

Measurement, Decoherence, Everett—and Tails

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The Measurement Problem

(A): The measurement problem is the threat of an indefinite macro-realm. The threat arises from the way that indefiniteness of values in the micro-realm can apparently be transmitted, during a quantum measurement, to the macro-realm.

A standard toy-model shows the problem. Consider a momentum measurement on an electron in a superposition of two momentum eigenstates: one for 1 unit of momentum, and the other for 2 units of momentum. Suppose we have a pointer (measurement apparatus), with ‘ready state’ $|r\rangle$, which reliably reads these eigenstates, in the sense that the composite system behaves as follows:–

$$|1\rangle|r\rangle \rightarrow |1\rangle|\text{reads } 1\rangle \quad ; \quad |2\rangle|r\rangle \rightarrow |2\rangle|\text{reads } 2\rangle \quad (0.1)$$

Then measuring an electron in a superposition of the two eigenstates yields (ignoring normalization):

$$\{|1\rangle + |2\rangle\}|r\rangle \rightarrow |1\rangle|\text{reads } 1\rangle + |2\rangle|\text{reads } 2\rangle. \quad (0.2)$$

But this final state is not an eigenstate of pointer position. So orthodoxy, i.e. the eigenvalue-eigenstate link, declares that the pointer has no definite position!

This problem can be generalized to much more realistic models of measurement. One standard model retains eq. 0.1’s assumption that the measurement is non-disturbing on eigenstates of the measured micro-quantity, but considers arbitrary initial superpositions. So we write, with $|\psi_i\rangle$ for these eigenstates, and $|\phi_i\rangle$ for eigenstates of pointer-position:

$$\{\Sigma_i c_i |\psi_i\rangle\} \otimes |r\rangle \rightarrow \Sigma_i c_i |\psi_i\rangle \otimes |\phi_i\rangle \quad (0.3)$$

(B): Beware! There is an apparent solution to this problem which is *wrong*—though it is repeated in some textbooks!

This apparent solution uses the quantum formalism that describes how the state of a composite system determines states for its component systems (these determined states are called ‘reduced states’, and are in almost all cases density matrices, not vectors). Using this formalism it *looks as if* the state determined for the pointer is what one wants: namely, a state representing a definite pointer-position, though one does not know which—and the various alternative positions have their orthodox Born-rule probabilities $|c_i|^2$.

More formally: the final state at the end of an ideal measurement on a micro-system in an initial superposition $\Sigma_i c_i |\psi_i\rangle$, $\Sigma_i c_i |\psi_i\rangle \otimes |\phi_i\rangle$ determines reduced states (density matrices) $\Sigma_i |c_i|^2 |\psi_i\rangle \langle \psi_i|$ and $\Sigma_i |c_i|^2 |\phi_i\rangle \langle \phi_i|$ —which *look* right.

There is still a problem! Indeed there are two: the first is fundamental, but the second can be solved by appealing to the fact that the pointer is not isolated (decoherence).

(1): These density matrices are not interpretable in the desired way, described above. (Jargon: they are not ignorance-interpretable). The reason is that if they *were* thus interpreted, then: (i) the composite system's final state $\sum_i c_i |\psi_i\rangle \otimes |\phi_i\rangle$ would also have to be thus interpretable; and (ii) it cannot be. Why?

(i): Because the ignorance-interpretation of a state means that an ensemble (a population) of systems described by the state is heterogeneous, i.e. has sub-ensembles differing in their experimental statistics (for some quantities or other). And heterogeneity amongst the component systems, implies *ipso facto* heterogeneity in the ensemble of composite systems. That is: the final composite system state $\sum_i c_i |\psi_i\rangle \otimes |\phi_i\rangle$ would also be ignorance-interpretable.

(ii): But the final composite system state is a vector, not a density matrix; which means it *cannot* be thus interpreted. For a heterogeneous ensemble is always described by a density matrix; and any vector and any density matrix differ in their predicted statistics for some quantity or other.

