Tim Maudlin, *New Foundations for Physical Geometry: The Theory of Linear Structures*, Oxford: Oxford University Press, 2014, pp. x + 363, £50.00.

The volume under review is the first of a projected pair in which the author will present a new approach to the geometry of physical space or spacetime. The present volume contains only the purely mathematical apparatus to be applied to physics in the sequel. This being so, to a large extent the evaluation of the book must await that sequel. The introduction announces the theses of the two volumes: (1) The most fundamental geometrical structure that organizes physical points into a space is the [straight or curved] line. (2) What endows spacetime with its geometry is time. How wide a range of approaches to physics can be accommodated from this perspective, and how comfortably, and how profitably? We will have to wait for the second volume to find out the scope and limits of the project.

Obviously, failure of the physical application would tend to reduce the mathematical apparatus to the status of a curiosity, or at any rate something of more mathematical than philosophical interest. Since I am writing under space limitations, for a philosophical and not a mathematical journal, I will not attempt to describe what the project looks like as pure mathematics, independent of the applications not yet shown. I do promise that readers of a mathematical bent should find the formal working out of theme (1) above intriguing. It is a type of project that some will always find themselves attracted to, and Maudlin has already recruited a number of enthusiastic collaborators thanked in connection with various parts of the project.

Inversely, success with the physics would largely excuse infelicities in the mathematics. And infelicities there are. To be sure, setting terminological matters aside, I have noticed no purely mathematical mistakes in the purely mathematical material making up the bulk of the volume. But terminological matters cannot be set aside. They are what give the book its distinctive flavor and appearance (all specifically Maudlinite technical terms being set in a special and very conspicuous typeface). More importantly, they reflect the main philosophical theme of this first volume, considered in itself independently of its sequel.

The book resembles Arend Heyting's *Intuitionism*, in the sense that many standard mathematical topics are reviewed, but done in a new way, motivated by a heterodox philosophical outlook. With both Heyting and Maudlin there are results that verbally contradict classical theorems. In both cases it turns out on examination that verbally identical formulations have different meanings for the heterodox and the classical mathematician, so that the conflict is really more in the definitions. There is a difference, however.

With Heyting this kind of conflict is unavoidable: There is a fundamental difference between intuitionistic and classical mathematicians over standards of proof. With Maudlin conflict could fairly easily have been avoided, but has been willfully sought. Maudlin has gone out of his way to pick a fight, by choosing to use standard terminology — much of it employed across the whole domain of mathematical geometry, of which the study of potential models for physical space is but one small corner — in non-standard ways, when it should not have been hard to coin new terms of his own. The appearance of dissidence always attracts a certain kind of philosopher, but it is less clear what the effect on mathematicians will be; and it is their collaboration that will be needed going forward.

Let me illustrate how conflict can arise. What look like smooth curves when viewed from the usual distance on a computer screen turn out, when viewed more closely, to be arrays of discrete pixels, of which there are only finitely many on the screen. The speculation is often thrown out, though never *worked* out, that physical space, too, may consist of only a very large finite number of maximally specific locations — it may be tendentious to call them "points". (One route to this thought is as follows. Quantum mechanics tells us a particle's location can be narrowly pinned down only at the cost leaving certain other factors, such as speed, very uncertain.

Relativity tells us there is an upper bound on what the speed of a particle can be, and therewith on the uncertainty of that speed. This suggests that there will be lower bound on how precisely the location of a particle can ever be pinned down. Some would infer from limitations on our knowledge of location limitations on what locations there are.) The desire to have ready a mathematical framework for physics that could be applied in developing such a speculation forms a notable part of Maudlin's motivation.

It seems to me that the most natural thing to say if the speculation is correct would be that there is no genuine continuity in physical reality, any more than there is any genuine simultaneity. To be sure, there are things that, when looked at on a macroscopic scale, *appear* to be continuous; and perhaps these will inherit much of the interest that attached to the continuous while we still thought there was such a thing. But would not the most natural thing to call them be something like *pseudo-continuous*? Maudlin, however, insists on using the standard term *continuous* in a non-standard way, so that even in a universe in which space is not a continuum in any ordinary or familiar sense, the notion of continuity still has non-trivial application. This usage of "continuous" is one of the two or three most important of several usages that lead to gratuitous verbal conflict with classical theorems.

