# INSTRUCTOR'S MANUAL

FOR

# *COMPUTABILITY AND LOGIC*

FIFTH EDITION

## PART A. FOR ALL READERS

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## Note

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*but instructors who adopt* Computability & Logic *as a textbook*
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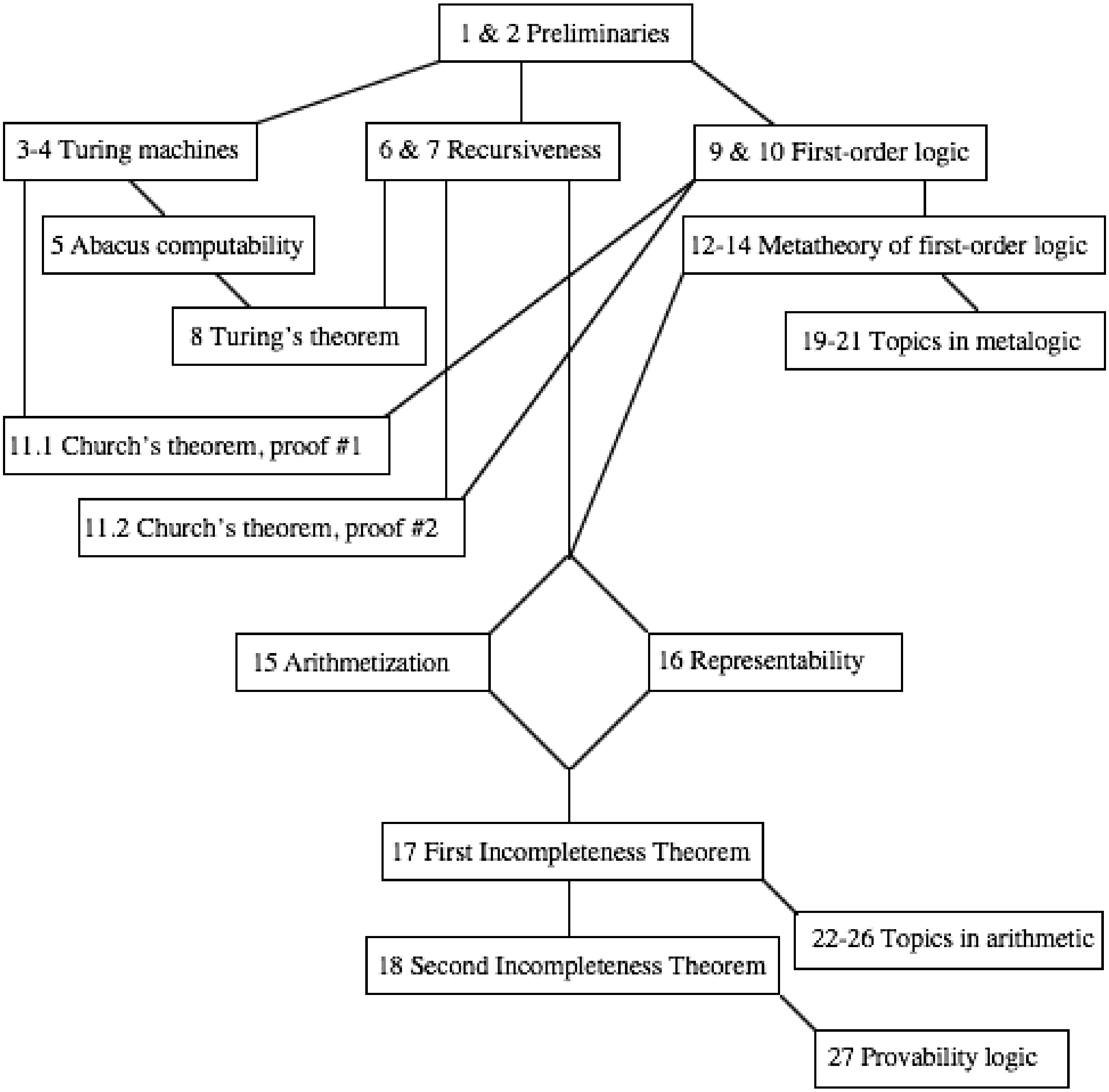
## Contents

# Dependence of Chapters



# General Remarks on Problems (for Students)

- The problems for each chapter should be read as part of that chapter, even those that are not assigned. They often add important information not covered in the text.
- The results of earlier problems, whether or not assigned, may be used in later problems. Many problems are parts of sequences.
- Before working the problems for any chapter, check to see whether there are any errata listed for that chapter, and especially for its problems.
- Hints are provided for odd-numbered problems in chapters 1-18. The hints for some problems are inevitably more substantial than those for others.

# Hints for Odd-Numbered Problems: Computability Theory (Chapters 1-8)

**Chapter 1**

**1.1** The converse assertion then follows from the first assertion by applying it to $f^{-1}$ and its inverse $f^{-1-1}$.

**1.3** For ($a$) consider the *identity* function $i(a) = a$ for all $a$ in $A$. For ($b$) and ($c$) use the preceding two problems, as *per* the general hint above.

**1.5** Show both sets are denumerable.

**1.7** If we can fix for each $i$ an enumeration of $A_i$

$$A_i = \{a_{i1}, a_{i2}, a_{i3}, \dots\}$$

Then we can enumerate $\bigcup A$, which is the set of all $a_{ij}$ for all $i$ and $j$ in the same way we enumerated pairs $(i, j)$ in Example 1.2. However, when we assume that for each $A_i$ there exists an enumeration of it, it follows that there exist many different such enumerations for each $A_i$; and when set theory is developed rigously, in order to conclude that there is a way of fixing *simultaneously* for *each i* some one, specific enumeration out of all the many different enumerations that exist, we need a principle known as the *axiom of choice*. As this is not a textbook of set theory, we are not going to go into such subtleties.

