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LOGIC & PHILOSOPHICAL METHODOLOGY

Introduction

For present purposes "logic" will be understood to mean the subject whose development is described in Kneale & Kneale [1961] and of which a concise history is given in Scholz [1961]. As the terminological discussion at the beginning of the latter reference makes clear, this subject has at different times been known by different names, "analytics" and "organon" and "dialectic", while inversely the name "logic" has at different times been applied much more broadly and loosely than it will be here. At certain times and in certain places — perhaps especially in Germany from the days of Kant through the days of Hegel — the label has come to be used so very broadly and loosely as to threaten to take in nearly the whole of metaphysics and epistemology. Logic in our sense has often been distinguished from "logic" in other, sometimes unmanageably broad and loose, senses by adding the adjectives "formal" or "deductive".

The scope of the art and science of logic, once one gets beyond elementary logic of the kind covered in introductory textbooks, is indicated by two other standard references, the *Handbooks* of mathematical and philosophical logic, Barwise [1977] and Gabbay & Guenthner [1983-89], though the latter includes also parts that are identified as applications of logic rather than logic proper. The term "philosophical logic" as currently used, for instance, in the *Journal of Philosophical Logic*, is a near-synonym for "nonclassical logic". There is an older use of the term as a near-synonym for "philosophy of language". This older usage is understandable, since so much of philosophy of language, and notably the distinction between sense and reference, did originally emerge as an adjunct to logical studies; but the older usage seems to be now obsolescent, and will be avoided here.

One side of the question of logic and philosophical methodology is that of the application of logic in philosophy. Since logic has traditionally been regarded as a methodological discipline, it is difficult or impossible to distinguish applications of logical *methods* from application of logical *results*, and no effort to maintain such a distinction will be made here. Distinctions and divisions within the topic of applications of logic in philosophy are to be made, rather, on the basis of divisions of logic itself into various branches. Mathematical logic comprises four generally recognized branches: set theory, model theory, recursion theory, and proof theory, to which last constructive mathematics, not in itself really a part of logic but rather of mathematics, is attached as a kind of pendant. Philosophical logic in the relevant sense divides naturally into the study of *extensions* of classical logic, such as modal or temporal or deontic or conditional logics, and the study of *alternatives* to classical logic, such as intuitionistic or quantum or partial or paraconsistent logics: The nonclassical divides naturally into the *extra*classical and the *anti*classical, though the distinction is not in every case easy to draw unambiguously.

It should not be assumed that "philosophical logic" will inevitably be more philosophically relevant than "mathematical logic". Through the early modern period logic as such was regarded as a branch of philosophy, but then that was equally the case for physics, and today the situation is quite different: Only a minority of professional logicians are housed in departments of philosophy, and this is true not just of specialists in “mathematical” logic but also of specialists in “philosophical” logic, many of whom are housed in departments either of mathematics or of computer science. Most nonclassical logics were initially introduced by philosophers, and with philosophical motives, but as their study has developed it has come to include the mathematical investigation of “logics” no one has ever advocated as accounts of the cannons governing deductive argumentation, just as geometry has come to include the mathematical study of “geometries” no one has ever seriously advocated as accounts of the structure of the physical space. For computer scientists, the literal truth of such philosophical ideas as may have played a role in motivating the original introduction of one or another logic is never what matters, but rather the heuristic suggestiveness and fruitfulness of such ideas, when taken in a perhaps metaphorical or unintended sense, for this or that technical application. The discussion to follow accordingly will not give special emphasis to philosophical logic merely because it is called “philosophical”.

Rather, the seven branches of logic that have been distinguished — (1) elementary logic, (2) set theory, (3) model theory, (4) recursion theory, (5) proof theory, (6) extraclassical logics, (7) anticlassical logics — will be given roughly equal coverage. As it happens, each of the seven topic areas listed has a somewhat different flavor: The bearing of some branches on philosophy is pervasive, while the bearing of other branches is localized; the influence of some branches on philosophy has been positive, while the influence of other branches has been problematic; the relevance of some branches to philosophy is widely recognized, while the relevance of other branches is less known and imperfectly understood. As a result there is great variation in the nature of the philosophical issues that the involvement of the different branches with philosophy have raised. And as a result the discussion below will be something of a potpourri.

