kosso 2000). This issue will not be treated here since my focus is on issues in the foundations of physics rather than on the epistemology and methodology of science. What cannot be avoided here is the issue of how to give content to the qualification "of special physical significance," since without a specification of content "spontaneous symmetry breaking" does not point to any definite set of phenomena requiring special treatment. While the physics literature on spontaneous symmetry breaking does not show unanimity, the focus tends to be on ground states, or vacuum states, or equilibrium states at a definite temperature, states which one might expect should exhibit a symmetry of the basic physical laws governing them.[5] This leads to the second strand of the discussion.

It is often said that cases of spontaneous symmetry breaking involve degeneracy of the ground state or the vacuum state. Such assertions are *prima facie* puzzling because they are intended to apply not only to classical physics but to ordinary quantum mechanics (QM) and quantum field theory (QFT) as well; but in the latter instances it is typically the case that the ground state, if it exists, is unique. The resolution of this apparent conundrum is simple but far-reaching. In QFT vacuum degeneracy is indeed a key feature of spontaneous symmetry breaking; but the relevant sense of degeneracy is radically different from that encountered in ordinary QM because it involves the existence of many unitarily inequivalent representations of the canonical commutation relations, within each of which resides a unique vacuum state. What sets spontaneous symmetry breaking in QFT apart from other commonplace example of spontaneous symmetry breaking is the fact that a symmetry of the laws of motion is not unitarily implementable–a feature that implies but is not implied by the the failure of the vacuum state to exhibit the symmetry. One of the main purposes of the present paper is to illuminate this novel feature which is largely untouched by the extant philosophical literature on spontaneous symmetry breaking.[6]

A third strand of spontaneous symmetry breaking makes further contact with Curie's Principle by adding a temporal dimension. It is sometimes said that spontaneous symmetry breaking concerns cases where an asymmetry emerges "spontaneously" in the sense that the breaking is sudden and seemingly without any precipitating asymmetric cause, e.g. a symmetric rod is subjected to an increasingly large symmetric load until it suddenly buckles asymmetrically. Such cases are in prima facie conflict with Curie's Principle. But if I am right about the status of this Principle, the cases in question cannot be counterexamples; rather, the principle tells us that there is more to such cases than first meets the eye, and it tells us where to look for the something more.

For better or for worse, the discussions of Curie's Principle and spontaneous symmetry breaking have become entangled with one another. My aim is to use the interaction of these topics to help to illuminate the important features of each of them. The paper is organized as follows. Section 2 provides my preferred way of understanding Curie's Principle. Section 3 considers a toy example of spontaneous symmetry breaking in classical physics. The *sine qua non* feature of spontaneous symmetry breaking–the failure of a symmetry of the laws of motion/field equations to be a symmetry of a physically significant state–can hold for the ground state in classical mechanics and (in a radically different sense) for the vacuum state in QFT. Section 4 explains why the analogue of this situation does not hold in ordinary QM. Section 5 introduces what most textbook presentations take to be the defining feature of spontaneous symmetry breaking in QFT–the existence of a symmetry of the Lagrangian that is not unitarily implementable. Section 6 uses the algebraic formulation of QFT to show that this seemingly mysterious situation is not only not mysterious but a mathematically straightforward commonplace. The discovery of spontaneous symmetry breaking in physics is then seen as the discovery that important physical systems instantiate this mathematical commonplace. Section 7 shows that a strengthened version of Curie's Principle is valid in QFT. At the same time it also indicates why the Principle is nearly vacuous in QFT. Section 8 takes up the fate of spontaneous symmetry breaking in the Higgs mechanism used to generate the masses of elementary particles while finessing an embarrassing consequence of symmetry breaking. Section 9 raises questions about the status of an (alleged) idealization used to generate spontaneous symmetry breaking in QFT. Concluding remarks are contained in Section 10.

2. A formulation of Curie's Principle

For present purposes[7] I propose to construe Curie's Principle as a conditional, asserting that *if*

> (CP1) the laws of motion/field equations governing the system are deterministic
>
> (CP2) the laws of motion/field equations governing the system are invariant under a symmetry transformation
>
> (CP3) the initial state of the system is invariant under said symmetry

then

> (CP4) the final state of the system is also invariant under said symmetry

Suppose for the moment that we understand determinism to mean that for any pair of evolutions allowed by the laws of motion/field equations, sameness of initial states implies sameness of final states. And suppose that for the moment we understand the invariance of laws of motion/field equations to mean that if an initial state is evolved for any chosen Δt to produce a final state and then the symmetry operation is applied to the final state, the resulting state is the same as obtained by first applying the symmetry operation to the initial state and then evolving the resulting state for the same Δt. Then it follows (as the reader can easily verify) that if the initial state is invariant under the said symmetry operation, so is the evolved state. Of course, the appearance of analyticity of the proposed version of Curie's Principle may be an illusion fostered by the vagueness necessitated by the generality of the level of discussion. So it is important that in instances where the relevant concepts of determinism and invariance are made precise, the proposed version of Curie's Principle is demonstrable. This I claim to be the case, and concrete example will be provided below.

Despite its analyticity, Curie's Principle is useful whenever we are in the market for a causal explanation of an asymmetry in some state of interest. The Principle tells us that the asymmetry is due to one (or more) of three factors: either the initial state is asymmetric; or the laws of motion/field equations do not respect the symmetry; or else determinism fails in a way that allows an asymmetry to creep in. Typically, it is the first and least interesting of these possibilities that in fact obtains. If the initial state does not exhibit any discernible asymmetry, the Principle assures us that there is a relevant asymmetry below the threshold of observability. And if a discernible asymmetry appears suddenly, then the Principle tells us that the initially indiscernible asymmetry was rapidly amplified so as to pass the threshold of observability. Before considering concrete examples of such phenomena, it will be helpful to introduce some apparatus that will get heavy usage here.

For the large majority of theories of modern physics, the equations of motion/field equations are derivable from an action principle: the dynamically possible motions are those that extremize the action $\mathfrak{A} = \int \mathcal{L}(\mathbf{u}, \mathbf{u}^{(n)}, \mathbf{x}) d\mathbf{x}$; these motions satisfy (generalized) Euler-Lagrange equations. The Lagrangian $\mathcal{L}$ has been written as a function of the independent variables $\mathbf{x}$, the dependent variables $\mathbf{u}$, and the derivatives $\mathbf{u}^{(n)}$, up to some order n, of the dependent variables with respect to the independent variables. A group $\mathcal{G}$ whose elements are mappings $g : (\mathbf{u}, \mathbf{x}) \rightarrow (\mathbf{u}', \mathbf{x}')$ is said to constitute a *variational symmetry* if (roughly speaking) it leaves the action $\mathfrak{A}$ invariant. A variational symmetry is necessarily a symmetry of the Euler-Lagrange equations that follow from the action principle, i.e. it carries solutions to solutions.[8] If $\mathcal{G}$ is a finite parameter Lie group, Noether's first theorem shows that a variational symmetry implies the existence of conserved currents (see Section

5 for an example).[9] A variational symmetry which yields unit Jacobian for the transformation of the independent variables $\mathbf{x}$ produces strict numerical invariance of the action. All of the examples of symmetries considered below satisfy this condition. For such cases I will use the standard (but imprecise) terminology in the physics literature and speak of a *symmetry of the Lagrangian*.

Specializing for sake of concreteness to the case of classical mechanics, the dependent variables are the configuration variables $\mathbf{q}$, and the one independent variable is time t. If the Lagrangian $\mathcal{L}(\mathbf{q}, \dot{\mathbf{q}}, t)$, $\dot{\mathbf{q}} := d\mathbf{q}/dt$, is nonsingular in the sense that the determinant of the Hessian $H_{ij} := \partial^2\mathcal{L}/\partial\dot{q}^i\partial\dot{q}^j$ does not vanish, then the initial conditions $(\mathbf{q}(0), \dot{\mathbf{q}}(0))$ pick out a unique solution to the Euler-Lagrange equations. Thus, (CP1) is secured. If the symmetry at issue is a symmetry of the Lagrangian, then (CP2) is secured. Thus, Curie's Principle implies that if the initial state $(\mathbf{q}(0), \dot{\mathbf{q}}(0))$ is invariant under the said symmetry, then so is any later state $(\mathbf{q}(t), \dot{\mathbf{q}}(t))$, $t > 0$, in the unique solution picked out by the initial data. When the Lagrangian is singular, arbitrary functions of t show up in the Euler-Lagrange equations and determinism apparently breaks down. A standard move for such cases is to maintain determinism by discovering "gauge freedom" in the state description (see Section 8).

3. Spontaneous symmetry breaking in classical physics

Greenberger (1978) has provided a toy model that neatly illustrates the seemingly spontaneous appearance of an asymmetry in a classical system. A bead, subject to the force of gravity, slides frictionlessly on a hoop that is rotating about its vertical diameter with an angular velocity Ω (see Fig. 1). The Lagrangian for the bead is

$$
\begin{aligned}
\mathcal{L}_1 &= \frac{1}{2}m(R^2\dot{\phi}^2 + \Omega^2R^2\sin^2\phi) - mgR(1-\cos\phi) \qquad (1)\\
&= \frac{1}{2}mR^2\dot{\phi}^2 - V_{eff}(\Omega)\\
V_{eff}(\Omega) &:= mgR\left[(1-\cos\phi) - \frac{1}{2}\frac{\Omega^2R}{g}\sin^2\phi\right]
\end{aligned}
$$

where m is the mass of the bead, R is the radius of the hoop, and ϕ is the angular displacement of the bead. The Lagrangian $\mathcal{L}_1$ is invariant under the transformation $\phi \to \phi' = -\phi$. When the angular velocity of the hoop is below the critical value $\Omega_c := \sqrt{g/R}$ there is a unique minimum of the effective potential $V_{eff}(\Omega)$ at $\phi = 0$ (see Fig. 2a) and, thus, a unique ground state of the system. But above Ω_c, $V_{eff}(\Omega)$ develops two local minima at finite $\phi = \pm\phi_o$ (see Fig. 2b) and, thus, there are two asymmetric ground states. Furthermore, $\phi = 0$ becomes an unstable point. So when $\Omega > \Omega_c$ we do not expect to see the bead remain at the bottom of the hoop but rather expect to see it slide up one side of the hoop or other, resulting in a ϕ-non-symmetric state.