**Chapter 2**

**2.1** Imitate the proof for the set of positive integers.

**2.3** You do *not* need to use trigonometry or give an analytical formula for the correspondence to do this problem; a simple geometric description of a correspondence will be enough.

**2.5** While this can be done using the preceding two problems, as *per* the general hint, for students who remember trigonometry, a correspondence can also be defined directly using the tangent function.

**2.7** Note that rational numbers whose denominator (when written in lowest terms) is a power of two have two binary representations, one ending in all 0's and the other in all 1's from some point on (as in $1/2 = .1000000\ldots = .0111111\ldots$), while *in every other case the binary representation is unique and does not involve all* 0's *or all* 1's *from any point on*.

**2.9** In addition to the immediately preceding problems, Problem 1.6 may be useful.

**2.11** Read carefully through the sequence of preceding problems.

**2.13** This is a philosophical rather than a mathematical question, and as such does not have a universally agreed answer, though there is a consensus that somehow *defining a set in terms of the notion of definability itself* is somehow to blame for the paradox.

**Chapters 3 & 4**

**3.1** One state will be required in (a), two in (b).

**3.3** Proceed as in Problem 3.1(b) but when reaching a blank in state 2 print a stroke and go into state 3. At this stage you will have a block of $n$ strokes followed by a blank followed by a block of $m + 1 + k$ strokes. In state 3 on a stroke move right and go into state 4. In state 4 on a stroke erase it. You will now have blocks of $n$, $m + 1$, and $k - 1$ strokes. Take it from there.

**3.5** Proceed in cycles, during each of which you erase the leftmost stroke of the first block and the rightmost stroke of the second block, and add a stroke to a third block to the right of them both. When one of the two original blocks has been completely erased, erase also the other. The trick is to keep track of when this happens.

**4.1** It is certainly not possible just *exploring* without *marking* the tape.

**4.3** It is not possible to preserve the original block unaltered while making a copy.

**4.5** A description of a function of the kind a universal machine would have to compute is implicit in the discussion of the diagonal function in the text.

**Chapter 5**

**5.1** Subtraction is to the predecessor function as addition is to the successor function.

**5.3** Use problem 5.1.

**5.5** Keep subtracting $y$ from $x$, while checking each time you do so that what is left is still $\geq y$.

**5.7** Manœuvres of just this kind take place the simulation of abacus machines by Turing machines.

**5.9** See preceding problems.

**5.11** See the proof of Theorem 4.1.

**Chapter 6**

**6.1** For instance, in (a), $g(x, y) = f(\mathrm{id}^2_2(x, y), \mathrm{id}^2_1(x, y))$.

**6.3** These can be done 'from scratch' or, generally more easily, by showing the indicated functions are compositions of functions already known to be primitive recursive.

**6.5** Proposition 6.5 may be useful.

**6.7** Each recursive function is denoted by some expression built up using Cn, Pr, and Mn from names for the zero, successor, and identity functions.

**6.9** Use the following fact: There is a recursive function $f$ such that $f(0) = 0$ but $f(x)$ is undefined for $x > 0$. (For instance, $f(x)$ = the least $y$ such that $|x - y| + y = 0$.)

**Chapter 7**

**7.1** Compare with Problem 6.1.

**7.3** Use Corollary 7.8.

**7.5** Consider the auxiliary function $g(n)$ = the least element of $A$ that is $> n$.

**7.7** Apply the preceding two problems to obtain a recursive function $a$ and use it and the original $f$ to define a suitable $g$.

**7.9** First show that the auxiliary function $g(n) = J(f(n), f(n+1))$ is primitive recursive, where $J$ is as in Problems 6.2 and 6.5.

**7.11** First introduce a suitable auxiliary function, as in Example 7.20.

**7.13** Suppose that $c_i$ and $d$ are the numbers associated with $g_i$ and $f$ respectively, so that

$$g_i(x_1, \ldots, x_n) < c_i \max(x_1, \ldots, x_n) + c_i,$$

$$f(y_1, \ldots, y_m) < d \max(y_1, \ldots, y_m) + d.$$

Show that $d(c + 1)$ will do as a number associated with $h$.

**7.15** This is the problem that requires most familiarity with mathematical induction, according to which, in order to prove that all $x$ and all $y$ have some property it is enough to show that

(1) 0 and 0 have the property

(2) if 0 and $j$ have the property, then 0 and $j + 1$ have the property

and that if $i$ is such that $i$ and $j$ have the property for all $j$, then

(3) $i + 1$ and 0 have the property

(4) if $i + 1$ and $k$ have the property, then $i + 1$ and $k + 1$ have the property.

**7.17** First show that the auxiliary function

$f(p, q)$ = the least $s$ that covers $(p, q)$

is a recursive total function.

**Chapter 8**

**8.1** Remember that the right numeral is obtained by reading *backwards*, so that if $x_1 = 2$ and $x_2 = 3$, say, then the right numeral is 11110111.

**8.3** Use the graph theorems.

**8.5** Use the fact, noted just before the statement of Theorem 8.5 that the graph relation of the universal function $F$ constructed in the proof of that theorem has the form $F(m, x) = y \leftrightarrow \exists t\, Qmxyt$ where $Q$ is *primitive* recursive.