Philosophy of logic is as much to be distinguished from logic proper, including philosophical logic, as history of linguistics is to be distinguished from linguistics proper, including historical linguistics. Another side of the question of logic and philosophical methodology is therefore that of the methodology of philosophy of logic, insofar as it has a methodology of its own, distinct from the methodology of philosophy at large. The first question about special methods peculiar to philosophy of logic as distinguished from other branches of philosophy is simply the question whether there are any such distinctive methods,

There is much to suggest that it ought to be answered in the negative. The scope and limits of philosophy of logic are quite differently understood by different philosophers of logic, as comparison of such classics as Strawson [1952] and Quine [1970], not to mention Haack [1978], soon reveals. But a not-too-controversial list of central topics in present-day philosophy of logic might include the following: *Should truth-bearers be taken to be sentence types, or sentence tokens, or propositions; and if the last, are these propositions structureless or structured; and if structured, are they coarse-grained and "Russellian" or fine-grained and "Fregean"? Are logical forms the same as grammatical forms, or perhaps the same as "deep" in contrast to "surface" grammatical forms; and whether or not they are, are they psychologically real, represented somehow in the mind or brain of the reasoner, or are they merely imposed by the analyst in the course of evaluating reasoning? Does the source of logical* *truth and logical knowledge lie in the meanings of the logical particles or elsewhere; and should that meaning be conceived of as constituted by truth conditions or by rules of use?* Obviously these central questions of philosophy of logic are very closely linked to central questions of philosophy of language and/or philosophy of linguistics. Indeed, they are so closely linked as to make it hard to imagine how there could be methods peculiar to philosophy of logic alone and not relevant also to these or other adjoining fields.

Yet upon further reflection it appears that there is after all at least one special methodological puzzle in philosophy of logic that may be without parallel elsewhere. The problem in question arises in connection with philosophical debates between propopents of anticlassical logics and defenders of classical logic, and it amounts to just this: What logic should be used in evaluating the arguments advanced by adherents of rival logics as to which logic is the right one? It is natural to suspect that both sides would soon become involved in circular reasoning; no doubt one side would be arguing in a vicious circle and the other side in a virtuous one, but still the reasoning would be circular on both sides. The question of how if at all noncircular debate over which is the right logic might be possible is perhaps the most readily identified distinctive methodological problem peculiar to philosophy of logic. It can, however, conveniently be subsumed under the question of the role of anticlassic logics in philosophy, which is already on the list of seven topics for exploration enumerated above.

1. Elementary Logic and Philosophy

Elementary logic, of which the half-dozen branches of advanced mathematical and philosophical logic that have been identified are so many specialized outgrowths, is concerned with the evaluation of arguments, but not just any kind of argument and not just any kind of evaluation. Its concern is with deductive arguments, arguments purporting to show that, assuming some things, something else then follows conclusively and not just probably. And its concern is with the formal validity of such arguments, with whether the forms of the premises and conclusion guarantee that if the former are true the latter is so as well, and not with their material soundness, with whether the premises are as a matter of actual fact true. Now as the present volume attests, in philosophy today the greatest variety of methods are employed. Nonetheless, deductive argumentation remains what it always has been, a very important and arguably the single most important philosophical method. Though it is impossible to collect precise statistics on such questions, undoubtedly philosophy remains among intellectual disciplines the second-heaviest user of deductive argumentation, next after mathematics but ahead of jurisprudence, theology, or anything else. And though formal validity is only one virtue to be demanded of deductive argumentation, it is a very fundamental and arguably the single most fundamental virtue, the *sine qua non*. Accordingly, it is widely agreed that every student of philosophy needs a least a rudimentary knowledge of logic, of how to assess the formal validity of deductive arguments. The point is perhaps not *universally* agreed: It would presumably be disputed by Andrea Nye, since Nye [1990] reaches the conclusion that "logic in its final perfection is insane"; but this is a radical — one may even say fringe — position.

What is more often disputed is not that students of philosophy should have a modicum of practical knowledge of logic, but rather how much is enough. How many concepts, how much terminology, must the student take in? Certainly the student needs to possess the concept of an *argument* in something like Monty Python's sense of "a connected series of statements intended to establish a proposition" as opposed to the colloquial sense of "a loud, angry exchange of opinions and insults". Surely the student also needs to understand the distinction between formal *validity* and material *soundness* — and it should be added, needs to appreciate the chief method for establishing *in*validity, that of exhibiting a parody, another argument of the same form whose premises are manifestly true and whose conclusion is manifestly false. (This is the method illustrated by the Mad Hatter when he replies to the assertion that "I mean what I say" and "I say what I mean" are the same, by objecting that one might as well say that "I see what I eat" and "I eat what I see" are the same. It is also the method used by Gaunilo replying to Anselm.) Ideally, the student should know some of the labels used in describing the logical forms of premises and conclusions, and for some of the most common kinds of valid arguments, and for some of the most egregious fallacies: Terms like *biconditional* and *modus ponens* and *many questions* should be in the student's vocabulary. (At the very least, the student should know enough to avoid the illiterate misuse of the expression "beg the question" that has become so annoyingly common of late.) But how much more should the student know? And is there any need to initiate the student into the mysteries of logical symbolism?

