

Discussion of  
**“Innovation, Firm Dynamics  
and International Trade”**

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## Simple Model: Envelop Theorem

- Free Entry Condition:

$$n_e = \int_{z_d} \left\{ [1 + I(z_x)D^{1-\rho}] \pi(z) - n_f - I(z_x)n_x \right\}$$

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- Direct versus Indirect Effects:

$$(\rho - 1)\hat{Z} = \underbrace{s_x \widehat{D^{1-\rho}}}_{\text{direct}} + \underbrace{[\hat{M} + \phi_d \hat{Z}_d + \phi_x \hat{Z}_x]}_{\text{indirect}=(1-\lambda)\hat{Z}}$$

## Comments: Limits of the Result

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$$(\lambda + \rho - 2)\hat{Z} = \tilde{s}_x \widehat{D}^{1-\rho}, \quad \tilde{s}_x \neq s_x$$

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- ④ Large changes in  $D$ :

$$ds_x/dD^{1-\rho} \quad \text{depends on the model}$$