**8.7** See the problems for chapter 7.

**8.9** Let $A$ be as in the proof of Corollary 8.8.

**8.11** Show that if this claim failed for some $f$, then $A$ would be recursive.

# Hints for Odd-Numbered Problems: Basic Metalogic (Chapters 9-18)

**Chapter 9**

**9.1** For readers who have not previously studied logic, or whose memories of their previous study of logic are rusty, there will be one subtlety here, over how to represent 'All Ms are Ss'. For an indication of the manner in which this construction is treated in modern logic, see displayed formulas (9) and (10) in section 9.1.

**9.3** Here 'in colloquial terms' would mean, for instance, saying 'grandparent' rather than 'parent of a parent'.

**9.5** Use induction on complexity.

**9.7** We do (c) as an example. If ($F$ & $B$) is to be anything less than the whole of ($F$ & $G$), then $B$) must be a left part of $G$, and hence by the Lemma 9.4(c) must have an excess of left over right parentheses. But this is impossible, since $B$, being a formula, has equally many parentheses of each kind, and therefore $B$) has one more right parenthesis than it has left parentheses.

**Chapter 10**

**10.1** First show that substituting $t$ for $c$ in a closed term does not change the denotation of the term.

**10.3** You will need to describe an interpretation, specifying its domain and the two-place relation on it that is to serve as the denotation of $R$. Reading $R$ as 'greater than' may help suggest one.

**10.5** In mathematics, 'All *As* are *Bs*' counts as 'vacuously' true if there are no *As*.

**10.7** Compare with Example 10.3(d).

**10.9** Compare with Example 10.5.

**10.11** For (c), think of replacing $A$ by $B$ as a two-step process: introduce a new atomic $C$, and first replace $A$ by $C$, then $C$ by $B$.

**10.13** For (a), the result for multiple variables is immediate from the result for a single replacement, on repeated application of the latter. To prove the result for a single variable, define a transformation * on formulas, eliminating bound occurrences of the variable $y$, by induction on complexity as follows. For an atomic formula $G$, let $G^* = G$. If $G = {\sim}F$, let $G^* = {\sim}F^*$, and if $G = (F_1 \,\&\, F_2)$, let $G^* = (F_1{}^* \,\&\, F_2{}^*)$, and similarly for $\vee$. If $G = \forall x F(x)$, where $x$ is a variable other than $y$, let $G^* = \forall x F^*(x)$, while if $G = \forall y F(y)$, let $G^* = \forall z F^*(z)$, where $z$ is the alphabetically first variable not already occurring, and similarly for $\exists$. It remains to prove $G$ and $G^*$ are equivalent for any sentence $G$.

**Chapter 11**

**11.1** Describe how to obtain a solution to the decision problem for implication from a solution to the decision problem for validity.

**11.3** Show that the second premiss implies

$\forall u \forall v \forall w((\exists y(Rwy \,\&\, Syv) \,\&\, Suv) \to Rwu)$

**11.5** There is no help for it but to reread the proof carefully.

**11.7** Re-examine the proof in section 11.1, then modify the definition of *standard interpretation* so that the domain consists only of the operating interval for the computation as defined in Problem 11.6.

**11.9** Try proving it first for $n = 0$, then for $n = 1$, then for $n = 2$, and so on.

**11.11** First show that the function stdh in the proof of Theorem 8.2 is a *three*-place function such that the set of pairs $(x, y)$ for which there exists a $z$ with stdh$(x, y, z)$ is non-recursive.

**Chapter 12**

**12.1** What does $A$ tells us about the relative numbers of elements in the domain satisfying $Px$ and satisfying $\sim Px$?

**12.3** This can be done with a language having a one-place predicate $Px$ and two one-place function symbols $f$ and $g$. The trick is to find a sentence saying that there are as many elements in the domain altogether as there are *pairs* of elements satisfying $Px$.

**12.5** Label the vertices in clockwise order $A, B, C, D$, and label the sides suggestively as $a = AB$, $b = BC$, $c = CD$, $d = DA$.

**12.7** In the days before modern computers and calculators, a shortcut used with multiplication problems was to turn them into addition problems. How was this done?

**12.9** If $\mathcal{M}$ is a model of $\Delta$, and if $j$ were an isomorphism from $\mathcal{M}$ to the standard model $\mathcal{N}$, what would be $j(c^{\mathcal{M}})$?

**12.11** Combine the methods of the appropriate parts of the preceding problem.

**12.13** Given a correspondence $f$ from $N$ to $X_1$, call one element $a$ of $X_1$ less than another element $b$ of $X_1$ if $f^{-1}(a)$ is less than $f^{-1}(b)$ in the usual order on natural numbers. Let $a_{0,0}$ be $f^{-1}(0)$, the least element of $X_1$. For each $k$ let $a_{k+1,0}$ be the least element of $X_1$ not $E_1$-equivalent to any $a_{i,0}$ for $i \leq k$. For each $m$ let $a_{k,m+1}$ be the least element of $X_1$ that is equivalent to $a_{k,0}$ and not identical to any $a_{k,i}$ for any $i \leq m$.

**12.15** See Problem 10.6

**12.17** Use the preceding problem and the observation that for any one, given denumerable nonstandard model or arithmetic, the set of sets of primes encrypted in that model is enumerable, since the set of elements of the domain available to encrypt sets of primes is.

**12.19** Look at the problems to follow.

**12.21** List the elements of the domain of $j$ in increasing $<_A$ order as $a_0, a_1, \ldots, a_n$, and let $b_i = j(a_i)$, so that $b_0 < b_1 < \ldots < b_n$ in the usual order on natural numbers. What the problem asks you to show is that, given any new $a$ in $A$ there will be a rational number $b$ such that $b$ is related to the $b_i$ in the usual order on rational numbers in the same way $a$ is related to the $a_i$.

