Key Workers and Funding Horizons*

Angus Foulis — Bank of England
Nobuhiro Kiyotaki — Princeton
John Moore — Edinburgh and LSE
Gabor Pinter — BIS
Shengxing Zhang — Carnegie Mellon

26th August 2026

Abstract

We build and test a dynamic model of a firm with key workers whose role is to develop and maintain an asset crucial to the firm's going-concern value. Our analysis may be particularly relevant for industries with low asset redeployability, where a firm has little value as a gone concern – a leading example being a firm whose capital is largely intangible. In our model, because key workers' human capital is inalienable and their remuneration can be renegotiated each period, hold-up limits the firm's ability to pledge distant future income, thus foreshortening the funding horizon. Firms that are more reliant on key workers are less sensitive to long-term interest rates, but are more exposed to short-term rates. We test these predictions using UK firm-level data and term structure shocks. Firms with more key workers – measured by various empirical proxies – exhibit maturity-specific investment and financing responses consistent with the model. Overall, our results highlight the importance of integrating labour contracting frictions, and the concomitant funding horizon heterogeneity, into models of firm dynamics and monetary transmission.

Keywords: key workers, funding horizons, firm investment, term structure, monetary policy

JEL codes: D22, D25, E22

*We would like to thank Jennifer La'O, John Leahy, Yueran Ma, and participants at conferences and seminars for helpful comments and suggestions. The views expressed in this paper are those of the authors, and not necessarily those of the Bank of England or its committees or those of the Bank for International Settlements.

1 Introduction

In this paper we build and test a model of a firm with key workers whose role is to develop and maintain an asset crucial to the firm's going-concern value. Our analysis may be particularly relevant for industries with low asset redeployability, where a firm has little value as a gone concern – a leading example being a firm whose capital is largely intangible. Intangible capital typically requires constant maintenance and development by a group of key workers and has little or no break-up value. But even a firm that primarily has physical assets could also be reliant on a group of key workers if the assets have low redeployability; e.g., if it has specialised machinery which only the key workers can operate.

In our intertemporal model, every period each of two or more key workers chooses an action, how hard to work. These actions cannot be disciplined by means of an upfront contract: the key workers' individual human capital is inalienable. Thus there is moral hazard vis a vis the owners of the enterprise, to whom returns accrue. The channel through which moral hazard operates is important: today's vector of actions chosen by the key workers affects returns tomorrow, on the day after tomorrow, and on all days thereafter. And, crucially, this process repeats: tomorrow, the key workers will take another vector of actions, which will affect returns on the following days; and so on. That is why it is helpful to think of the role of the key workers as developing and maintaining an *asset*. What is not affected by their actions today is today's return — there is no 'within-period moral hazard'. For any given history, owners directly receive *all* of the enterprise's current return. There is no question of splitting up control rights and/or return streams across different types of investor: the usual considerations of corporate finance do not apply. An obvious complication here is if key workers are, at least in part, themselves owners – we return to this later in the Introduction.

Existing dynamic models with inalienable human capital put within-period moral hazard at their heart. For example, in Hart and Moore (1994), an insider to a project can, each period, threaten to shirk so as to deny to outsiders the project's return that period. But unless the outsiders irreversibly pull the plug on the project — liquidate — today's shenanigans (shirk or work) don't affect the project's future potential. In Hart and Moore (1989, 1998), project returns accrue directly to the insider — he or she can steal them — but, again, liquidation aside, there is no technological linkage between moral hazard today (theft or no theft) and the project's potential in the future.

The nature of the moral hazard in our model – the fact that it is an *asset* which is vulnerable to the key workers' actions – introduces a twist. Starting at any point in time, the further ahead owners look, the more opportunities they see there will have been for key workers to have taken a bite of the cherry. Beginning at date t, say, owners know that, come date $t+s$, there will have been $s-1$ more intervening periods during which key workers can threaten not to work and so can extract a share of the date $t + s$ return: that $t + s$ cherry will have had $s - 1$ more bites taken out of it. The net effect is to foreshorten the owners' investment horizons: they realise that the overall fraction of the project's return coming their way – viewed from the day of their initial investment – shrinks through time. This is not to say that the environment is inherently non-stationary; nor that projects with an ongoing opportunity cost should eventually stop. For example, given a project with a permanent stable return stream, once the initial investment is done the foreshortening effect would be the same irrespective of when one starts to look, because any previous bites of the cherry that have been taken by the key workers are, from an owner's perspective, sunk costs.

This twist provides us with our road in to an empirical test of the model. The term structure of interest rates matters differently to different firms. Movements in long-term versus short-term interest rates have differential effects on how much funding can be raised against an investment project, depending on how vulnerable the owner of the project will be to its key workers.

Let us introduce the notion of 'funkiness'. A *funky* firm, defined within a set of comparably sized firms, has a number n of key workers greater than the median for that set. The remainder of firms in the set are *non-funky*. Specifically, we measure n by the ratio of the number of directors – the individuals legally responsible for running the firm – to the total number of employees. The idea is that the owner of a funky firm is more vulnerable to its key workers – simply because there are proportionately more of them.

Our model predicts that the investors in funky firms have more foreshortened horizons. Accordingly, we test the hypothesis that movements in *long-term* interest rates have *less* impact on the investment of funky firms than non-funky. (Our model also predicts that movements in *short-term* interest rates have *more* impact on the investment of funky firms than non-funky. But to understand this would take us too far into the details at this juncture.)

We bring the model to firm-level data and test its predictions using variation in interest rate shocks across the yield curve, identified by high-frequency monetary policy shocks. We find that funky firms exhibit sharply maturity-asymmetric investment responses. In response to a 100 basis point increase in the 5-year interest rate, funky firms reduce investment by 9.6% after two years, noticeably less than the 12.9% for non-funky firms. (In contrast, when the 1-year yield rises by the same amount, funky firms reduce investment by 6.5%, whereas non-funky firms reduce it by just 2.8%.) Not only are these effects economically significant in absolute terms, the *differences* in the responses between funky and non-funky firms are also statistically significant. Together, they support our model's prediction that funding horizons shape firms' sensitivity to the maturity of interest rate shocks.[1]

[1] These magnitudes exceed estimates from studies using firm-level data on listed firms (e.g., Cloyne, Ferreira, Froemel, and Surico, 2023; Jeenas and Lagos, 2024). As we discuss in the text, this difference appears to be due to our sample being dominated by small and medium-sized enterprises, which are far smaller than typical public firms.

To arrive at these results, we use panel data on UK firms from May 1998 to December 2017, which combine information on financial accounts, employment composition, and governance structures. In the empirical analysis, we measure investment using the cumulative growth rate of real fixed assets; throughout, references to firms' investment responses should be understood in this sense. Our proxy for funkiness – the ratio of the number of directors to employees – reflects the concentration of firm-specific human capital tied to oversight and continuity. To avoid conflating organisational structure with firm size, we compute this ratio within size buckets, capturing intensity rather than scale. We classify a firm as funky if its director-to-employee ratio is above the median within its firm size bucket. To validate this classification, we test robustness to several alternative definitions. These include defining funky firms within bins defined jointly by firm size and: (i) industry (SIC1), (ii) firm age (split around 15 years), and (iii) region (NUTS1).[2] In each case, we continue to define funky firms as those above the within-bucket median. We also construct an extended measure of funkiness, augmenting our baseline measure with a wage-based measure, using the ratio of director remuneration to total remuneration, again normalised within firm size buckets. Across all definitions, the maturity-dependent investment responses remain consistent and statistically significantly different, confirming that key worker dependence is a robust predictor of heterogeneous monetary transmission.

Directors often own shares, or rights to acquire shares. In our data there is a simple flag to indicate that a particular director owns shares, but not how many shares. Given the context of our model and our funkiness measure, a flagged director thus wears two hats: she is a key worker *and* an owner. What difference might this two-hats issue make, theoretically and practically? In theory, the fact that a key worker bargains for remuneration with the owners means that a share-owning director is partly bargaining with herself. But unless she is a 100% owner, there remains a conflict with the other shareholders. Every extra £1 she gets in remuneration from the bargain is offset by *less* than £1 she gets in profit, so arguably the former driver is the more powerful. Thanks to this argument, our model's qualitative predictions stay intact – at no point do we attempt a quantitative test. The possibility that this remuneration might be in the form of new shares rather than money is immaterial to our theory. In practice, rewarding a key worker with shares, or options to acquire shares, may be a temporary matter: in the medium run, she may sell the shares. We find that if we restrict our funkiness measure to *shareholding* directors, our results are very similar. This also holds if we focus on larger firms, which is compelling, since an individual director in these firms is less likely to have a sizeable shareholding and so the two hats are less of an issue.[3]

To identify interest rate shocks by maturity, we use high-frequency changes in UK govern-

[2]There are 12 NUTS1 regions in the UK. For details see: https://www.ons.gov.uk/methodology/geography/ukgeographies/eurostat.

[3]These firms are still relatively small given that our data is dominated by small and medium-sized enterprises. The median number of employees for firms in our sample is 40, while the 90th percentile is 210.

ment bond yields around Bank of England monetary policy announcements. We separately construct short- and long-maturity shocks and link them to firm-level investment responses using variation in firms' accounting year-ends, which determines their exposure to interest rate surprises between reporting dates. This allows us to estimate the differential effect of monetary policy shocks across the yield curve while holding fixed firm-level fundamentals.

We also examine how *financing* responds to these shocks and find patterns consistent with our model's predictions. Compared to non-funky firms, borrowing by funky firms is less sensitive to long rates (but their borrowing reacts slightly more strongly to short rates). These borrowing responses reinforce the investment evidence.

Overall, our results highlight the importance of integrating labour contracting frictions, and the concomitant funding horizon heterogeneity, into models of firm dynamics and monetary transmission.

The paper is organised as follows. The next section introduces our microeconomic model. Section 3 embeds this into a macroeconomic framework. Section 4 presents our main empirical evidence, and the final section relates this to the empirical literature. The Appendix contains additional empirical results and proofs.

2 Model

This section analyzes the partial equilibrium of an economy with two types of risk neutral agents, engineers and savers. Time is discrete, $t = 0, 1, 2, \ldots$. Agents have a common discount rate $1/R_t$ so that the gross rate of interest equals R_t. Consider perturbations of R_t in the neighbourhood of a steady state R.

At any date, new production facilities are constructed by groups of engineers who share an investment opportunity. A group comprises $n \geq 2$ engineers, who we think of as "key workers". Their investment technology has constant returns to scale. To construct one unit of plant requires one building and x units of goods. Following construction, the plant produces goods in all subsequent periods, with an initial productivity of $z = 1$ which is the maximum productivity level.

$$\left.\begin{array}{r} x \text{ goods} \\ 1 \text{ building} \end{array}\right\} \rightarrow \left\{ \begin{array}{c} 1 \text{ unit of plant with initial productivity } z = 1 \\ \text{capacity for } n \text{ key workers to maintain the plant} \end{array}\right. \tag{1}$$

As a by-product of plant construction, the n engineers each acquire the capacity to maintain the productivity of the plant. Think of this acquisition of human capital – maintenance capacity – as learning-by-doing through investment, as in Arrow (1962). In every period, a unit of plant with productivity $z \leq 1$ yields output $y = az$. After production, the size of plant and building shrinks by factor $\lambda < 1$ due to depreciation. We will restrict attention to values of parameters for which the technology has a rate of return strictly greater than the interest rate.

To finance their investment, a group of n engineers raises as much funding as possible by simply selling ownership of the plant to a saver, or to several savers; by assumption, they cannot sell their human capital. A saver who buys the plant knows that in subsequent periods she will need the original group of engineers to maintain the plant's productivity. They are key workers because only they know how to maintain this plant. However there is moral hazard: in every period, each of these engineers independently has freedom to choose whether to pull his weight (work) or not (shirk). At any date t, the evolution of the plant's productivity, starting at $z_t = z \leq 1$, depends on how many of the n engineers work or shirk.

If *no-one shirks* at date t, assume productivity next period is

$$z_{t+1} = z' = \rho z + 1 - \rho, \tag{2}$$

where $0 < \rho < 1$ measures the persistence of the current productivity z. Notice that if in the current period $z = 1$, the maximum level, and no-one shirks, then in the next period productivity stays at 1. If in the current period $z < 1$, and no-one shirks, then in the next period productivity climbs back up towards 1.

If just *one engineer shirks* at date t, assume productivity next period is lower:

$$z_{t+1} = \widehat{z} = \left[1 - \frac{1-\theta}{n}\right] z', \tag{3}$$

where $0 < \theta < 1$. Notice that if one engineer were to shirk for a long time, then productivity would keep falling, but never below

$$\underline{z} = \frac{(1-\rho)\left[1 - \frac{1-\theta}{n}\right]}{1 - \rho\left[1 - \frac{1-\theta}{n}\right]}. \tag{4}$$

So if, in every period, either no-one shirks or just one engineer shirks, productivity z stays within bounds: $z \in (\underline{z}, 1]$, where the infimum value $\underline{z}$ is strictly positive.

If *two or more engineers shirk* at date t, assume the plant is permanently wasted: for all $s \geq 1$,

$$z_{t+s} = 0.$$

The gap between z' and $\widehat{z}$ is an engineer's marginal contribution to maintenance when none of his colleagues shirks:

$$z' - \widehat{z} = \frac{1-\theta}{n} z' = \frac{1-\theta}{n}(\rho z + 1 - \rho). \tag{5}$$

The factor $1-\theta$ measures the *combined* marginal contributions of all n engineers:

$$n \times (z' - \widehat{z}) = (1-\theta)(\rho z + 1 - \rho).$$

For the empirical section of the paper, we have in mind that $1-\theta$ may be increasing in n.

We will be interested in an equilibrium where no-one ever shirks. Productivity will stay at the maximum value: at date t, z_t will equal 1 and, given no shirking, z_{t+1} will also equal 1.

Let q_t be the price of one building, in terms of goods, at the end of date t. Take the rental price, f say, of a building to be constant, so

$$q_t = \frac{f}{R_t} + \frac{f\lambda}{R_t R_{t+1}} + \frac{f\lambda^2}{R_t R_{t+1} R_{t+2}} + ... + \frac{f\lambda^\tau}{R_t R_{t+1} R_{t+\tau}} + ... \tag{6}$$

After production, plant can be liquidated and revert to being a plain building at value λq_t per unit. Let $V_t(z)$ be the value of a unit of plant with productivity z to the plant owner, a saver, at the beginning of date t. The value function satisfies Bellman equation

$$V_t(z) = az + \max\left\{\lambda q_t, -nw_t(z) + \frac{\lambda}{R_t} V_{t+1}(z')\right\} \tag{7}$$

where $w_t(z)$ is negotiated wage rate paid to each engineer per unit of plant. The first term in the right hand side is current output. The first term inside the curly bracket is the value of liquidation. The second term is the value of continuation: minus the current wage bill plus the present value of the plant next period with new productivity z'.

In the wage negotiation between the plant owner and her group of n engineers, at each date t, each engineer makes a take-it-or-leave-it offer to the owner, taking other engineers' offers as given. In Nash equilibrium,

$$w_t(z) = \frac{\lambda}{R_t}\left[V_{t+1}(z') - V_{t+1}(\hat{z})\right]. \tag{8}$$

We focus on an equilibrium where, unless two or more workers have simultaneously shirked, owners never want to liquidate. To this end, cap the rental price of a building:

$$f < a\frac{(1-\rho)\theta}{R-\lambda\theta\rho}\frac{R-\lambda\rho\left(1-\frac{1-\theta}{n}\right)}{1-\rho+\rho\frac{1-\theta}{n}}. \qquad (\textbf{Assumption 1})$$

We now guess and verify an equilibrium.