This point is not controversial, but it is subtle—and sometimes missed in the textbooks. D'Espagnat (1976) emphasised the point, and introduced helpful jargon. He calls a density matrix that can be ignorance-interpreted 'proper', and one that cannot be thus interpreted 'improper'. He also attaches both these adjectives to the noun 'mixture', rather than 'density matrix'; (but I think 'mixture' strongly connotes 'heterogeneous ensemble' and so 'proper mixture'). There are also other jargons for the distinction.

To sum up problem (1), in d'Espagnat's jargon:—Beware! The reduced states determined by the final composite state at the end of a measurement are improper mixtures. They may look mathematically right, as solutions to the measurement problem. But they cannot be interpreted in the way one would like, i.e. as a matter of probability-weighted ignorance about which of a set of various alternatives (pointer-positions) is definite.

(2): The final composite system state we have so far discussed, $\sum_i c_i |\psi_i\rangle \otimes |\phi_i\rangle$, is special. For:

(i): the general composite system vector-state, when expanded using arbitrary bases $\{|\psi_i\rangle\}$, $\{|\phi_j\rangle\}$ in the two factor-spaces, involves a double summation, over both i and j . That is: the general state is: $\Psi = \sum_{ij} c_{ij} |\psi_i\rangle \otimes |\phi_j\rangle$.

(ii): Agreed: given the general state Ψ , one can find other bases $\{|\eta_i\rangle\}$, $\{|\zeta_j\rangle\}$ such that Ψ can be expressed in terms of a single sum. That is: such that Ψ can be written as $\sum_k d_k |\eta_k\rangle \otimes |\zeta_k\rangle$ (where the range of the index k is at most the minimum of the two factor-spaces' dimensions). We say that Ψ has a *Schmidt* or *biorthogonal* expansion.

(iii): So, setting aside problem (1), the general composite vector-state $\Psi = \sum_k d_k |\eta_k\rangle \otimes |\zeta_k\rangle$ determines as reduced states of the micro-system, and pointer: $\sum_k |d_k|^2 |\eta_k\rangle \langle \eta_k|$ and $\sum_k |d_k|^2 |\zeta_k\rangle \langle \zeta_k|$.

So problem (2) is: One cannot in general get a final pure composite state that is biorthogonal in the pointer-quantity.

Decoherence

Of the two problems of measurement just discussed, decoherence solves problem (2) but not (1). Idea: the pointer is not isolated system: it is continually interacting with its environment of e.g. air molecules or photons. In the physics of this interaction, the pointer's position is crucial: for various models, the final state of the total system, electron+pointer+environment, determines for the pointer a mixture very close to the “desired” (mathematically, but not interpretatively) right mixture of position eigenstates.

That is: the final reduced state of the apparatus is nearly diagonalized in the representation for pointer-position; i.e. the off-diagonal terms of the density matrix, often called ‘tails’, are nearly zero.

Jargon: Zurek says *einselection* of states (and of the quantities of which they are eigenstates), where *ein* stands for ‘environmentally induced’.

The fact that problem (1) is not solved is shown in the terminology: ‘coherence’ \approx interference terms; ‘decoherence’ \approx diffusion of coherence. So what philosophers will call the main conceptual problem remains: how to understand the “selection” of a single component of the improper mixture. (Here ‘understand’ of course includes the Everettian alternative, ‘deny’: i.e. deny that a single component is “selected”, and maintain instead that all are equally real.)

That decoherence does not solve problem (1) is uncontroversial.

(A): *Measurement is not special!* Similarly for other macroscopic systems (‘macro-systems’) that have a small number of macroscopic or slow or massive degrees of freedom (like the position of the centre of mass), that interact with an environment with very many microscopic or fast or light degrees of freedom such as air molecules or photons. The macrosystem can even be tiny, e.g. a dust particle 10^3 cm. in radius. Plausible models of many such cases (*quantum Brownian motion*) lead to the macrosystem state being very close to a mixture of position eigenstates.

So some talk of the environment making a “continuous measurement” on the system, or of “monitoring” the system: but these words now have no anthropocentric connotation—of the sort Bell condemned!

(B): *Not just position; and not just the “final state”!* One can deduce the approximate validity of the classical equations of motion of a dust-particle from the underlying equations for the quantum system, together with a description of the decoherence process. That is: think of each component of the improper mixture as a wave-packet, rather than a projector or “approximate projector”: the wave-packet is driven along an approximately classical trajectory.