There is then also a further question about *how* the student should acquire the range of knowledge called for, whatever its extent may be. Undergraduate concentrators in mathematics, who all at some fairly early stage in their training need to "learn what a proof is", generally do so not through the explicit study of logic, but in connection with a course on some core branch of mathematics, perhaps on number theory, perhaps real analysis (calculus done rigorously); if they undertake a formal study of mathematical logic, as most do not, it will be at some later stage. Perhaps, then, the modicum of logical vocabulary and theory needed by students of philosophy should likewise be imparted, not in a separate course, but in conjunction with some kind of introductory topics-in-philosophy course. Or perhaps it should be left to writing courses, except that one hears horror stories about what students are told in such courses ("Your writing is much too clear") by instructors from literature departments who are under the baleful influence of certain fashionable theoreticians. In short, while surely some course in elementary, introductory-level logic should be offered, what is debatable is whether it should, for prospective philosophy concentrators, be made a requirement or left as an elective.

There is then also a further question about what the content of such a course, whether required or not, should optimally be, and in particular, what additional material should be included beyond the modicum of formal, deductive logic that is absolutely essential. Should it just be more formal, deductive logic? Or should it be a bit of what is called "informal logic", or critical thinking? Or should it be a bit of what is called "inductive logic", or probabilistic reasoning? Or should it be "deviant logic", or anticlassical positions? Or should it be a little of this and a little of that? The appearance of the present volume suggests still yet another alternative, that of folding instruction in elementary logic into a general "methods of philosophy" course. The main point is that the most obvious issues raised by the role of elementary logic in philosophy are curricular issues, affecting the philosopher *qua* teacher of philosophy more than the philosopher *qua* philosopher.

2. Set Theory and Philosophy

Sophisticated developments in higher axiomatic set theory (as described in Part B of Barwise [1977]) have influenced philosophy of mathematics, but treatment of the matter will be postponed so that it may be discussed in conjunction with the influence of proof theory on that same specialized branch of philosophy. Leaving all that aside for the moment, more elementary set theoretic results — or if not results, at least notation and terminology — are quite commonly used in a variety of branches of philosophy, as they are quite commonly used in a variety of branches of many other disciplines. The most elementary set-theoretic material, including such concepts as those of *element*, *subset*, *intersection*, *union*, *complement*, *singleton*, *unordered pair* and *ordered pair*, the material whose use is the most widespread in philosophy, has penetrated instruction in mathematics down to the primary school level, and can be presumed to be familiar to students of philosophy without much need for separate discussion, except perhaps a very brief one to fix notation, which has not been absolutely standardized. In many branches of analytic philosophy, however, a bit more of set theory is involved. One may go on to use some marginally more advanced notions, perhaps those pertaining to certain special kinds of binary relations such as *functions* and *orders* and *equivalences*, along with attendant concepts like those of *injectivity* and *surjectivity* and *bijectivity*, or of *reflexivity* and *symmetry* and *transitivity*. There may also be some need or use for the notion of *ancestral* from what is called "second-order logic", a part of the theory of sets or classes that sometimes passes for a branch of logic. And not all of these matters can be counted on to have been already absorbed by students in primary or secondary school mathematics. But the problems raised by the role of set theory in philosophy are not exclusively the kinds of curricular issues that we have seen to arise in connection with the role of elementary logic (though indeed *if* the introductory logic curriculum is to be rethought, the possible introduction of a bit more set theory than is customarily covered at present might be one issue to be considered).

One significant problem raised by philosophers' use of set-theoretic notions and notations is an embarrassment that arises for philosophers of a certain bent, those inclined towards "nominalism" in the modern sense. For views of this kind have no patience with and leave no room for sorts of entities for which it makes questionable sense to ask after their location in time and space, and no sense to ask after what they are doing or what is being done to them. And sets are paradigmatic examples of entities that are of such a sort, often pejoratively called "Platonic", historically absurd though this usage is, or more neutrally called "abstract". Philosophers inclined to nominalistical views will, it seems, need to watch out and take care that they do not, in the very exposition and development of those views, fall into violations of their professed principles by making mention, in the way that is so common among philosophers, of abstract, so-called Platonic apparatus from set theory. For opponents of nominalism have often argued that if would-be nominalists can be caught themselves frequently using set-theoretic notions, then such notions cannot really be so intellectually disreputable as nominalist doctrine would maintain, and acquiring knowledge of them cannot really be so impossible as popular epistemological arguments for nominalism insist. The conflict between the widespread use of set theory within and outside logic and nominalist challenges to abstract ontology is taken to be *the* main problem in philosophy of logic in Putnam [1971], the *locus classicus* for the "indispensability argument", according to which, set theory being useful and used in logic, mathematics, science, and philosophy to the point that one could hardly do without it, one ought simply to accept it. But the issues seem today by no means so clear-cut as they did to Putnam.