Proposition 1. *For R_t in the neighbourhood of R, there is an equilibrium in which no one ever shirks and the value function is affine:*[4]

$$V_t(z) = c_t + \nu_t z, \text{ where} \tag{9}$$

[4]There are other equilibria. We have constructed one where the value function has an infinite set of jumps. But we conjecture that within the class of polynomials, the equilibrium value function in Proposition 1 is unique.

$$\nu_t = a + \frac{\lambda\theta\rho}{R_t}\nu_{t+1} \text{ and } c_t = \frac{\lambda}{R_t}c_{t+1} + \frac{\lambda\theta(1-\rho)}{R_t}\nu_{t+1}.$$

Plant owners do not want to liquidate, even if, off the equilibrium path, a single key worker has shirked for a long time and the current productivity z_t has dropped close to the infimum $\underline{z}$:

$$\lambda q_t < -nw_t(\underline{z}) + \frac{\lambda}{R_t}V_{t+1}(\rho\underline{z} + 1 - \rho).$$

Turn now to the choice of investment and ask how the scale of the investment depends on the funding that can be raised from savers. When a group of investing engineers sells plant at date t, per unit they can raise

$$b_t = \frac{1}{R_t}V_{t+1}(1), \tag{10}$$

which, as shown in the Appendix, can be expressed

$$\begin{aligned} b_t = {} & \frac{a}{R_t} + \frac{a\lambda\theta}{R_tR_{t+1}} + \frac{a\lambda^2}{R_tR_{t+1}R_{t+2}}\left[1-(1-\theta)(1+\theta\rho)\right] + ... \\ & + \frac{a\lambda^\tau}{R_tR_{t+1}...R_{t+\tau}}\left[1-(1-\theta)\frac{1-(\theta\rho)^\tau}{1-\theta\rho}\right] + ... \end{aligned} \tag{11}$$

Claims 2 and 3 assemble some useful facts about b_t and q_t from (11) and (6).

Claim 2. (a) The lower is θ below 1, the smaller is b_t and the shorter is the duration of b_t.[5]

(b) The duration of b_t is shorter than that of q_t.

(c) The τ-period-ahead cash flow gap between b_t and q_t, $a\lambda^\tau\left[1-(1-\theta)\frac{1-(\theta\rho)^\tau}{1-\theta\rho}\right] - f\lambda^\tau$, is positive for low enough τ, but becomes negative for high enough τ iff $f > \frac{(1-\rho)\theta}{1-\theta\rho}a$.

An increase in the future interest rate $R_{t+\tau}$ lowers both b_t and q_t only through its impact on the present value of cash flow terms from date $t+\tau$ onwards. Making use of Claim 2:

Claim 3. An increase in a future interest rate $R_{t+\tau}$

(a) lowers b_t less when θ is lower;

(b) lowers $(b_t - q_t)$ for low enough τ, but raises $(b_t - q_t)$ for high enough τ iff $f > \frac{(1-\rho)\theta}{1-\theta\rho}a$.

Notice that Claim 2(c) and Claim 3(b) make use of a critical value $\frac{(1-\rho)\theta}{1-\theta\rho}a$ of the rental price of buildings f. Given $n \geq 2$, this critical value is less than the upper bound in Assumption 1.[6]

[5] Appealing to Macaulay (1938), we define the duration of cash flows y_{t+1}, y_{t+2}, y_{t+3}, $\ldots$ as $\frac{\frac{1}{R_t}y_{t+1} + \frac{2}{R_tR_{t+1}}y_{t+2} + \frac{3}{R_tR_{t+1}R_{t+2}}y_{t+3} + ...}{\frac{1}{R_t}y_{t+1} + \frac{1}{R_tR_{t+1}}y_{t+2} + \frac{1}{R_tR_{t+1}R_{t+2}}y_{t+3} + ...}$.

[6] They are the same when $n = 1$ and the upper bound grows with n.

The investment technology has constant returns to scale. To ensure that it is profitable yet not entirely self-financing, assume

$$\frac{a}{R-\lambda}\left[1-\frac{\lambda(1-\theta)}{R-\lambda\theta\rho}\right] < x+\frac{f}{R-\lambda} < \frac{a}{R-\lambda}. \qquad \textbf{(Assumption 2)}$$

Here, the left hand expression is the steady state value of b_t. The left hand inequality says that, in the neighbourhood of steady state, there is a shortfall between the per unit funding capacity, b_t, and the unit cost of investment, $x + q_t$. The right hand inequality says that the investment technology has a rate of return strictly greater than the interest rate. Given the upper bound on f in Assumption 1, the steady state value of b_t strictly exceeds the steady state value of q_t.[7] Taken together, then, Assumptions 1 and 2 require x to be not too small, but not so big as to render investment unprofitable.

A group of engineers with an investment opportunity use all their worth net of consumption, N_t say, to make up the shortfall between b_t and $x + q_t$. Thus the size of their investment is

$$I_t = \frac{N_t}{x+q_t-b_t}. \tag{12}$$

Consider the impact of an increase in a future interest rate $R_{t+\tau}$. To see how investment responds, look at the semi-elasticities:

$$\frac{1}{I_t}\frac{\partial I_t}{\partial R_{t+\tau}} = \frac{1}{N_t}\frac{\partial N_t}{\partial R_{t+\tau}} + \frac{1}{x+q_t-b_t}\frac{\partial (b_t-q_t)}{\partial R_{t+\tau}}. \tag{13}$$

By Claim 3(b), for firms with θ in the region above $f/\left[(1-\rho)\,a+\rho f\right]$,[8] the second term on the right hand side of (13) is negative. An increase in interest rate $R_{t+\tau}$ lowers funding capacity b_t more than it lowers investment cost $x + q_t$, so that, as is usual, investment shrinks. But for firms with θ in the region below $f/\left[(1-\rho)\,a+\rho f\right]$, if the increase is to a distant interest rate (τ is high enough), the second term is positive – costs are lowered more than funding, and investment *expands*.[9]

This is suggestive of the following conclusion: investment by firms that are more vulnerable to their key workers – firms that have lower θ – is less impacted by shocks to distant interest rates (and, possibly, more impacted by shocks to near term interest rates). But the full argument is more intricate: to establish whether the second term is monotonic in θ within either of these

[7] Take n to infinity raising the right hand side of Assumption 1, and divide by $R - \lambda$, to obtain a simple upper bound on $f/(R-\lambda)$ that is in turn strictly less than the left hand side of Assumption 2.

[8] That is, $\frac{(1-\rho)\theta}{1-\theta\rho}a > f$.

[9] The notion that a persistent lowering of interest rates might serve to contract investment and reduce growth is the subject of Kiyotaki, Moore, and Zhang (2021). See also Asriyan, Laeven, Martin, Van der Ghote, and Vanasco (2025).

two regions needs more work.

We have other work to do too. Discovering the overall effects of a term structure shock on investment needs a theory of the net worth N_t, the first term on the right hand side of (13). And, most importantly, we need to build a framework in which both kinds of firm, those with low and high θ, can co-exist in the long run.

3 Funky and Non-funky Firms

Section 2 modelled the dynamic interaction between a group of key workers and external investors. There is a technology for which, at every date following investment in new plant, the key workers are able to exploit their hold-up power: they can extract their combined marginal contribution to maintaining the plant, measured as a fraction $1-\theta$ of total productivity. The remainder of the paper tests this model using firm-level data. In particular, we wish to study how the investment response to short and long run interest rate shocks varies across firms with different degrees of hold-up power $1-\theta$.

There are two challenges, one empirical, one theoretical. Empirically, we do not have any direct measure of θ. As a proxy, we attempt to measure n, the number of key workers, by the number of directors a firm has – more precisely, by the ratio of the number of directors to the total number of employees. For otherwise similar firms (size, industry, age, ...), we label firms with a value of n above the median as "funky"; the rest are "non-funky". And because $1-\theta$ measures the n key workers' *combined* marginal contribution to maintaining plant productivity, we make the plausible assumption that $1-\theta$ is increasing in n; so funky firms have a lower θ than non-funky.

The theoretical challenge to studying how, ceteris paribus, the investment response to interest rate shocks varies across funky and non-funky firms, is that if the two types of firm differed only in respect of their θ (via their n), then funky firms – being more vulnerable to exploitation by key workers – would eventually be driven out by non-funky firms. We cannot simply look at the cross derivatives (with respect to interest rate and θ) of the investment formula (12) from Section 2: we need to dig a little deeper into that ceteris paribus. This is the main purpose of this section, which, regrettably, has to be somewhat algebraically heavy.

Suppose there are two types of competitive firm, funky F and non-funky N, whose respective outputs are imperfect substitutes in the production of a final good. Funky and non-funky firms share the investment technology from Section 2 – with common values of x, λ, ρ, and physical productivity – except that funky firms have a larger number of key workers:

$$n_F > n_N.$$

In consequence, external investors in funky firms are more exposed to exploitation:

$$\theta_F < \theta_N.$$

Nonetheless, a funky firm is able to coexist alongside a non-funky firm because, *endogenously*, its product price is higher. As we shall see in the first part of Lemma 1 below, the firms' value productivities (product price times physical productivity) are ranked:

$$a_F > a_N.$$

For the rest of this section, it will be algebraically convenient to identify a firm by its θ, with corresponding $n(\theta)$ and $a(\theta)$, and to differentiate with respect to θ as if it were a continuous variable.

Although the empirical examination does not depend upon a particular production function for the final good, it may help to bear in mind a concrete example, the CES case. Consider the final good Y being produced from the two intermediate goods, (y_F, y_N), according to a constant returns to scale production function

$$Y = \left[y_F{}^{\frac{\epsilon-1}{\epsilon}} + y_N{}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}$$

where $\epsilon > 1$ is the elasticity of substitution. The value productivity of firm θ depends upon relative price $p(\theta)$ and physical productivity $\overline{a}$ as

$$a(\theta) = p(\theta)\,\overline{a},$$

where the final goods price P (normalized to unity) depends upon the price of each intermediate goods as

$$1 = P = \left[(p(\theta_F))^{1-\epsilon} + (p(\theta_N))^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}.$$

The relative price depends upon the ratio of sector outputs y_F and y_N to the aggregate output Y as

$$p(\theta_F) = \left(\frac{y_F}{Y} \right)^{-\frac{1}{\epsilon}} \text{ and } p(\theta_N) = \left(\frac{y_N}{Y} \right)^{-\frac{1}{\epsilon}}$$

When a group of new engineers has an investment opportunity, all their combined net worth is used to fill the gap between the per unit investment cost, $x + q_t$, and the external funding capacity, $b_t(\theta)$. Let $N_t(\theta)$ be the combined net worth of the $n(\theta)$ new engineers investing in sector θ. The investment of sector θ at date t equals

$$I_t(\theta) = \frac{N_t(\theta)}{x + q_t - b_t(\theta)} \tag{14}$$

Engineers who have invested in the past – old engineers – do not invest any more: they live off their (declining) wage income.

Crucially, assume that, although the external funding capacity, $b_t(\theta)$, depends on θ, rates of return for new engineers are equalized across types in the long run. In the steady state, the rate of return for engineers to invest, R^E, satisfies

$$\begin{aligned} q + x - b(\theta) &= \frac{w(\theta)\, n(\theta)}{R^E} + \frac{\lambda w(\theta)\, n(\theta)}{(R^E)^2} + \frac{\lambda^2 w(\theta)\, n(\theta)}{(R^E)^3} + \ldots \\ &= \frac{w(\theta)\, n(\theta)}{R^E - \lambda}. \end{aligned}$$

From Proposition 1, together with equation (8), engineers' combined compensation in the steady state is

$$w(\theta)\, n(\theta) = \frac{\lambda(1-\theta)a(\theta)}{R - \lambda\theta\rho}.$$

Thus the steady state rate of return on investment

$$R^E = \frac{1}{q + x - b(\theta)} \frac{\lambda(1-\theta)a(\theta)}{R - \lambda\theta\rho} + \lambda. \tag{15}$$

Again from Proposition 1, together with equation (10), the external funding capacity in the steady state

$$b(\theta) = \frac{1}{R}\left(c(\theta) + \nu(\theta)\right) = a(\theta)\widetilde{b}(\theta),$$

where function $\widetilde{b}(\theta)$ is defined by

$$\widetilde{b}(\theta) = \left[1 + \frac{\lambda\theta(1-\rho)}{R-\lambda}\right] \frac{1}{R - \lambda\theta\rho}.$$

These relationships can be used to study how the value productivity and the external funding capacity depend on θ:

Claim 4. In the steady state,

$$\frac{a'(\theta)}{a(\theta)} = \frac{R - \lambda\rho}{(1-\theta)(R - \lambda\theta\rho)} \left[1 - \frac{a(\theta)}{(R-\lambda)(q+x)}\right] < 0, \tag{16}$$

$$\frac{b'(\theta)}{b(\theta)} = \frac{q+x-b}{q+x} \frac{R - \lambda\rho}{(1-\theta)\left[R - \lambda + \lambda\theta(1-\rho)\right]} > 0. \tag{17}$$

Intuitively, because a firm with a higher θ has a larger borrowing capacity, the intermediate good output of the sector is larger, and the relative price is lower: $a(\theta)$ decreases with θ.

But the indirect effect of a lower relative output price does not overturn its larger borrowing capacity: $b(\theta)$ increases with θ.

Along a steady state growth path, the rates of return for new investing engineers are equalized across types through the fraction, $\varphi(\theta)$ say, choosing type θ. That is, at any date t, the combined net worth, $N_t(\theta)$, of those engineers investing in type θ equals $\varphi(\theta)N_t$, where N_t denotes the total net worth of the entire investing cohort.

Because along the steady state growth path,

$$K_t(\theta) = \lambda K_{t-1}(\theta) + \frac{\varphi(\theta)N_t}{x+q-b(\theta)}, \tag{18}$$

it follows that

$$\frac{K_t(\theta)}{N_t} \propto \frac{\varphi(\theta)}{x+q-b(\theta)},$$

where the constant of proportionality does not depend on θ. The external funding capacity $b(\theta)$ and the fraction of investing engineers $\varphi(\theta)$ both affect the relative size of the total capital stock of the firms of type θ.

In the CES production function example above, $a(\theta)$ and the relative size of capital stock are related as

$$a(\theta) = \overline{a}p(\theta) \propto \left[\frac{K_t(\theta)}{N_t}\right]^{-\frac{1}{\epsilon}}.$$

So from equation (15),

$$a(\theta) = \frac{(q+x)(R-\lambda\rho\theta)(R^E-\lambda)}{\lambda(1-\theta)+(R^E-\lambda)\left[1+\frac{\lambda\theta(1-\rho)}{R-\lambda}\right]} \propto \left[\frac{\varphi(\theta)}{x+q-b(\theta)}\right]^{-\frac{1}{\epsilon}}.$$

$\varphi(\theta)$ adjusts so that this proportionality is satisfied across θ's: think of budding engineers deciding their type far in advance, at the start of their education which takes many periods to complete.

Along a steady state path with growth rate G, the relative price of intermediate goods, and hence $a(\theta)$, are constant because the investment rate

$$\frac{I_t(\theta)}{K_{t-1}(\theta)} = G - \lambda \tag{19}$$

is constant and independent of θ.