**12.23** It will suffice to build a sequence of finite partial isomorphisms $j_i$ as in Problem 12.22. Problem 12.21 can be used to get from $j_i$ to $j_{i+1}$, but some care will be needed to arrange that every element of $A$ gets into the domain of some $j_i$ eventually.

**12.25** Proceed as in Problem 12.23, but this time also take care to arrange that every rational number gets into the range of some $j_i$.

**12.27** The preceding problems do not yet cover all the possibilities.

**Chapter 13**

**13.1**, **13.3**, **13.5**, **13.7** Hints are given in the text of section 13.5.

**13.9** Imitate the proof of the isomorphism lemma, Proposition 12.5.

**13.11** For (a) use the preceding problem; for (b) first note that if $B(c)$ implies $A$ and $c$ does not appear in $A$, then $\exists x\, B(x)$ implies $A$. (For if $\exists x B(x)$ does not imply $A$, then $\{\exists x B(x), \sim A\}$ is satisfiable, and then by Example 10.5(b) so is $\{B(c), \sim A\}$, and $B(c)$ does not imply $A$.)

**13.13** See Problem 13.12.

**13.15** See Problem 12.18.

**Chapter 14**

**14.1** The compactness theorem is relevant.

**14.3** Look how we got from (2) to (7) in Example 14.4.

**14.5** Look how we got from (2) to (6) in Example 14.12.

**14.7** Remember that you may use the results of earlier problems.

**14.9** As in Example 14.13, all rides on making a suitable choice of formula $A(x)$ to apply (R8) to.

**14.11** Imitate the proof of the inversion lemma for negation.

**14.13** To show the effect of (R11) can be obtained using (R12), use the relevant inversion lemmas.

**Chapter 15**

**15.1** The length is the number of digits in the decimal expansion of $e$ that are $< 8$.

**15.3** Apply Corollaries 12.17 and 15.7 to $T \cup \{A\}$ and $T \cup \{\sim A\}$ where neither $A$ nor $\sim A$ is a theorem of $T$.

**15.5** How many of the $A_i$ would it take to deduce all the $B_j$?

**15.7** The idea is just to 'check for each $n$ through all possible models of size $n$', or more precisely, through a set of possible models containing at least one representative of each isomorphism type of models of size $n$. Generalize the preceding problem appropriately to show the set of isomorphism type representatives for a fixed $n$ can be taken to be finite.

**15.9** Let $R$ be a recursive relation such that $a$ is the code number of theorem of $T$ if and only if $\exists n\, Ran$. Consider the set of sentences $B$ such that for some $A$ and $n$, $B$ is the conjunction of $n$ copies of $A$, and $Ran$ holds, where $a$ is the code number of $A$.

**Chapter 16**

**16.1** Use Lemma 16.6.

**16.3** Use Theorem 16.13.

**16.5** See the proof of Corollary 15.6(a).

**16.7** Use Proposition 7.17 and the remark following.

**16.9** Again use Proposition 7.17.

**16.11** Recall that we have proved $0 + y = y$ and $1 + y = y'$ have been proved in Examples 16.18 and 16.19.

**16.13** For (*c*) first note that there is a least *n* with the property 'there is a sequence of length *n* with property *P*.'

**16.15** To make (Q1)-(Q2) and (Q7)-(Q10) true, the denotation of **0** should be taken to be the least pair (in the $<_2$-order) and the denotation of ' the function that given any pair as argument yields as value the least pair (in the $<_2$-order) among the pairs greater (in the $<_2$-order). It remains to devise a suitable addition function.

**16.17** Use 'induction in the metalanguage,' proving the result first for $m = 0$, then for $m = n'$ assuming it holds for $n$.

**16.19** For (a) again use 'induction in the metalanguage,' proving the result first for $b = 0$, then for $b = c'$.

**16.21** The first half of the problem is to show how, using induction and the axioms of **Q**, to obtain the two axioms of **R** that are not axioms of **Q**. But one of these, (Q0), has already been done as Example 16.17, so it only remains to do (Q11). The other half of the problem is to show how, using induction and the axioms of **R**, to obtain the four axioms of **Q** that are not axioms of **R**. But two of these, (Q7) and (Q9), have already been done in section 16.4, so it only remains to do (Q8) and (Q10). For the first half of the problem note that according to Problems 16.10 and 16.11, we can get the commutative law for addition using induction and axioms common to **Q** and **R**.

**Chapter 17**

**17.1** Use Theorem 16.16 and Problem 16.9.

**17.3** Imitate the proof of the diagonal lemma, beginning as follows: For formulas $E_1(x, y)$ and $E_2(x, y)$ with code numbers $e_1$ and $e_2$, let the first and second double diagonals be

$\exists x \exists y(x = \mathbf{e}_1 \,\&\, y = \mathbf{e}_2 \,\&\, E_1(x, y))$, logically equivalent to $E_1(\mathbf{e}_1, \mathbf{e}_2)$ and

$\exists x \exists y(x = \mathbf{e}_1 \,\&\, y = \mathbf{e}_2 \,\&\, E_2(x, y))$, logically equivalent to $E_2(\mathbf{e}_1, \mathbf{e}_2)$.

**17.5** The logical equivalence of $A(t)$ and $\exists y(y = t \,\&\, A(y))$ will be useful.

**17.7** To begin with $R$ be the Rosser sentence of $T$, $T_0 = T + \{\sim R\}$, $T_1 = T + \{R\}$. Let $R_e$ for $e = 0$ or 1 be the Rosser sentence of $T_e$. Let $T_{00} = T_0 + \{\sim R_0\}$, $T_{01} = T_0 + \{R_0\}$, $T_{10} = T_1 + \{\sim R_1\}$, $T_{11} = T_1 + \{R_1\}$, and continue in this way.