For proponents of nominalism today often imagine there is some cheap and easy solution to the difficulties of the philosopher who would like to pose as a hard-headed nominalist without having to give up the use of any of the customary set-theoretic and mathematical methods that have penetrated into contemporary analytic philosophy. The supposed solution is to be found in some kind of instrumentalism that will allow them to use set-theoretic language when speaking out of one side of their mouths, while continuing to deny the existence of sets when speaking out of the other side; or else in some kind of distinction that will allow them to say sincerely that it is literally true that sets exist, while still denying that they have thereby undertaken any "ontological commitment" to sets. Sympathizers with nominalism now point to new "fictionalist" possibilities, permitting one use in practice whatever is useful while still rejecting it in principle — while critics complain that waters previously clear have been muddied by obscurantism about a supposed gap between "existential implications" and "ontological commitments". But these contentious issues are all too familiar to those who follow the literature in metaphysics and philosophy of mathematics, and need not be enlarged upon further here.

3. Model Theory and Philosophy

Alfred Tarski's work on model theory (the foundation stone of the subject as expounded in Part A of Barwise [1977]) arose out of his famous definition of truth. The strategy used in Tarski [1956] was the method of *giving a characterization of what it is for a sentence of a language to be true under a given interpretation by induction on the syntactic complexity of the sentence*, for instance, defining truth for a conjunction in terms of truth for its conjuncts. The same method is adopted and adapted also in Kripke [1963] to give a model theory for modal and related logics, which involves introducing a system of indices, picturesquely called "possible worlds", and the relativizing of the notion of truth to that of "truth at a possible world". Both the method of definition by induction on complexity and the notion of possible world have become immensely influential, the former especially in the philosophy of language of Donald Davidson, the latter especially in the metaphysics of David Lewis. Yet the closer one looks at the original work of Tarski and Kripke, the more dubious becomes the supposed connection between that work and the developments in philosophy of language and metaphysics that it has somehow given rise to. In the case of the metaphysics of possible worlds, the looseness of the connection is generally recognized, since Kripke notoriously very explicitly and emphatically repudiated anything like the Ludovician conception of possible worlds as something like distant planets way off in logical space. In the case of philosophy of langauge, the looseness of the connection between "formal semantics" or model theory and "linguistic semantics" or meaning theory is perhaps not so widely understood.

It seems to be not so widely recognized as it might that truth-conditional theories of meaning as developed by Davidson and others represent an inversion rather than an application of the Tarskian standpoint: Tarski took *truth* to be the problematic notion, rendered suspect by the well-known paradoxes, whose meaning needed explanation or definition, and took as understood and available for use in his definition the meanings of the expressions of the language for which truth was being defined; whereas Davidson takes the notion of *truth* more or less for granted as an unanalyzed and undefined primitive, and attempts to use it to characterize the meanings of expressions of the object language. Davidson himself was quite selfconsciously turning Tarski on his head, but Davidson's followers have perhaps not always recognized that truth-conditional semantics is not Tarski rightside up but Tarski upside down. Tarski did call model theory "semantics", as indeed did Kripke; but what Tarski meant by "semantics" is not at all what linguists and philosophers of language today mean by it, as should be clear enough, even without going into the complicated history of the usage of the term, from the fact that Tarski's list of paradigmatically "semantic" notions includes *truth* but not *synonymy*. The thought that truth-conditional semantics of a Davidsonian kind (or with variations of a Kaplanian kind) is anything like a direct application of "formal semantics" of a Tarskian kind (or with variations of a Kripkean kind) is simply mistaken, and represents a kind of fallacy of equivocation on the ambiguous term "semantics". Whether the departure from or inversion of the model-theoretic perspective that has led to truth-conditional theories of meaning was a good thing or a bad thing is too large an issue to be entered into here; but departure or inversion it unquestionably was: Logic can neither take the credit nor bear the blame for truth-conditional semantics.