In our empirical analysis, we consider small and transitory shocks to the term structure of interest rates. To streamline, assume that neither $\varphi(\theta)$ nor $a(\theta)$ is affected by these shocks.

Assumption 3. *The fraction $\varphi(\theta)$ of budding engineers choosing type θ is decided too far in advance to react to transitional shocks to the term structure of interest rates.*

Assumption 4. *For each θ, the capital stock $K_{t-1}(\theta)$ is large relative to investment $I_t(\theta)$, so that the relative price of intermediate goods, and hence the value productivity $a(\theta)$, are unaffected by transitional shocks to the term structure of interest rates.*

We have data on the fixed capital stock $K_t(\theta)$ in year t for individual firms of type θ. The dependent variable in our regressions is the rate of change of fixed capital

$$L_t(\theta) = \frac{K_t(\theta) - K_{t-1}(\theta)}{\left[K_t(\theta) + K_{t-1}(\theta)\right]/2},$$

using the method of, e.g., Törnqvist, Vartia, and Vartia (1985) and Davis, Haltiwanger, and Schuh (1998), to approximate growth in the presence of zeros. The change of the fixed capital of firm θ is related to investment of that firm and the depreciation as

$$K_t(\theta) - K_{t-1}(\theta) = I_t(\theta) - (1-\lambda)K_{t-1}(\theta).$$

Although we use the average of fixed capital at the end of years t and $t-1$ as the denominator, it is not too different from the fixed capital of year $K_{t-1}(\theta)$ in our data. Therefore the dependent variable becomes

$$\begin{aligned} L_t(\theta) &\simeq \frac{I_t(\theta)}{K_{t-1}(\theta)} - (1-\lambda) \\ &= \frac{\varphi(\theta)N_t/K_{t-1}(\theta)}{x + q_t - b_t(\theta)} - (1-\lambda). \end{aligned}$$

Turn now to the interest rate shocks. We examine in particular the effect on investment of small transitory changes in interest rates near the steady state growth path. Suppose a single unexpected shock hits at date t, which causes the interest rate τ periods ahead, $R_{t+\tau}$, to deviate from R. The partial effect of this shock on the rate of change of capital stock in the neighbourhood of the steady state is given by

$$\begin{aligned} \frac{\partial L_t(\theta)}{\partial R_{t+\tau}} &= \frac{\varphi(\theta)N_t/K_{t-1}(\theta)}{x+q-b(\theta)}\left[\frac{1}{N_t}\frac{\partial N_t}{\partial R_{t+\tau}} + \frac{\frac{\partial}{\partial R_{t+\tau}}\left(b_t\left(\theta\right) - q_t\right)}{x+q-b(\theta)}\right] \\ &= (G-\lambda)\left[\frac{1}{N_t}\frac{\partial N_t}{\partial R_{t+\tau}} + \frac{\frac{\partial}{\partial R_{t+\tau}}\left(b_t\left(\theta\right) - q_t\right)}{x+q-b(\theta)}\right]. \end{aligned} \tag{20}$$

Here Assumption 3 has been used to get the first line and the second line follows from equation (19). Notice that (20) is akin to (13), but not in semi-elasticity form.

We examine the effects on investment of a shock to the entire term structure of interest rates. For parsimony, take the term structure to be approximately affine. Then, shocks to the term structure can be captured by two changes. In our empirical study, we make use of the 1-year interest rate and the 5-year average interest rate.[10] Let s_t and r_t be the deviations from steady state of these two rates at date t. Then, given the affine term structure, for small deviations from steady state, the deviation of the τ year average interest rate is

$$\Delta_{\tau,t} \;=\; \frac{5-\tau}{4}s_t + \frac{\tau-1}{4}r_t, \tag{21}$$

as long as the maturity τ is not extremely long, say $\tau \leq T$ for some large number T. Because the change of the interest rates is transitory, the economy will eventually go back to the steady state. (Check: $\Delta_{1,t} = s_t$, $\Delta_{5,t} = r_t$.)

Propositions 5 and 6 make use of Assumption 4 to find the overall effect of these term structure shocks, as long as θ is not too small (equations giving the lower bounds can be found in the Appendix).[11]

Proposition 5. *Suppose θ is greater than some lower bound $\underline{\theta}^r$, which may be zero. Given the affine term structure (21), a shock r_t that raises the 5-year average interest rate (but keeps the 1-year interest rate unchanged) reduces a firm's investment rate, $L_t(\theta)$, less when the firm's type θ is lower.*

Proposition 6. *Suppose θ is greater than some lower bound $\underline{\theta}^s$. Also suppose $\lambda > (2/3)R$. Given the affine term structure (21), a shock s_t that raises the 1-year interest rate (but keeps the 5-year average interest rate unchanged) reduces a firm's investment rate, $L_t(\theta)$, more when the firm's type θ is lower.*

In broad terms, these two propositions say that investment by firms that are more vulnerable to their key workers – firms that have lower θ – is less impacted by shocks to distant interest rates, but more impacted by shocks to near term interest rates. The reason is that those firms are funded primarily from their near term revenues.

4 Testing the Theory

4.1 Towards Empirics: More Heterogeneity

Our two-type model in Section 3 needs to be generalized to allow for more heterogeneity.

[10] The 5-year average interest rate equals $(R_t R_{t+1} \ldots R_{t+4})^{1/5}$, which is approximately $\frac{1}{5}(R_t + R_{t+1} + \ldots + R_{t+4})$ for small deviations from steady state.

[11] Strictly speaking, in Propositions 5 and 6 we compute the sensitivity of investment to interest rate shocks assuming the transition period T is long enough to be approximated by infinity. More accurate computation should take into account the finiteness of T.

Suppose now there are many sectors. By "sector", we might be thinking about firms of a particular size, or industry, or age, or some combination. Within each sector, there are two types of firm, funky and non-funky – just as in Section 3. That is, the funky and non-funky firms *within a sector* share a common investment technology (they have common values of x, λ, ρ, and physical productivity $\overline{a}$); they share a common rate of return R^E for engineers; and they share a common growth rate G. But *across sectors* there can be variation in all these.

We do not want to clutter up the algebra, but it should be clear how to construct, for example, a larger CES model in which the outputs of different sectors are imperfect substitutes in the production of a final good. The elasticities of substitution across the outputs of different sectors need not be the same as those within sectors.

4.2 Measuring Funkiness

In our theory, the combined bargaining power of key workers, $1-\theta$, increases with the number of them, n. In our empirical work, we proxy key workers by directors. The dichotomy between a funky firm (low θ) and a non-funky firm (high θ) will rest on whether the ratio of the number of directors to the total number of its employees lies above or below the median for otherwise similar firms (size, industry, age, ...). For each firm i, we compute the ratio

$$\phi_i = \frac{D_i}{E_i}, \tag{22}$$

where D_i and E_i are the time-averages (over the sample period) of the number of directors and employees of firm i, respectively.

To ensure that our measurement of funkiness is not simply picking up the effect of firm size, we sort firms evenly into ten buckets by size (measured by their time-average level of fixed assets). In our baseline specification, firm i is classified as funky if ϕ_i is above the median of its bucket and non-funky otherwise.

To further avoid our measurement of funkiness being tainted by the industry (respectively age) of a firm, we later extend our baseline specification by sorting firms evenly into 10-size×7-industry buckets (respectively 10-size×2-age buckets) and reclassifying firm i as funky if and only if its ϕ_i is above the median of its new smaller bucket.

4.3 Data

To empirically test Propositions 5 and 6, we will use UK firm-level data to measure the ratio (22) separately for each firm and estimate its effects on firms' investment sensitivities to shocks to short-term and long-term interest rates. Before developing our empirical design, we briefly describe the firm-level data and the identification strategy for interest rate shocks.

Firm-Level Data In the U.K. all firms must file annual accounts with Companies House. We source this data via the data provider Bureau van Dijk (BvD). To provide comprehensive historical coverage, we compile 31 historical snapshots of BvD data, following the methodology of Bahaj, Foulis, and Pinter (2020). This dataset covers both public and private firms, and all sectors of the economy, and contains financial accounts as well as the number of employees. Whilst reporting requirements vary by firm size, our final dataset is dominated by small and medium-sized enterprises.

Under U.K. company law, all companies, of any size, must have at least one director.[12] These individuals are legally responsible for running the firm and ensuring that it works in the interest of shareholders. BvD collect information on the appointment and resignation dates of every directorship (in addition to other characteristics such as age and occupation), allowing us to accurately calculate the number of directors working in the firm through time. Firms also report the remuneration of directors (as a whole), as well as the total remuneration for employees the firm.

We restrict our analysis to for profit public and private companies in Great Britain, and exclude firms operating in selected industries.[13] We exclude subsidiaries, to ensure that the directors have full control over their firm, and omit firms that never report our key variables of interest (including accounting items "Fixed Assets", "Number of Employees", and "Remuneration"). We also restrict our sample to observations where we can observe the response of "Fixed Assets" 0,1,2 and 3 years out & to firms that are "Live", with regular annual accounting windows (between 11 and 13 months). Finally, we omit the smallest firms, whose average real "Fixed Assets" over our sample are less than £25,000.[14]

Interest Rate Shocks To approximate short and long-term borrowing costs for firms we use the yields on 1- and 5-year UK government bonds, respectively, which are available as a consistent series throughout our sample period. To isolate unanticipated changes in these yields, we use the high-frequency monetary policy shock series of Braun, Miranda-Agrippino, and Saha (2023) as instruments, which are available from June 1997. We use the changes in financial variables in a 30-minute window around the interest rate announcement of the Bank

[12] There were 2.8 million active company directors in 2014 in the U.K. (Bahaj, Foulis, and Pinter 2020).

[13] Specifically, we exclude those in Agriculture (2003 UK SIC Code 0111-0202); Fishing (2003 UK SIC Code 0501-0502); Mining and Quarrying (2003 UK SIC Code 1010-1450); Utilities (2003 UK SIC Code 4011-4100); Finance and Insurance (2003 UK SIC Code 6511-6720); Real Estate (2003 UK SIC Code 7011-7032); Public Administration (2003 UK SIC Code 7511-7530); Education/Health/Charity (2003 UK SIC Code 8010-8532); Other Communities/Social/Personal Service Activities (UK 2003 UK SIC Code 9000-9305); Private Households & Extra-Territorial Organisations (9500-9900).

[14] We deflate "Fixed Assets" with the CPI index from the UK Office for National Statistics, with 2015 as the base year.

of England.[15]

Whilst UK firms must file annual accounts, and at the same time each year, there is no restriction on which month accounts must be filed in. As a result, firm accounting months are distributed throughout the year (with nodes in March and December). As a result, whilst the underlying accounting data is annual, as firms file in different months, they will experience different series of interest rates changes and monetary policy shocks, allowing us to exploit this monthly variation. For a firm filing their accounts at time t, we compute the change in 1- and 5-year government bond yields over the last 12 months, and instrument this with the sum of the relevant monetary shock series over the same time period.

Specifically, our instruments are the cumulative high-frequency surprises in the 1-year and 5-year gilt yields, together with the surprise in the fourth quarterly Short Sterling futures contract (FSScm4), all measured in 30-minute windows around Bank of England policy announcements.[16]

Summary Statistics Our sample period runs from May 1998 (as the shock series begins in June 1997), and runs to December 2017.[17] Table 1 provides summary statistics on our baseline sample, which contains 460,238 observations on 45,032 firms. The table highlights that our sample (as with the full firm population) is dominated by small and medium-sized enterprises. The median firm in our sample has £3m in real assets, £733k in real fixed assets, is 13 years old, and has 40 employees. The median firm has 2.8 directors.

4.4 Empirical Specifications

Our baseline regressions estimate the response of firm investment to changes in short-term and long-term interest rates, and consider how this differs for funky vs non-funky firms. To proxy investment, we use the change in real fixed assets, $FA_{i,t}$, of firm i, at time t. Our baseline specification uses local projections (Jorda 2005) and considers the change in fixed assets between several accounting periods:

$$\frac{FA_{t+h,i} - FA_{t-1,i}}{0.5\,(FA_{t+h,i} + FA_{t-1,i})} = \sum_{g=1}^{G} \beta_g^h \times Dg_i \times \Delta r_t^S + \sum_{g=1}^{G} \gamma_g^h \times Dg_i \times \Delta r_t^L + \alpha_i + v_{i,t}^h, \tag{23}$$

where the left-hand side of (23) is the growth rate measure that has become standard in the firm dynamics literature because it carries some useful properties of log differences but also

[15]The monetary policy surprise series by Braun, Miranda-Agrippino, and Saha (2023) applies to the UK the methodology developed by Gürkaynak, Sack, and Swanson (2005) for the US.

[16]The FSScm4 contract is linked to expectations of 3-month sterling money-market rates roughly one year ahead; over our 1998–2017 sample it is Libor-based.

[17]We calculate a firm's ϕ_i using observations where the firm reports both the number of directors and the number of employees. Our measure of the number of directors is available from 1998m1 to 2015m8.

accommodates firm entry and exit; $h \in \{0, 1, 2, 3\}$ indexes a set of regressions at different horizons, running from 0 to 3 years, capturing the cumulative change in fixed assets since the year prior to the shocks, over these different horizons.[18] The terms Δr_t^S and Δr_t^L are the changes in the 1 year and 5 year government bond yields over the 12 months prior to the firm's accounting date. These changes are instrumented with the cumulative sum of monetary policy shocks over the same period. To allow for heterogeneous responses, the term Dg_i is a dummy variable that takes a value of 1 when firm i is part of a particular group g (e.g. funky) and 0 otherwise.[19] The impulse responses to a change in the short-term and long-term interest rates for a particular group are then given by the vectors of coefficient estimates $\left\{\beta_g^h\right\}_{h=0}^3$ and $\left\{\gamma_g^h\right\}_{h=0}^3$, respectively. The term α_i is a firm fixed effect which controls for any time-invariant factors specific to firm i (possibly affecting the firm's growth rate).

We also consider an alternative specification including time-fixed effects:

$$\frac{FA_{t+h,i} - FA_{t-1,i}}{0.5\left(FA_{t+h,i} + FA_{t-1,i}\right)} = \sum_{g=1}^{G-1} \beta_g^h \times Dg_i \times \Delta r_t^S + \sum_{g=1}^{G-1} \gamma_g^h \times Dg_i \times \Delta r_t^L + \alpha_i + \delta_t + v_{i,t}^h, \quad (24)$$

where δ_t is a month fixed effect which absorbs any shocks affecting all firms equally that file in the same month. In this specification we have to omit the coefficient estimate for one group (to avoid collinearity with the month fixed effect) and throughout we drop the estimate for *funky* firms. This regression then shows the estimated effect for non-funky firms *relative* to funky firms. Throughout, these regressions on firm-outcomes are two-way clustered by month and firm.

4.5 Baseline Results

First Stage Regression Table 2 shows the first stage regression linking the change in 1-year and 5-year government bond yields during over the firms' annual accounts to the cumulative sum of monetary policy shocks during the same period. Column 1 of Table 2 shows that a cumulative increase of 100bps in the 1 year government bond yield during the shock windows results in an annual increase of 420bps in the 1 year yield. This overshoot could be due to government bond yields taking longer than 20 minutes to react to Bank Rate announcements. The relationship with the other shocks is weaker and statistically insignificant. Column 2 shows that the annual change in the 5 year yield is driven both by shocks to the 1 and 5 year yields, during the shock windows.[20]

[18] We do not winsorize the outcome variables as, by construction, they are bounded between ±200%.