In fact, however, if the initial state is *exactly* symmetric–$\phi(0) = 0$ and $\dot{\phi}(0) = 0$–then cranking up the angular velocity from a value less than Ω_c to a value greater than Ω_c will not result in an asymmetric state–the bead will stay at the bottom of the hoop. To break the symmetry some asymmetry is required. The most natural thing is to postulate an ensemble of small departures from the initially symmetric state, with the statistical distribution of states in the ensemble adjusted appropriately to explain the actually observed frequency with which the bead goes up the $+\phi$ side vs. the $-\phi$ side.

Thus far the toy model of symmetry breaking treats the angular velocity Ω of the hoop as an exogenous variable under the control some external mechanism. But nothing prevents us from adding another term to the Lagrangian so as to obtain a deterministic dynamics for Ω as well as ϕ. As long as the total Lagrangian respects the $\phi \to -\phi$ symmetry–as can easily be arranged–we obtain a fully self-contained, deterministic story of why the bead went up one one side of the hoop rather than the other.

The change of shape in the potential from that in Fig. 2a to that in Fig. 2b is analogous to what happens in a *phase transition*. Properly speaking such terminology is applied to collective phenomena where the concepts of temperature T and entropy S are given a meaning, and the relevant change of shape is that of the free energy $F = U - TS$, where U is the internal energy of the system. However, the limited goal here is to describe features of spontaneous symmetry breaking in fundamental theories where the basic variables do not include T and S.[10] Extending by analogy the term phase transition to the present cases, the point becomes that phase transitions produce spontaneous symmetry breaking in the sense of the rapid emergence of a discernible asymmetry. But, at the risk of belaboring what should be obvious, phase transitions are not needed to secure the core features of spontaneous symmetry breaking–the failure of states of special interest to reflect symmetries of the equations of motion, the degeneracy of grounds states, etc.

4. Curie's Principle and spontaneous symmetry breaking in ordinary QM

In cases where an application of my preferred form of Curie's Principle is valid, a violation of (CP4) in the form of a final state that does not respect the symmetry at issue requires a violation of one or more of (CP1)-(CP3). A violation of (CP1)–the assumption of a deterministic evolution–is contemplated in accounts of the quantum measurement process which envision a momentary interruption of the deterministic Schrödinger dynamics by a "collapse" of the state vector into an eigenstate of the observable being measured.[11] Such a collapse assuredly can produce a transition from a symmetric initial state to an asymmetric final state since a symmetric superposition can be built out of asymmetric states. Fortunately, there is no need here to purse this matter into the morass of the quantum measurement problem since, as will be argued in Section 6, measurement collapse cannot produce the key features of spontaneous symmetry breaking in QFT; in particular, a symmetric vacuum state cannot be built as a superposition of degenerate, asymmetric vacuum states. It would be a worthwhile project to develop a statistical form of Curie's Principle that could apply in cases where strict determinism fails but statistical determinism holds, but such a project is beyond the scope of this paper.[12]

The core sense of spontaneous symmetry breaking–the failure of a state of special interest to exhibit a symmetry of the equations of motion–can, of course, occur without the help of measurement collapse. But it is worth pointing out that the degeneracy of the ground state is absent in the quantum mechanical treatment of simple one-dimensional systems that are analogous to the bead-and-hoop model. (A quantum system is said to have a *ground state* just in case the energy spectrum is at least partially discrete and there is a lowest energy eigenvalue.) Consider, for example, a particle confined to one spatial dimension and moving in an external potential $V(q)$. The Lagrangian has the form $\mathcal{L}_2 = \frac{1}{2}m\dot{q}^2 - V(q)$. Suppose that $V(q)$ is differentiable and bounded from below and that $V(q) \to +\infty$ as $q \to \pm\infty$. Then when the system is quantized it is found that–regardless of whether or not $V(q)$ has local minima away from 0–not only is the lowest energy eigenvalue nondegenerate, but the entire energy eigenvalue spectrum is discrete and non-degenerate. Furthermore, if the potential is symmetric about the origin (analogously to the effective potential in Fig. 2b), i.e. $V(q) = V(-q)$, and thus $q \to -q$ is a symmetry of $\mathcal{L}_2$, then the eigenfunctions $|E\rangle$ of energy have a definite parity, i.e. $\hat{P}|E\rangle = \pm|E\rangle$ where $\hat{P}$ is the parity operator. Thus, $\langle E|\hat{q}|E\rangle = \langle E| \pm q \pm |E\rangle = \langle E|\hat{P}^{-1}\hat{q}\hat{P}|E\rangle = \langle E| - \hat{q}|E\rangle$, which means that $\langle E|\hat{q}|E\rangle = 0$ and, *a fortiori*, the expectation value of $\hat{q}$ in the ground state is 0.

The non-degeneracy of the ground state holds quite generally in QM. For instance, if the Hamiltonian operator has the form $-\Delta + V(q)$, with V continuous and bounded below, and is essentially self-adjoint, then the ground state, if it exists, is unique up to phase (see Glimm and Jaffe 1987, Sec. 3.3). Nor can ordinary QM display degeneracy of the ground state in the sense we are about to encounter in QFT since unitarily inequivalent representations of the canonical commutation relations cannot arise for systems with a finite number of degrees of freedom. The precise statement of this result is given in the Stone-von Neumann theorem.[13] It is the breakdown of this theorem for systems with an infinite number of degrees of freedom that makes the existence of unitarily inequivalent representations a characteristic feature of QFT.

5. Spontaneous symmetry breaking in QFT

If one thinks of the vacuum as a state of "nothingness," then it is difficult to imagine how the vacuum can fail to share a symmetry of the Lagrangian or the Hamiltonian. But once physicists' imagination was expanded to encompass this possibility, they began to see spontaneous symmetry breaking all over the map, from condensed matter physics to cosmology and much in between. Only simple toy models will be considered here. Although these models lack realism, they serve to isolate some of the key features of spontaneous symmetry breaking in QFT that appear in the much more complicated models and they nicely illustrate the foundations issues of interest to philosophers.[14]

A standard example used to illustrate the defining feature of spontaneous symmetry breaking in QFT involves a zero mass scalar field φ on Minkowski spacetime.[15] The Lagrangian is $\mathcal{L}_3 = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi$, which is invariant under the transformations of the field $\varphi \to \varphi' = \varphi + \chi$, where $\partial_\mu\chi = 0$. As a result of the equation of motion $\Box\varphi = 0$ the current $j^\mu := \partial^\mu\varphi$ obeys the conservation law $\partial_\mu j^\mu = 0$. Now suppose that when the field is quantized, there is a state $|0\rangle$ that is identified as the vacuum state and that this state is invariant under spatial translations.[16] And suppose for *reductio* that there is a self-adjoint operator $\hat{Q}$ associated with the global charge $Q := \int j^0 d^3x$, where the '^{0}' indicates the time component and where the integration is taken over all space. If it existed $\hat{Q}$ would be the generator of a one-parameter group of unitary operators $\hat{U}(\chi) := \exp(i\chi\hat{Q})$ implementing the symmetry $\hat{\varphi} \rightarrow \hat{\varphi}' = \hat{\varphi} + \chi$ for the quantized field. From the invariance of the vacuum state under spacetime translations, whose generators are the four-momenta $\hat{P}^\mu$, and from $[\hat{P}^\mu, \hat{Q}] = 0$, it follows that

$$\begin{aligned} \langle 0|\hat{\jmath}^0(x)\hat{Q}|0\rangle &= \langle 0|\exp(-\hat{P}^\mu \cdot x)\hat{\jmath}^0(0)\exp(+\hat{P}^\mu \cdot x)\hat{Q}|0\rangle \\ &= \langle 0|\hat{\jmath}^0(0)\hat{Q}|0\rangle \end{aligned} \tag{2}$$

Thus, the norm-square of the total charge operator $\hat{Q}$ in the vacuum state is

$$\begin{aligned} \langle 0|\hat{Q}\hat{Q}|0\rangle &= \int \langle 0|\hat{\jmath}^0(x)\hat{Q}|0\rangle d^3x \\ &= \int \langle 0|\hat{\jmath}^0(0)\hat{Q}|0\rangle d^3x \end{aligned} \tag{3}$$