**17.9** Use the Craig reaxiomatization lemma, Problem 15.9.

**17.11** To obtain $\mathcal{N}$, take the set of elements satisfying $N(x)$ as the domain $|\mathcal{N}|$. Let $R^{\mathcal{N}}$ hold for elements of the domain $|\mathcal{N}|$ if and only if $R^{\mathcal{M}}$ does, let $c^{\mathcal{N}} = c^{\mathcal{M}}$ for each constant $c$ (as we may since it is given that $c^{\mathcal{M}}$ satisfies $N(x)$, that is, belongs to the domain $|\mathcal{N}|$), and let the value of $f^{\mathcal{N}}$ for elements of the domain $|\mathcal{N}|$ be the same as the value of $f^{\mathcal{N}}$ (as we may since it is given that if elements satisfy $N(x)$ so does the value of $f$ for those elements).

**17.13** It is enough to find a formula $N(x)$ that is satisfied in $\mathcal{Z}$ by an integer if and only if that integer is non-negative, for then by Problem 17.12, relativization will be a translation, and by Problem 17.10 the set $T$ of sentences true in $\mathcal{Z}$ will be undecidable since the set $S$ of sentences true in $\mathcal{N}$ is undecidable. If we have <, we can simply use $x = \mathbf{0} \vee \mathbf{0} < x$ for $N(x)$. To find an $N(x)$ that does *not* involve $<$ requires a major result of number theory, but one that has been mentioned more than once in this book.

**Chapter 18**

No problems.

# Errata

*Please report any discovered to jburgess@princeton.edu*

Thanks to Mauro Allegranza, Tsvi Benson-Tilsen, E. Biedermann, Nan Chen, John Corcoran, Sinan Dogramaci, Sean Duggan, Michael Forbes, David Furey, Cosmo Grant, Richard Heck, David Hitchcock, David Keyt, Cyras Kristijonas, Ran Lanzet, Yuval Masory, James Mattingly, Derek Mesman-Hallman, Cory Nichols, Christopher Pincock, Marcus Säbel, Peter Smith, Jamie Tappenden, Bram van Heuveln.

p. xii, line 10, read "first incompleteness" for "first completeness"

p. xiii, line 4, read "Saul Kripke" (as in all earlier editions) for "Paul Kripke"

p. 6, two lines after displayed definition of $s(n)$: read "of other sorts" for "of others sorts"

p. 15, problem 1.5(a), first line: read "positive rational numbers less than one" for "rational numbers"; last line: add "and $m < n$" before final period.

p. 24, line 7: read "write smaller and smaller" for "writer smaller and smaller"
p. 24, line 15 from bottom: read "for most of us" for "for most us"

p. 26, line 13: read "of the box" for "of box"

p. 27, line 4 from bottom: reader "shortened" for "shorted"

p. 31, two lines above specification **(a)**: read "values are represented" for "values represented"

p. 37, three lines below **Theorem 4.1**: read "we were able to compute" for "we were able compute"

p. 38, line 11 from bottom: read "standard" for "stardard"

p. 39, line 14: delete the remark in parentheses. [Hints are not longer at the back of the book, but are now in the Instructor’s Manual.]

p. 42, Example 4.6: read "$\geq 2n$" for "$\geq 2p(n)$"

p. 42, statement of Example 4.6, read "$\geq 2n$" for $\geq 2p(n)$"

p. 43 Figure 4-3: replace each of the three occurrences of "$k$" by "$j$"
[alternatively, leave the figure as is, and change the text of the last paragraph on p. 43 through the end of the proof on p. 44, replacing each "$j$" by "$k$" and the three occurrences of "$k$" in the last three lines of the proof by "$j$"]

p. 45, Abstract, line 3 from bottom: read "an abacus" for "a abacus"

p.51, middle, paragraph (b), first line: read "stones in some" for "stones is some"

p. 52, two lines below displayed equation: 1 should be the subscript, not superscript; $r$ should be the superscript, not subscript.

four lines below displayed equation: read "stones" for "strokes"

p. 58, four lines from bottom: read "function $f$" for "functions $f$"

p. 60, lines 5 and 6 from bottom: read $i$ for $j$ both places

p. 60, line 2 from bottom: read "$f(x, 0) = 0$" for "$f(x, 1) = 0$"

p.62, problem 5.8, first line: read "given" for "give"

p. 64, line 11: read "any" for "any any"

p. 72, problem 6.6, the first minus sign should be a plus sign

p. 75, 2nd displayed formula, read $f_m$ for $f_n$

p.77, end of proof of Theorem 7.4: replace the last words "only replace y by y-1" by "to make some slight changes"

p. 78, Example 7.7, 2nd line after displayed formula:
add "or ($y = 0$ & $z = x$)" after "$y \cdot z < x$"

p.78, Corollary 7.8, first line read "regular primitive recursive function" for "regular primitive function"

p. 79, line 10 from bottom: read "argue" for "agrue"

p. 82, line 7 read $R_2x$ *for* $R_2y$
line 11 read $R_1x$ *for* $R_1y$

p.86, Problem 7.3, line 6:
add "$z > 0$ and " before $x \mid z$"
and line 7:
add before period at end of next-to-last sentence:
"[except that, by convention, we let lcm($x$, 0) = lcm(0, $y$) = 0]"

p.87, problem 7.15, three displayed lines (definition of β-code):
first line: read ≤ for <
second line: add + 1 at the end

p. 91 displayed 3-line definition, last line: read "a ≥ 3" for "a = 3"

p. 92, displayed definition of newstat: add at end before period: "· sg($q$)"

p.93 displayed equation defining stdh: the last term should be:
nstd(left(conf($m$, $x$, $t$)), right(conf($m$, $x$, $t$))

p.97, proof of Corollary 8.8: Replace the third sentence by "If it were recursive, its characteristic function $c$ would be a recursive function." and delete the words "the complement of" from the fifth sentence.