[19] Where we analyses heterogeneous responses by groups, we use as instruments the interaction of these dummy variables with the three monetary policy shock series.

[20] Note that whilst the coefficient on FSScm4 is negative in both columns, this variable has a positive relationship with both the change in the 1 and 5 year yields when included in the regression alone, without the

The results in Table 2 are implicitly weighted by the distribution of firm observations over time, including across years, but also the distribution of filing months within years. As an alternative, Appendix Table A1 shows results in the pure time series. The patterns are qualitatively similar, and the shocks to the 5 year yield now have a statistically significant impact on the change in the 5 year yield.

Average Effects Table 3 shows the average effect of unexpected changes to short and longer-term interest rates on investment. The three columns show the cumulative investment response over 0,1,2,3 years in response to monetary policy shocks which raise the 1 year yield and the 5 year yield by 100bps over 12 months since the lagged set of firm accounts. The Kleibergen and Paap (2006) F-statistic of 14 alleviates concerns around weak instruments. The results show that after 2 years, cumulative real investment is 11.2% lower in response to a 100bps increase in the 5 year yield and 4.7% lower in response to the same increase in the 1 year yield (though this latter change is not statistically significant).

Our baseline estimates are quantitatively large relative to canonical estimates: standard benchmarks for aggregate VARs (Christiano, Eichenbaum, and Evans, 2005) or publicly listed firms (Cloyne, Ferreira, Froemel, and Surico, 2023; Jeenas and Lagos, 2024) suggest an aggregate investment decrease of 1-2% per 100 bps increase in rates, consistent across identification strategies such as recursive ordering, and high-frequency instruments. However, these aggregate elasticities mask significant heterogeneity. Even within public firms, Cloyne, Ferreira, Froemel, and Surico (2023) document that younger, non-dividend paying firms (proxying those facing financial constraints) exhibit responses closer to 4%. Our sample is dominated by firms that are orders of magnitude smaller and likely more financially constrained than the typical Compustat firm—our median firm holds around £3 million in total assets, compared to approximately $165 million for the median firm in Jeenas and Lagos (2024).

Funky vs Non-Funky Firms Table 4 shows the investment sensitivity of funky and non-funky firms to unexpected changes in 1 year and 5 year interest rates. In this table funky firms are defined as those with a director to employee ratio above the median, within their firm size bucket (measured by 10 buckets of average real fixed assets). Panel a) shows results without a month-fixed effect, with the specification from Equation (23). The results show that funky firms are more responsive to unexpected changes in 1 year interest rates: in response to an unexpected 100bps increase in the 1-year rate since the firms last accounts, funky firms reduce investment by 6.5% after 2 years, whilst non-funky firms only reduce it by 2.8%. By contrast, the funky firms are less responsive to changes in the 5 year yield: following an unexpected 100bps increase in the 5-year rate, funky firms reduce investment by 9.6%, whilst non-funky

other two shock series.

firms reduce it by 12.9%. These relative responses are consistent with the predictions of the theory.

To test whether the differential effects of the two interest rate shocks (across funky and non-funky firms) are statistically significant, Panel b) of Table 4 shows the estimated results with month fixed effects included in the regressions.[21] These results show the response of non-funky firms relative to funky firms. In response to an unexpected 100bps increase in 1 year yields, investment decreases 2.7pp. less for non-funky firms after 2 years; whilst in response to an increase in 5 year yields, non-funky firms decrease investment by 3.5pp. more. Both of these differences are highly statistically significant.

Shareholder-Directors In many firms, the more important key workers may be awarded equity, to encourage retention. To explore this, in Table 5 we restrict our definition of key workers to those company directors who are *ever* shareholders at the given firm.[22] Funky firms are then those with an above median ratio of key workers to employees, within their size-bucket. The results show a very similar pattern to our baseline measure: across all horizons, the investment of non-funky firms responds materially less to surprise changes in the 5-year rate, and the differences are slightly larger at most horizons. As discussed in the Introduction, sharing-holding directors are partial owners, which mitigates moral hazard. But if they are the more important key workers, then the moral hazard problem is exacerbated. Empirically, these two forces seem to offset each other.

4.6 Alternative Definitions of Funky Firms

Firm Industry Our baseline definition of funky firms could be correlated with the industry a firm is in and industry-specific sensitivities to monetary policy (e.g. due to demand) could be driving our results. To check this concern, we use the same directors/employees ratio from Equation (22), but split within 10 size buckets for each of 7 different SIC 1 industries (that is, 70 buckets in total). Appendix Table A2 shows that the results are very similar when using this more granular definition of funky firms.

Firm Age Another concern is that director to employee ratios could be correlated with firm-age, which is a key determinant of firms' sensitivity to monetary policy (Bahaj, Foulis, Pinter, and Surico 2022). To address this we define funky firms within firm age x 10 size buckets, where the firm age buckets are split around above/below 15 years old. Appendix Table A3

[21]We note that in this relative effect specification the F-Statistic for instrument strength is above the standard critical thresholds.

[22]We link observations on the same individual over time by creating an ID key based on their name and date of birth, in a similar manner to Bahaj, Foulis, and Pinter (2020).

shows very similar results with this measure, suggesting that our results are not simply picking up the effects of firm age.

Firm Region A separate potential concern is that the ratio of a firm's directors to employees could be correlated with the firm's location, and our results could be driven by heterogeneous geographical responses to monetary policy shocks. To address this, within each of Great Britain's 11 NUTS 1 regions, we generate 10 buckets of firm size and then define a firm as funky if the directors/employees ratio is above the median within each of the 110 buckets. Appendix Table A4 shows very similar results with this alternative definition of funky firms (in fact, the heterogeneity is greater).

Wage-Augmented Definition These measures all rely on the ratio of the number of directors to total employment within the firm. However, key workers may be paid relatively more, so as an alternative, we consider an expanded definition of funky firms which takes this into account. Here a firm is classified as funky if, within their firm size bucket, either their director to employee ratio or their director remuneration to total remuneration ratio is above the median. Table 6 shows the results of this regression, with funky firms, under this expanded definition, shown to respond relatively less to changes in 5 year yields, with this difference statistically significant at every horizon.

4.7 Further Results

Effect Across the Size Distribution We now return to the baseline funky firm definition of Table 4 and consider which firms are driving the response. We re-run this regression within 10 separate firm size buckets. Table 7 shows the level effects (the responses to the change in the 1 year yield are included in the regression but omitted from the table to save space). Appendix Table A5 repeats this with the relative specification (including the month fixed effect). We see that, throughout most of the size distribution, the investment of funky firms responds relatively less to changes in 5 year yields. Our baseline results are not driven by one particular firm size group, but are apparent across the size distribution.

Effect Across Industries Table 8 (level regression) and Appendix Table A6 (relative regression) repeat this exercise across different SIC1 industries. Whilst Table 8 highlights different sensitivities of investment to monetary policy across industries, we consistently see that the investment of funky firms responds relatively less to the change in 5 year yields.

Firm Liabilities A key prediction of the theory is that the borrowing responses of funky and non-funky firms will mirror the same heterogeneity as firm investment. To explore this, Table

9 shows the response of real Total Liabilities to monetary policy. This uses the baseline funky measure, based on the director to employee ratio within size buckets. The first table shows the level effects: after 2 years, real total liabilities have contracted by 7.3% for non-funky firms, and 5.7% for funky firms, in response a 100bps increase in the 5 year yield over the last 12 months. The second table shows the relative effects, confirming that non-funky firms respond more to the increase in the 5 year yield, with this difference statistically significant at the 2-year horizon.

Overall, our results show that the borrowing and investment of funky firms are less sensitive to surprise changes in long-term interest rates, consistent with the predictions of our theory.

5 Relation to the Empirical Literature

A large empirical literature documents heterogeneous firm-level responses to conventional monetary policy, typically along dimensions that proxy financial constraints. Classic work emphasises firm size, leverage, and collateral (Gertler and Gilchrist, 1994; Bernanke, Gertler, and Gilchrist, 1999). Recent contributions have revisited the roles of these factors (Otonello and Winberry, 2020; Crouzet and Mehrotra, 2020) as well as highlighted the role of age and opacity (Durante, Ferrando, and Vermeulen, 2022; Cloyne, Ferreira, Froemel, and Surico, 2023; Bahaj, Foulis, Pinter, and Surico, 2022), liquidity and balance-sheet strength (Ippolito, Ozdagli, and Perez-Orive, 2018; Jeenas, 2024), and debt maturity or refinancing risk (Jungherr, Meier, Reinelt, and Schott, 2024). In addition, it has been established that monetary policy transmission is heterogeneous across regions (Carlino and Defina, 1998; Beraja, Fuster, Hurst, and Vavra, 2019) and industries (Barth and Ramey, 2002; Peersman and Smets, 2005; Dedola and Lippi, 2005).

Our contribution is to introduce the importance of key workers – "funkiness" – as a distinct source of heterogeneity. Firms that rely on key workers face tighter internal financing constraints because cumulative moral hazard shortens the horizon over which future returns can be pledged.

A natural concern is that our measure of funkiness may simply proxy for firm size and age, or correlate with other factors such regional effects or industry composition. We have addressed this directly by constructing our funkiness measure within firm-size groups, and verified robustness to double-sorting by age×size, region×size, and industry×size. The fact that funky firms consistently exhibit attenuated responses to long-maturity shocks (and amplified responses to short-maturity shocks) assuages this concern.

At the aggregate level, prior work distinguishes between short-rate policy, forward guidance, and quantitative/qualitative easing, showing that they move the macroeconomy differentially (e.g., Gürkaynak, Sack, and Swanson, 2005; Swanson, 2021). However, firm-level evidence exploiting this structure is limited. A recent paper by Gürkaynak, Karasoy Can, and Lee

(2022) finds that the stock market distinguishes sharply between short rate and slope surprises – with balance sheet features (such as floating-rate debt) shaping the cross-section of responses.[23] While their analysis highlights the empirical relevance of maturity-differentiated transmission, it is reduced form and does not supply a structural mechanism governing why shocks at different maturities should translate into different real outcomes. Our contribution is complementary: we provide a theory that endogenously links key workers to firms' differential sensitivity to shocks along the yield curve.

[23] In a similar spirit, English, Van den Heuvel, and Zakrajsek (2018) looks at the effects of shocks to both the level and slope of the term structure on banks.

Tables

Table 1: Summary Statistics

	(1) mean	(2) p10	(3) p25	(4) p50	(5) p75	(6) p90	(7) sd	(8) N
Real Total Assets (2015 £000's)	55,496	295	1,119	3,042	6,868	18,366	2,214,081	45,032
Real Fixed Assets (2015 £000's)	28,862	59	180	733	2,434	7,470	942,911	45,032
Firm Age (Years)	19.96	5.17	7.92	13.36	25.71	43.42	18.17	45,032
Number of Employees	261.49	4.00	11.75	40.00	90.33	209.79	4,460.04	45,032
Number of Directors	3.19	1.22	2.00	2.79	4.00	5.50	1.87	45,032

Note: The table shows summary statistics based on our sample of 45,032 firms over the period 1998m5-2017m12, as described in Section 4.3. All variables are firm-level averages over our sample period.

Table 2: IV First Stage: Gilt Yields and Monetary Policy Shocks: Firm-Level

	Δ1 Year Yield (1)	Δ5 Year Yield (2)
$\sum$ Shocks to 1 Year Yield	4.20***	3.44***
	(1.16)	(0.86)
$\sum$ Shocks to 5 Year Yield	-1.38	1.52
	(1.92)	(1.40)
$\sum$ Shocks to FSScm4	-0.88	-3.33***
	(1.35)	(0.90)
N	460238	460238
R^2	0.05	0.09
F-Statistic	15.09	11.80

Note: This table shows the "first stage" regression linking the annual change in UK government bond yields with the cumulative sum of monetary policy shocks over the same period. The regression sample is as described in Section 4.3 and runs over the period 1998m5-2017m12. These regressions are run at the firm-level and are based on the distribution of firm accounting periods throughout the year. $\sum$ Shocks to 1 Year Yield is the cumulative sum of shocks over the 12-month period corresponding to the firm's accounting period that occurred to 1 year government bond yields in 30 minute windows around the interest rate announcements of the Bank of England. $\sum$ Shocks to 5 Year Yield is similarly defined for 5 year government bond yields, whilst FSScm4 refers to the 3M Libor 4th quarterly contract. All regressions include firm fixed effects. Standard errors, two-way clustered by month and firm, are shown in parentheses. Asterisks denote significance levels (* p<0.1, ** p<0.05, *** p<0.01).

Table 3: The Average Effect of Interest Rates on Investment

	Change in Fixed Assets (%)			
Horizon (Years)	0	1	2	3
Δ1 Year Yield	-2.22	-3.20	-4.65	-6.51
	(2.18)	(3.71)	(4.88)	(5.88)
Δ5 Year Yield	-4.98*	-8.38*	-11.22*	-12.74
	(2.81)	(4.80)	(6.49)	(8.02)
N	460238	460238	460238	460238
R^2	-0.02	-0.05	-0.07	-0.08
K-Papp F-Stat	14.24	14.24	14.24	14.24

Note: This table shows the average response of firm investment over the 0-3 year horizon to unexpected changes in 1 and 5 year interest rates. The regression sample is as described in Section 4.3 and runs over the period 1998m5-2017m12. Δ1 Year Yield is the change in the 1 year UK government bond yield over the firm's accounting period. Δ5 Year Yield is similarly defined for the 5 year government bond yield. Both series are instrumented with the high-frequency changes in 1 year government bond yields, 5 year government bond yields, and the 3M Libor 4th quarterly contract (FSScm4), in 30 minute windows around the Bank Rate announcements of the Monetary Policy Committee. The monthly shock series are cumulated over a 12-month period corresponding to firms' accounting periods. All regressions include firm fixed effects. Standard errors, two-way clustered by month and firm, are shown in parentheses. Asterisks denote significance levels (* p<0.1, ** p<0.05, *** p<0.01).