4. Recursion Theory and Philosophy

The consortium of disciplines collectively known as "cognitive studies" includes among other components philosophy of mind, neurology, and several branches of computer science. The whole subject of computer science is, on its theoretical side, an outgrowth of the branch of logic called "recursion theory" (as expounded in Part C of Barwise [1977]), now sometimes alternatively called "computability theory", along with its offshoot complexity theory. As a result, acquaintance with the basics of this branch of logic — with the notion of *Turing* *machine*, above all — is desirable if not indispensable background for philosophers involved in cognitive studies. One needs this kind of background simply to read a lot of the current literature, both the large positive literature that endorses and in various ways applies a "computational theory of mind" and the much smaller negative literature that argues there are deep conceptual confusions in "machine-state functionalism".

The positive literature is too vast to be intelligently surveyed in the space available here; nor is the present author the best person to undertake such a survey. The relation of computer science to philosophical methodology really deserves a chapter of its own, or perhaps even two chapters (with the branch known as "artificial intelligence" getting separate treatment). Central to the much smaller negative literature is the discussion of "Kripkenstein's skeptical paradox", and even that is too large a subject to be gone into seriously here. The fundamental problem is just this: What is it for some physical object to constitute a realization of some abstract algorithm, or for some material organ such as a brain to be an embodiment of an idealized machine? The same object may be construed as an imperfect realization of any number of different abstract algorithms, and there is nothing *in the object* to make one construal correct and the others erroneous. Or so Kripke and followers argue. As Kripke has noted, this problem, if it is a genuine one, creates difficulties not only for functionalist philosophy of mind and functionalist cognitive psychology, but also for much of contemporary philosophy of language and contemporary linguistics, which following Chomsky makes use of a notion of "competence" that is not to be identified with observable "performance" but is nonetheless supposed to be "psychologically real". The background in the pertinent branch of logic, recursion theory, that one needs to follow the discussion in either the positive or the negative literature is not extensive, and perhaps could be obtained from popularizations, without the need to enter deeply into technicalities; but background there is. But recursion theory's centerpiece, the Church-Turing thesis, is relevant to philosophical methodology in quite another way.

The Church-Turing thesis is important not only as an analytical tool, but also as a paradigm of the successful solution to a difficult problem of analysis. The problem Alonzo Church and Alan Turing addressed was the following. Impossibility results in mathematics have a certain utility in telling us not to waste time attempting certain tasks, though how much value such a warning has will depend on what use is made of the time saved. Negative, impossibility results almost always require more background analysis that positive, possibility results. If one wants to show it is possible to construct a given figure with ruler and compass, it is enough to give instructions for the construction, and a proof that it works as advertized. If one wants to show a construction *im*possible, however, one needs some sort of analysis of what constructibility amounts to. As with constructibility, so with computability. If one wants to show a function is computable, it is enough to present the instructions for computing it, and a proof that they work. If one wants to show a function is *un*computable, however, one needs some sort of analysis of what computability amounts to. Church and Turing each undertook, independently of the other, this task of analysis, seeking to find a rigorously-definable mathematical notion that would be coextensive with the intuitive notion of computability. Church proposed to identify computability with (something called lambda-calculability, and later with) recursiveness, and Turing with computability by one of his machines. It was quickly seen the functions computable by a Turing machine are precisely the recursive functions, so that the theses of Church and Turing are in a sense equivalent. They are now almost universally accepted by experts. There is, however, a certain difference.

There is no hope of giving a fully, formally rigorous mathematical proof of the coincidence between some rigorously-defined notion and some intuitive notion, since all the notions involved in a fully, formally rigorous mathematical proof must be rigorously-defined and not intuitive ones. (That is one reason why there are so few fully, formally rigorous proofs in philosophy as compared to mathematics.) So neither Church nor Turing offered a fully, formally rigorous mathematica proof for his thesis, nor did either claim that "computable" just *means* recursive or Turing computable, or that his thesis was analytic. Turing, however, did offer a heuristic argument, based on the thought that all one ever does in a computation is make and arise marks and move around the page, and that making and erasing marks could be done one stroke at a time, and moving around the page one step at a time. Church, by contrast, did little more than cite the nonexistence of any obvious counterexamples (and in the seven or eight decades since his day, no on has since found a plausible one). Church's thesis and Turing's thesis thus represent apparently extensionally successful analyses that can hardly be claimed to be analytic, and Church's work and Turing's work exhibit two different ways, one more *a posteriori* and one more *a priori*, in which one could hope to argue for such an analysis. There is a lesson for analytic philosophers in all this, about the scope and limits of the method of analysis, but it is perhaps one that, requiring as it does familiarity with some rather technical material, has not as yet been as widely understood or as seriously taken much to heart as it should be.