Example of a mixture with 2 components: the canonical two-hump wave-form in 1 spatial dimension, with the humps moving in opposite directions. (But NB the wave-form represents an improper mixture, not a position-representation wave-function.)

That is: We want, and models provide, good behaviour not only in position (or a quantity “close” to position), but also in momentum.

So in terms of continuous measurement: the environment is making a continuous

joint approximate measurement of position and momentum on the system.

And narrow wave packets approximately follow classical trajectories provided the external potential does not vary significantly across the packet (Ehrenfest theorem).

Indeed: some models give improper mixtures whose components are nearly coherent states: “coherent states from decoherence”.

(C): *Going beyond quantum Brownian motion!* Recent work on decoherence in quantum fluids, where there is no clear distinction between system and environment, shows that decoherence selects certain quantities (roughly, hydrodynamic variables) as “behaving classically”. Again, one can deduce the approximate validity of the classical equations of motion for a fluid. (For details, cf. Halliwell (1999): ‘The Emergence of Hydrodynamic Equations from Quantum Theory: A Decoherent Histories Analysis’, available at: <http://xxx.soton.ac.uk/abs/quant-ph/9912037>.)

(D): *Amazing efficiency!* Joos & Zeh (1985) consider (among other examples) a dust particle of radius 10^{-3} cm. in air. They consider the interference terms distinguishing a superposition, of two positions for the (centre of mass of the) dust particle, just 10^{-4} cm. apart, from the corresponding mixture. They show that these terms converge to 0 like $\exp -(t/10^{-36}\text{sec.})!$ And they stay very tiny for very long time-scales (10^{10} years).

(E): *Open-endedness* This is an active research area: much to be done, in terms both of models and theorems. It is also controversial among physicists, quite apart from the main conceptual problem on which philosophers concentrate, i.e. the “selection” of a single component of the improper mixture.

Some of this controversy illustrates the usual tensions between calculating in a model (“how generic is that?!”) and proving a theorem (“are your assumptions physically realistic?”). But there are also conceptual aspects...

How satisfactory is it that? ...

(i): There is in general no natural unique definition of ‘the macrosystem’ to be considered, since there will be rapid and efficient decoherence for any of a wide range of definitions.

(ii): Recent work on hydrodynamics apart, one needs a system-environment split to be put in “by hand”.

(iii): There is no general characterization of which states or basis get selected by decoherence. So beware: talk of decoherence ‘selecting a preferred basis or quantity’ is short-hand. But agreed: models suggest some general ideas about the characterization. Namely:

(a): for quantum Brownian motion: states that get *least* entangled with the environment (cf end of (B) above). One way to make this more precise is: states whose time-evolute (if they *were* the system’s state!) has less von Neumann entropy than others (Zurek’s *predictability sieve*).

(b): for hydrodynamics: the local densities of energy, momentum, number, charge etc. Reason (Halliwell): These are almost conserved if averaged over small volumes, and so are close to exactly conserved quantities ‘which obey superselection rules’.

(F): *Vagueness* Progress on the physics of items (i)-(iii) in (E) will still leave various aspects as a matter of a spectrum, a range: the ‘problem of tails’.

(i): How satisfactory is that? The answer will no doubt depend on the interpretation of quantum mechanics adopted: e.g. pilot-wave, modal, GRW, “Copenhagen” or “Everett”. I myself think that in particular, the Everettian has no problem: they explicitly accept the reality of the “branches” represented by tails, i.e. by regions of configuration space assigned a very low amplitude. See “Everett” below.

(ii): And is there any useful link to philosophical theories of vagueness in language and thought?

“Everett”

The amazing idea: *each component of the universal state Ψ that represents a definite macrorealm is actual/real.*

I will develop this idea in three stages. It will be clearest to postpone discussion of time till the second, and especially the third, stages. First, I make four general comments (A)-(D), including introducing Everett’s notion of the *relative state*. The second stage, (E)-(F) urges two philosophical merits of the Everettian proposal. The third stage, (G), returns us to the treatment of time.

(A): *Jargon* ‘world’, ‘branch’, ‘perspective’ in the ‘multiverse’.