It follows that $\langle 0|\hat{Q}\hat{Q}|0\rangle = \infty$ unless $\hat{Q}|0\rangle = 0$. The latter cannot hold, for if it did it would follow that $\hat{U}(\chi)|0\rangle = |0\rangle$, resulting in $\langle 0|\hat{U}(\chi)\hat{\varphi}\hat{U}^{-1}(\chi)|0\rangle = \langle 0|\hat{\varphi}|0\rangle$. But also $\langle 0|\hat{U}(\chi)\hat{\varphi}\hat{U}^{-1}(\chi)|0\rangle = \langle 0|\hat{\varphi} + \chi|0\rangle = \langle 0|\hat{\varphi}|0\rangle + \chi$, which together require $\chi = 0$. There still remains the possibility that $\hat{Q}$ is defined on some dense domain of the Hilbert space that does not include the vacuum state. The gap is is filled by Streater (1965) where it is shown that the non-existence of a unitary $\hat{U}(\chi)$ follows from the facts that the vacuum state is the unique Poincaré invariant state and that, if it exists, $\hat{U}(\chi)$ commutes with Poincaré transformations. But the above calculation is important because it shows that a frequently used characterization of spontaneous symmetry breaking is, at best, heuristic. This characterization uses the condition $\hat{Q}|0\rangle \neq 0$ or the consequence that $\langle 0|\hat{\varphi}'|0\rangle \neq 0$, where $\hat{\varphi}' := [\hat{Q}, \hat{\varphi}]$. Strictly speaking, these equations are not meaningful since $|0\rangle$ is not in the domain of $\hat{Q}$.[17]

The upshot is that the symmetry $\varphi \rightarrow \varphi' = \varphi + \chi$ of the Lagrangian is not represented by a one-parameter (χ) group of unitary operators, contrary to the usual experience with continuous symmetries in QM.[18] In the following section this consequence will be recast in the algebraic formulation of QFT, and it will be shown that the symmetry $\varphi \rightarrow \varphi' = \varphi + \chi$ connects unitarily inequivalent representations of the algebra of field observables, each with its own vacuum state–this is the precise sense in which spontaneous symmetry breaking in QFT involves degeneracy of the vacuum. But before turning to that discussion there are a few more points that need to be made with the help of the current apparatus.

The symmetry $\varphi \to \varphi' = \varphi + \chi$ of the Lagrangian $\mathcal{L}_3$ is referred to as "internal" since it concerns the transformation of the dependent variables of the action. It is also called "global" because χ does not depend on the space-time variables. The invariance of the Lagrangian under such transformations invokes Noether's *first* theorem, which implies that if the global symmetries form a finite parameter Lie group then there is an associated "proper" conservation law in which a current is conserved as a consequence of the laws of motion–as is exemplified in the above model. There are other examples where the symmetry transformations are "local" in that they involve arbitrary functions of the independent variables. These examples invoke Noether's *second* theorem which implies that the Euler-Lagrange equations are not independent and, thus, that there is an apparent violation of determinism.[19] The implications of such cases for Curie's Principle and spontaneous symmetry breaking will be taken up in Section 8.

Suppose now that the above model is expanded to include a potential term, giving the modified Lagrangian $\mathcal{L}_3' = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)$. If $V(\varphi)$ is not a constant function, $\mathcal{L}_3'$ will not be invariant under the continuous transformations $\varphi \to \varphi' = \varphi + \chi$ with $\partial_\mu\chi = 0$. If , however, $V(\varphi)$ is an even function of φ, then $\mathcal{L}_3'$ is invariant under the discrete inversion transformation $\mathcal{I} : \varphi \to \varphi' = -\varphi$. But since Noether's theorems do not apply to such symmetries, the above argument for the non-unitary implementability of the symmetry is hamstrung. Attempts to fill the gap with heuristic considerations of the following kind can be found at several places in the physics literature. Suppose that the potential $V(\varphi)$ has a shape similar to that of Fig. 2b. Then the classical field equation that results from $\mathcal{L}_3'$ has solutions $\varphi = \pm v \neq 0$ corresponding to the local minima of $V(\varphi)$. This is taken to suggest that the vacuum expectation value of the quantized field $\hat{\varphi}$ will not be zero but will have one of the two values $\pm v$ (see Goldstone 1961, 162). If the suggestion is accepted, then it cannot be the case that the inversion symmetry $\mathcal{I}$ is implemented by a unitary operator $\hat{U}(\mathcal{I})$ and that the vacuum is invariant (up to phase) under $\hat{U}(\mathcal{I})$. For otherwise $v = \langle 0|\hat{\varphi}|0\rangle = \langle 0|\hat{U}^{-1}(\mathcal{I})\hat{\varphi}\hat{U}(\mathcal{I})|0\rangle = \langle 0| - \hat{\varphi}|0\rangle = -v$, contradicting the assumption that $v \neq 0$.[20]

The upshot of this heuristic argument is not quite the full sense of the spontaneous breaking of the inversion symmetry since the contradiction can be escaped by holding onto the unitary implementability of the inversion symmetry and rejecting the invariance of the vacuum under this symmetry. The latter alternative does capture one feature of spontaneous break down of symmetry–namely, the symmetry of the vacuum is lower than the symmetry of the Lagrangian–but this is not the full official sense of spontaneous symmetry breaking for QFT. In any case, the heuristics being employed are problematical, as can be illustrated by the example from Section 3 of the particle moving in one spatial dimension under the influence of a potential $V(q)$ with $V(q) = V(-q)$. If $V(q)$ has a shape similar to that of Fig. 2b, then the classical equations of motion have solutions with $q = \pm const$ corresponding to the local minima of $V(q)$. But when this system is quantized the ground state (or any energy eigenstate) is non-degenerate, and the expectation value of the "field" $\hat{q}$ in the ground state (or any energy eigenstate) is 0. Of course, one knows from the beginning that systems in ordinary QM cannot exhibit full spontaneous breaking. So perhaps the analogical reasoning going from the classical field to the quantum field is valid while its counterpart going from the classical particle to its quantization in ordinary QM is not. But one would like more than a *perhaps*.

In sum, while it may be the case that discrete symmetries are spontaneously broken, a convincing argument for the case is lacking.

6. Spontaneous symmetry breaking in algebraic QFT[21]

For many physicists the algebraic formulation of QFT offers little aside from pedantry. True or not, the pedantry is helpful to philosophers seeking to understand interpretational issues. What makes the algebraic apparatus especially helpful in the present context is that it makes pellucid the features of QFT that are concerned with the existence of unitarily inequivalent representations of the canonical commutation relations (CCR), a phenomenon that cannot occur in the ordinary QM of systems with a finite number of degrees of freedom. It is a common mantra in QFT that the choice of a representation of the CCR depends on the dynamics ("Haag's theorem")[22]. Spontaneous symmetry breaking in QFT is concerned with cases where the dynamics does *not* determine the representation in the sense that inequivalent representations exist for the *same* dynamics. Thus, a full understanding of spontaneous symmetry breaking in QFT cannot be gained by beavering away within any one representation of the CCR–as was done in the preceding section–but must take into account structural features of QFT that cut across different representations. It is exactly these structural features which the algebraic approach is designed to illuminate, and any other approach which does offer this illumination will be adequate for present purposes.

In the algebraic approach a quantum system is described by C^*-algebra $\mathcal{A}$ and a state ω specified by a positive linear functional on $\mathcal{A}$.[23] Think of $\mathcal{A}$ as encoding the the structure of (bounded) observables of a system that is common to all Hilbert space representations, whether they are unitarily equivalent or not, and think of ω as assigning expectation values to the elements of $\mathcal{A}$. This can be made more precise as follows. A *representation* $(\mathcal{H}, \pi)$ of $\mathcal{A}$ consists of a separable Hilbert space $\mathcal{H}$ and a structure preserving map $\pi : \mathcal{A} \to \mathcal{B}(\mathcal{H})$ of $\mathcal{A}$ into the algebra of bounded operators $\mathcal{B}(\mathcal{H})$ on $\mathcal{H}$. A fundamental theorem due to Gelfand, Naimark, and Segal (GNS) shows that any state ω on $\mathcal{A}$ determines a representation $(\mathcal{H}_\omega, \pi_\omega)$ of $\mathcal{A}$ and a vector $|\Psi_\omega\rangle \in \mathcal{H}_\omega$, such that $\omega(A) = \langle\Psi_\omega|\pi_\omega(A)|\Psi_\omega\rangle$ for all $A \in \mathcal{A}$; and further, the vector $|\Psi_\omega\rangle$ is *cyclic*, i.e. $\pi_\omega(\mathcal{A})|\Psi_\omega\rangle$ is a dense subset of $\mathcal{H}_\omega$. The GNS triple $(\mathcal{H}_\omega, \pi_\omega, |\Psi_\omega\rangle)$ is unique up to unitary equivalence. An algebraic state is said to be *mixed* if it can be written as a non-trivial convex combination $\lambda\omega_1 + (1-\lambda)\omega_2$, $0 < \lambda < 0$, $\omega_1 \neq \omega_2$; otherwise the state is said to be *pure*. A basic property of GNS representations is that a state ω is pure iff its GNS representation is irreducible.

In this setting, a *symmetry* of the system is specified by an automorphism of $\mathcal{A}$. An automorphism θ of $\mathcal{A}$ can also be viewed as acting on states; viz., given a state ω on $\mathcal{A}$, θ produces a new state $\widehat{\theta\omega} := \omega \circ \theta$. If there is a state ω of some special physical significance–such as a vacuum state–that is not θ-invariant, i.e. $\widehat{\theta\omega} \neq \omega$, then one might say that the symmetry θ is spontaneously broken in the state ω. However, physicists typically reserve this label for cases where θ is not unitarily implementable in the state ω. That θ is *unitarily implementable* in the state ω means that on the Hilbert space $\mathcal{H}_\omega$ of the GNS representation determined by ω there is a unitary operator $\hat{U}$ such that $\pi_\omega(\theta(A)) = \hat{U}\pi_\omega(A)\hat{U}^{-1}$ for all $A \in \mathcal{A}$. It is easy to see (and, in fact, follows from Lemma 1 below) that if θ is not unitarily implementable with respect to the state ω then the symmetry θ is broken in the state ω; but the converse is not necessarily true–unitary implementability with respect to ω does not imply that ω is θ-invariant.