Table 4: Investment Responses By Funky Firm Split: Funky Defined Within Size Buckets

	Change in Fixed Assets (%)			
Horizon (Years)	0	1	2	3
Non-funky#Δ1 Yr Yield	-1.48	-2.29	-2.83	-3.89
	(2.17)	(3.69)	(4.63)	(5.45)
Funky#Δ1 Yr Yield	-2.95	-4.09	-6.45	-9.11
	(2.22)	(3.75)	(5.16)	(6.35)
Non-funky#Δ5 Yr Yield	-6.03**	-9.64**	-12.89**	-14.83**
	(2.80)	(4.68)	(6.06)	(7.32)
Funky#Δ5 Yr Yield	-3.93	-7.12	-9.58	-10.71
	(2.87)	(4.96)	(6.96)	(8.76)
N	460238	460238	460238	460238
R^2	-0.02	-0.05	-0.07	-0.08
K-Papp F-Stat	6.97	6.97	6.97	6.97

(a) Level Effects

	Change in Fixed Assets (%)			
Horizon (Years)	0	1	2	3
Non-funky#Δ1 Yr Yield	1.09*	1.12	2.71***	4.10***
	(0.61)	(0.79)	(0.81)	(1.20)
Funky#Δ1 Yr Yield				
Non-funky#Δ5 Yr Yield	-2.28***	-2.71***	-3.49***	-4.34***
	(0.71)	(0.86)	(1.00)	(1.41)
Funky#Δ5 Yr Yield				
N	460238	460238	460238	460238
R^2	-0.00	-0.00	-0.00	-0.00
K-Papp F-Stat	14.24	14.24	14.24	14.24

(b) Relative Effects

Note: This table shows the heterogeneous response of firm investment over the 0-3 year horizon to unexpected changes in 1 and 5 year interest rates, with results split by funky/non-funky firms. In this regression a funky firm is defined as one with above median director to employees ratio, within 10 buckets of firm size. The regression sample is as described in Section 4.3 and runs over the period 1998m5-2017m12. Δ1 Year Yield is the change in the 1 year UK government bond yield over the firm's accounting period. Δ5 Year Yield is similarly defined for the 5 year government bond yield. Both series are instrumented with the high-frequency changes in 1 year government bond yields, 5 year government bond yields, and the 3M Libor 4th quarterly contract (FSScm4), in 30 minute windows around the Bank Rate announcements of the Monetary Policy Committee. The monthly shock series are cumulated over a 12-month period corresponding to firms' accounting periods. All regressions include firm fixed effects. Sub-Table a) shows results using specification 23, whilst Sub-Table b) shows results using specification 24. Standard errors, two-way clustered by month and firm, are shown in parentheses. Asterisks denote significance levels (* p<0.1, ** p<0.05, *** p<0.01).

Table 5: Investment Responses By Funky Firm Split: Funky Defined For Shareholder Directors

	Change in Fixed Assets (%)			
Horizon (Years)	0	1	2	3
Non-funky#Δ1 Yr Yield	-1.50	-1.81	-2.39	-3.36
	(2.08)	(3.46)	(4.35)	(5.02)
Funky#Δ1 Yr Yield	-2.79	-4.30	-6.41	-8.96
	(2.31)	(3.97)	(5.39)	(6.65)
Non-funky#Δ5 Yr Yield	-5.83**	-9.47**	-12.51**	-14.26**
	(2.62)	(4.33)	(5.61)	(6.67)
Funky#Δ5 Yr Yield	-4.29	-7.53	-10.28	-11.68
	(3.03)	(5.26)	(7.30)	(9.21)
N	460238	460238	460238	460238
R^2	-0.02	-0.05	-0.07	-0.08
K-Papp F-Stat	6.85	6.85	6.85	6.85

(a) Level Effects

	Change in Fixed Assets (%)			
Horizon (Years)	0	1	2	3
Non-funky#Δ1 Yr Yield	0.78	1.45*	2.54**	3.79***
	(0.61)	(0.87)	(1.07)	(1.34)
Funky#Δ1 Yr Yield				
Non-funky#Δ5 Yr Yield	-2.42***	-3.14***	-3.62***	-4.23***
	(0.69)	(0.92)	(1.19)	(1.52)
Funky#Δ5 Yr Yield				
N	460238	460238	460238	460238
R^2	-0.00	-0.00	-0.00	-0.00
K-Papp F-Stat	14.38	14.38	14.38	14.38

(b) Relative Effects

Note: This table shows the heterogeneous response of firm investment over the 0-3 year horizon to unexpected changes in 1 and 5 year interest rates, with results split by funky/non-funky firms. In this regression a funky firm is defined as one with above median shareholder-director to employees ratio, within 10 buckets of firm size. The regression sample is as described in Section 4.3 and runs over the period 1998m5-2017m12. Δ1 Year Yield is the change in the 1 year UK government bond yield over the firm's accounting period. Δ5 Year Yield is similarly defined for the 5 year government bond yield. Both series are instrumented with the high-frequency changes in 1 year government bond yields, 5 year government bond yields, and the 3M Libor 4th quarterly contract (FSScm4), in 30 minute windows around the Bank Rate announcements of the Monetary Policy Committee. The monthly shock series are cumulated over a 12-month period corresponding to firms' accounting periods. All regressions include firm fixed effects. Sub-Table a) shows results using specification 23, whilst Sub-Table b) shows results using specification 24. Standard errors, two-way clustered by month and firm, are shown in parentheses. Asterisks denote significance levels (* p<0.1, ** p<0.05, *** p<0.01).

Table 6: Investment Responses By Funky Firm Split: Funky Definition Including Wage Measure

	Change in Fixed Assets (%)			
Horizon (Years)	0	1	2	3
Non-funky#Δ1 Yr Yield	-1.72	-2.58	-3.09	-3.98
	(2.29)	(3.90)	(4.89)	(5.72)
Funky#Δ1 Yr Yield	-2.38	-3.25	-5.05	-7.40
	(2.15)	(3.62)	(4.87)	(5.94)
Non-funky#Δ5 Yr Yield	-5.96**	-9.71**	-13.00**	-15.13**
	(2.92)	(4.89)	(6.32)	(7.59)
Funky#Δ5 Yr Yield	-4.56	-7.99*	-10.69	-11.93
	(2.79)	(4.76)	(6.56)	(8.17)
N	441228	441228	441228	441228
R^2	-0.03	-0.05	-0.07	-0.08
K-Papp F-Stat	7.15	7.15	7.15	7.15

(a) Level Effects

	Change in Fixed Assets (%)			
Horizon (Years)	0	1	2	3
Non-funky#Δ1 Yr Yield	0.38	0.22	1.36	2.67**
	(0.56)	(0.82)	(0.96)	(1.18)
Funky#Δ1 Yr Yield				
Non-funky#Δ5 Yr Yield	-1.53**	-1.94**	-2.60***	-3.49***
	(0.67)	(0.83)	(0.96)	(1.19)
Funky#Δ5 Yr Yield				
N	441228	441228	441228	441228
R^2	-0.00	-0.00	-0.00	-0.00
K-Papp F-Stat	14.17	14.17	14.17	14.17

(b) Relative Effects

Note: This table shows the heterogeneous response of firm investment over the 0-3 year horizon to unexpected changes in 1 and 5 year interest rates, with results split by funky/non-funky firms. In this regression a funky firm is defined as one with above median director to employees ratio within 10 firm size buckets or above median director remuneration to total remuneration ratio within 10 firm size buckets. The regression sample is as described in Section 4.3 and runs over the period 1998m5-2017m12. Δ1 Year Yield is the change in the 1 year UK government bond yield over the firm's accounting period. Δ5 Year Yield is similarly defined for the 5 year government bond yield. Both series are instrumented with the high-frequency changes in 1 year government bond yields, 5 year government bond yields, and the 3M Libor 4th quarterly contract (FSScm4), in 30 minute windows around the Bank Rate announcements of the Monetary Policy Committee. The monthly shock series are cumulated over a 12-month period corresponding to firms' accounting periods. All regressions include firm fixed effects. Sub-Table a) shows results using specification 23, whilst Sub-Table b) shows results using specification 24. Standard errors, two-way clustered by month and firm, are shown in parentheses. Asterisks denote significance levels (* p<0.1, ** p<0.05, *** p<0.01).

Table 7: The Level Effects of 5Y Interest Rate Shocks on Investment: By Size

	Change in Fixed Assets (%)			
Horizon (Years)	0	1	2	3
Size 1: Non-funky#Δ5 Yr Yield	-6.10*	-10.03**	-12.86**	-13.94**
Size 1: Funky#Δ5 Yr Yield	-5.49	-10.54*	-12.28	-12.44
Size 2: Non-funky#Δ5 Yr Yield	-8.47**	-14.75***	-18.82***	-19.18**
Size 2: Funky#Δ5 Yr Yield	-8.30**	-9.58*	-10.50	-10.31
Size 3: Non-funky#Δ5 Yr Yield	-8.63**	-12.89**	-13.99**	-16.31**
Size 3: Funky#Δ5 Yr Yield	-5.87*	-8.87*	-11.39*	-10.47
Size 4: Non-funky#Δ5 Yr Yield	-7.76*	-11.11*	-13.78*	-14.48
Size 4: Funky#Δ5 Yr Yield	-2.88	-5.53	-8.78	-10.07
Size 5: Non-funky#Δ5 Yr Yield	-5.38**	-8.09*	-13.69**	-14.52**
Size 5: Funky#Δ5 Yr Yield	-3.43	-6.88	-9.13	-9.90
Size 6: Non-funky#Δ5 Yr Yield	-6.02**	-9.35**	-12.27**	-13.70**
Size 6: Funky#Δ5 Yr Yield	-1.43	-6.10	-8.47	-10.32
Size 7: Non-funky#Δ5 Yr Yield	-4.46**	-6.59*	-10.28**	-11.14*
Size 7: Funky#Δ5 Yr Yield	-3.25	-5.38	-8.08	-8.56
Size 8: Non-funky#Δ5 Yr Yield	-3.96	-4.92	-7.43	-10.73
Size 8: Funky#Δ5 Yr Yield	-2.86	-5.47	-6.96	-10.20
Size 9: Non-funky#Δ5 Yr Yield	-3.64	-6.25	-8.56	-12.47
Size 9: Funky#Δ5 Yr Yield	-5.59	-8.63	-12.02	-12.18
Size 10: Non-funky#Δ5 Yr Yield	-5.54	-12.92**	-17.50**	-22.73**
Size 10: Funky#Δ5 Yr Yield	-2.68	-7.39	-11.40	-15.37

Note: This table shows the heterogeneous response of firm investment over the 0-3 year horizon to unexpected changes in 1 and 5 year interest rates, with results split by funky/non-funky firms and run separately for 10 firm size buckets. The response to Δ1 Year Yield is also included in the regression but not shown, for compactness. The results in this table follow specification 23. In this regression a funky firm is defined as one with above median director to employees ratio, within 10 buckets of firm size. The regression sample is as described in Section 4.3 and runs over the period 1998m5-2017m12. Δ1 Year Yield is the change in the 1 year UK government bond yield over the firm's accounting period. Δ5 Year Yield is similarly defined for the 5 year government bond yield. Both series are instrumented with the high-frequency changes in 1 year government bond yields, 5 year government bond yields, and the 3M Libor 4th quarterly contract (FSScm4), in 30 minute windows around the Bank Rate announcements of the Monetary Policy Committee. The monthly shock series are cumulated over a 12-month period corresponding to firms' accounting periods. All regressions include firm fixed effects. Standard errors, not-shown, are two-way clustered by month and firm. Asterisks denote significance levels (* p<0.1, ** p<0.05, *** p<0.01).

Table 8: The Level Effects of 5Y Interest Rate Shocks on Investment: By Industry

	Change in Fixed Assets (%)			
Horizon (Years)	0	1	2	3
Manufacturing: Non-funky#Δ5 Yr Yield	-5.73**	-9.11**	-12.04***	-14.36***
Manufacturing: Funky#Δ5 Yr Yield	-4.07*	-6.58*	-9.30*	-11.55*
Construction: Non-funky#Δ5 Yr Yield	-4.55	-9.66	-12.76	-13.80
Construction: Funky#Δ5 Yr Yield	-4.46	-7.66	-9.17	-9.23
Wholesale/Retail: Non-funky#Δ5 Yr Yield	-5.53*	-8.41	-10.80	-12.52
Wholesale/Retail: Funky#Δ5 Yr Yield	-3.43	-6.34	-7.80	-7.45
Hotel & Restaurant: Non-funky#Δ5 Yr Yield	-4.21	-7.73**	-12.17**	-16.00**
Hotel & Restaurant: Funky#Δ5 Yr Yield	-3.69	-4.46	-6.43	-8.27
Transport and Comm.: Non-funky#Δ5 Yr Yield	-5.75**	-8.34*	-11.18**	-11.85*
Transport and Comm.: Funky#Δ5 Yr Yield	-2.38	-7.07	-8.09	-10.97
Business: Non-funky#Δ5 Yr Yield	-9.06**	-13.56**	-18.24***	-20.39**
Business: Funky#Δ5 Yr Yield	-5.40	-9.67	-14.17	-15.81

Note: This table shows the heterogeneous response of firm investment over the 0-3 year horizon to unexpected changes in 1 and 5 year interest rates, with results split by funky/non-funky firms and run separately for the different industry groups. The response to Δ1 Year Yield is also included in the regression but not shown, for compactness. The results in this table follow specification 23. In this regression a funky firm is defined as one with above median director to employees ratio, within 10 buckets of firm size. The regression sample is as described in Section 4.3 and runs over the period 1998m5-2017m12. Δ1 Year Yield is the change in the 1 year UK government bond yield over the firm's accounting period. Δ5 Year Yield is similarly defined for the 5 year government bond yield. Both series are instrumented with the high-frequency changes in 1 year government bond yields, 5 year government bond yields, and the 3M Libor 4th quarterly contract (FSScm4), in 30 minute windows around the Bank Rate announcements of the Monetary Policy Committee. The monthly shock series are cumulated over a 12-month period corresponding to firms' accounting periods. All regressions include firm fixed effects. Standard errors, not shown, are two-way clustered by month and firm. Asterisks denote significance levels (* p<0.1, ** p<0.05, *** p<0.01).

Table 9: Total Liabilities Response By Funky Firm Split: Funky Defined Within Size Buckets

	Change in Total Liabilities (%)			
Horizon (Years)	0	1	2	3
Non-funky#Δ1 Yr Yield	-1.01	-1.56	-4.26	-6.42
	(1.47)	(2.64)	(3.66)	(4.31)
Funky#Δ1 Yr Yield	-1.96	-2.22	-5.13	-7.68
	(1.53)	(2.57)	(3.71)	(4.69)
Non-funky#Δ5 Yr Yield	-2.54	-5.28	-7.29	-6.60
	(1.96)	(3.43)	(5.05)	(6.00)
Funky#Δ5 Yr Yield	-1.40	-4.31	-5.67	-5.15
	(2.14)	(3.50)	(5.32)	(6.86)
N	458324	458324	458324	458324
R^2	-0.01	-0.02	-0.04	-0.06
K-Papp F-Stat	6.97	6.97	6.97	6.97

(a) Level Effects

	Change in Total Liabilities (%)			
Horizon (Years)	0	1	2	3
Non-funky#Δ1 Yr Yield	0.65	0.15	0.13	0.35
	(0.53)	(0.66)	(0.81)	(0.91)
Funky#Δ1 Yr Yield				
Non-funky#Δ5 Yr Yield	-1.25*	-1.14	-1.84*	-1.65
	(0.74)	(0.96)	(1.04)	(1.22)
Funky#Δ5 Yr Yield				
N	458324	458324	458324	458324
R^2	-0.00	-0.00	-0.00	-0.00
K-Papp F-Stat	14.28	14.28	14.28	14.28

(b) Relative Effects

Note: This table shows the heterogeneous response of total firm liabilities over the 0-3 year horizon to unexpected changes in 1 and 5 year interest rates, with results split by funky/non-funky firms. In this regression a funky firm is defined as one with above median director to employees ratio, within 10 buckets of firm size. The regression sample is as described in Section 4.3 and runs over the period 1998m5-2017m12. Δ1 Year Yield is the change in the 1 year UK government bond yield over the firm's accounting period. Δ5 Year Yield is similarly defined for the 5 year government bond yield. Both series are instrumented with the high-frequency changes in 1 year government bond yields, 5 year government bond yields, and the 3M Libor 4th quarterly contract (FSScm4), in 30 minute windows around the Bank Rate announcements of the Monetary Policy Committee. The monthly shock series are cumulated over a 12-month period corresponding to firms' accounting periods. All regressions include firm fixed effects. Sub-Table a) shows results using specification 23, whilst Sub-Table b) shows results using specification 24. Standard errors, two-way clustered by month and firm, are shown in parentheses. Asterisks denote significance levels (* p<0.1, ** p<0.05, *** p<0.01).