I like Wallace’s proposal:

(i) that we use ‘world’ for what is represented by *any* component of Ψ . So ‘world’ is precise; but wholly arbitrary, and so in general useless and instantaneous—no useful notion of persistence or trans-temporal identity.

(i) that we use ‘branch’ for a definite macrorealm. So ‘branch’ is approximate, anthropocentric and persisting: since ‘definite macrorealm’ has these features.

(B) “*Mentalism*” not just “*anthropocentrism*”? Some authors allow: a pointer can be indefinite in position, even within a branch, provided that it *appears* definite.

(C) *Vagueness*. I take it that vagueness is a matter of language and thought, not of reality. Further, I believe the orthodox view that vagueness is semantic indecision.

But defining ‘branch’ vaguely is OK, since decoherence is so ubiquitous and efficient that the vagueness cannot be noticed by macroscopic physics. That is: the different precisifications of the vagueness all lie (within a branch) well within the error-bars of a macroscopic description of the branch.

(D) The idea of *relative state*: “what the other component system looks like, assuming the given component system is in a certain state”. More precisely: the state prescribed for the other component system, by a selective Luder’s-rule measurement on the given component system.

Given $\Psi = \sum_{ij} |\phi_i\rangle |\psi_j\rangle$ as the state of the composite system $S_1 + S_2$, the state of S_2 , relative to $|\phi_i\rangle$ on S_1 —for short: the relative state of $|\phi_i\rangle$ —is $\sum_j |\psi_j\rangle$ (up to normalization).

So the Everettian defines a branch (at a time) by:—

Factorize the universal Hilbert space into $N + 1$ factor spaces; one factor for each of the N macrosystems (dust-particles etc.), and one factor for the (vastly more complicated) “rest of the universe”. Then: for all N macro-systems, take a component of its “post-decoherence” improper mixture (i.e. a component in the quantity selected by decoherence), and then take the relative state of the rest of universe.

In other words, we expand the universal state as

$$\Psi = \sum c_{i_1 i_2 \dots i_N j} |\phi_{i_1}\rangle |\theta_{i_2}\rangle \dots |\eta_{i_N}\rangle |\psi_j\rangle. \quad (0.4)$$

Three comments about this definition of ‘branch’ (at a time). Of which the third is longest (and most philosophical).

(D.1): *The role of decoherence* This definition of branch appeals to decoherence, twice over. First, and most obviously: in taking for each macrosystem, a component of its “post-decoherence” improper mixture (i.e. a component in the quantity selected by decoherence). Second, less obviously: as discussed in decoherence (E), there is hope that the physics of decoherence will enable us to avoid taking ‘macrosystem’ as a primitive concept.

Contrast the traditional Everettian appeal (by DeWitt, Graham), to primitive notions of ‘apparatus’ (instead of our ‘macrosystem’), and ‘pointer-position’ (instead of our ‘preferred quantity or vaguely defined group of quantities, selected by decoherence’).

(D.2): *Against requiring precision once-for-all* Any precise specification of a preferred quantity, e.g. position or energy, “once and for all” for all systems, will very probably make ordinary objects such as a dust-particle or a cat a collective feature of many branches (thus defined), and not in any one of them. E.g. chemical bonds need delocalized electrons.

So it seems wiser to have the quantity specified system-by-system (more generally, case by case) by present and future results in the physics of decoherence. (This point is strengthened by (D.3)’s point that we need to allow for state-dependent factorizations...)

(D.3): *State-dependent factorizations; Objects as Patterns* We should allow the factorization of the universal state-space to depend on Ψ . After all, think of how the particles treated by elementary quantum theory emerge from quantum field theory: *some* states of the underlying quantum fields have suitable properties (in particular, energy-momentum being appropriately localized for a time-interval that is long compared to micro-physical time-scales).

Indeed, also within elementary quantum theory, we learn that ordinary objects are best conceived of as *patterns*. Very different ordinary objects, such as a dog and cat, or even a dog and a brick, can be thought of as different states of a Hilbert space. Indeed, there are two main points here, (1) and (2): of which the second, (2), supports the Everettian; as follows.