This characterization allows one to see how and in what sense spontaneous symmetry breaking in QFT involves vacuum degeneracy. Say that two states ω and ω' on $\mathcal{A}$ are *spatially equivalent* ($\omega \sim \omega'$) iff their respective GNS representations $(\mathcal{H}_\omega, \pi_\omega)$ and $(\mathcal{H}_{\omega'}, \pi_{\omega'})$ are unitarily equivalent, i.e. there is a isomorphism $E : \mathcal{H}_\omega \to \mathcal{H}_{\omega'}$ such that $\pi_{\omega'}(A) = E\pi_\omega(A)E^{-1}$ for all $A \in \mathcal{A}$. The question now becomes, under what conditions are $\widehat{\theta\omega}$ and ω and spatially equivalent? The answer is given by

> *Lemma 1.* An automorphism θ of $\mathcal{A}$ is unitarily implementable in state ω iff $\widehat{\theta\omega} \sim \omega$.

It follows that if the automorphism θ is not unitarily implementable with respect to the state ω, then $\widehat{\theta\omega}$ and ω are not spatially equivalent. And if the Hilbert spaces of the GNS representations determined by $\widehat{\theta\omega}$ and ω possess vacuum states, then there is a "degeneracy of the vacuum" in that the vacuum state vectors corresponding to $\widehat{\theta\omega}$ and ω belong to unitarily inequivalent representations of the algebra of observables. This, of course, is compatible with the uniqueness of the vacuum in the sense that within each of the GNS representations there is only one (up to phase) vector satisfying the conditions taken to characterize the vacuum.

Consider again the model of the previous section with Lagrangian $\mathcal{L}_3 = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi$, i.e. the massless Klein-Gordon field. The *Weyl algebra* $\mathcal{A}_{\mathcal{L}_3}$ is a C^*-algebra that codes in algebraic form the Weyl form of the CCR.[24] Let θ_χ be an automorphism of $\mathcal{A}_{\mathcal{L}_3}$ induced by a symmetry $\varphi \to \varphi' = \varphi + \chi$ of the Lagrangian $\mathcal{L}_3$, and let Θ be an automorphism induced by the Poincaré symmetry that is characteristic of the free field vacuum. Since θ_χ and Θ commute, if the state ω is Θ-invariant, so is $\omega_\chi := \widehat{\theta_\chi\omega}$; and if in addition θ_χ is not unitarily implementable with respect to ω, then ω and ω_χ are Θ-invariant states whose GNS representations are unitarily inequivalent. In this way Streater (1965) demonstrated that for each value of χ, there is a Poincaré invariant state ω_χ with positive energy, and for $\chi \neq \chi'$, $\omega_{\chi'}$ and ω_χ determine unitarily inequivalent representations of the Weyl algebra $\mathcal{A}_{\mathcal{L}_3}$.

Since the difference between automorphisms that are and are not unitarily implementable may seem abstruse, it is important to note that it can be given operational significance. For any automorphism θ of $\mathcal{A}$ and any pure state ω on $\mathcal{A}$ the following inequality holds for any $A \in \mathcal{A}$: $0 \leq |\omega(A) - \omega(\theta(A))| \leq 2||A||$.[25] When θ is unitarily implementable with respect to ω the upper bound becomes $||A||$. But when θ is not unitarily implementable with respect to ω there is a self-adjoint element $X \in \mathcal{A}$ such that $||X|| = 1$ and such that the upper bound saturates, with the upshot is that $|\omega(X) - \omega(\theta(X))| = 2$ (see Fabri et al. 1967). Thus, as Fabri et al. (1967) comment, when a symmetry θ is spontaneously broken in state ω in the sense that θ is not unitarily implementable with respect to ω, then θ is never broken "just a little bit" for all observables but is maximally broken with respect to some observable. And further, the experimental detection of the breaking does not require measuring instruments of exquisite accuracy–anything better than 50% accuracy suffices.

In ordinary QM or QFT a judicious choice of coefficients can produce a symmetric superposition from asymmetric states. However, if "symmetric" and "asymmetric" are understood in the senses of spontaneous symmetry breaking, this trick cannot be used to produce a symmetric vacuum state from a linear superposition of the asymmetric "degenerate" vacuum states. If one tries to think of the different degenerate vacuum states as belonging to the same Hilbert space, then these states must lie in different "superselection sectors" between which a meaningful superposition is impossible.[26] By the same token, measurement collapse of a superposition cannot produce an asymmetric vacuum state from a symmetric one. In the algebraic formulation the point can be made with the help of the concept of disjointness of states. The *folium* $\mathfrak{F}(\omega)$ of a state ω on a C^*-algebra $\mathcal{A}$ is the set of all states that can be expressed as density matrices on the Hilbert space of the GNS representation of $\mathcal{A}$ determined by ω. The states ω_1 and ω_2 are said to be *disjoint* iff $\mathfrak{F}(\omega_1) \cap \mathfrak{F}(\omega_2) = \emptyset$. Disjointness has two equivalent characterizations. First, ω_1 and ω_2 are disjoint iff the GNS representation $\pi_{\omega_1+\omega_2}$ determined by $\omega_1 + \omega_2$ is the direct sum of the GNS representations π_{ω_1} and π_{ω_2} determined by ω_1 and ω_2, i.e. $\pi_{\omega_1+\omega_2} = \pi_{\omega_1} \oplus \pi_{\omega_2}$, $\mathcal{H}_{\omega_1+\omega_2} = \mathcal{H}_{\omega_1} \oplus \mathcal{H}_{\omega_2}$ and $|\Psi_{\omega_1+\omega_2}\rangle = |\Psi_{\omega_1}\rangle \oplus |\Psi_{\omega_2}\rangle$, where as usual $(\pi_\omega, \mathcal{H}_\omega, |\Psi_\omega\rangle)$ is the GNS triple associated with ω. Second, ω_1 and ω_2 are disjoint iff there is a projection operator $\hat{P} \in \pi_{\omega_1+\omega_2}(\mathcal{A})'$ such that $\omega_1(A) = \langle \Psi_{\omega_1+\omega_2} | \hat{P} \pi_{\omega_1+\omega_2}(A) | \Psi_{\omega_1+\omega_2} \rangle$ and $\omega_2(A) = \langle \Psi_{\omega_1+\omega_2} | (\hat{I} - \hat{P}) \pi_{\omega_1+\omega_2}(A) | \Psi_{\omega_1+\omega_2} \rangle$ for all $A \in \mathcal{A}$.[27] The point then is that for pure algebraic states, spatial inequivalence is coextensive with disjointness. For mixed states the story is a bit more complicated. Algebraic states ω_1 and ω_2 are said to be *quasi-spatially equivalent* just in case $\mathfrak{F}(\omega_1) = \mathfrak{F}(\omega_2)$, which is the same as saying that the states are spatially equivalent up to multiplicity (see Bratteli and Robinson 1987, p. 80). For pure states, whose GNS representations are irreducible, quasi-spatial equivalence and spatial equivalence coincide. As for mixed states, whose GNS representations are reducible, spatial equivalence up to multiplicity is the appropriate substitute for spatial equivalence. Mixed states can fail to be quasi-spatially equivalent without being disjoint. But the most important type of mixed states encountered in physical applications are *factor states*[28] which are either disjoint are quasi-spatially equivalent.

Finally, in preparation for the discussion in Section 8 below, a few words about the notion of spontaneous symmetry breaking for gauge symmetries are in order. The algebraic approach provides a sense for this notion that is exactly parallel to that for spontaneous symmetry breaking for non-gauge symmetries; namely, the automorphism θ corresponding to the gauge symmetry is not unitarily implementable with respect to some state ω of interest and, consequently, $\omega \circ \theta \neq \omega$. But if the root notion of a gauge transformation is that of a transformation that connects different descriptions of the same physical state, it follows that whereas Nature can break a non-gauge symmetry θ by choosing a θ-non-symmetric state, She cannot break a gauge symmetry in the same fashion. We, not Nature, break the gauge symmetry by choosing a particular gauge condition. Consequently the automorphism θ induced by the gauge symmetry must be construed as acting on a field algebra $\mathcal{F} \supset \mathcal{A}$ that is larger than the algebra $\mathcal{A}$ of genuine, gauge independent observables since otherwise $\omega \circ \theta$ and ω would be genuinely different states.

7. Curie's Principle in algebraic QFT

Now let us turn to the status and the usefulness of Curie's Principle in QFT. If the "Heisenberg dynamics" from time t_o to t_1 is given by an automorphism α of the algebra of observables $\mathcal{A}$, then $\omega \to \widehat{\alpha\omega} := \omega \circ \alpha$ gives the corresponding "Schrödinger dynamics" from time t_o to t_1, and vice versa. Curie's Principle has a straightforward implementation in the algebraic setting: it says that if the dynamics from time t_o to t_1 is given by an automorphism α [(CP1)], if α is invariant under θ (i.e. $\theta\alpha\theta^{-1} = \alpha$) [(CP2)], and if the initial state ω_o is θ-invariant [(CP3)], then the evolved state $\omega_1 := \widehat{\alpha\omega_o}$ is also θ-invariant [(CP4)]. As with its classical counterpart, this version of Curie's Principle is a necessary truth (see Prop. 2 in the Appendix).

If the initial state ω_o is θ-invariant then, trivially, θ is unitarily implementable in state ω_o; and by Curie's Principle, if the dynamics α is θ-invariant, then the evolved state $\omega_1 := \widehat{\alpha\omega_o}$ is also θ-invariant and again, trivially, θ is unitarily implementable in the evolved state.