5. Proof Theory and Philosophy

Proof theory in the narrow and strict sense (in which it is understood in Part D of Barwise [1977]) consists of certain specific types of theorems about certain specific types of formalisms: cut-elimination theorems for sequent calculi in the style of Gerhard Gentzen and normalization theorems for systems of natural deduction in the style of Dag Prawitz. Its techniques and its technicalities have to a degree been brought by Michael Dummett and his followers into debates about classical *vs* intuitionistic logic, but it is not that side of proof theory that I wish to consider here. I will be concerned rather with proof theory in a broader and looser sense, the kind of study that begins with Kurt Gödel's two famous incompleteness theorems, which are main goal of any intermediate-level course in logic, and are treated in many textbooks (often in conjunction with the Church-Turing thesis), besides being the subject of a large literature of popularization of very mixed quality. The second of the two theorems says, roughly speaking, that if one restricts oneself to the most constructive and least controversial means of proof, then one cannot prove any *absolute* consistency results for any interesting mathematical axiom systems: One cannot prove that any such system is free from contradiction. One can, however, often prove *relative* consistency results, to the effect that system B is consistent relative to system A, meaning that if system A is free from contradiction, then so is system B, or contrapositively, if there is a contradiction in system B, then there is a contradiction in system A. Proof theory in the broader or looser sense is concerned with comparing the "consistency strengths", where B counts as being of the same consistency strength as A if B can be proved consistent relative to A and vice versa, while B counts as being of lesser consistency strength than A if B can be proved consistent relative to A, but not vice versa.

Logicians have shown that virtually all systems that have been seriously proposed as foundations for mathematics fall somewhere on a linearly ordered scale of consistency strengths that leads from a very weak but still nontrivial system called "Robinson arithmetic", to a very strong system called "Zermelo-Frankel set theory plus rank-into-rank large cardinals". This itself is a striking result, since it is very easy to contrive artificial examples of systems that are of incomparable consistency strength. Why then should there be no naturally-occurring examples? This basic result, the work of many hands, is accompanied by any number of other striking theorems. Some of these results, due to various workers, show that the bulk of the mathematics that finds serious applications in science and engineering can be developed in systems quite low down in the scale, where dwell most of the systems proposed by dissident "constructivist" mathematicians. Another one of these results, due to Yuri Matiyasevich building on work of several predecessors, shows that every time one moves up a notch on the scale, more theorems of number theory of a very simple type (asserting the non-existence of solutions to a certain Diophantine equation) become provable. The most sophisticated methods of proof theory in the narrow and strict sense are used in the study of the lower end of this scale, while sophisticated methods of a quite different, set-theoretic kind (notably Paul Cohen's method of forcing) are used at the upper end.

Many would say the results obtained by such methods are of considerable potential relevance to issues about mathematics that are much debated among philosophers. They would add that it is unfortunate that knowledge of such results, which in itself does not require deep involvement in the technicalities of their proofs, is perhaps not as widespread as it ought to be (to the extent that philosophers can sometimes be found writing as if they believed, contrary to the Matiyasevich theorem, that pie-in-the-sky set-theoretic assumptions about "large cardinals" just *cannot* have any impact on anything so down-to-earth as number theory). Unsurprisingly, given that in philosophy everything is potentially disputable and almost everything is actually disputed, there are others who would discourage study of the results from mathematical logic I have been discussing, or for that matter any other results from mathematical logic at all. They will perhaps quote *dicta* of Wittgenstein about the "disastrous invasion" of mathematics by logic, and about "the so-called mathematical foundations of mathematics" being merely a painted rock under a painted tower. As in other cases, I can here only note the existence of a disagreement over the role of logic in philosophy here, without attempting to resolve it.

6. Extraclassical Logics and Philosophy

The traditional logic, based on Aristotle's syllogistic, was inadequate to the task of analyzing serious mathematical proofs, mainly because it lacked any treatment of relations. The logic that has displaced traditional syllogistic and that is now called "classical" was developed, mainly by Gottlob Frege, precisely for the purpose of analyzing mathematical reasoning. Classical logic goes beyond traditional logic by just as much as is needed to analyze mathematical arguments — just as much, and no more. It takes no note of grammatical mood or tense, of epistemic or deontic modalities, or of subjunctive or counterfactual conditionals, since none of these matter for mathematics or are to be found in purely mathematical language. By contrast, they are very much to be found in philosophical language, and do matter for philosophy. Hence there would seem to be much room for philosophically relevant extensions of classical logic (as treated in Gabbary & Gunthner [1984]), enriching its formal language with the sorts of things just enumerated.

