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# Bubbles and Crashes

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# Story of a typical technology stock

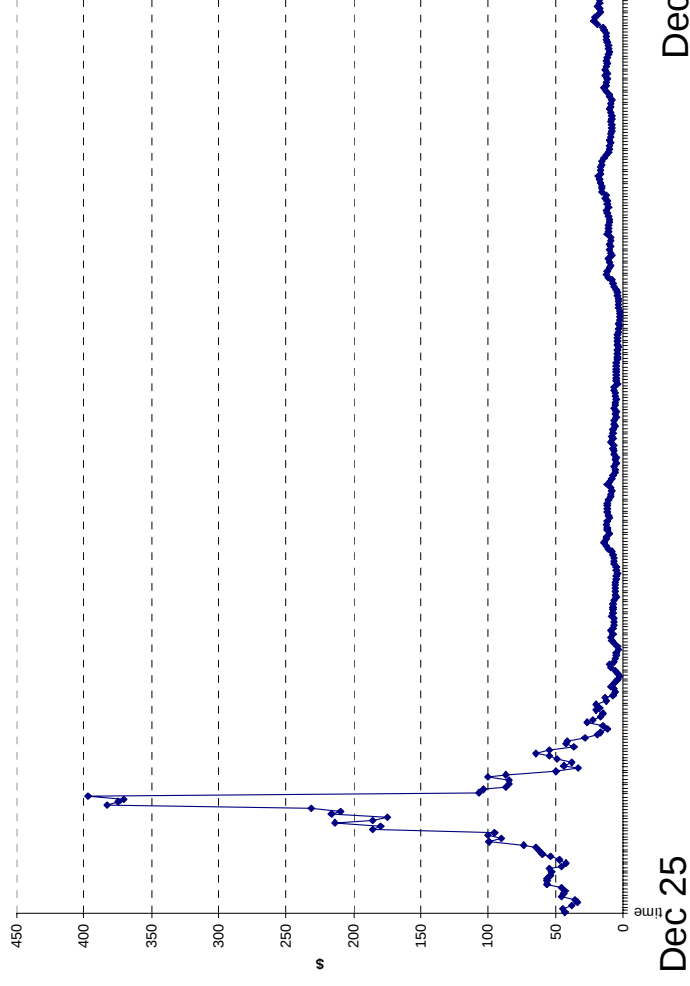
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- Company X introduced a revolutionary wireless communication technology.
- It not only provided support for such a technology but also provided the informational content itself.
- It's IPO price was \$1.50 per share. Six years later it was traded at \$ 85.50 and in the seventh year it hit \$ 114.00.
- The P/E ratio got as high as 73.
- The company never paid dividends.

# Story of RCA - 1920's

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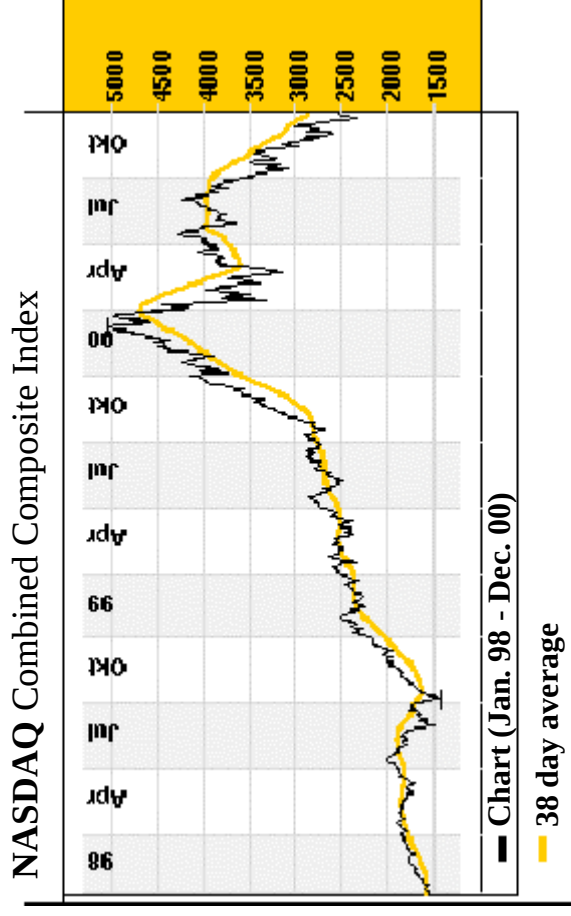
- Company: *Radio Corporation of America (RCA)*
- Technology: *Radio*
- Year: *1920's*