References

Arrow, K. J. (1962): "The Economic Implications of Learning by Doing," *The Review of Economic Studies*, 29(3), 155–173.

Asriyan, V., L. Laeven, A. Martin, A. Van der Ghote, and V. Vanasco (2025): "Falling interest rates and credit reallocation: lessons from general equilibrium," *Review of Economic Studies*, 92(4), 2197–2227.

Bahaj, S., A. Foulis, and G. Pinter (2020): "Home Values and Firm Behaviour," *American Economic Review*, 110(7).

Bahaj, S., A. Foulis, G. Pinter, and P. Surico (2022): "Employment and the residential collateral channel of monetary policy," *Journal of Monetary Economics*, 131, 26–44.

Barth, M. J., and V. A. Ramey (2002): "The Cost Channel of Monetary Transmission," .

Beraja, M., A. Fuster, E. Hurst, and J. Vavra (2019): "Regional Heterogeneity and the Refinancing Channel of Monetary Policy," *The Quarterly Journal of Economics*, 134(1), 109–183.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999): "The Financial Accelerator in a Quantitative Business Cycle Framework," *Handbook of Macroeconomics*, 1, 1341–1393.

Braun, R., S. Miranda-Agrippino, and T. Saha (2023): "Measuring Monetary Policy in the UK: the UK Monetary Policy Event-Study Database," *Bank of England Staff Working Papers*, 1050.

Carlino, G., and R. Defina (1998): "The Differential Regional Effects Of Monetary Policy," *The Review of Economics and Statistics*, 80(4), 572–587.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113(1), 1–45.

Cloyne, J., C. Ferreira, M. Froemel, and P. Surico (2023): "Monetary Policy, Corporate Finance, and Investment," *Journal of the European Economic Association*, 21(6), 2586–2634.

Crouzet, N., and N. R. Mehrotra (2020): "Small and Large Firms over the Business Cycle," *American Economic Review*, 110(11), 3549–3601.

Davis, S. J., J. C. Haltiwanger, and S. Schuh (1998): "Job creation and destruction," *MIT Press Books*, 1.

Dedola, L., and F. Lippi (2005): "The monetary transmission mechanism: Evidence from the industries of five OECD countries," *European Economic Review*, 49(6), 1543–1569.

Durante, E., A. Ferrando, and P. Vermeulen (2022): "Monetary policy, investment and firm heterogeneity," *European Economic Review*, 148, 104251.

English, W. B., S. J. Van den Heuvel, and E. Zakrajsek (2018): "Interest rate risk and bank equity valuations," *Journal of Monetary Economics*, 98(C), 80–97.

Gertler, M., and S. Gilchrist (1994): "Monetary Policy, Business Cycles, and the Behavior of Small Manufacturing Firms," *The Quarterly Journal of Economics*, 109(2), 309–340.

Gürkaynak, R., H. G. Karasoy Can, and S. S. Lee (2022): "Stock Market's Assessment of Monetary Policy Transmission: The Cash Flow Effect," *Journal of Finance*, 77(4), 2375–2421.

Gürkaynak, R., B. Sack, and E. Swanson (2005): "Do Actions Speak Louder Than Words? The Response of Asset Prices to Monetary Policy Actions and Statements," *International Journal of Central Banking*, 1(1).

Hart, O., and J. Moore (1989): "Default And Renegotiation: A Dynamic Model Of Debt," Working papers 520, Massachusetts Institute of Technology (MIT), Department of Economics.

——— (1994): "A Theory of Debt Based on the Inalienability of Human Capital," *The Quarterly Journal of Economics*, 109(4), 841–879.

——— (1998): "Default and Renegotiation: A Dynamic Model of Debt," *The Quarterly Journal of Economics*, 113(1), 1–41.

Ippolito, F., A. K. Ozdagli, and A. Perez-Orive (2018): "The transmission of monetary policy through bank lending: The floating rate channel," *Journal of Monetary Economics*, 95, 49–71.

Jeenas, P. (2024): "Firm Balance Sheet Liquidity, Monetary Policy Shocks, and Investment Dynamics," Working Papers 1409, Barcelona School of Economics.

Jeenas, P., and R. Lagos (2024): "Q-Monetary Transmission," *Journal of Political Economy*, 132(3), 971 – 1012.

Jorda, O. (2005): "Estimation and Inference of Impulse Responses by Local Projections," *American Economic Review*, 95(1), 161–182.

Jungherr, J., M. Meier, T. Reinelt, and I. Schott (2024): "Corporate Debt Maturity Matters for Monetary Policy," International Finance Discussion Papers 1402, Board of Governors of the Federal Reserve System (U.S.).

Kiyotaki, N., J. Moore, and S. Zhang (2021): "Credit Horizons," now considerably updated and titled "Funding Horizons, Interest Rates, and Growth" Working Paper 28742, National Bureau of Economic Research.

Kleibergen, F., and R. Paap (2006): "Generalized reduced rank tests using the singular value decomposition," *Journal of Econometrics*, 133(1), 97–126.

Macaulay, F. R. (1938): *Some Theoretical Problems Suggested by the Movements of Interest Rates, Bond Yields and Stock Prices in the United States since 1856*, vol. None of *NBER Books*. National Bureau of Economic Research, Inc, none edn.

Otonello, P., and T. Winberry (2020): "Financial Heterogeneity and the Investment Channel of Monetary Policy," *Econometrica*, 88(6), 2473–2502.

Peersman, G., and F. Smets (2005): "The Industry Effects of Monetary Policy in the Euro Area," *Economic Journal*, 115(503), 319–342.

Swanson, E. (2021): "Measuring the effects of federal reserve forward guidance and asset purchases on financial markets," *Journal of Monetary Economics*, 118(C), 32–53.

Törnqvist, L., P. Vartia, and Y. Vartia (1985): "How Should Relative Change Be Measured?," *The American Statistician*, 39(1), 43–46.

A Appendix Tables

Table A1: IV First Stage: Gilt Yields and Monetary Policy Shocks: Time Series

	Δ1 Year Yield	Δ5 Year Yield
	(1)	(2)
$\sum$ Shocks to 1 Year	2.23**	1.78**
	(1.07)	(0.78)
$\sum$ Shocks to 5 Year	-0.07	2.28***
	(1.16)	(0.85)
$\sum$ Shocks to FSScm4	0.33	-2.12***
	(0.99)	(0.72)
N	236	236
R^2	0.09	0.12
F-Statistic	8.56	11.20

Note: This table shows the "first stage" regression linking the annual change in UK government bond yields with the cumulative sum of monetary policy shocks over the same period. This regression is run at the monthly-level (not the firm-level) over the period 1998m5-2017m12. $\sum$ Shocks to 1 Year Yield is the cumulative sum of shocks over the past 12 months that occurred to 1 year government bond yields in 30 minute windows around the interest rate announcements of the Bank of England. $\sum$ Shocks to 5 Year Yield is similarly defined for 5 year government bond yields, whilst FSScm4 refers to the 3M Libor 4th quarterly contract. Standard errors are shown in parentheses. Asterisks denote significance levels (* p<0.1, ** p<0.05, *** p<0.01)

Table A2: Investment Responses By Funky Firm Split: Funky Defined Within Size x Industry

	Change in Fixed Assets (%)			
Horizon (Years)	0	1	2	3
Non-funky#Δ1 Yr Yield	-1.48	-2.27	-2.70	-3.78
	(2.21)	(3.72)	(4.63)	(5.47)
Funky#Δ1 Yr Yield	-2.88	-4.02	-6.46	-9.09
	(2.19)	(3.74)	(5.17)	(6.32)
Non-funky#Δ5 Yr Yield	-6.13**	-9.78**	-12.97**	-14.92**
	(2.83)	(4.72)	(6.09)	(7.37)
Funky#Δ5 Yr Yield	-3.95	-7.18	-9.70	-10.83
	(2.84)	(4.93)	(6.94)	(8.71)
N	458816	458816	458816	458816
R^2	-0.02	-0.05	-0.07	-0.08
K-Papp F-Stat	6.99	6.99	6.99	6.99

(a) Level Effects

	Change in Fixed Assets (%)			
Horizon (Years)	0	1	2	3
Non-funky#Δ1 Yr Yield	0.99	0.99	2.73***	4.05***
	(0.61)	(0.75)	(0.84)	(1.18)
Funky#Δ1 Yr Yield				
Non-funky#Δ5 Yr Yield	-2.33***	-2.70***	-3.34***	-4.15***
	(0.66)	(0.83)	(1.02)	(1.41)
Funky#Δ5 Yr Yield				
N	458816	458816	458816	458816
R^2	-0.00	-0.00	-0.00	-0.00
K-Papp F-Stat	14.25	14.25	14.25	14.25

(b) Relative Effects

Note: This table shows the heterogeneous response of firm investment over the 0-3 year horizon to unexpected changes in 1 and 5 year interest rates, with results split by funky/non-funky firms. In this regression a funky firm is defined as one with above median director to employees ratio within 10 firm size buckets within each UK SIC 1 industry. The regression sample is as described in Section 4.3 and runs over the period 1998m5-2017m12. Δ1 Year Yield is the change in the 1 year UK government bond yield over the firm's accounting period. Δ5 Year Yield is similarly defined for the 5 year government bond yield. Both series are instrumented with the high-frequency changes in 1 year government bond yields, 5 year government bond yields, and the 3M Libor 4th quarterly contract (FSScm4), in 30 minute windows around the Bank Rate announcements of the Monetary Policy Committee. The monthly shock series are cumulated over a 12-month period corresponding to firms' accounting periods. All regressions include firm fixed effects. Sub-Table a) shows results using specification 23, whilst Sub-Table b) shows results using specification 24. Standard errors, two-way clustered by month and firm, are shown in parentheses. Asterisks denote significance levels (* p<0.1, ** p<0.05, *** p<0.01).

Table A3: Investment Responses By Funky Firm Split: Funky Defined Within Size x Age

	Change in Fixed Assets (%)			
Horizon (Years)	0	1	2	3
Non-funky#Δ1 Yr Yield	-1.53	-2.45	-3.03	-4.06
	(2.18)	(3.72)	(4.65)	(5.47)
Funky#Δ1 Yr Yield	-2.90	-3.95	-6.27	-8.97
	(2.21)	(3.73)	(5.15)	(6.33)
Non-funky#Δ5 Yr Yield	-6.00**	-9.51**	-12.59**	-14.60**
	(2.79)	(4.70)	(6.07)	(7.33)
Funky#Δ5 Yr Yield	-3.95	-7.23	-9.86	-10.92
	(2.87)	(4.93)	(6.95)	(8.76)
N	460238	460238	460238	460238
R^2	-0.02	-0.05	-0.07	-0.08
K-Papp F-Stat	6.99	6.99	6.99	6.99

(a) Level Effects

	Change in Fixed Assets (%)			
Horizon (Years)	0	1	2	3
Non-funky#Δ1 Yr Yield	1.02*	0.86	2.37***	3.86***
	(0.61)	(0.81)	(0.82)	(1.22)
Funky#Δ1 Yr Yield				
Non-funky#Δ5 Yr Yield	-2.22***	-2.45***	-2.89***	-3.88***
	(0.68)	(0.88)	(1.00)	(1.44)
Funky#Δ5 Yr Yield				
N	460238	460238	460238	460238
R^2	-0.00	-0.00	-0.00	-0.00
K-Papp F-Stat	14.24	14.24	14.24	14.24

(b) Relative Effects

Note: This table shows the heterogeneous response of firm investment over the 0-3 year horizon to unexpected changes in 1 and 5 year interest rates, with results split by funky/non-funky firms. In this regression a funky firm is defined as one with above median director to employees ratio within groups defined by 10 firm size buckets for firms with average age below 15 years and 10 firm size buckets for firms with average age above 15 years. The regression sample is as described in Section 4.3 and runs over the period 1998m5-2017m12. Δ1 Year Yield is the change in the 1 year UK government bond yield over the firm's accounting period. Δ5 Year Yield is similarly defined for the 5 year government bond yield. Both series are instrumented with the high-frequency changes in 1 year government bond yields, 5 year government bond yields, and the 3M Libor 4th quarterly contract (FSScm4), in 30 minute windows around the Bank Rate announcements of the Monetary Policy Committee. The monthly shock series are cumulated over a 12-month period corresponding to firms' accounting periods. All regressions include firm fixed effects. Sub-Table a) shows results using specification 23, whilst Sub-Table b) shows results using specification 24. Standard errors, two-way clustered by month and firm, are shown in parentheses. Asterisks denote significance levels (* p<0.1, ** p<0.05, *** p<0.01).

Table A4: Investment Responses By Funky Firm Split: Funky Defined Within Size x Region

	Change in Fixed Assets (%)			
Horizon (Years)	0	1	2	3
Non-funky#Δ1 Yr Yield	-1.43	-2.16	-2.61	-3.69
	(2.19)	(3.71)	(4.67)	(5.51)
Funky#Δ1 Yr Yield	-2.98	-4.21	-6.63	-9.27
	(2.21)	(3.74)	(5.12)	(6.29)
Non-funky#Δ5 Yr Yield	-6.10**	-9.82**	-13.22**	-15.15**
	(2.78)	(4.67)	(6.09)	(7.37)
Funky#Δ5 Yr Yield	-3.89	-6.97	-9.28	-10.41
	(2.89)	(4.97)	(6.93)	(8.71)
N	460160	460160	460160	460160
R^2	-0.02	-0.05	-0.07	-0.08
K-Papp F-Stat	6.98	6.98	6.98	6.98

(a) Level Effects

	Change in Fixed Assets (%)			
Horizon (Years)	0	1	2	3
Non-funky#Δ1 Yr Yield	1.15*	1.37	3.11***	4.46***
	(0.66)	(0.83)	(0.92)	(1.28)
Funky#Δ1 Yr Yield				
Non-funky#Δ5 Yr Yield	-2.41***	-3.07***	-4.17***	-5.03***
	(0.73)	(0.92)	(1.08)	(1.47)
Funky#Δ5 Yr Yield				
N	460160	460160	460160	460160
R^2	-0.00	-0.00	-0.00	-0.00
K-Papp F-Stat	14.25	14.25	14.25	14.25

(b) Relative Effects

Note: This table shows the heterogeneous response of firm investment over the 0-3 year horizon to unexpected changes in 1 and 5 year interest rates, with results split by funky/non-funky firms. In this regression a funky firm is defined as one with above median director to employees ratio within 10 firm size buckets within each UK Nuts 1 region. The regression sample is as described in Section 4.3 and runs over the period 1998m5-2017m12. Δ1 Year Yield is the change in the 1 year UK government bond yield over the firm's accounting period. Δ5 Year Yield is similarly defined for the 5 year government bond yield. Both series are instrumented with the high-frequency changes in 1 year government bond yields, 5 year government bond yields, and the 3M Libor 4th quarterly contract (FSScm4), in 30 minute windows around the Bank Rate announcements of the Monetary Policy Committee. The monthly shock series are cumulated over a 12-month period corresponding to firms' accounting periods. All regressions include firm fixed effects. Sub-Table a) shows results using specification 23, whilst Sub-Table b) shows results using specification 24. Standard errors, two-way clustered by month and firm, are shown in parentheses. Asterisks denote significance levels (* p<0.1, ** p<0.05, *** p<0.01).