Suppose then that the initial state ω_o is not θ-symmetric. It is still possible that θ is unitarily implementable in ω_o, in which case call ω_o *semi-θ-symmetric*. Is it possible in addition that the semi-θ-symmetric initial state ω_o evolves to a state $\omega_1 := \widehat{\alpha\omega_o}$ that fails to be semi-θ-symmetric? Not if the dynamical automorphism α is θ-symmetric. That is the content of the following lemma, which may be thought of as an extension of the basic form of Curie's Principle.

> *Lemma 2.* Suppose that the automorphism θ of $\mathcal{A}$ is unitarily implementable with respect to an initial state ω_o, and also that the dynamical automorphism α is θ-invariant in that $\theta\alpha\theta^{-1} = \alpha$. Then θ is unitarily implementable with respect to the evolved state $\widehat{\alpha\omega_o}$.

Note that Lemma 2 does not require that the dynamical automorphism α be unitarily implementible.

Although Curie's Principle in algebraic QFT has a certain utility, it is nearly vacuous. It is not completely vacuous since for any automorphism θ of a C^*-algebra there is some state ω which is θ-invariant and, thus, with respect to which θ is unitarily implementable (see Arageorgis et al. 2002). But the Principle can be vacuous if vacuum representations are demanded. Say that a state ω on a C^*-algebra $\mathcal{A}$ determines a *vacuum representation* just in case (a) the GNS representation determined by ω is unitarily equivalent to a representation $(\pi, \mathcal{H})$ such that the action of the automorphism group of $\mathcal{A}$ generated by Poincaré transformations is realized by a unitary group acting on $\mathcal{H}$, and (b) there is a cyclic vector $|0\rangle \in \mathcal{H}$ that is the unique (up to phase) vector invariant under said unitary group.[29] In the case of the model with Lagrangian $\mathcal{L}_3$ (recall Section 5) what was shown in algebraic terms is that for the automorphism θ of $\mathcal{A}_{\mathcal{L}_3}$ induced by the symmetry of $\mathcal{L}_3$, if ω determines a vacuum representation for $\mathcal{A}_{\mathcal{L}_3}$, then θ is not unitarily implementable with respect to ω. Thus, under the demand for vacuum representations, there simply are no states on $\mathcal{A}_{\mathcal{L}_3}$ which are θ-symmetric or even θ-semi-symmetric. And so Curie's Principle in either of the forms given above is vacuous for this case since the antecedent condition of an initially symmetric or semi-symmetric state is never fulfilled. Perhaps a better label for such cases than spontaneous symmetry breaking would be ubiquitous or automatic symmetry breaking.

This ubiquity need not entirely obviate the desire for a causal explanation of a spontaneously broken symmetry. In analogy with the toy example from Section 3 of spontaneous symmetry breaking in classical physics, one can imagine a parameterized family of Lagrangians for a field such that there is a symmetry of the Lagrangian that is unitarily implementable for the quantized field until, but not after, the parameter attains a critical value. A request for a dynamical explanation of the change of value in the parameter is perfectly in order.[30]

8. Curie's Principle, spontaneous symmetry breaking, and gauge symmetries in QFT

Rework the model of the scalar field from Section 5 so that φ becomes complex valued and acquires a potential $V(\varphi)$. The Lagrangian now becomes $\mathcal{L}_4 = \partial_\mu\varphi\partial^\mu\varphi^* - V(\varphi)$, $V(\varphi) := \lambda(\varphi\varphi^*)^2 + \varkappa^2(\varphi\varphi^*)$, where λ is a positive constant.[31] $\mathcal{L}_4$ is invariant under the transformation $\varphi \to \varphi' = \exp(-i\xi)\varphi$, where $\partial_\mu\xi = 0$. When $\varkappa^2 < 0$ the potential $V(\varphi)$ has minima at all values of φ lying on the circle $|\varphi| = \sqrt{-\varkappa^2/\lambda}$ in the the complex φ-plane. (The graph of $V(\varphi)$ above the complex φ-plane is analogous to what is obtained from rotating the classical potential of Fig. 2b about the vertical axis to produce a Mexican hat or wine bottle shape.) Thus, there is an infinity of ground states for the classical field. Each ground state is asymmetric, i.e. is non-invariant under the transformations $\varphi \to \varphi' = \exp(-i\xi)\varphi$, but the entire circle of ground states can be generated by starting with any one such state and applying the transformations $\varphi \to \varphi' = \exp(-i\xi)\varphi$ with different values of the spacetime constant ξ. One expects, therefore, that the quantized field exhibits the characteristic features of spontaneous symmetry breaking in QFT. An application of Noether's first theorem implies that there is a conserved current, which in the present case is $j^\mu := i[\varphi^*\partial^\mu\varphi - \partial^\mu\varphi^*\varphi]$. Then the same type of argument used in Section 5 shows that the associated global Noether charge is not represented by a self-adjoint operator and, hence, that the symmetry of the Lagrangian $\mathcal{L}_4$ is not unitarily implementable.

In the heyday of gauge theories it was often said that relativity theory demands that "global" symmetries be made "local," and it was thought that the implementation of this demand provided a magic route to the discovery of the laws governing interactions.[32] In the present example the demand would be that the "global" symmetry $\varphi \to \varphi' = \exp(-i\xi)\varphi$, where $\partial_\mu\xi = 0$, be replaced by a "local" symmetry $\varphi \to \varphi' = \exp(-i\xi(x))\varphi$, where $\xi(x)$ is now an arbitrary function of spacetime position. Implementing this demand is accomplished by the introduction of a vector field A_μ coupled to the φ field to produce the Lagrangian $\mathcal{L}_5 := -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + [(\partial_\mu + ieA_\mu)\varphi^*][(\partial_\mu - ieA_\mu)\varphi] - V(\varphi)$, where $F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu$, e is the coupling constant, and the potential $V(\varphi)$ is the same as in $\mathcal{L}_4$. $\mathcal{L}_5$ is invariant under $\varphi \to \varphi' = \exp(-i\xi(x))\varphi$ if the vector field A_μ is assumed to transform as $A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \frac{1}{e}\partial_\mu\xi(x)$.

One result of this maneuver is to bring to bear Noether's *second* theorem: when the variational symmetries form an infinite dimensional Lie group whose parameters are arbitrary functions of the independent variables, the resulting Euler-Lagrange equations are not independent but must satisfy a set of mathematical identities. Thus, the maneuver has resulted in a case of underdetermination. The second result of the maneuver is to undercut the familiar interpretation of conservation laws. The "global" transformations $\varphi \to \varphi' = \exp(-i\xi)\varphi$, $\partial_\mu \xi = 0$, $A_\mu \to A'_\mu = A_\mu$, are symmetries of the expanded Lagrangian $\mathcal{L}_5$. Therefore, Noether's first theorem applies so that there must be an associated conserved current. There is indeed; it is now given by $j^\mu := ie[\varphi^* \partial^\mu \varphi - \partial^\mu \varphi^* \varphi] - 2e^2 A^\mu \varphi^* \varphi$. However, the conservation law $\partial_\mu j^\mu = 0$ for $\mathcal{L}_5$ is no longer "proper." Noether showed that when a "local" symmetry group of the Lagrangian admits a "global" symmetry group as a proper subgroup, then the conserved current associated with the latter can be written as a linear combination of the Euler-Lagrange expressions and the divergence of an anti-symmetric "superpotential" (see Barbashov and Nesterenko 1983, Sec. 5). Thus, for any solution of the Euler-Lagrange equations the vanishing of the divergence of the current associated with the global symmetry is a mathematical identity. Noether dubbed such conservation laws "improper"; in the modern mathematics literature they are called "trivial" (see, for example, Olver 1993, Ch. 5). Improper or trivial they may be, but such conservation laws do have a content: they express the fact that in any solution to the equations of motion, the amount of Noether charge in a spatial volume is equal to the flux through the surface bounding the volume, i.e. the conservation law is equivalent to a form of Gauss' theorem.

The first result would seem to render Curie's Principle inoperative by negating the first antecedent condition (CP1)–determinism–and, thus, opening a loophole through which an asymmetric state can emerge from a symmetric one. This loophole is almost never utilized. Instead it is closed by saying that the apparent violation of determinism is merely apparent, the false appearance being due to the many-one relation between the descriptions of a physical situation and the situation itself. Or, in more portentous terms, the "local" symmetry transformations $\varphi \to \varphi' = \exp(-i\xi(x))\varphi$ are regarded as gauge transformations connecting the descriptions that all correspond to the same physical situation. This reading of the first result impacts on the second result, making the meaning of conserved current and its corresponding Noether charge even more opaque since neither of these quantities are gauge independent. And in general the redundant descriptive apparatus signaled by the presence of non-trivial gauge freedom constitutes a veil though which the intrinsic gauge-independent content of the model can be dimly glimpsed.