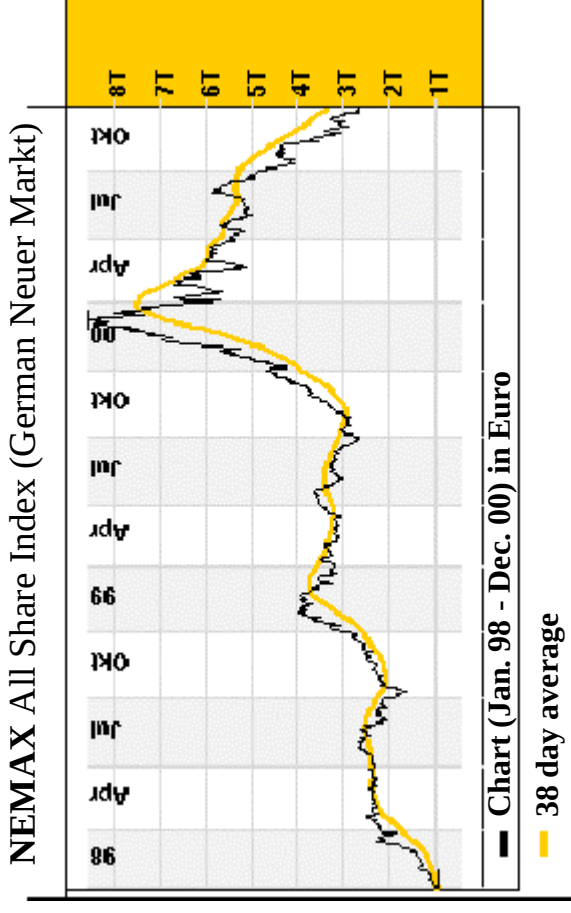
- It peaked at \$ 397 in Feb. 1929, down to \$ 2.62 in May 1932,

# Internet bubble?

# - 1990's



Loss of ca. **60 %**  
from high of \$ 5,132



Loss of ca. **85 %**  
from high of Euro 8,583

- Are bubbles *recurrent*?
- What happened in March 2000?
- Further evidence of bubble
  - ▶ crash was not accompanied by fundamental news.
  - ▶ excess volatility

# Do (rational) professional ride the bubble?

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- South Sea Bubble (1710 - 1720)
  - ▶ *Isaac Newton*
    - 04/20/1720 sold shares at £7,000 profiting £3,500
    - re-entered the market later - ended up losing £20,000
    - “I can calculate the motions of the heavenly bodies, but not the madness of people”
- Internet Bubble (1992 - 2000)
  - ▶ *Druckenmiller* of Soros' Quantum Fund didn't think that the party would end so quickly.
    - “We thought it was the eighth inning, and it was the ninth”
  - ▶ *Julian Robertson* of Tiger Fund refused to invest in internet stocks

# Pros' dilemma

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- ▶ “The moral of this story is that irrational market can kill you ...
- ▶ Julian said ‘This is irrational and I won’t play’ and they carried him out feet first.
- ▶ Druckenmiller said ‘This is irrational and I will play’ and they carried him out feet first.”

Quote of a financial analyst, *New York Times*

*April, 29 2000*

# Classical Question

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- ▶ Suppose behavioral trading leads to mispricing.
- **Can mispricings or bubbles persist in the presence of rational arbitrageurs?**
- What type of information can lead to the bursting of bubbles?