Table A5: The Relative Effects of 5Y Interest Rate Shocks on Investment: By Size

	Change in Fixed Assets (%)			
Horizon (Years)	0	1	2	3
Size 1: Non-funky#Δ5 Yr Yield	-0.71	0.75	-0.49	-1.71
Size 1: Funky#Δ5 Yr Yield				
Size 2: Non-funky#Δ5 Yr Yield	-0.14	-5.05**	-7.89***	-8.69***
Size 2: Funky#Δ5 Yr Yield				
Size 3: Non-funky#Δ5 Yr Yield	-3.39	-4.38	-2.75	-5.78*
Size 3: Funky#Δ5 Yr Yield				
Size 4: Non-funky#Δ5 Yr Yield	-5.74***	-6.71***	-6.33***	-6.16***
Size 4: Funky#Δ5 Yr Yield				
Size 5: Non-funky#Δ5 Yr Yield	-1.97	-1.01	-4.42*	-4.30*
Size 5: Funky#Δ5 Yr Yield				
Size 6: Non-funky#Δ5 Yr Yield	-4.62**	-3.31*	-3.81**	-3.31
Size 6: Funky#Δ5 Yr Yield				
Size 7: Non-funky#Δ5 Yr Yield	-0.99	-0.91	-1.83	-2.05
Size 7: Funky#Δ5 Yr Yield				
Size 8: Non-funky#Δ5 Yr Yield	-1.32	0.30	-0.86	-1.24
Size 8: Funky#Δ5 Yr Yield				
Size 9: Non-funky#Δ5 Yr Yield	1.73	2.10	3.12*	-0.74
Size 9: Funky#Δ5 Yr Yield				
Size 10: Non-funky#Δ5 Yr Yield	-3.06**	-5.63***	-5.89***	-7.53***
Size 10: Funky#Δ5 Yr Yield				

Note: This table shows the heterogeneous response of firm investment over the 0-3 year horizon to unexpected changes in 1 and 5 year interest rates, with results split by funky/non-funky firms and run separately for 10 firm size buckets. The response to Δ1 Year Yield is also included in the regression but not shown, for compactness. The results in this table follow specification 24. In this regression a funky firm is defined as one with above median director to employees ratio, within 10 buckets of firm size. The regression sample is as described in Section 4.3 and runs over the period 1998m5-2017m12. Δ1 Year Yield is the change in the 1 year UK government bond yield over the firm's accounting period. Δ5 Year Yield is similarly defined for the 5 year government bond yield. Both series are instrumented with the high-frequency changes in 1 year government bond yields, 5 year government bond yields, and the 3M Libor 4th quarterly contract (FSScm4), in 30 minute windows around the Bank Rate announcements of the Monetary Policy Committee. The monthly shock series are cumulated over a 12-month period corresponding to firms' accounting periods. All regressions include firm fixed effects. Standard errors, not shown, are two-way clustered by month and firm. Asterisks denote significance levels (* $p<0.1$, ** $p<0.05$, *** $p<0.01$).

Table A6: The Relative Effects of 5Y Interest Rate Shocks on Investment: By Industry

	Change in Fixed Assets (%)			
Horizon (Years)	0	1	2	3
Manufacturing: Non-funky#Δ5 Yr Yield	-1.87*	-2.80*	-2.99	-3.11
Manufacturing: Funky#Δ5 Yr Yield				
Construction: Non-funky#Δ5 Yr Yield	-0.31	-2.23	-3.76*	-4.88**
Construction: Funky#Δ5 Yr Yield				
Wholesale/Retail: Non-funky#Δ5 Yr Yield	-2.30**	-2.18*	-3.07**	-4.99***
Wholesale/Retail: Funky#Δ5 Yr Yield				
Hotel & Restaurant: Non-funky#Δ5 Yr Yield	0.15	-2.24	-4.13	-6.01
Hotel & Restaurant: Funky#Δ5 Yr Yield				
Transport and Comm.: Non-funky#Δ5 Yr Yield	-3.50*	-1.92	-3.90	-1.93
Transport and Comm.: Funky#Δ5 Yr Yield				
Business: Non-funky#Δ5 Yr Yield	-3.53**	-3.51*	-3.52	-3.91
Business: Funky#Δ5 Yr Yield				

Note: This table shows the heterogeneous response of firm investment over the 0-3 year horizon to unexpected changes in 1 and 5 year interest rates, with results split by funky/non-funky firms and run separately for the different industry groups. The response to Δ1 Year Yield is also included in the regression but not shown, for compactness. The results in this table follow specification 24. In this regression a funky firm is defined as one with above median director to employees ratio, within 10 buckets of firm size. The regression sample is as described in Section 4.3 and runs over the period 1998m5-2017m12. Δ1 Year Yield is the change in the 1 year UK government bond yield over the firm's accounting period. Δ5 Year Yield is similarly defined for the 5 year government bond yield. Both series are instrumented with the high-frequency changes in 1 year government bond yields, 5 year government bond yields, and the 3M Libor 4th quarterly contract (FSScm4), in 30 minute windows around the Bank Rate announcements of the Monetary Policy Committee. The monthly shock series are cumulated over a 12-month period corresponding to firms' accounting periods. All regressions include firm fixed effects. Standard errors, not shown, are two-way clustered by month and firm. Asterisks denote significance levels (* $p<0.1$, ** $p<0.05$, *** $p<0.01$).

B Theoretical Appendix

B.1 Proof for Proposition 1

Proof. We prove the proposition using guess-and-verify approach. Conjecture that $V_t(z) = c_t + \nu_t z$ is affine. From the wage equation (8),

$$w_t(z)n = \frac{\lambda}{R_t}\left[V_{t+1}(z') - V_{t+1}(\hat{z})\right]n = \frac{\lambda\nu_{t+1}}{R_t}(z' - \hat{z})n = \frac{\lambda\nu_{t+1}(1-\theta)}{R_t}(\rho z + 1 - \rho)$$

Substituting into the Bellman equation (7) (assuming continuation is optimal), we obtain

$$\begin{aligned} V_t(z) &= az - nw_t(z) + \frac{\lambda}{R_t}V_{t+1}(z') \\ &= az - \frac{\lambda\nu_{t+1}(1-\theta)}{R_t}(\rho z + 1 - \rho) + \frac{\lambda}{R_t}[c_{t+1} + \nu_{t+1}(\rho z + 1 - \rho)] \\ &= az + \frac{\lambda\nu_{t+1}\theta}{R_t}(\rho z + 1 - \rho) + \frac{\lambda c_{t+1}}{R_t}. \end{aligned}$$

Matching coefficients, we get, for the constant term, $c_t = \frac{\lambda c_{t+1}}{R_t} + \frac{\lambda\nu_{t+1}\theta(1-\rho)}{R_t}$, and, for the coefficient on z, $\nu_t = a + \frac{\lambda\nu_{t+1}\theta\rho}{R_t}$. These are the recursions stated in the proposition.

Uniqueness of Affine Solution. Suppose two affine solutions exist. For the coefficient on z, both must satisfy $\nu_t = a + \frac{\lambda\theta\rho}{R_t}\nu_{t+1}$. In steady state with $R_t = R$, this gives $\nu = a + \frac{\lambda\theta\rho}{R}\nu$, or

$$\nu = \frac{a}{1 - \frac{\lambda\theta\rho}{R}} = \frac{aR}{R - \lambda\theta\rho}.$$

Since $R_t > 1$ for all t and $0 < \lambda < 1$, we have $\lambda\theta\rho < 1 < R_t$, so the denominator $R_t - \lambda\theta\rho > 0$ and ν_t is uniquely determined by forward iteration from any terminal condition satisfying the transversality condition.

Similarly, given the unique sequence $\{\nu_s\}_{s\geq t}$, the recursion for c_t uniquely determines $\{c_s\}_{s\geq t}$ by forward iteration. Hence the affine solution is unique.

No-Liquidation Condition. We verify that continuation is preferred to liquidation even when productivity has fallen to its infimum $\underline{z}$ from (4). The owner prefers continuation iff

$$\lambda q_t < -nw_t(\underline{z}) + \frac{\lambda}{R_t}V_{t+1}(\rho\underline{z} + 1 - \rho).$$

Substituting the affine forms, we get

$$
\begin{aligned}
&- nw_t(\underline{z}) + \frac{\lambda}{R_t} V_{t+1}(\rho \underline{z} + 1 - \rho) \\
&= \frac{\lambda}{R_t}[c_{t+1} + \nu_{t+1}(\rho \underline{z} + 1 - \rho)\theta].
\end{aligned}
$$

From equation (10) and the steady state expressions for b_t and q_t near R, (**Assumption 1**) ensures that

$$
\lambda q < \frac{\lambda}{R}[c + \nu(\rho \underline{z} + 1 - \rho)\theta]
$$

holds for all $z \in [\underline{z}, 1]$, and in particular at $z = \underline{z}$. Therefore, the owner strictly prefers continuation to liquidation even at the worst case productivity, verifying the claim in the proposition. □

B.2 Derivation of equation (11)

From Proposition 1

$$
\begin{aligned}
\nu_t &= a + \frac{\lambda\theta\rho}{R_t}\nu_{t+1} \\
&= a + \frac{a\lambda\theta\rho}{R_t} + \frac{a(\lambda\theta\rho)^2}{R_t R_{t+1}} + ... + \frac{a(\lambda\theta\rho)^{\tau+1}}{R_t R_{t+1} \cdots R_{t+\tau}} + ...,
\end{aligned}
$$

$$
\begin{aligned}
c_t &= \frac{\lambda\theta(1-\rho)}{R_t}\nu_{t+1} + \frac{\lambda}{R_t}c_{t+1} \\
&= a\lambda\theta(1-\rho)\left[\frac{1}{R_t} + \frac{\lambda}{R_t R_{t+1}}(1+\theta\rho) + ... + \frac{\lambda^\tau}{R_t R_{t+1} \cdots R_{t+\tau}} \frac{1-(\theta\rho)^{\tau+1}}{1-\theta\rho} + \cdots\right].
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
b_t &= \frac{1}{R_t}V_{t+1}(1) = \frac{1}{R_t}(\nu_{t+1} + c_{t+1}) \\
&= \frac{a}{R_t} + \frac{a\lambda\theta\rho}{R_t R_{t+1}} + ... + \frac{a(\lambda\theta\rho)^\tau}{R_t R_{t+1} \cdots R_{t+\tau}} + ... \\
&+ \lambda\theta(1-\rho)\left[\frac{a}{R_t R_{t+1}} + \frac{\lambda}{R_t R_{t+1} R_{t+2}}(1+\theta\rho) + ... + \frac{\lambda^{\tau-1}}{R_t R_{t+1} \cdots R_{t+\tau}} \frac{1-(\theta\rho)^\tau}{1-\theta\rho} + \cdots\right] \\
&= \frac{a}{R_t} + \frac{a\lambda}{R_t R_{t+1}}[1-(1-\theta)] + \frac{a\lambda^2}{R_t R_{t+1} R_{t+2}}[1-(1-\theta)(1+\theta\rho)] + ... \\
&+ \frac{a\lambda^\tau}{R_t R_{t+1} ... R_{t+\tau}}\left[1-(1-\theta)\frac{1-(\theta\rho)^\tau}{1-\theta\rho}\right] + ...
\end{aligned}
$$

This is (11) in the text.

B.3 Proof of Claim 2.

To prove parts (a) and (b) of the claim, we think of the durations of q_t and b_t, D_{qt} and $D_{bt}(\theta)$, as the expectation of a random variable.

$$D_{b,t}(\theta) = \mathbb{E}_{\boldsymbol{\pi}_t(\theta)}[\tau] = \sum_{\tau=1}^{\infty} \tau \pi_{\tau,t}(\theta),$$
$$D_{q,t} = \mathbb{E}_{\boldsymbol{\pi}_t(1)}[\tau] = \sum_{\tau=1}^{\infty} \tau \pi_{\tau,t}(1),$$

where

$$\pi_{\tau,t}(\theta) \propto \frac{\lambda^{\tau-1}}{\prod_{s=0}^{\tau-1} R_{t+s}} \left[1 - (1-\theta)\frac{1-(\theta\rho)^{\tau-1}}{1-\theta\rho}\right]$$

and the constant of proportionality is independent of τ. We can show that as θ increases, the distribution $\pi(\theta)$ shifts to the right in the sense of first-order stochastic dominance. A sufficient condition is the monotone likelihood ratio property (MLR) in τ: for any $\theta_2 > \theta_1$, the ratio $\pi_\tau(\theta_2)/\pi_\tau(\theta_1)$ is increasing in τ. To see that, show $\frac{\pi_{\tau+1}(\theta)}{\pi_\tau(\theta)}$ is increasing in θ, which implies $\left(\frac{\pi_{\tau+1}(\theta_2)}{\pi_{\tau+1}(\theta_1)}\right)/\left(\frac{\pi_\tau(\theta_2)}{\pi_\tau(\theta_1)}\right) = \left(\frac{\pi_{\tau+1}(\theta_2)}{\pi_\tau(\theta_2)}\right)/\left(\frac{\pi_{\tau+1}(\theta_1)}{\pi_\tau(\theta_1)}\right) > 1$. Since i is increasing, $\mathbb{E}_{\boldsymbol{\pi}_t(\theta)}[\tau]$ is increasing in θ, i.e.,$D_{b,t}(\theta)$ is increasing in θ and $D_{b,t}(\theta) \leq D_{q,t} = D_{b,t}(1)$.

Part (c) of the claim is immediate.

B.4 Proof of Claim 3.

Let

$$\begin{aligned}\frac{b_t}{a} &= \sum_{i=0}^{\tau-1} \frac{\lambda^i}{\left(\prod_{j=0}^{i} R_{t+j}\right)} \left[1 - (1-\theta)\frac{1-(\theta\rho)^i}{1-\theta\rho}\right] + \sum_{i=\tau}^{\infty} \frac{\lambda^i}{\left(\prod_{j=0}^{i} R_{t+j}\right)} \left[1 - (1-\theta)\frac{1-(\theta\rho)^i}{1-\theta\rho}\right] \\ &= b_{t,\tau}^1 + b_{t,\tau}^2\end{aligned}$$

where

$$b_{t,\tau}^1 = \sum_{i=0}^{\tau-1} \frac{\lambda^i}{\left(\prod_{j=0}^{i} R_{t+j}\right)} \left[1 - (1-\theta)\frac{1-(\theta\rho)^i}{1-\theta\rho}\right]$$
$$b_{t,\tau}^2 = \sum_{i=\tau}^{\infty} \frac{\lambda^i}{\left(\prod_{j=0}^{i} R_{t+j}\right)} \left[1 - (1-\theta)\frac{1-(\theta\rho)^i}{1-\theta\rho}\right]$$

We now have

$$\frac{R_{t+\tau}}{b_t}\frac{\partial b_t}{\partial R_{t+\tau}} = -\frac{b_{t,\tau}^2}{b_t/a}$$

When the steady state where $R_t = R$ for all t, we get

$$\frac{b_{t,\tau}^{2}}{b_t/a} = \frac{\frac{\theta(1-\rho)}{R-\lambda}\left(\frac{\lambda}{R}\right)^{\tau} + \frac{1-\theta}{R-\lambda\theta\rho}\left(\frac{\lambda\theta\rho}{R\theta\rho}\right)^{\tau}}{\frac{\theta(1-\rho)}{R-\lambda} + \frac{1-\theta}{R-\lambda\theta\rho}}$$
$$= \left[1-\omega\left(\theta\right)\right]\left(\frac{\lambda}{R}\right)^{\tau} + \omega\left(\theta\right)\left(\frac{\lambda\theta\rho}{R}\right)^{\tau}$$

where

$$\omega\left(\theta\right) = \frac{\frac{1-\theta}{R-\lambda\theta\rho}}{\frac{\theta(1-\rho)}{R-\lambda} + \frac{1-\theta}{R-\lambda\theta\rho}} = \frac{(1-\theta)(R-\lambda)}{(1-\theta\rho)\left[R-\lambda(1-\theta+\theta\rho)\right]}$$

In the neighborhood of the steady state, we get

$$\frac{R_{t+\tau}}{b_t}\frac{\partial b_t}{\partial R_{t+\tau}} = -(1-\omega(\theta))\left(\frac{\lambda}{R}\right)^{\tau} - \omega(\theta)\left(\frac{\lambda\theta\rho}{R}\right)^{\tau}$$
$$= -\left\{1-\omega(\theta)\left[1-(\theta\rho)^{\tau}\right]\right\}\left(\frac{\lambda}{R}\right)^{\tau}$$

Given that b_t is increasing in θ and $\omega(\theta)$ is decreasing in θ, $\frac{\partial b_t}{\partial R_{t+\tau}} < 0$ is decreasing in θ. So b_t falls less when θ is lower. We thus proved part (a) of the claim.