p. 104, Example 9.2, line 3: delete second "of the language"

p.110, line 6: read "Then note that if" for "Then note that".

p.112, problem 9.2, first line: read "at" for "of at"

p.113, problem 9.3 (b): add before last parenthesis "& $y \neq z$"
p.113, problem 9.8: read "How could we change the definition of formula" for "How could we change the definition".

p.119, line 2: read "of terms" for "of term"

p. 126, line 4 from end of italicized chapter summary: Read "not making any avoidable use of Turing's or Church's thesis" for "not depending on Turing's or Church's thesis"

p. 128, displayed formula (5): read $m > 0$ for $m \neq 0$

p. 129, equation (12): read $\mathbf{Q}_1$ for $\mathbf{Q}_0$

p. 130, displayed formula on line 7 from bottom: delete one left parenthesis and insert **0** before the second $x$ and before the third $x$
p. 130, displayed formula on line 5 from bottom:
insert subscript $a$ on first **S** and subscript $p$ on second **S**

p. 132, line 5 of §11.2: read "contains the constant" for "contains the constants"
p. 132, line 2 from bottom: read "from 0 to $r$" for "from 1 to $r$"

p. 133, third line below displayed formual (5b), read **f** for $f$
p. 133, line 7: read "$\geq 0$" for "$> 0$".
p. 133, line 7 from bottom: read "where all sentences" for "where also sentences"

p. 134, displayed formula (6a): read "$\mathbf{f}_j(\mathbf{a})$" for "$\mathbf{f}_j(\mathbf{a}, \mathbf{0})$"
p. 134, displayed formula (7b): read " $\mathbf{f}_i(\mathbf{a}, \mathbf{p})$" for "$\mathbf{f}i(\mathbf{a}, \mathbf{p})$"
p. 134, line 14 from bottom: read "$\exists y$" for "$\forall y$"
p. 135, problem 11.5: read "Theorem 11.1" for "Theorem 11.2".

p. 136, problem **11.10** read "Problem 8.12" for "Problem 8.13"
p. 136, problem **11.16** read "Theorem 11.14" for "Theorem 11.16"

p. 152, problem **12.21**, line 4: read " $j(a_1) <_B j(a_2)$" for " $j(a_1) <_A j(a_1)$"

p. 152, problem **12.22**, first line: insert "show that" after "Continuing the preceding problem,"

p. 153, displayed formula **(S5)**: delete brackets from "$\{\exists x B(x)\}$" and delete "or in $\exists x B(x)$"

p. 154, statement of Lemma **13.2**, second line: read "all sets of sentences" for "all sets of formulas"

p. 154, proof of Lemma **13.2**,

line 1, end: "is a" for "is"

line 9: read "$\Gamma_0 \cup \{B\}$" for "$\Gamma \cup \{B\}$"

ine 10: read "$\Gamma \cup \{B\}$" for "$\Gamma_0 \cup \{B\}$"

p. 159, second indented biconditional: read "[c1]*****, ..., [cn]*****" for "[c1], ..., [cn]"

p. 160, paragraph 2 of §13.4, last line: read (S1) for (S0)

p. 162, line 6 from bottom: read "for every sentence" for "for every formula"

p. 164, Problem 13.8, line 2, read "12.1" for "13.1"

p. 164, Problem 13.8, line 5, read "and" for "and and"

p. 169, last paragraph, line 4: read "one or another" for "one of another"

p. 171, last full paragraph on page, line 4: read "string of symbols constitutes" for "string symbols constituties"

p. 172, Example 14.5, annotation to line (2): read (R2a) for (R2b)

p. 172, Example 14.6, annotation to line (2): read (R2a) for (R2b)

p. 172, Example 14.6, annotation to line (4): read (R2b) for (R2a)

p. 172, Example 14.8, annotation to line (2): read (R2a) for (R2b)

p. 172, Example 14.8, annotation to line (5): read (R2b) for (R2a)

p. 173, Example 14.10, annotation to line (5): read (R2a) for (R2b)

p. 177, line 8: read "satisfaction properties" for "satisfiability properties"

p. 178, verification of (S2): exchange annotations (R2a), (R2b)
p. 178, verification of (S4), last annotation: read (R2b) for (R2a)

p. 179, verification of (S6), last annotation: read (R2a) for (R2b)
p. 179, verification of (S8), second annotation: read (R2a) for (R2b)

p. 188, line 2 after table: read "**7**, '" for "**7**', " (i.e. transpose accent and comma)

p. 189, 1st full ¶, line 5 from bottom: read $\Gamma$ for $\Gamma_0$

p. 190, 1st line of proof of Cor. 15.5: read "(soundness and) completeness" for "completeness"

p. 191, 3rd displayed formula, insert space between **n** and *D*.
p. 191, last ¶, 2nd line: read *T* for T

p. 192, 3rd line from end of proof of Cor. 15.7: read "recursive total function" for "recursive function"

p. 193, lines 12-13 below table: replace all four occurrences of 36 by 8
p. 193, lines 13 below table: replace 89 by "over 50"

p. 195, second paragraph, lines 9-10: read "less than" for "less that"

p. 201, line 2: the first subscript in the displayed formula should be 2, not 3, and the last subscript should be 3, not 2.
p. 201, line 4: the first subscript should be 3, not 2

p. 203, lines 9 and 8 from bottom: read N + 1 for N both places

p. 204, Theorem 16.6 (b) read "recursive or semirecursive set or relation" for "recursive set".