# Main Literature

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- **Keynes (1936)**                       $\Rightarrow$       **bubble can emerge**
  - ▶ “It might have been supposed that *competition between expert professionals*, possessing judgment and knowledge beyond that of the average private investor, would correct the vagaries of the ignorant individual left to himself.”
- **Friedman (1953), Fama (1965)**  
**Efficient Market Hypothesis  $\Rightarrow$       no bubbles emerge**
  - ▶ “If there are many sophisticated traders in the market, they may cause these “bubbles” to burst before they really get under way.”
- **Limits to Arbitrage**
  - ▶ arbitrageurs are myopic/short-lived and risk averse (DeLong et al. [DSSW], 1990a)
  - ▶ fund managers (arbitrageurs) face liquidation risk due to principal-agent problem (Shleifer & Vishny, 1997)
  - ▶ arbitrageurs exploit delayed reaction of feedback traders (DeLong et al. [DSSW], 1990b)

# Timing Game - Synchronization

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- (When) will behavioral traders be overwhelmed by rational arbitrageurs?
- *Collective* selling pressure of arbitrageurs *more than suffices* to burst the bubble.
- Rational arbitrageurs understand that an *eventual* collapse is inevitable.  
But when?
- Delicate, difficult, dangerous ***TIMING GAME!***

# Elements of the Timing Game

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1. *Coordination*      at least  $\kappa > 0$  arbs have to be ‘out of the market’
2. *Competition*      only *first*  $\kappa < 1$  arbs receive pre-crash price.
3. *Profitable ride*      ride bubble (stay in the market) as long as possible.

## 4. *Sequential Awareness*

arbs understand that for a variety of reasons (dispersion of ‘viewpoints’, risk exposure, etc.) they will individually come up with different solutions when to exit the market.

### *A Synchronization Problem arises!*

- ▶ Absent of sequential awareness  
competitive element dominates  $\Rightarrow$  and bubble burst immediately.
- ▶ With sequential awareness  
incentive to TIME THE MARKET leads to  $\Rightarrow$  “delayed arbitrage” and  
persistence of bubble.