Modal logic in the broad sense — comprising temporal logic, epistemic and deontic logic, conditional logic, and more, as well as modal logic in the narrow sense of the logic of "necessarily" and "possibly", which is the part of the larger subject that will be of most concern here — has aspired to provide just such philosophically relevant extensions of classical logic. It was largely developed in hopes of making itself philosophically useful. Performance, however, has not lived up to promise, and it is not going too far to say that at times modern modal logic has done more to darken rather than to enlighten our understanding. Some of the most glaring deficiencies of conventional modal logic were early pointed out by the hostile critic Quine, but unfortunately the reaction of modal logic's champions to Quine's critique was highly defensive and often uncomprehending. More was done in the way of developing elaborate technical constructions to prove various conventional systems to be formally consistent, than was done in the way of analyzing and explaining the notions of necessity and possibility and their representation in language in order to show the systems in question intuitively intelligible, or where they were not so, to replace them by novel systems that would be.

One reason modal logic has been little able to provide guidance to philosophers engaged in modal reasoning is that modal logicians have never been able to agree as to which modal logic is the right one. And no wonder, since prior to Kripke [1972] they were generally hopelessly confused about the nature of necessity and possibility: The possible in the sense of what potentially could have been was conflated with the possible in the sense of what can without self-contradiction be said actually to be. As we now say, "metaphysical" modality was conflated with "logical" modality. But even today, when the importance of that distinction has been widely though by no means universally recognized, modal logic is still full of *dubia*, even at the sentential level. The state of quantified modal logic (QML) is much worse. Until Kripke [1963] modal logicians did not even know how to develop systems of QML that would avoid — in the sense of making them optional extras, that one can assume or not as one chooses, rather than something built in to the basic formalism — the dubious "Barcan formulas", which imply that anything that could possibly have existed actually does exist, and that nothing that actually does exist could possibly have failed to exist. Even with that problem out of the way, however, the basic syntax of conventional QML is out of alignment with the way in which modal distinctions are expressed in natural language. For the formalism treats a modality as operating on a whole clause, including all of its subordinate clauses, while in natural language modal distinctions operate on verbs, allowing the grammatical mood of a subordinate clause to differ from that of the main clause to which it is subordinate. The result is that with the conventional formalism it is difficult or impossible to express something like "If all those who wouldn't have come here if they hadn't been obliged to do so now leave, there will be no one left" or "I could have been a lot thinner than I am"; nor is the addition of "actuality operators" to the language a sufficient remedy. There are a number of philosophical logicians at work today trying to improve matters, but it is too early to say whether a genuinely philosophically useful modal logic is going to emerge from their efforts.

To look briefly on the bright side, workers in theoretical computer science do seem to have found a number of modal systems useful in *non*-philosophical ways. Also, the category of extraclassical logics is not exhausted by modal logic, even when "modal logic" is taken in the broadest sense, and a number of extraclassical but nonmodal logics, including plural logic and predicate-functor logic, have occasionally figured in interesting ways in philosophical projects, though there is no space to go into such matters in detail here.

7. Anticlassical Logics and Philosophy

A. W. Kinglake is said to have proposed that every church should bear over its doors the inscription "important if true". Whatever one thinks of that suggestion, there is no question but that the three words of the proposed inscription apply to the claims made by proponents of anticlassical logics (as surveyed in Gabbary & Guenthner [1985] as well as Haack [1978]). The advocates of paraconsistent logics may go furthest in making claims about how many philosophical problems would be easily solved if their principles were adopted, but advocates of other anticlassical logics are not far behind. Further, for a number of such logics, some applications of their technical formalisms have been suggested that would *not* require literal belief in the underlying motivating philosophical ideas, giving another potential reason to study the formalisms even if one rejects the ideas. Moreover, a large number of technical results, some of them quite impressive as pure mathematics, have accumulated concerning such logics. A curious phenomenon, however, may be observed in the technical literature on the metatheory of anticlassical logics.