These issues are of more than idle curiosity since, as the knowledgeable reader will have recognized, the above model is an illustration of the Higgs mechanism which has become a standard feature of the Standard Model of particle physics.[33] It achieved this status by neatly solving two problems in one fell swoop: it resolved an embarrassment connected with spontaneous symmetry breaking while explaining how particles got their masses. The embarrassment is the result of Goldstone theorem which shows that the example of the massless scalar field that was used to motivate the idea of spontaneous symmetry breaking in QFT is not just an example. In more detail, the theroem demonstrates that a broken "global" continuous symmetry (invoking Noether's first theorem and the existence of a conserved current), together with some additional cherished assumptions of QFT (local commutativity, translation symmetry, and positivity of energy) imply the existence of spinless zero mass bosons. The result is robust, being provable in both conventional QFT (see Goldstone et al. 1962) and algebraic QFT (see Kastler et al. 1966). It temporarily dampened the enthusiasm for spontaneous symmetry breaking since the experimental evidence was against the existence of Goldstone bosons. The enthusiasm was restored by moving to the Higgs model with Lagrangian $\mathcal{L}_5$. This model can be taken to involve two scalar fields ζ and Φ, which come respectively from the imaginary and real parts of φ. The field ζ possesses zero mass, but this field can can be made to disappear by an appropriate choice of gauge (the "unitary gauge"). Thus, the Higgs model is not a counterexample to Goldstone's theorem; rather it shows that by introducing an appropriate interacting field which induces gauge invariance, the theorem can be finessed in that the zero mass modes can be ignored since they do not describe any real (i.e. gauge independent) phenomenon. In the same gauge which suppresses Goldstone bosons the other scalar field Φ (the "Higgs field") and the vector field A_μ have positive masses.

This is undeniably one of the more brilliant accomplishments of modern physics. But what exactly is accomplished is hidden behind the veil of gauge redundancy. The popular presentations use the slogan that the vector field has acquired its mass by "eating" the Higgs field. But, as the authors of the standard treatises well know but rarely bother to warn the unwary reader, talk of *the* Higgs field has to be carefully qualified since by itself, the value of Φ does not have gauge invariant significance. The popular slogan can be counterbalanced by the cautionary slogan that neither mass nor any other genuine attribute can be gained by eating descriptive fluff.

None of this need be any concern for practicing physicists who know when they have been presented with a fruitful idea and are concerned with putting the idea to work. But it is dereliction of duty for philosophers to repeat the physicists' slogans rather than asking what is the content of the reality that lies behind the veil of gauge. Here an analogy may be helpful. Consider Maxwell electromagnetism written in terms of electromagnetic potentials. It is found that the initial value problem for Maxwell's equations does not have a unique solution–the initial values of the potentials and their time derivatives can evolve into many different values at any given future time. This is not viewed as a real violation of determinism since, it is said, the potentials involve unphysical degrees of freedom. The imposition of a gauge condition can be used to quash these unphysical degrees of freedom; and, in particular, the imposition of the Lorentz gauge turns Maxwell's equations into second order hyperbolic equations which do admit a well-posed initial value problem. This is fine as far as it goes. But it remains to ask: Is the procedure just described a merely ad hoc maneuver to save determinism, or is there a systematic way to identify the gauge freedom? And when the gauge freedom is removed, what are the genuine (= gauge invariant) degrees of freedom of the electromagnetic field?

P. A. M. Dirac developed an apparatus that, in principle, gives answers to both of these questions, and it applies to any theory whose equations of motion are derivable from a Lagrangian. A system whose Lagrangian falls under Noether's second theorem–the case now at issue–corresponds to a constrained Hamiltonian system.[34] The *primary constraints* are those that follow from definition of the canonical momenta.[35] The *secondary constraints* are those that follow from the demand that the primary constraints be preserved by the laws of motion. The total family of constraints defines a subspace of the Hamiltonian phase space called the *constraint surface*. A *first class constraint* is one which "weakly commutes" with all the constraints in the sense that its Poisson bracket with any constraint vanishes on the constraint surface. These first class constraints generate the transformations that are taken to be gauge. To get rid of the redundant structure one can pass to the reduced phase space by quotienting out the gauge orbits generated by the first class constraints, producing an unconstrained Hamiltonian system in which the reduced phase space variables are gauge invariants. Mathematical obstructions can derail this reduction procedure, e.g. the reduced phase space may not form a manifold; but in the absence of such obstructions, normal methods for quantizing unconstrained Hamiltonian systems can then be applied.[36]

What is the upshot of applying this reduction procedure to the Higgs model and then quantizing the resulting unconstrained Hamiltonian system? In particular, what is the fate of spontaneous symmetry breaking? To my knowledge the application has not been carried out. For purposes of discussion I will assume that the reduction process leads to local fields and that when these fields are quantized they satisfy the standard assumptions of QFT, such as Poincaré invariance, locality, etc. If this assumption were false, the implementation of the Higgs mechanism would necessitate a radical revision of the way business is conducted in QFT. Next, if the reduced model admits a finite dimensional Lie group of internal symmetries, then Noether's first theorem and the argumentation of Section 5 apply, leading to the conclusion that a symmetry is still subject to spontaneous breaking. But Goldstone's theorem also applies and Goldstone bosons have not been quashed after all. If, on the other hand, the reduced model admits no non-trivial internal symmetries or else only a discrete group of internal symmetries, then Goldstone's theorem does not apply and the spectre of Goldstone bosons has been definitively dispatched. But if there are no non-trivial internal symmetries there is *ipso facto* no possibility of symmetry breaking. If there are discrete internal symmetries there is the possibility of symmetry breaking; but as noted in Section 5, Noether's first theorem does not apply and, consequently, the demonstration of symmetry breaking based on the non-existence of a self-adjoint global charge operator does not apply. This does not preclude there being some other demonstration of the spontaneous breakdown of the hypothesized discrete symmetries; but such a demonstration–as opposed to heuristic considerations–is wanting.

While there are too many what-ifs in this exercise to allow any firm conclusions to be drawn, it does suffice to plant the suspicion that when the veil of gauge is lifted, what is revealed is that the Higgs mechanism has worked its magic of suppressing zero mass modes and giving particles their masses by quashing spontaneous symmetry breaking. But confirming the suspicion or putting it to rest require detailed calculations, not philosophizing.

9. The role of idealizations

In this section I want to air some skeptical doubts that arise from the (alleged) role of idealizations in spontaneous symmetry breaking. While idealizations are useful and, perhaps, even essential to progress in physics, a sound principle of interpretation would seem to be that no effect can be counted as a genuine physical effect if it disappears when the idealizations are removed. This principle might be taken to call into question the novel features of spontaneous symmetry breaking in QFT–the non-unitary implementability of a symmetry and the degeneracy of the vacuum–since they emerge only for infinitely large systems. For instance, the demonstration of the non-unitary implementability of the symmetry of the Lagrangian $\mathcal{L}_3$ given in Section 5 took the form of showing that the global charge operator does not exist as a well-defined self-adjoint operator. Actually, the charge operator $\hat{Q}_V$ associated with a finite volume V of space does exist; it is only the infinite volume limit $\lim_{V\to\infty} \hat{Q}_V$ that fails to exist. But (the complaint continues) actual systems exhibiting spontaneous symmetry breaking–such as ferromagnets and superconductors–are finite.

One reaction would be to ask about the prospects that a broken symmetry can retain some approximation to the features of a spontaneously broken symmetry without going all the way the limit of the infinite volume limit. A symmetry θ can be broken by a state ω (i.e. ω is not θ-invariant) even though θ is unitarily implementable with respect to ω. In the GNS representation $(\pi_\omega, \mathcal{H}_\omega)$ determined by such a ω the symmetry θ is, by hypothesis, implemented by a unitary $\hat{U}_\theta$. It can happen that there are states $|\psi\rangle \in \mathcal{H}_\omega$ such that $|\psi\rangle$ and its θ-image $\hat{U}_\theta|\psi\rangle$ become as close to orthogonal as can be desired as the volume is increased, e.g. $|\langle\psi|\hat{U}_\theta|\psi\rangle|$ diminishes exponentially fast in V. This may or may not be good enough to account for the observed features of spontaneous symmetry breaking. As explained in Section 6, the difference between a unitarily implementable automorphism and a non-unitarily implementable automorphism is at least as great as the difference between "1" and "2" in the expectation value of some observable. If this observable is important in explaining observed features of spontaneous symmetry breaking, then the limit of infinite volume is not a dispensable idealization.

Another reaction would be to deny that there is any idealization involved in taking an infinite volume limit.[37] The impression to the contrary is based on intuitions that are trained in classical physics to think of physical systems as consisting of hunks of spatially localized matter. If, however, QFT is true, then matter is nothing but excitations in quantum fields. If the theory is empirically adequate it will explain how circumstances can conspire to give the impression of spatial localization of matter. But even if one is concerned solely with observables associated with spacetime regions all of which share the same finite spatial support, the full quantum field theoretic description involves global features. Of course, the philosophy of local QFT is that a complete description of a local spacetime region is given by the subalgebra of observables associated with that region and by the restriction of the state on the global algebra to the said subalgebra. But the global state is not fully characterized by its behavior on any finite spatial volume no matter how large. For example, even the humblest of vacuum states–the Minkowski vacuum–entails correlations between spacetime regions having an arbitrarily large a spatial separation. Thus, intuitions retrained to fit QFT would think of all physical systems as being infinitely large while recognizing that for practical purposes some systems can be treated as spatially finite objects.

Of the two responses, I favor the "no-idealization" one, but I freely admit that there is room for dispute.

10. Conclusion

Curie's Principle has proved to be very resilient. My explanation is that this Principle is analytic but nonetheless useful. However, its usefulness is diminished by QFT where it becomes nearly vacuous. QFT also necessitates a reappraisal of spontaneous symmetry breaking. The core notion of this concept–the failure of some state of special physical importance to exhibit a symmetry of the Lagrangian–is applicable in QFT. But the application involves wholly novel features such as the non-unitary implementability of the symmetry and the degeneracy of the vacuum. Because these features are connected to a number of fundamental interpretational issues in QFT, spontaneous symmetry breaking serves as a good entry point to the mysteries of QFT. But by the same token, a full understanding of spontaneous symmetry breaking is possible only after these issues have received satisfactory resolutions.