(1): Any two Hilbert spaces of the same dimension are isomorphic. So if two objects, say a dog and a brick, have the same number of degrees of freedom, we can “in principle” use a Hilbert space isomorphism to map the quantum-theoretic description of one object onto the other. But more is true: under some conditions, such an isomorphism involves a physically natural correspondence of quantities—and under such an isomorphism, all the differences between the objects seem to be a matter of differing patterns in a wave-function.

Thus imagine that a dog and a brick—or more likely, a dog and a cat—have exactly the same number of atoms (say 6×10^{24}), with the same proportions of the chemical elements (hydrogen, oxygen, carbon, nitrogen, iron etc.). Working with a non-relativistic quantum-theoretic description, in which each atom consists of electrons, protons and neutrons, we can imagine the average atom has 40 such particles. So the

dog and cat each have $N := 40 \times 6 \times 10^{24}$ particles, and its quantum state has a position-representation as a wave-function in $L^2(\mathbb{R}^{3N})$. Among all the countless Hilbert-space isomorphisms between the copies of $L^2(\mathbb{R}^{3N})$ we use to represent the dog's and cat's states, we can then restrict attention to those that give a physically natural mapping of quantities. At a minimum, we will want the position/energy/momentum etc. of an electron/proton/neutron to be mapped to the same quantity, position etc., on a particle of the same species, electron etc. (Nevermind what more we might want.) Under such a natural mapping, all the differences between the dog and the cat seem to be a matter of differing patterns in a wave-function. (Differences that show up in the dog's and cat's states not being images of each other under the mapping.)

(2): Agreed: so far, dog and cat each have their own Hilbert space (with no common factor-spaces), and are thereby distinct (indeed disjoint) systems, notwithstanding the “differences seeming to be just a matter of pattern”. But as we have seen, a full quantum-theoretic description of the unitary evolution of a Schrödinger-cat experiment (until after the action of the “infernal device”) will yield, as the final reduced state of the cat, an improper mixture (with weights about 50-50) of ‘cat alive’ and ‘cat dead’.

Let us again talking, for simplicity, in terms of wave-functions on configuration space. That is: we will now assume that position (“configuration”) is the preferred quantity selected by decoherence, and that we can thereby pass from talk of improper mixtures to talk of wave-functions (after all, their statistics match for position itself). So: each of these two alternatives, ‘cat alive’ and ‘cat dead’, is represented by a large-amplitude peak over a region of configuration space: the first peak being over the myriadly many configurations that instantiate (in a common philosophical jargon: “realize”) a walking/purring/metabolizing cat, and the second peak being over the myriadly many configurations that instantiate a cat lying down and decaying.

So the Everettian proposal that both alternatives are equally actual/real corresponds precisely to a view of objects like cats (or living cats or dead cats) as patterns. Where ‘corresponds precisely’ amounts to an ‘iff’: the Everettian’s proposal implies taking the cat to be a pattern; and taking cats and other macroscopic objects as patterns in a quantum state implies that after a Schrödinger-cat experiment, there are indeed *two cats*.

Two final comments about (2):—

(i): This “functionalist” view of objects, especially of examples of natural kinds, or more generally of examples of scientific concepts, can be summed up as: a cat is whatever behaves enough like a cat for our purposes of description, prediction and explanation. This view has been defended by Dennett (e.g. ‘Real Patterns’, *Journal of Philosophy* 1991)); and developed for the Everettian interpretation by Wallace.

(ii): The discussion in (2) took the live and dead cats-as-pattern to each have about 50% probability. But the discussion of objects as pattern is surely independent of small vs. large probability (or complex amplitude). So I think the Everettian should accept objects as patterns, even when the probabilities (weights) are tiny. Hence my remark in (F) of decoherence that I think that tails present no problem to the Everettian

(except perhaps a problem of being credible!!). That is: I think the Everettian can and should explicitly accept the reality of the “branches” represented by tails, i.e. by regions of configuration space assigned a very low amplitude.

... I turn to presenting two philosophical merits of the Everettian proposal...

(E): *Populations and probability* We do *not* need a whole population of systems in each component of Ψ (in a preferred quantity). For (very briefly):

1): Do not ground probability in counting measures!

