

AST 553. Plasma Waves and Instabilities

Syllabus

Introduction

❖ Basic concepts

- Waves in plasmas as oscillations of electromagnetic field
- Maxwell's equations, Gauss's laws as initial conditions
- Model of a linear medium, conductivity operator
- Conductivity in the spatial representation
- Ohm's law, spatial and temporal dispersion
- Susceptibility operator, dielectric operator
- Maxwell's equations for linear media
- Frequency operator, wavevector operator
- Dispersion operators for electrostatic and electromagnetic waves

❖ Waves in homogeneous linear media

- Model of a homogeneous linear medium
- Spectral representation of the conductivity operator
- Convolution theorem, physical meaning of $\sigma(\omega, \mathbf{k})$
- Kramers–Kronig relations
- Spectral representation of the susceptibility operator
- Dielectric tensor, dispersion tensor, refractive index
- Applicability of the electrostatic approximation
- Dispersion relation, dispersion branches, scalar and vector waves
- Wave polarization, polarization vectors
- Quasimonochromatic waves
- Phase velocity and group velocity
- Dispersive and dispersionless waves
- Schrödinger equation for the wave envelope

❖ Nonmagnetized cold plasma as a linear medium

- Linearized fluid equations
- Conductivity, susceptibility, and dielectric operators
- Conductivity in the coordinate and spectral representation

- Dispersion tensor, dispersion branches
- Applicability, infinitesimally weak collisions

❖ **Waves in homogeneous nonmagnetized cold plasma**

- Static magnetic-field mode
- Electron Langmuir oscillations
- Transverse electromagnetic waves

❖ **Basic wave transformations**

- WKB approximation
- Field structure near cutoff, Airy model
- Reinstating the WKB approximation near cutoffs
- Oblique incidence on a cutoff: TE wave
- Oblique incidence on a cutoff: TM wave

General theory of quasimonochromatic waves

❖ **Asymptotic expansion of a dispersion operator**

- Geometrical-optics (GO) parameter
- Wavevector and wavevector operator in spacetime
- Waves as vectors in Hilbert space
- Wigner–Weyl transform (WWT) and its basic properties
- Envelope dispersion operator
- Envelope equation
- Dispersion operator: symbol vs. dispersion tensor

❖ **Equations of geometrical optics**

- GO ordering, approximate envelope equation
- Scalar equation for the complex amplitude
- Local dispersion relation, local polarization
- Consistency relations
- Ray equations, quantum interpretation
- Wave-action density \mathcal{I} and flux density \mathcal{J}
- Connection between the group velocity, \mathcal{I} , and \mathcal{J}
- Equation for the action density, \mathcal{I} on a ray
- Spin Hall effect of light

❖ **Wave action, energy, and momentum**

- Action conservation, action as an adiabatic invariant
- \mathcal{I} and \mathcal{J} for electromagnetic waves
- Wave energy, transport equation
- Wave energy flux
- Absorbed power, Ohmic heating
- Wave momentum, transport equation
- α -channeling as momentum conservation

Waves in plasmas: fluid theory

❖ Waves in cold magnetized plasma

- CCS magnetized plasma: symbol of $\hat{\epsilon}$ vs. dielectric tensor
- Expression for ϵ in Stix's notation (S, D, R, L, P)
- High-frequency and low-frequency limits of ϵ
- General dispersion relation
 - Biquadratic equation for the refraction index N
 - Equation for $\tan \theta$
 - Cutoffs and resonances
 - Lower-hybrid and upper-hybrid frequencies
 - Electrostatic dispersion relation
- Parallel propagation
 - Cutoffs, resonances, polarization
 - L and R waves, whistlers
- Perpendicular propagation
 - O wave: dispersion, polarization, cutoff
 - X wave: dispersion, cutoffs
- Propagation at a general angle, level repulsion
- Characteristic frequencies in magnetically confined fusion plasmas

❖ Waves in warm fluid plasma

- Fluid equations for warm plasma
- Waves in warm nonmagnetized plasma
 - Perpendicular and parallel susceptibility
 - Warm species, Langmuir waves
 - Hot species: Debye shielding
 - Ion acoustic waves, ion plasma waves
- Waves in warm magnetized plasma (qualitatively)