introduction

**model setup**

preliminary analysis

persistence of **bubbles**

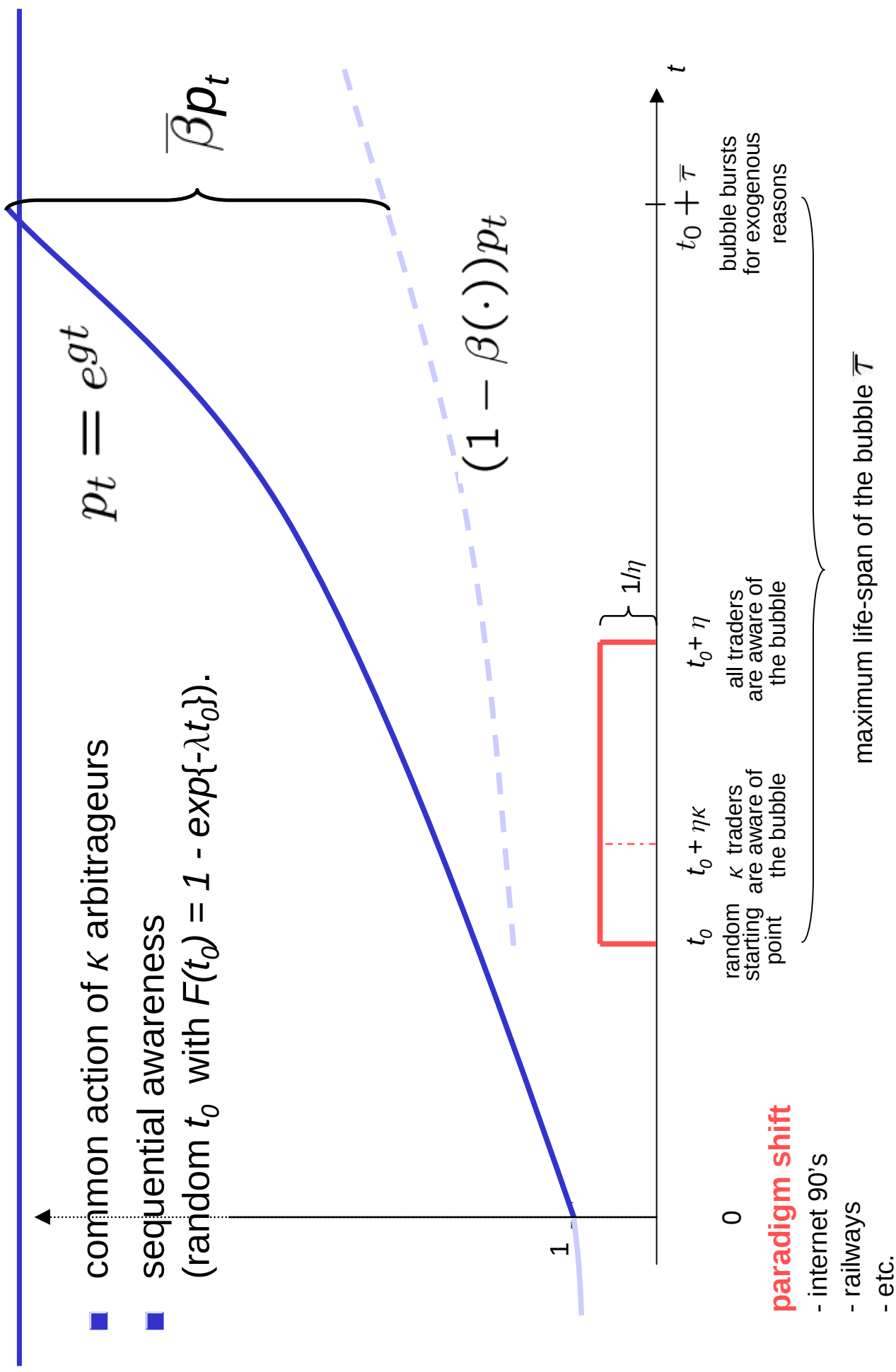
public events

price cascades and rebounds

conclusion

# Model setup

- common action of  $\kappa$  arbitrageurs
- sequential awareness  
(random  $t_0$  with  $F(t_0) = 1 - \exp\{-\lambda t_0\}$ ).



# Payoff structure

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## ■ Cash Payoffs (difference)

- ▶ Sell 'one share' at  $t-\Delta$  instead of at  $t$ .

$$p_{t-\Delta} e^{r\Delta} - p_t$$

where  $p_t = \begin{cases} e^{gt} & \text{prior to the crash} \\ (1 - \beta(t - t_0))e^{gt} & \text{after the crash} \end{cases}$

- ▶ Execution price at the time of bursting.

$$p_t^{\text{burst}} = \begin{cases} e^{gt} & \text{for first random orders up to } \kappa \\ (1 - \beta(t - t_0))e^{gt} & \text{all other orders} \end{cases}$$

# Payoff structure (ctd.), Trading

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- Reputational penalty  $zp_t$  for attacking if bubble does not burst
  - ▶ relative performance evaluation
  - ▶ draw downs
- Small transactions costs  $ce^r$
- Risk-neutrality but max/min stock position
  - ▶ max long position
  - ▶ max short position
  - ▶ due to capital constraints, margin requirements etc.

## ■ **Definition 1:** *trading equilibrium*

- ▶ Perfect Bayesian Nash Equilibrium
- ▶ Belief restriction: trader who attacks at time  $t$  believes that all traders who became aware of the bubble prior to her also attack at  $t$ .

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## **Preliminary analysis**

preemption motive - trigger strategies

sell out condition

**persistence of bubbles**

public events

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# Trigger Strategies

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- Bursting date  $T^*(t_0) = \min\{T(t_0 + \eta\kappa), t_0 + \bar{\tau}\}$
- Role of Preemption Motive
  - Rules out coordinated sell out on Friday July 13<sup>th</sup>.
  - Bubble never bursts with strictly positive prob. at some  $t^{13}$ .
    - Suppose it would, then selling pressure would exceed  $\kappa$  with prob  $> 0$ .
    - Hence, price would drop already at  $t^{13} \Rightarrow$  incentive to sell out earlier
  - well defined density of bursting date  $\pi(t|t_i)$  for each arb.