2): Recent theorems (Deutsch and Wallace) deduce the Born rule from Everettian decision theory.

Rather we should say that a single system *splits* when the number of components of Ψ increases. Cf. the end of (D.3) above.

(F): *Analogy: worlds for Everettian are like times for the eternalist (aka: the tenseless theorist, the “B-theorist”, “block universe” theorist).*

In both cases, the items (worlds and times respectively) are:—

(i): Useful, or even indispensable: how else can the Everettian interpret the wave-function; or the eternalist describe time-evolution?.

(ii): In principle definable in an arbitrary way, though in practice, there is just one or a few useful choices: cf. the Everettian’s contrast between (in Wallace’s jargon, cf (A)) worlds and branches; and the eternalist’s contrast between an arbitrary foliation (or more generally: local spacetime coordinate system), and one that is useful (a good “choice of variables”) for the problem at hand.

(iii): Not a significant ontological commitment, additional to the main one; (the universal quantum state; the “block-universe” respectively).

(iv): Most objects, and their properties, are a collective features of many of the items (worlds/times).

... I turn to discussing the treatment of time in Everettian interpretations ...

(G): *Time in Everett* There are two main points to make.

(i): In general, persistence over time (aka: trans-temporal identity) makes no sense for (systems in) arbitrary worlds. But it makes vague and approximate sense for systems in branches.

(ii): How should we understand *splitting*? Is it a matter of:

(a): one pointer with contrary positions relative to distinct worlds/branches?

Or

(b): many pointers (“copies” of one another) with different positions, one from another?

In discussing this, an analogy can be made with two rival metaphysical accounts of persistence. (a) is like (a'): *endurance*. (b) is like (b'): *perdurantism*.

Two points in favour of option (b):—

1): Option (b) does *not* involve violations of conservation laws for conserved quantities such as mass (non-relativistically) or charge: these quantities do not increase at a split. Cf. how perdurantism does not involve double-counting of quantities like mass and charge.

2) Option (b) is supported by (D.3's) idea of *objects as patterns*. For:

First: a macroscopic pointer might be subject to a Schrödinger-cat like experiment, in which it is vaporized in one of the two ensuing branches (but say, reading '5' in the other). In such a case, it seems wrong to say that after the branching, there is one pointer, vaporized in one branch, but intact and reading '5' in the other.

And second: Suppose you reply that this is not so wrong, since in the branch where the pointer is vaporized, it can be identified with its microphysical constituents, dispersed though they be. The rejoinder is that as (D.3) pointed out, even microphysical constituents such as electrons, protons and neutrons are really patterns in underlying quantum fields: and as such, the constituents might exist in only one of the two ensuing branches.

Everybody’s Problem: “Tails”

I first describe, in (I), the problem of tails in a way that is formally very simple, but applicable to discussion of several interpretations, including the modal, GRW and Everett interpretations. Then in (II), I describe what I take to be the Everettian view of tails; and how it leads the Everettian to a straightforward, but flexible, answer to (I)’s formulation of the problem.

(I): *The problem in general*

The problem is that:

(i) at the start of developing the interpretation, we desire certain quantities on certain systems to have definite values; but

(ii) the developed interpretation does not give us this; but at best, definite values (equal to, or at least close to, the desired ones) for quantities “close” to the desired ones, on the desired systems (or perhaps similar systems).

Discussion has revolved around the mismatch as regards quantities; not as regards values, or the systems concerned. I suppose there are two reasons for this.

(a): A reason of happenstance. The problem is mostly discussed for the modal and GRW interpretations; where the mismatch of quantities is much more prominent. (The mismatch of systems does not occur.)

(b): A reason of principle. Maybe the mismatch of quantities is more unsettling than the others. To be sure, several authors write that we can be confident, at least once we adopt a general line of interpretation, of our desiderata in (i) about which quantities are to be definite: at least more confident than about (i)’s desiderata about values and systems.

(To set up the problem, one can allow that in (i) we are vague, or unopinionated, about exactly which quantities, values and systems. There will still be a problem, if in (ii) the developed interpretation delivers definiteness (for quantities, values, systems) that mismatches the desiderata so grossly as to fall outside the allowances of vagueness or lack of opinion.)