Waves in plasmas: kinetic theory

❖ Introduction to kinetic theory of plasma waves

- Probability distribution, variable transformations
- Canonical distribution, Liouville theorem
- Vlasov equation, linearized Vlasov equation
- Longitudinal conductivity
- Phase mixing
- Case–van Kampen modes
- Initial-value problem, quasimodes

❖ Waves in nonmagnetized plasma

- Dielectric tensor
- Landau's rule
- Transverse waves in isotropic plasma
- Longitudinal waves in isotropic plasma
- Approximate formulas for ω_r and ω_i

❖ Electrostatic waves in Maxwellian plasma

- Plasma dispersion function Z
- Z as an integral over the Landau contour
- Asymptotics of Z
- Langmuir waves in Maxwellian plasma
- Ion acoustic waves in Maxwellian plasma

❖ Landau damping and kinetic instabilities

- Passing and trapped trajectories
- Bounce frequency as a function of energy
- Width of the trapping island
- Mechanism of Landau damping
- Nonlinear saturation of Landau damping
- Inverse Landau damping
- BGK waves
- Nyquist theorem
- Stability of distributions with one and two peaks

❖ Dispersion and dissipation in magnetized plasma

- Solution of the linearized Vlasov equation on characteristics
- General structure of the dielectric tensor, cold limit
- Conditions of resonant absorption in not-too-hot plasma
- Resonance condition in the single-particle picture
- Landau damping: basic properties
- Transit-time magnetic pumping
 - Physical mechanism, analogy with Landau damping
 - Basic properties, effect of cross terms
- Cyclotron damping
 - Basic properties
 - Diffusion paths
 - Nonlinear saturation

❖ Waves in magnetized plasma

- Kinetic dielectric tensor of magnetized isotropic Maxwellian plasma
- O and X waves
- Electrostatic electron Bernstein waves (EBW)
- EBW coupling with X wave and DK waves
- Plasma heating with EBW:
 - CMA diagram for X wave
 - X–B conversion
 - O–X–B conversion
- Electrostatic ion Bernstein waves (EBW)
- Accessibility of the lower-hybrid resonance

❖ Quasilinear theory

- Introduction
 - Interaction with one wave: fractional nonlinearities
 - Interaction with two waves: Chirikov criterion
 - Interaction with many waves: statistical approach
- Simplified quasilinear theory for homogeneous plasma
 - Assumptions, quasilinear (QL) approximation
 - Diffusion coefficient: basic properties
 - Equation for the spectral energy density
 - Dispersion relation
 - Conservation laws of QL theory
- Broadband bump-on-tail instability

- Formation of a quasilinear plateau
- Momentum conservation
- Dressed particles, ponderomotive effects, and amended QL theory
 - Oscillation centers (OCs), ponderomotive energy
 - OC Hamiltonian
 - Variational principle for GO waves
 - Quasilinear diffusion for the OC distribution
 - Wave kinetic equation for on-shell fields
 - Conservation laws: OCs + waves vs. particles + fields

Homeworks (tentative)

❖ Homework 1

- Electrostatic approximation
- Photon wave function in cold magnetized plasma
- Beam-plasma instability (cold electrostatic limit)
- Surface waves

❖ Homework 2

- Single-mode dynamics within geometrical optics
- Coupling of resonant waves, mode conversion

❖ Homework 3

- Interferometry, reflectometry, Faraday rotation
- Wave transformation in the ionosphere
- MHD waves in warm fluid plasma

❖ Homework 4

- Alfvén resonance
- Wave propagation: initial-value problem
- Longitudinal waves in Lorentzian plasma

❖ Homework 5

- Two-stream instability in Lorentzian plasma
- Weibel instability in non-magnetized plasma
- Plasma susceptibility in magnetic field

❖ Homework 6

- Kinetic waves propagating parallel to magnetic field
- Kinetic whistler waves
- Cyclotron damping