## ■ Proposition 1: Trigger strategies.

- Given  $c > 0$ , arb  $t_i$  never sells out only for an instant.  
He stays out of the market at least until  $t_i + \epsilon$  sells out.
- Arb  $t_i + \epsilon$  stays out until  $t_i + 2\epsilon$  exits and so on.
- By trading equilibrium, arb  $t_i$  stays out until  $t_i + \eta\kappa$  exits.

# Sell out condition for $\Delta \rightarrow 0$ periods

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- sell out at  $t$  if

$$\underbrace{\Delta h(t|t_i) E_t[\text{bubble}|\cdot]}_{\text{benefit of attacking}} \geq \underbrace{(1 - \Delta h(t|t_i)) [(g - r) + z] p_t \Delta}_{\substack{\text{appreciation rate} \\ \text{reputational penalty} \\ \text{cost of attacking}}}$$

$$h(t|t_i) \geq \frac{(g-r)+z}{\beta(t-T^*-1(t))}$$

RHS converges to  $\searrow [(g-r) + z]$  as  $t \rightarrow \infty$

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**persistence of bubbles**

exogenous crashes

endogenous crashes

lack of common knowledge

public events

price cascades and rebounds

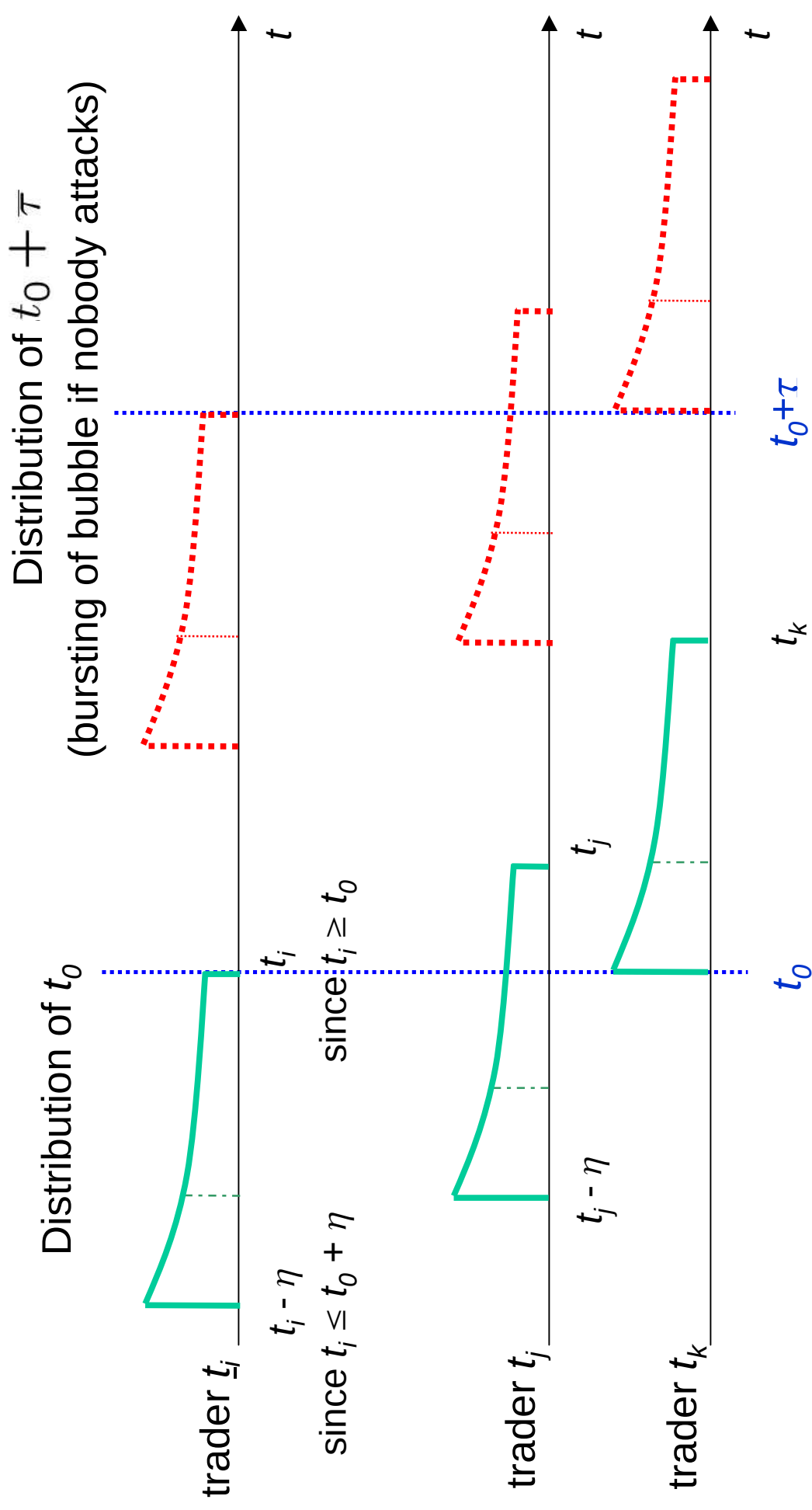
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# Persistence of Bubbles