So consider the following “model” of the problem-situation, which focusses only on a mismatch of quantities:—

(A): Let us say that physical quantities R and R' are *close enough* when according to our ambient theory, physical and philosophical, we will accept definiteness of the one (with value $r \in \mathbb{R}$ on system S) as a substitute for a desideratum that the other be definite (with value r on S).

(This could be made precise in various ways, from a philosophical viewpoint. Eg “functional-role semantics” would say: R and R' are close enough when we take both (i) the grounds for ascribing a value, and (ii) the consequences of ascribing a value, to be very similar for R and for R' .)

(B): Consider two self-adjoint operators O and O' on (for simplicity) a (finite-dimensional) Hilbert space: with the same (pure discrete) spectrum, and matching degeneracies. Let us say that O and O' are *Hilbert-close* when their eigenspaces for a

common eigenvalue are close. Here we take eigenspaces U, V of O, O' respectively to be close if we can find an orthobasis u_i of U , and an orthobasis v_i of V , which with a suitable ordering yield $\langle u_i | v_j \rangle \approx \delta_{ij}$.

(I suppose this can be developed, and yet made precise, for more general cases. Maybe the spectra are not the same, but only close. Maybe the degeneracies do not match. Maybe we can consider infinite-dimensional Hilbert spaces. Maybe we can define closeness of eigenspaces, and so quantities, without using a basis.)

In general terms, “problem of tails”, for quantities, is then the interpretative question:

Are we happy to say that that physical quantities R and R' are *close enough*, in (A)’s sense, when their representing operators O and O' are Hilbert-close, *a la* (B)?

This is clearly liable to be an interpretation-dependent matter! Cf (II).

Besides: It seems clear that one could give a similar formulation of the problem, for values and for systems. For

Closeness of values is only desirable if the quantities, or at least corresponding eigenspaces, are close in the above sense from (B)

Closeness of systems can be expressed by taking the two systems as sub-systems of a third, larger system; and then considering the closeness in (B)’s sense of the factor Hilbert spaces representing the two systems.

(II): *Tails for the Everettian*

The abstract general question at the end of (I), viz.

Are we happy to say that that R and R' are close enough, in (A)’s sense, when their representing operators O and O' are Hilbert-close, *a la* (B)?

becomes more vivid when we consider the Everettian proposal, in particular the Everettian’s idea of objects as patterns, as discussed at the end of (D.3) (in comments (2) and (2.ii) within D.3).

I will argue that broadly speaking, the Everettian answers this question ‘Yes’: the underlying reason being that decoherence only defines a range of “preferred quantities”, indeed an approximate and even vague range. But we will see some worthwhile details about the relation between tails and how to define a branch. We will also see that for the Everettian the “real mystery” of quantum mechanics is the one we first encounter when learning the theory—viz. the interpretation of superposition.

To bring out these points, we can specialize to a single eigenspace of the quantities and operators involved.

Imagine that R is a quantity that encodes, within macroscopically attainable precision, the positions (and perhaps also velocities) of the macroscopically distinguishable parts of the cat. So R is one of the approximate and even vague range of quantities on the cat selected by decoherence. (We again imagine the R has pure discrete spectrum, say by being a discretization of position (and maybe momentum) for non-relativistic quantum theory.)

Let U be an eigenspace of R which corresponds to ‘cat alive’ in the sense that U is the sum (span) of a family of orthogonal rays (1-dimensional subspaces, U_1, U_2, \dots), all

of whom represent a walking/purring/metabolizing cat. (So: (i) U will in general have very high dimension; (ii) we can allow that there are many other eigenspaces U', U'', \dots of R , of course orthogonal to $U \equiv \oplus_i U_i$, which also correspond to ‘cat alive’.)

Let $|\psi\rangle$ be a vector orthogonal to U , representing (one highly specific version of!) ‘cat dead’. (So $|\psi\rangle \notin U', U'', \dots$) Let D be the ray spanned by $|\psi\rangle$.