p. 204 Add at end of proof of Theorem 16.6: The semirecursive case requires adding a quantifier $\exists$.

p. 204, example **16.7**, displayed formula: replace all four instances of **<** by **≤**

p. 205, add at end of statement of Lemma 16.8: Likewise any semirecursive relation.

p. 206, , add at end of statement of Lemma 16.11: Likewise any semirecursive relation.

p. 208, 1st ¶ of proof of Theorem 16.13, line 3: read "successor" for "sucessor"

p. 212, proof of Theorem 16.16: the boldface **0** on the last line should be **1**.

p. 212, Problem 16.9: omit the words "containing **Q**".

p. 214, lines 3-4, read "can be proved by mathematical induction" for "follows from the principle of mathematical induction"

p. 215, last line: read **m' ≠ n** for **m ≠ n**

p. 217, problem 16.4, read "16.13" for "16.3".

p. 219, Problem 16.18. The italicized sentence at the end is not part of the problem and should be on a separate line

p. 223, next to last line of proof of Theorem 17.6: read $f(a)$ for $f(n)$

p. 223, last line of proof of Theorem 17.6: 17.5 for 17.4

p. 223, third line after proof of Theorem 17.6: read "semidecidable" for "decidable"

p. 223, line 2 from bottom, delete "as it is in the axioms of **Q**,"

p. 224, section 17.2, first paragraph, line 8: read "Theorem 17.5" for "Theorem 17.4"

p. 224, bottom line: read $\text{Disprf}_T$ for Disprf

p. 225, first line of proof of **17.8**. Read "then there is some *n*" for "then there is some *a*"

p. 226, paragraph after proof of **17.9**: corners around $G_{\mathbf{Q}}$ are missing in all three instances of "$\mathrm{Prf}_{\mathbf{Q}}(G_{\mathbf{Q}},\ldots$"

p. 229, problem 17.2, line 3: read "in *T*" for "in P".

p. 234, line 9 from bottom: read "Theorem 18.3" for "Theorem 18.1"
p. 234, line 7 from bottom: read $\mathrm{Prv}_T$ for Prv

p. 244, lin 10 from bottom, read "is applied" for "is applies"
p. 244, statement of **Proposition 19.2** It should read: Any formula built up by ~ and & and v from given formulas is logically equivalent to one in disjunctive normal form.

p. 245, line 1, the first sentence should read: First move negations inside junctions as in the proof of Proposition 19.1.
p.245, line 3, ((B v D) & (C v D)) should read: ((B & D) v (C & D))
p. 245, first line after proof of Proposition 19.2: read "each disjunct" for "each disjunction"

p. 246, proof of Theorem 19.5, lines 8-9: read "so no variable appearing bound in one conjunct or disjunct appears in the other" for "so the conjuncts or disjuncts have no variables in common"

p. 247, first line of section 19.2: read "quantifiers" for "quantifies"
p. 247, line 7 from bottom: read "satisfy" for "satify"

p. 249, displayed formula (S1): Read $b_1$ for ( $b_1$) and $b_n$ for ( $b_n$)
p. 249, displayed formula (S2): delete parentheses
p. 249, next-to-last line: read $|A|$ for $A$

p. 250, line 8: read "(S2)" for "(S1)"
p. 250, Example 19.8, line 4 from bottom (= two lines below second displayed formula): read $(a + b)/2$ for $(a - b)/2$

p. 251, proof of Theorem 19.9, line 6: read "there will be an enumerable" for "there will be enumerable"

p. 252, displayed material, last line: delete initial $\forall z$
p. 252, line 9 from bottom: read "if there is no set" for "if there is set"

p. 254, line 19: delete first occurrence of "objects"
p. 254, second paragraph before **19.11**, first line: read "Proposition 19.7" for "Proposition 12.7"ß

p. 255, line 20: read $m^k$ for $k^m$

p. 260 chapter summary, read "no nonlogical symbols but such as occur both in A and in C" for "both in A and in B"

p. 261, second to last line: read "What we are going to do" for "What we going do"

p. 262 (S5): read "then" for "and then", delete final "or $\exists xF(x)$"
p. 262, 1st paragraph after (S6), line 2: read "S(0)-S(6)" for "S(0)-S(1)"
p. 262, 1st paragraph after (S6), line 10: read "an unbarred" for "and unbarred"

p. 263, line 8: subscript on 2nd $D$ in line should be 1, not 2.
p. 263, line 12: read "both" for "both both"
p. 263, *Proof, Part II*, line 4: read "equivalence" for "equality"

p. 265, Example 20.8, next-to-last line: delete ", while $L$ is".
p. 265, twelve lines from end of section 20.2: read "language" for "langauge"

p. 266, 3rd line after 2nd displayed formula: read "left" for "right"

p. 267, line 6 of proof of Lemma 20.9: in "K = M + N", "M" and "N" should be script font

p. 270, line 3 from bottom: read "problem for full" for "problem full"

p. 274 "*Proof of Lemma 21.7*" should be "*Proof of Lemma 21.8*"
p. 274, Proof of Lemma 21.12, line 3: read "appears in $B$" for "appears in $F$"

p. 275, 1st line after 1st displayed formula: read "occur free" for "occur"

p. 275, 3rd line after 1st displayed formula: read "atomic or negated atomic" for "atomic"
p. 275, both displayed formulas: subscripts $r$ should be $t$

p. 276, Proof of Lemma 21.3: read "$v_1, \ldots , v_k$" for "$u_1, \ldots , u_k$"
p. 276, Proof of Lemma 21.4, line 4: delete subscript $i$ on $P$