Appendix

A *C^*-algebra* $\mathcal{A}$ is an algebra, over the field $\mathbb{C}$ of complex numbers, with an involution $*$ satisfying: $(A^*)^* = A$, $(A+B)^* = A^* + B^*$, $(\lambda A)^* = \bar{\lambda} A^*$ and $(AB)^* = B^* A^*$ for all $A, B \in \mathcal{A}$ and all complex λ (where the overbar denotes the complex conjugate). In addition, a C^*-algebra is equipped with a norm, satisfying $\|A^* A\| = \|A\|^2$ and $\|AB\| \leq \|A\| \, \|B\|$ for all $A, B \in \mathcal{A}$, and is complete in the topology induced by that norm. It is assumed here that

$\mathcal{A}$ contains a unit $\mathbf{1}$ such that $\mathbf{1}A = A\mathbf{1} = A$ for all $A \in \mathcal{A}$. Observables are identified with self-adjoint elements of $\mathcal{A}$, i.e. elements A such that $A^* = A$. A *state* on $\mathcal{A}$ is a linear functional ω that is normed ($\omega(\mathbf{1}) = 1$)) and positive ($\omega(A^*A) \geq 0$ for all $A \in \mathcal{A}$).

A *representation* of a C^*-algebra $\mathcal{A}$ is a mapping $\pi : \mathcal{A} \to \mathcal{B}(\mathcal{H})$ from the abstract algebra into the concrete algebra $\mathcal{B}(\mathcal{H})$ of bounded linear operators on a Hilbert space $\mathcal{H}$ such that $\pi(\lambda A + \mu B) = \lambda\pi(A) + \mu\pi(B)$, $\pi(AB) = \pi(A)\pi(B)$, and $\pi(A^*) = \pi(A)^\dagger$ for all $A, B \in \mathcal{A}$ and all $\lambda, \mu \in \mathbb{C}$. Two representations $(\pi_1, \mathcal{H}_1)$ and $(\pi_2, \mathcal{H}_2)$ of a C^*-algebra $\mathcal{A}$ are said to be *unitarily equivalent* just in case there is an isomorphism $V : \mathcal{H}_1 \to \mathcal{H}_2$ such that $V\pi_1(A)V^{-1} = \pi_2(A)$ for all $A \in \mathcal{A}$.

The proofs of Lemmas 1 and 2 and Prop. 2 are all trivial, but for sake of completeness they are given here. Lemma 1 follows easily from

Prop. 1. Let $\mathcal{A}$ be a C^*-algebra with unit $\mathbf{1}$, ω a state on $\mathcal{A}$, and θ an automorphism of $\mathcal{A}$. Let $(\mathcal{H}_\omega, \pi_\omega, |\Psi_\omega\rangle)$ and $(\mathcal{H}_{\widehat{\theta\omega}}, \pi_{\widehat{\theta\omega}}, |\Psi_{\widehat{\theta\omega}}\rangle)$ be the GNS triples determined by θ and $\widehat{\theta\omega}$ respectively. Then there is an isomorphism $E : \mathcal{H}_\omega \to \mathcal{H}_{\widehat{\theta\omega}}$ such that $E\pi_\omega(\theta(A))E^{-1} = \pi_{\widehat{\theta\omega}}(A)$ for all $A \in \mathcal{A}$. *Proof*: Define the linear map $E : \mathcal{H}_\omega \to \mathcal{H}_{\widehat{\theta\omega}}$ by

$$E\pi_\omega(\theta(A))|\Psi_\omega\rangle = \pi_{\widehat{\theta\omega}}(A)|\Psi_{\widehat{\theta\omega}}\rangle \tag{4}$$

and extending by continuity ($\pi_\omega(\theta(A))|\Psi_\omega\rangle = \pi_\omega(\mathcal{A})|\Psi_\omega\rangle$ is dense in $\mathcal{H}_\omega$). Equation (4) can be written as $E\pi_\omega(\theta(A))[\mathbf{1}]_\omega = \pi_{\widehat{\theta\omega}}(A)[\mathbf{1}]_{\widehat{\theta\omega}}$, i.e.

$$E[\theta(A)]_\omega = [A]_{\widehat{\theta\omega}} \tag{5}$$

Hence,

$$E[A]_\omega = [\theta^{-1}(A)]_{\widehat{\theta\omega}} \tag{6}$$

So E is linear bi-jective and preserves inner products. $\langle E[A]_\omega | E[B]_\omega\rangle = \langle [\theta^{-1}(A)]_{\widehat{\theta\omega}} | [\theta^{-1}(B)]_{\widehat{\theta\omega}}\rangle = \widehat{\theta\omega}((\theta^{-1}(A))^*\theta^{-1}(B)) = \widehat{\theta\omega}(\theta^{-1}(A^*B)) = \omega(A^*B) = \langle [A]_\omega | [B]_\omega\rangle$. So E is a Hilbert space isomorphism. Now for all $A, B \in \mathcal{A}$, $\pi_{\widehat{\theta\omega}}(A)[B]_{\widehat{\theta\omega}} = [AB]_{\widehat{\theta\omega}} \Rightarrow \pi_{\widehat{\theta\omega}}(A)E[\theta(B)]_\omega = E[\theta(AB)]_\omega \Rightarrow \pi_{\widehat{\theta\omega}}(A)E[\theta(B)]_\omega = E\pi_\omega(\theta(A))[\theta(B)]_\omega$. But B and $\theta(B)$ are arbitrary, so $\pi_{\widehat{\theta\omega}}(A)E = E\pi_\omega(\theta(A)) \Rightarrow \pi_{\widehat{\theta\omega}}(A) = E\pi_\omega(\theta(A))E^{-1}$.

Proof of Lemma 2. $\widehat{\theta\omega} \sim \omega$ by hypothesis. So $\widehat{\theta\omega}(A) = \omega(\theta(A)) = \omega(UAU^*)$ for some unitary $U \in \mathcal{A}$ and every $A \in \mathcal{A}$. Now $\widehat{\theta\widehat{\alpha\omega}}(A) = \widehat{\alpha\omega}(\theta(A)) = \omega(\alpha(\theta(A)) = \omega(\theta(\alpha(A))$, where the last equality follows from the hypothesis that $\theta\alpha\theta^{-1} = \alpha$. But $\omega(\theta(\alpha(A)) = \omega(U\alpha(A)U^*) = \omega(\alpha[\alpha^{-1}(U)A\alpha^{-1}(U^*)])$. Since α^{-1} is an automorphism, $\alpha^{-1}(U) = V$ for some unitary element $V \in \mathcal{A}$, and $\alpha^{-1}(U^*) = (\alpha^{-1}(U))^* = V^*$. Thus, $\widehat{\theta\widehat{\alpha\omega}}(A) = \omega(\alpha(VAV^*)) = \widehat{\alpha\omega}(VAV^*)$, i.e. θ is unitarily implementable in state $\widehat{\alpha\omega}$.

In the algebraic setting Curie's Principle can be stated as:

Prop. 2 (Curie's Principle). Suppose that the initial state ω_o is θ-symmetric (i.e. $\widehat{\theta\omega_o} := \omega_o \circ \theta = \omega_o$) and that the dynamics α is also θ-symmetric (i.e. $\theta\alpha\theta^{-1} = \alpha$). Then the evolved state $\omega_1 := \widehat{\alpha\omega_o}$ is θ-symmetric. *Proof*: $\widehat{\theta\omega_1} = \widehat{\theta\widehat{\alpha\omega_o}} = \widehat{\alpha\omega_o} \circ \theta = \omega_o \circ \alpha \circ \theta = \omega_o \circ \theta \circ \alpha = \widehat{\theta\omega_o} \circ \alpha = \widehat{\alpha\widehat{\theta\omega_o}} = \widehat{\alpha\omega_o} = \omega_1$.

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Notes

1. "Lorsque certains effects révèlent une certaine dissymétrie, cette dissymétrie doit se retrouver dans les causes qui lui ont donné naissance." An English translation of Curie's paper can be found in Rosen (1982, pp. 17-25).
2. For cases where the equations of motion/field equations are derivable from an action principle the point becomes that there is a symmetry of the Lagrangian that is not a symmetry of the physically significant state. This will be discussed in detail below.
3. I take this qualifier from Roberts (1992, p. 404).
4. "C'est la dissymétrie qui crée le phénonène."
5. See Roberts (1992, p. 404). Weinberg (1996) quips that "we do not have far to look for examples of spontaneous symmetry breaking," e.g. most of the objects we encounter in daily life have a definite orientation in space although the laws governing the motions and interactions of the atoms that compose these objects are rotationally symmetric (p. 163). But Weinberg immediately goes on to discuss how spontaneous symmetry breaking in quantum field

theory differs from such homely examples, and the key difference concerns the behavior of the quantum vacuum state.

6. Attempts by philosophers of science to come to grips with the implications spontaneous symmetry breaking can be found in Morrison (1995) and (2000, sec. 4.4); Cao (1997, Ch. 10); Kosso (2000); Liu (2001); and Balashov (2002); and Part III ("Symmetry breaking") of Brading and Castellani (2003). Historical surveys are to be found in Radicati (1987) and Brown and Cao (1991).

7. I leave open the question of whether the present proposal captures Curie's original intentions. Ismael (1997), who gives a similar reading of Curie's Principle, offers textual evidence about Curie's intentions.

8. For a precise statement and proof of these assertions, see Olver (1993, Sec. 4.2). A symmetry of the equations of motion need not be a variational symmetry.