Define a subspace V as “very close to U , but with a slight component along $|\psi\rangle$ ”. Of course, there is a lot of choice about how to do this: which of U ’s 1-dimensional subspaces, U_1, U_2, \dots , should be “perturbed” i.e. replaced in the definition of V by a ray with a “little” non-zero projection on D ? Let us take U_1 to be thus “perturbed”.

That is: we choose a small real number $\epsilon > 0$; then with U_1 spanned by, say $|\phi\rangle$, we define V_1 as the ray spanned by $(1 - \epsilon)|\phi\rangle + \epsilon|\psi\rangle$. Then we define V as the sum of the following family of mutually orthogonal rays: V_1, U_2, U_3, \dots

By construction V is close to U , in the sense explained in (B) of (I) above.

So the problem of tails, in the formulation of (I), applied to the Everettian, is the question whether:—

Assuming the cat is in a state that not only lies in V but also has non-zero projection on D (not just on V_1), is the Everettian happy to say that the cat is alive (i.e. despite the non-zero, though small, projection on D)?

More precisely: the assumption of the question should be that the cat is in an improper mixture, one component of which is a ray with non-zero projection on D . So the question is: considering this component, is the Everettian happy to say that the cat is alive “in this branch”?

The answer to this question obviously depends on the definition of ‘this branch’, in particular on how the definition relates to the subspaces U and V . There are two options, which I label (i) and (ii). But I think the approximateness (and even vagueness) of the notion of a branch means the Everettian does not need to choose between them. In some contexts, the first option will seem more natural; in others the second. (I agree that at first sight, the first seems more natural, since it ties ‘branch’ to the first subspace I introduced, viz. U .)

(i): Since we took U as an eigenspace of the “preferred quantity”, it is natural to take ‘this branch’ to *exclude* this admixture of “cat dead”, i.e. it is natural to take ‘this branch’ to be orthogonal to D . In which case, the answer to the above question is surely Yes. For one main idea of the Everettian strategy is to maintain the orthodox eigenvalue-eigenstate link *within* each world or branch.

But if we say this, we must also admit that there is a non-zero amplitude for *another* branch (represented by a subspace, in general of high dimension, of which the ray D is a subspace). And in this other branch, the cat is dead. Cf. the Everettian’s idea of objects as patterns, as discussed at the end of (D.3) (in comments (2) and (2.ii) within D.3).

Thus we return to the basic vision of the Everettian: and to the puzzle of understanding superposition. The Everettian can admit that this is an especially vivid puzzle for macro-systems. But on this version of Everett, there is no *further* problem

of tails.

(ii): On the other hand, suppose we take V to define ‘this branch’; so that the non-zero amplitude for (admixture of) “cat dead” is in this branch.

This option may seem to violate the original Everettian desideratum that within a branch macro-systems should have definite values, at least for familiar quantities like position and momentum; and that cats should be definitely alive or dead. Indeed, combining the Everettian’s idea of objects as patterns, discussed at the end of (D.3) (in comments (2) and (2.ii)), with the idea of V defining a branch implies that this branch *does* contain a dead cat (albeit of low amplitude) as well as a live one.

But maybe we should take the approximate and flexible nature of branches to allow for branches in which macro-systems do not have definite values: i.e. we should be willing to give up the original Everettian desideratum. It seems to me that the Everettian should accept this: after all, it reflects the vagueness of the distinction between ‘world’ and ‘branch’ (in Wallace’s jargon). So the idea will be that we can afford to allow for such branches since they will be “near” other branches that *do* represent a definite macrorealm (i.e. in which all macro-systems have definite values for familiar quantities). Here “nearness” of branches can be spelt out in terms of the closeness in Hilbert space of their corresponding subspaces (as in (B) of (I) above); and can be required to be stringent enough that differences in values (or probabilities) assigned to quantities fall within the error-bars of macroscopic physics.

I think that on this option, the problem of tails, i.e. the question posed above, looks much as it did in (I) for other interpretations, i.e. the modal and GRW interpretations. But with the difference that just because the Everettian only expects an approximate, even vague, definition of “branch” (and as of today, can only get a rough and tentative definition), they can resort to option (i).

To sum up: I think the Everettian does not need to choose between defining ‘this branch’ *a la* (i), and *a la* (ii).