With the exception of the mathematical intuitionists, advocates of anticlassical logics generally *make no serious effort to conform their own metatheoretic reasoning to patterns that are valid according to their own professed views*. Their own deductive behavior thus suggests that they secretly believe in the classical logic they profess to reject. This phenomenon was perhaps first noted by Kripke (unpublished) in the work of "relevance" or "relevant" logicians, who officially declare the inference "P or Q, but not P, so Q" to be "a simple inferential mistake such as only a dog would make," but who nonetheless were caught by Kripke using that very forbidden form of inference in the proof of a major metatheorem. How far it is legitimate for defenders of classical logic to invoke this curious phenomenon in debating with attackers is itself a debatable question. On the one hand, many writers on informal logic or critical thinking would hold it to be a fallacy to argue that since the heretics can't themselves live up to their doctrines, those doctrines must be in error; and it is indeed not entirely inconceivable that human thought should be drawn irresistably into certain patterns of inference that are nonetheless hopelessly wrong. In a way and in a sense, some of the psychological literature on heuristics points in something like such a direction. Nonetheless, it is less common for advocates of anticlassical logics to respond in this way to Kripke-style objections than for them to respond that even though classical logic is not to be relied upon in general, there are special reasons why it may be relied upon in certain special areas, including the metatheory of anticlassical logic. Extended but inconclusive debates about "classical recapture" then ensue.

On the other hand, given the great difficulty, alluded to in the introductory remarks at the outset above, of non-question-begging debate over logical principles, it is very natural to slide from the question "Which logic is right?" to the question "Which logic should we follow?" And in connection with the latter question the observation that the logic one's opponents are proposing we should follow is one that they are incapable of following themselves is a perfectly cogent objection. After all, "ought" implies "can", does it not? In practice debate often slides further from the prescriptive question "Which logic *should* we follow?" to the descriptive question “Which logic *do* we follow?” as both sides appeal to common sense. Be all that as it may, we still have the curious phenomenon noted before us.

With the exception of the mathematical intuitionists, most advocates of anticlassical logics are vulnerable to an objection, or anyhow subject to an observation, that Solomon Feferman has emphasized in connection with partial logic (a three-valued logic with “truth value gaps”): “Nothing like sustained ordinary reasoning can be carried out” using the logic they propose. The reason why this should be so is not immediately clear, but that it is so is what experience in working with these logics, along with the evidence of anticlassical logicians’ metatheoretic behavior, suggests. Even the mathematical intuitionists may be no real exception. The intuitionists have been able to develop a substantial intuitionist or constructivist mathematics entirely in conformity with their principles, with all proofs conforming to intuitionistic logic; and of this intuitionistic or constructivistic mathematics the intuitionists’ work on the metatheory of intuitionistic logic forms one chapter. But the intuitionists have made no serious, sustained attempt to develop an intuitionist empirical science, and it is difficult to make out what logic, precisely, their underlying principles would imply to be the appropriate one for empirical reasoning.

For intuitionists or neo-intuitionists, in explaining why they adhere to the logic they do in mathematics, cite issues about proof: They take the very meaning of mathematical statements to be constituted by their proof-conditions. But in the empirical domain there is generally no question of apodictic proof. Evidence may establish a presumption, but presumptions are always defeasible, giving empirical reasoning a nonmonotonic character: What one is warranted in asserting given certain evidence may become unwarranted given more evidence. Moreover, performing the operations that would be needed to verify or falsify one empirical claim may make it impossible to perform the operations needed to verify or falsify another, and this phenomenon is by no means confined to the microscopic world or to quantum interference. These features, which have no counterparts in mathematics, suggest that a different logic from the one intuitionists use in mathematics would be require; but the details of the required logic have not been worked out.

Even if one is not at all tempted to adopt an anticlassical logic, there mere thought that someone might do so, or that some planet may harbour intelligent extraterrestrials who have done so, and whose mathematics and science must therefore be very different from ours, is philosophically intriguing, suggesting as it does a very radical form of "underdetermination of theory by evidence" or "conventionality". This is one way in which anticlassical logics can exercise a fascination even over philosophers who are by no means willing to give up classical logic.

Conclusion

The foregoing hodge-podge of affirmations, denials, and interrogatives cannot be summed up in any simple slogan. From the miscellany of observations, reflections, and evaluations offered above, no single clear message, no overarching moral about logic and philosophy readily emerges. Examination of the many areas where the large and diversified field of logic impinges on the even larger, even more diversified field of philosophy merely confirms what was said at the outset, that the role of some parts of logic in philosophy is pervasive and positive, while that of others is peripheral, and that of yet others problematic. But at least one may say this: The interaction of logic with philosophy remains after more than two millenia a lively on-going process. And one may also add this: If logic is to have the fullest possible positive influence, there will be a need both for existing achievements of logic to be more effectively communicated to philosophers and for logicians to expand and extend, and in some cases alter and amend, their own work.

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