p. 278, paragraph beginning "Also", line 3: read "$4n + 1$" for "$4m + 1$"
p. 278, last paragraph of proof, 1st line: read $Sxb$ for $Swb$

p. 283, next-to-last paragraph of proof, next-to-last line: read "**m** be $m$" for "**m** by $m$"

p. 285, problem 22.2, displayed formula: add a left parenthesis immediately to the right of the left bracket, and a right parenthesis immediately to the left of the final arrow

p. 286, line 2 of abstract: read "language of arithmetic" for "language in arithmetic"
p. 286, 3rd line of main text, read "true" for "ture"

p. 287, first line of fact (5): read "code number q" for "code numbers q"

p. 288, 2nd paragraph, line 2: read "either $a$ is" for "either $k$ is"
p. 288, seven lines from bottom: read "on expanding L by adding" for "on expanding L be adding"

p. 289, displayed item (5): last $S$ should be $B$.

p. 293, line 9: read "essentially" for "essential"

p. 295, displayed formula: insert prime ' at end of left hand side of equation

p. 296, 4th paragraph, line 3: read "a quantifier-free" for "and quantifier-free"
p. 296, 5th paragraph, line 2: read "no additional" for "no addition"

p. 297, item (12), displayed formulas (ii) and (iii): read boldface "**<**" for lightface "<" in all three occurrences
p. 297, item (12), 2nd last line: read "that if (i) or (ii) holds" for "that (i) or (ii) holds"
p. 297, item (14), line 1: read "preceding two steps" for "preceding three steps"

p. 298, item (21), first displayed line: last $k$ should be a subscript like the others
p. 298, last line: read $\mathbf{D}_m(i - j)$ for $\mathbf{D}_n(i - j)$

p. 303, next-to-last paragraph, first sentence: the first "NUMBER" (before comma) should be "number", and the last "number" (before period) should be "NUMBER".

p. 304, line 5 from bottom: read "and $a$ an integer" for "and $a$ and integer".

p. 306, line just before displayed formula: the word "theory" should be "set"

p. 307, 1st full paragraph, last line: read "for" for "for for"

p. 310, Proof of Theorem 25.4b, 2nd line: read "Chapter 17" for "Chapter 16"
p. 310, Proof of Theorem 25.4b, 1st displayed line in proof: read $\sim\exists z \leq y \beta(x, z)$ for $\sim\exists z \leq y \beta(x, y)$ and $\sim\exists z \leq y \alpha(x, z)$ for $\sim\exists z \leq y \alpha(x, y)$

p. 311, 2nd displayed formula: should end with comma (,) not period (.)

p. 312, line 11 from bottom: read "$N$** of $L$**" for "$N$** of $L$*"
p. 312, line 9 from bottom: both occurrences of $L$ should be $L$*

p. 314, item (3), read "formula $F(x)$ of $L$**" for "formula $F(x)$ of $L$*"

p. 315, next-to-last paragraph, line 3: read "in $F(x)$" for "in $F(X)$"

p. 320, statement of Ramsey's theorem, line 3: read "subset $Y$ of size at least $n$" for "size-$n$ subset $Y$"

p. 323, line 6: read "nine nodes" for "ten nodes"

p. 327, §27.1, first paragraph, next to last line: after "sentences are sentences" insert "(also called formulas)"
p. 327, §27.1, 2nd paragraph, line 3: read "sentences" for "formulas"

p. 328, lines 9, 10, 11 from bottom: each $\wedge C$ should be $\wedge\Gamma$

p. 330, proof of Lemma 27.3, line (2): read 27.2(1) for 25.2(1)
p. 330, line 3 from bottom: read "of whose disjuncts" for "of whose conjuncts"

p. 331, 2nd full paragraph, lines 3-4, read "*B*, *C*, and *D* are subsentences of *A* or (in the case of *D*) possibly ⊥" for "*C* and *D* are subsentences of *A*"
p. 331, line 3 from bottom" read "□*B* and *B* are both" for "□*B* are both *B*"

p. 332, end of proof of Theorem 27.1: read "□*C* is in *w*" for *w* ⊨ □*C*"
p. 332, last displayed item in proof of Proposition 27.4: a dot should precede the *first A*, and none should precede the *second A*

p. 334, Proof of Theorem 27.8, fifth paragraph: read "main other part" for "only other part"
p. 334, Proof of Theorem 27.8. End of proof read "□*C* is in *w*" for *w* ⊨ □*C*"
Also, add at end "(We leave it to the reader to show that if *A* is not a theorem, then ~*A* belongs to some *w* in *W*.)"

p. 337, first line below table: read $\pi_\alpha$ for γ

p. 338, lines 10-11 read "then this is a theorem, hence so are *D* and □*D*, which may be replaced by ~ ⊥" for "then this is a theorem, so we may replace *D* by ⊥"
p. 338, displayed formulas (1)-(3) the superscripts are wrong:
for (1) they should be $n + 1$ and $m$
for (2) and (3) they should be $m$ and $n$ and $n$ +1 and $n$

p. 339, lines 7-8: one negations should be dropped on each line, the last of the two on line 7, and the middle one of the three on line 8
p. 339, proof of Lemma 27.15, line 2: a □ is missing before the second $C_i(p)$
p. 339, proof of Lemma 27.16, line 4: the double arrow ↔ should be single →
p. 339, proof of Lemma 27.17, line 2: read "$v > u$" for "$v \geq u$" and "$u < v$" for "$u \leq v$"
p. 339, proof of Lemma 27.17, line 4: read "and so by Lemma 27.15" for "and so by that lemma"