9. For a readable treatment of the Noether theorems and their applications in physics, see Barbashov and Nesterenko (1983).

10. The quantum treatment of phase transitions belongs to quantum statistical mechanics. One of the features that emerges from this treatment is the necessity to invoke unitarily inequivalent representations of the canonical commutations relations which, as will be seen below, is a key to spontaneous symmetry breaking in quantum field theory. For an accessible introduction to some of the issues involved in quantum phase transitions, see Ruetsche (2003)

11. Needless to say, the idea that collapse of the state vector is an objective physical process is highly controversial. See Albert (1998) for an overview of different ways of treating the measurement problem in QM.

12. See Ismael (1997) for a start on this project.

13. With $U_m(s) := \exp(iq_m s)$ and $V_n(t) := \exp(ip_n t)$, the Weyl form of the canonical commutation relations $p_n q_m - q_m p_n = -i\delta_{nm}$, etc., is given by $V_n(t)U_m(s) = U_m(s)V_n(t)\exp(ist\delta_{nm})$ and $U_m(s)U_n(t) - U_n(t)U_m(s) = 0 = V_m(s)V_n(t) - V_n(t)V_n(s)$. This form of the commutation relations avoids problems about domains of definition for unbounded operators. When the ranges of m and n, are finite, the Stone-von Neumann theorem says that all irreducible representations of these relations by continuous unitary groups on Hilbert space are unitarily equivalent. For an infinite number of degrees of freedom–in particular, for field theories–the theorem no longer applies.

14. There are many good reviews of the physics of spontaneous symmetry breaking. Among the ones I found most helpful are Coleman (1985, Ch. 5)

and Guralnik et al. (1968).
15. A particularly clear treatment is found in Aitchison (1982, Ch. 6). The discussion assumes that the dimension of space is two or greater; for the reasons that this assumption is needed, see Coleman (1973).
16. In different approaches to QFT the vacuum is expected to satisfy different conditions. In Fock representations, the vacuum state is the zero-particle state. In most approaches it is postulated–and sometimes proved–that the vacuum is the unique (up to phase) state that is Poincaré invariant.
17. This result is peculiar to the zero mass Klein-Gordon field. Streater (1966) shows that when $m > 0$ spontaneous symmetry breaking cannot occur for the Klein-Gordon field.
18. Commentators will sometimes say that a spontaneously broken symmetry is represented in a "non-Wigner" mode. This terminology invites confusion. Wigner (1959) was concerned with ray mappings of a separable Hilbert space $\mathcal{H}$ that preserve transition probabilities, and he showed that the action of such a mapping is given by a vector mapping that is either linear unitary or antilinear antiunitary. So the notion that a spontaneously broken symmetry is represented in a "non-Wigner" mode has led some commentators to conclude that a spontaneously broken symmetry does not conserve probability (see, for example, Fonda and Ghirardi 1970, p. 446). But as will be seen below, 'symmetry' in spontaneous symmetry breaking does not invoke the ray or vector maps at the basis of Wigner's theorem.
19. For an account of what Noether (1918) did in her classic paper on variational symmetries and her purpose in stating two theorems, see Brading and Brown (2003).
20. Using similar heuristic reasoning, T. D. Lee (1973) discusses the spontaneous breaking of P, CP, and T. For an application of the (assumed) spontaneous breakdown of CP symmetry to cosmology, see Zel'Dovich et al. (1975).
21.My 'Rough guide to spontaneous symmetry breaking' (2003b) gave a brief and (alas!) all-too-rough introduction to the use of the algebraic formalism for QFT to illuminate features of symmetry breaking.
22. Haag's theorem shows, for example, that irreducible representations of a free scalar field and a self-interacting scalar field are unitarily inequivalent.
23. See the Appendix for more details. The mathematics of algebraic QFT is developed in Bratteli and Robinson (1987, 1996).
24. For a construction of the Weyl algebra for the Klein-Gordon field, see Wald (1994).

25. Here $||\bullet||$ is the norm of the C^*algebra. Note that $||\theta(A)|| = ||A||$ for all $A \in \mathcal{A}$.
26. And in the case where there are an uncountable infinity of degenerate vacuum states, the Hilbert space that accommodates them all would be non-separable.
27. $\pi_{\omega_1+\omega_2}(\mathcal{A})'$ denotes the *commutant* of $\pi_{\omega_1+\omega_2}(\mathcal{A})$, i.e. the set of all bounded operators on $\mathcal{H}_{\omega_1+\omega_2}$ that commute with each element of $\pi_{\omega_1+\omega_2}(\mathcal{A})$. The equivalence of the definitions of disjointness is proved in Bratteli and Robinson (1996).
28. A state ω on a C^*-algebra $\mathcal{A}$ is said to be a *factor* iff the von Neumann algebra $\pi_\omega(\mathcal{A})''$ (where "$''$" indicates the double commutant) has a trivial center, i.e. $\pi_\omega(\mathcal{A})'' \cap \pi_\omega(\mathcal{A})'$ consists of multiples of the identity.
29. One might also require of a vacuum representation that it be unitarily equivalent to a Fock representation with $|0\rangle$ being the no-particle state. But this is not essential for the present point.
30. he reader is invited to compare the present discussion with the "Panel discussion" that followed Simon Saunders question about the possibility of a dynamical representation of spontaneous symmetry breaking; see Cao (1999, pp. 382-383).
31. This model is taken from Goldstone (1961).
32. For a nice antidote to this hype, see Martin (2001).
33. For an account of the discovery of this mechanism, see Higgs (1997).
34. For details about constrained Hamiltonian systems and techniques for quantizing them, see Henneaux and Teitelboim (1992). For an introduction designed to be accessible to philosophers of science, see Earman (2003a).
35. In classical mechanics the Legendre transformation takes one from the Lagrangian variables $(\mathbf{q}, \dot{\mathbf{q}})$ to the Hamiltonian variables $(\mathbf{q}, \mathbf{p})$, where the canonical momenta $\mathbf{p}$ are given by $\partial\mathcal{L}/\partial\dot{\mathbf{q}}$. For unconstrained Hamiltonian systems, $\mathbf{p}$ will be $m\dot{\mathbf{q}}$.
36. There is another approach to quantizing constrained Hamiltonian systems called *Dirac constraint quantization.* It promotes the classical Hamiltonian constraints to operators on a Hilbert space and identifies the physical sector as consisting of vectors that are annihilated by the operator constraints. It is mathematically possible that reduced phase space and Dirac constraint quantization yield physically inequivalent results. As far as I am aware this possibility is not realized in the types of examples at issue.
37. Assuming, of course, that the actual universe is spatially infinite. The latest evidence from cosmological observations is that this assumption is cor-

rect.

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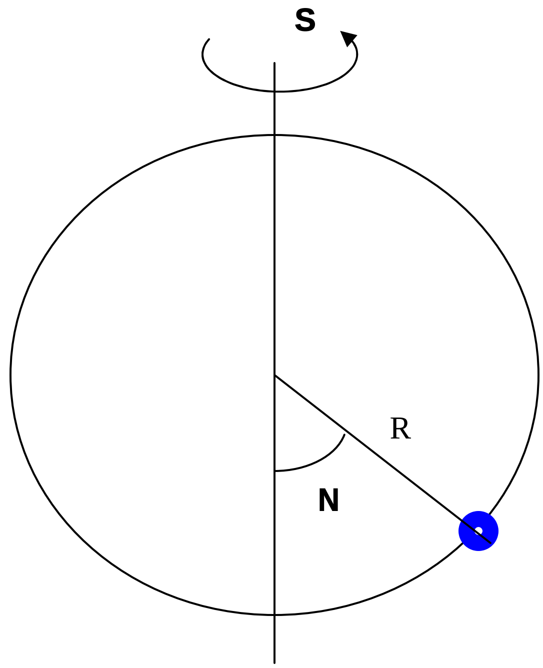
Figure captions

Fig. 1: Greenberger’s hoop-and-bead model

Fig: 2: Effective potential for angular momentum below the critical value

Fig. 3 Effective potential for angular momentum above the critical value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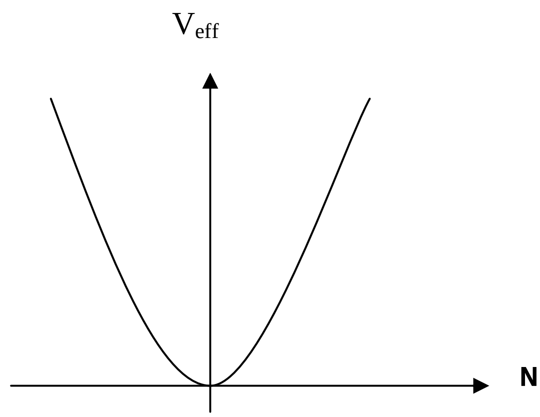

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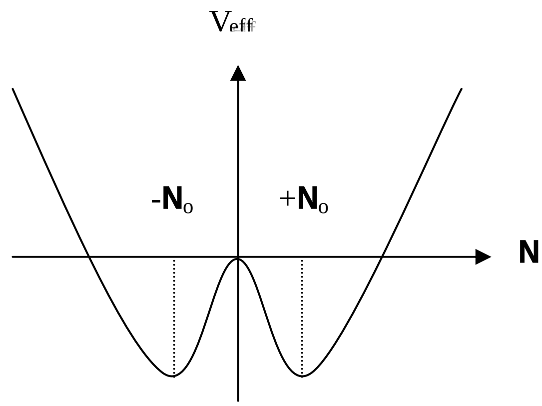

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