When $\tau = 0$, an increase in R_t lowers $(b_t - q_t)$. The rest of part (b) is immediate.

B.5 Proof of Claim 4

In the steady state, the rate of return for new engineer group to invest is equalized across types. Thus, the RHS of

$$R^E - \lambda = \frac{1}{q+x-a\left(\theta\right)\widetilde{b}\left(\theta\right)}\frac{\lambda(1-\theta)a\left(\theta\right)}{R-\lambda\theta\rho}$$

is independent of θ in the long run. Thus

$$0 = \frac{\partial}{\partial\theta}\ln RHS = -\frac{1}{1-\theta} + \frac{\lambda\rho}{R-\lambda\theta\rho} + \frac{a'\left(\theta\right)}{a\left(\theta\right)} + \frac{b}{q+x-b}\left[\frac{a'\left(\theta\right)}{a\left(\theta\right)} + \frac{\widetilde{b}'\left(\theta\right)}{\widetilde{b}\left(\theta\right)}\right], \text{ or}$$

$$\frac{a'\left(\theta\right)}{a\left(\theta\right)} = \frac{R-\lambda\rho}{\left(1-\theta\right)\left(R-\lambda\theta\rho\right)}\left[1-\frac{a}{(R-\lambda)(q+x)}\right],$$

as in the claim. Because the inside the square bracket in RHS is negative due to the profitability of investment $a > (R-\lambda)(q+x)$, we learn $\frac{a'(\theta)}{a(\theta)} < 0$.

Alternatively, because $a(\theta) = b(\theta)/\widetilde{b}(\theta)$, we have

$$\begin{aligned} R^E - \lambda &= \frac{b(\theta)}{q + x - b(\theta)} \frac{\lambda(1-\theta)}{(R - \lambda\theta\rho)\widetilde{b}(\theta)} \\ &= \frac{b(\theta)}{q + x - b(\theta)} \frac{\lambda(1-\theta)(R-\lambda)}{R - \lambda + \lambda\theta(1-\rho)}. \end{aligned}$$

Because the RHS of this equation is independent of θ in the long run, we have

$$0 = \frac{d}{d\theta} \ln RHS = \left(\frac{1}{b} + \frac{1}{q+x-b}\right)\frac{\partial b}{\partial \theta} - \frac{1}{1-\theta} - \frac{\lambda(1-\rho)}{R-\lambda+\lambda\theta(1-\rho)}, \text{ or}$$

$$\frac{b'(\theta)}{b(\theta)} = \frac{q+x-b}{q+x} \frac{R - \lambda\rho}{(1-\theta)\left[R - \lambda + \lambda\theta(1-\rho)\right]},$$

as in the claim.

B.6 Proof of Proposition 5.

Proof. We show that the cross-partial $\frac{\partial^2 L_t(\theta)}{\partial\theta\partial r_t}$ is negative for sufficiently high θ.[24]

From Proposition 1 and Assumption 4,

$$V_t(1;\theta) = a(\theta)\left\{1 + \sum_{\tau=1}^{\infty}\left(\frac{\lambda}{R_{\tau,t}}\right)^{\tau}\left[\frac{\theta(1-\rho)}{1-\rho\theta} + \frac{1-\theta}{1-\rho\theta}(\rho\theta)^{\tau}\right]\right\},$$

and hence

$$b_t(\theta) = \frac{1}{R_{1,t}} V_{t+1}(1;\theta) = a(\theta)\widetilde{b}_t(\theta), \qquad \widetilde{b}_t(\theta) = \sum_{\tau=1}^{\infty} \frac{\lambda^{\tau-1}}{(R_{\tau,t})^{\tau}}\left[\frac{\theta(1-\rho)}{1-\rho\theta} + \frac{1-\theta}{1-\rho\theta}(\rho\theta)^{\tau-1}\right].$$

Using (21) and expanding around $(s_t, r_t) = (0, 0)$, term-by-term differentiation yields

$$\frac{\partial\widetilde{b}_t(\theta)}{\partial r_t} = -\frac{\lambda\theta}{2(1-\rho\theta)}X_r, \quad X_r = \frac{1-\rho}{(R-\lambda)^3} + \frac{\rho(1-\theta)}{(R-\lambda\rho\theta)^3}, \tag{25}$$

$$\frac{\partial\widetilde{b}_t(\theta)}{\partial s_t} = -X_s, \qquad X_s = \frac{1}{2(1-\rho\theta)}\left[\frac{\theta(1-\rho)(2R-3\lambda)}{(R-\lambda)^3} + \frac{(1-\theta)(2R-3\lambda\rho\theta)}{(R-\lambda\rho\theta)^3}\right], \tag{26}$$

where we used $\tau(5-\tau) = 4\tau - \tau(\tau-1)$ and the standard series identities

$$\sum_{\tau=1}^{\infty} \tau x^{\tau-1} = \frac{1}{(1-x)^2}, \qquad \sum_{\tau=2}^{\infty} \tau(\tau-1)x^{\tau-2} = \frac{2}{(1-x)^3}, \qquad |x| < 1.$$

[24] Throughout, denominators are well-defined under the maintained restrictions in the main text, in particular $q + x - b(\theta) > 0$, $R > \lambda$, and $R > \lambda\rho\theta$.

Analogously,
$$\frac{\partial q_t}{\partial r_t} = -f \cdot \frac{\lambda}{2(R-\lambda)^3}, \qquad \frac{\partial q_t}{\partial s_t} = -f \cdot \frac{2R-3\lambda}{2(R-\lambda)^3}.$$

Differentiating (25) w.r.t. θ gives

$$\frac{\partial^2 \widetilde{b}_t(\theta)}{\partial\theta\partial r_t} = -\frac{\lambda}{2}\left[\frac{X_r}{(1-\rho\theta)^2} + \frac{\rho\theta}{1-\rho\theta}\frac{\lambda\rho(3-2\theta)-R}{(R-\lambda\rho\theta)^4}\right].$$

Using (16), we can simplify the cross partial derivative as

$$\begin{aligned}\frac{\partial^2 b_t(\theta)}{\partial\theta\partial r_t} &= a(\theta)\left[\frac{\partial^2 \widetilde{b}_t(\theta)}{\partial\theta\partial r_t} + \frac{a'(\theta)}{a(\theta)}\frac{\partial \widetilde{b}_t(\theta)}{\partial r_t}\right] \\ &= -\frac{\lambda a(\theta)}{2}\left\{\frac{X_r}{(1-\rho\theta)^2} + \frac{\rho\theta}{1-\rho\theta}\frac{\lambda\rho(3-2\theta)-R}{(R-\lambda\rho\theta)^4}\right. \\ &\qquad \left. + \frac{\theta}{1-\rho\theta}X_r\frac{R-\lambda\rho}{(1-\theta)(R-\lambda\theta\rho)}\left[1-\frac{a}{(R-\lambda)(q+x)}\right]\right\}.\end{aligned}$$

Collecting terms in the first-order expansion of $\frac{\partial^2 L_t(\theta)}{\partial\theta\partial r_t}$ and factoring out a strictly positive scalar (given the maintained restrictions in the footnote), we can write

$$\frac{\partial^2 L_t(\theta)}{\partial\theta\partial r_t} \propto \frac{f}{f+(R-\lambda)x} - C(\theta), \text{ where}$$

$$C(\theta) = \frac{1-\theta}{1-\rho\theta}\left\{\frac{1-\rho}{R-\lambda\rho}\left[\frac{R-\lambda\rho\theta}{1-\rho\theta} + \frac{\theta(R-\lambda\rho)}{1-\theta}\right] + \frac{\rho(1-\theta)(R-\lambda)^3}{(R-\lambda\rho\theta)^3(R-\lambda\rho)}\left[\lambda\rho\theta + \frac{R-\lambda\rho^2\theta^2}{1-\rho\theta}\right]\right\}.$$

A direct differentiation shows $C'(\theta) > 0$ on $\theta \in [0,1]$ and $C(1) = 1$. Hence, if $C(0) \leq \frac{f}{f+(R-\lambda)x}$, there exists a unique $\underline{\theta}^r \in (0,1]$ solving

$$C(\underline{\theta}^r) = \frac{f}{f+(R-\lambda)x}.$$

Otherwise, set $\underline{\theta}^r = 0$. For all $\theta \geq \underline{\theta}^r$ we have $\frac{f}{f+(R-\lambda)x} - C(\theta) \leq 0$ (strictly < 0 for $\theta > \underline{\theta}^r$), implying $\frac{\partial^2 L_t(\theta)}{\partial\theta\partial r_t} < 0$ and establishing Proposition 5. □

B.7 Proof of Proposition 6.

Proof. We show that the cross-partial $\frac{\partial^2 L_t(\theta)}{\partial\theta\partial s_t}$ is positive for sufficiently high θ.

From (26),

$$\frac{\partial^2 \widetilde{b}_t(\theta)}{\partial\theta\partial s_t} = -\frac{\rho}{1-\rho\theta}X_s - \frac{1}{2(1-\rho\theta)}\left[(1-\rho)\frac{2R-3\lambda}{(R-\lambda)^3} + \frac{-2R-3\lambda\rho+6\lambda\rho\theta}{(R-\lambda\rho\theta)^3} + \frac{3\lambda\rho(1-\theta)(2R-3\lambda\rho\theta)}{(R-\lambda\rho\theta)^4}\right].$$

Using (16) and $\frac{\partial \widetilde{b}_t(\theta)}{\partial s_t} = -X_s$, we have

$$\begin{aligned}\frac{\partial^2 b_t(\theta)}{\partial\theta\partial s_t} &= a(\theta)\left[\frac{\partial^2 \widetilde{b}_t(\theta)}{\partial\theta\partial s_t} - \frac{a'(\theta)}{a(\theta)}X_s\right] \\ &= a(\theta)\left[\frac{\partial^2 \widetilde{b}_t(\theta)}{\partial\theta\partial s_t} - \frac{R-\lambda\rho}{(1-\theta)(R-\lambda\theta\rho)}\left(1-\frac{a}{(R-\lambda)(q+x)}\right)X_s\right].\end{aligned}$$

Proceeding as in the proof of Proposition 5, collecting terms in the first-order expansion of $\frac{\partial^2 L_t(\theta)}{\partial\theta\partial s_t}$ and factoring out a strictly positive scalar (under the maintained restrictions ensuring denominators are positive), we find

$$\frac{\partial^2 L_t(\theta)}{\partial\theta\partial s_t} \;\propto\; D(\theta) - \frac{f}{f+(R-\lambda)x},$$

where we define $D(\theta) \equiv \widetilde{D}(\theta)/\widetilde{D}(1)$ with

$$\begin{aligned}\widetilde{D}(\theta) = {} & \frac{(1-\theta)(R-\lambda\theta\rho)}{2(1-\rho\theta)}\left[\frac{\rho}{1-\rho\theta} + \frac{R-\lambda\rho}{(1-\theta)\,(R-\lambda\theta\rho)}\right]\left[\frac{\theta(1-\rho)(3\lambda-2R)}{(R-\lambda)^3} + \frac{(1-\theta)\,(3\lambda\rho\theta-2R)}{(R-\lambda\rho\theta)^3}\right] \\ & + \frac{(1-\theta)(R-\lambda\theta\rho)}{2(1-\rho\theta)}\left[(1-\rho)\frac{3\lambda-2R}{(R-\lambda)^3} + \frac{2R+3\lambda\rho-6\lambda\rho\theta}{(R-\lambda\rho\theta)^3} + \frac{3\lambda\rho(1-\theta)(3\lambda\rho\theta-2R)}{(R-\lambda\rho\theta)^4}\right].\end{aligned}$$

Direct simplification yields

$$\widetilde{D}(0) = -\frac{\left(1-\rho\frac{\lambda}{R}\right)\left(2-\frac{\lambda}{R}\right)}{2(R-\lambda)} < 0, \qquad \widetilde{D}(1) = \frac{(R-\lambda\rho)(3\lambda-2R)}{2(R-\lambda)^3} > 0.$$

The derivative of $\widetilde{D}(\theta)$ is then

$$\widetilde{D}'(\theta) = \frac{\lambda^3\rho(1-\theta)}{(R-\lambda)^2\,(R-\lambda\rho\theta)^4}\Big(R^2(1-\rho) + \theta\,\rho\,[\,3R^2 - 4R\lambda - 2R\lambda\rho + 3\lambda^2\rho\,]\Big).$$

Let $Q(\theta) \equiv R^2(1-\rho) + \theta\rho[\,3R^2 - 4R\lambda - 2R\lambda\rho + 3\lambda^2\rho\,]$. Since $Q(\theta)$ is affine in θ, it suffices to check $Q(0) > 0$ and $Q(1) > 0$. We have $Q(0) = R^2(1-\rho) > 0$, and

$$Q(1) = R^2(1+2\rho) - 4R\lambda\rho + \rho^2\lambda(3\lambda - 2R),$$

which is a convex quadratic in ρ because $3\lambda - 2R > 0$ (implied by $\widetilde{D}(1) > 0$) and satisfies $Q(1)\big|_{\rho=0} = R^2 > 0$ and $Q(1)\big|_{\rho=1} = 3(R-\lambda)^2 > 0$. Hence $Q(1) > 0$ for $\rho \in [0,1]$, and therefore $\widetilde{D}'(\theta) > 0$ for $\theta \in [0,1)$.

Since $\widetilde{D}'(\theta) > 0$, the normalized function $D(\theta)$ is strictly increasing with $D(1) = 1$ and

$D(0) = \widetilde{D}(0)/\widetilde{D}(1) < 0$. Therefore there exists a unique $\underline{\theta}^s \in (0, 1]$ solving

$$D(\underline{\theta}^s) = \frac{f}{f + (R - \lambda)x},$$

and for all $\theta \geq \underline{\theta}^s$ we have $\frac{\partial^2 L_t(\theta)}{\partial\theta\partial s_t} > 0$, establishing Proposition 6. $\square$