Maudlin's justification for proceeding as he does is a claim that terms like “continuous” — or to give another key example, “open” — have certain intuitive meanings to which mathematical usage ought to be faithful, in a way that the mathematical usage of “field” or “ring” does not need to be faithful to the usage of farmers or jewelers. This seems the main philosophical theme of this first volume considered in isolation and not as build-up towards the successor volume. But claims about what the supposed intuitive meaning, presumably going back to before the evolution of the usage now called classical, of this or that term includes or excludes are not supported by citing historical sources, say using the invaluable website “Earliest Known Uses of Some of the Words of Mathematics”, «http://jeff560.tripod.com/mathword.html».

At most we find a two-stage procedure. First, some source like Wikipedia is quoted, where the writer is obviously trying to give an informal, heuristic idea of what is meant by some mathematical term *as classically used*. Second, it is claimed that this informal, heuristic description better fits Maudlin’s heterodox notion than the classical notion undoubtedly intended. Such a procedure cannot be expected to produce compelling results.

Perhaps the strangest case is that of “open subset of a space”. In 1899 René Baire, working with n-dimensional Euclidean space, introduced the terms “open sphere” and “closed sphere” just as they are used working with *n*-dimensional Euclidean space today. He also introduced “open *n*-dimensional set” for what today would still be called an “open subset of *n*-dimensional Euclidean space”, claiming that the general definition captures what is distinctive about the open as opposed to a closed sphere. The definition was then extended from Euclidean to metric spaces, and finally to topological spaces, as part of a story too long to fit in a short review article.

Going back to the Euclidean case, under the classical definitions deriving from Baire, an *m*-dimensional open sphere is an open subset of *m*-dimensional Euclidean space, but not of *n*-dimensional Euclidean space if $n > m$. This Maudlin finds unacceptable. He claims that there is an intuitive sense of “open set” that was is violated by the standard definition even in the case of Euclidean spaces. The correct definition, according to Maudlin, would not be of a relative notion “open subset of such-and-such a space” but of an absolute notion of “open set”. Even if the open *m*-sphere sits in *n*-space for $n > m$, it should be called an open set. All this, about *the very coiner of the term* supposedly misusing it, is simply asserted without argument.

Maudlin does coin one term of his own, “submetrical”, but I find it unhelpful and potentially misleading. Consider the plane. *Metric* geometry for it considers the notion of distance and others definable from it, whatever is invariant under rigid motions (translations, rotations, reflections). *Euclidean* geometry allows also dilatations or uniform expansions or contractions, and treats only relative and not absolute distance, shape and not size. *Affine* geometry allows also sheering motions, with the result that angle measure and similarity of figures are lost, but collinearity and parallelism remain. The subject called *analysis situs* in the eighteenth and nineteenth centuries, and *topology* in the twentieth and twenty-first, allows bendings and stretchings, so that straightness is lost and all that are left are certain "positional" properties. (An example of a "positional" problem, the oldest famous problem belonging at the distinctively topological level, is the Seven Bridges of Königsberg.) Maudlin's usage of "submetrical", never precisely defined, tends to insinuate that it is a defect of topology that it treats only topological, positional properties, and not other properties pertaining to levels below the metrical, intermediate between it and the topological.

I have complained that more attention ought to be given to word-histories, but I must say that the historical interpretations that are given in the work, mainly at the beginning, cannot be trusted. Student readers need to be warned against two, especially. On p. 16 it is asserted that Newton would not have recognized irrational or negative numbers. This shows that Maudlin has not read even the first few pages of the *Universal Arithmetick*. For there Newton rejects Euclid's definition of number as a multiplicity of units, allowing only whole numbers; he insists that any ratio of magnitudes such a lengths may be considered a number, explicitly pointing out that this includes "surds" as the ratios of incommensurables; and a bit later he discusses negative numbers, indicating that to represent them geometrically one must take account of direction. On p. 227 Euclid's second postulate "to extend a given finite straight line continuously in a straight line," is misquoted, omitting the word "finite", and then it is argued that since an infinite line can't be further extended, Euclid cannot be asserting the existence of an extension, but only its uniqueness if it exists; whereas ancient commentators already complained that Euclid needs a uniqueness assumption that no postulate explicitly acknowledges.

Given the predominantly negative tone of the foregoing remarks, let me close by reiterating my opening statement that to a large extent the evaluation of the book must await the appearance of its sequel.

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