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- **Proposition 2:** Suppose  $\frac{\lambda}{1-e^{-\lambda\eta\kappa}} \leq \frac{g-r}{\beta}$ .
  - ▶ existence of a unique trading equilibrium
  - ▶ traders begin attacking after a delay of  $\tau^1 < \bar{\tau}$  periods.
  - ▶ bubble does *not* burst due to endogenous selling prior to  $t_0 + \bar{\tau}$ .

# Sequential awareness

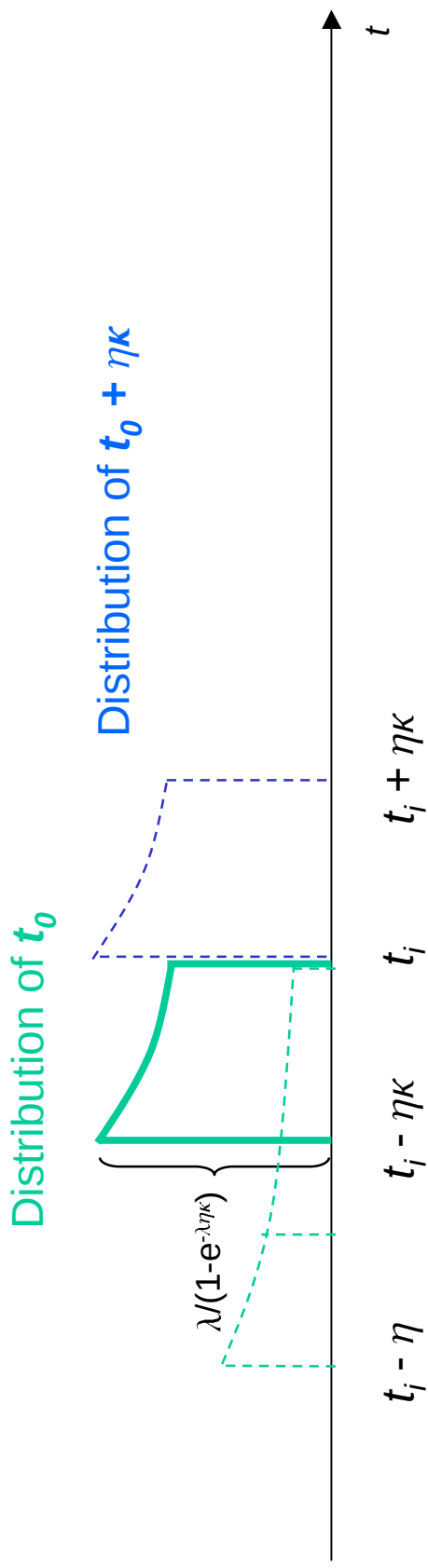


# Conjecture 1: Immediate attack

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$\Rightarrow$  **Bubble bursts at  $t_0 + \eta\kappa$**

when  $\kappa$  traders are aware of the bubble

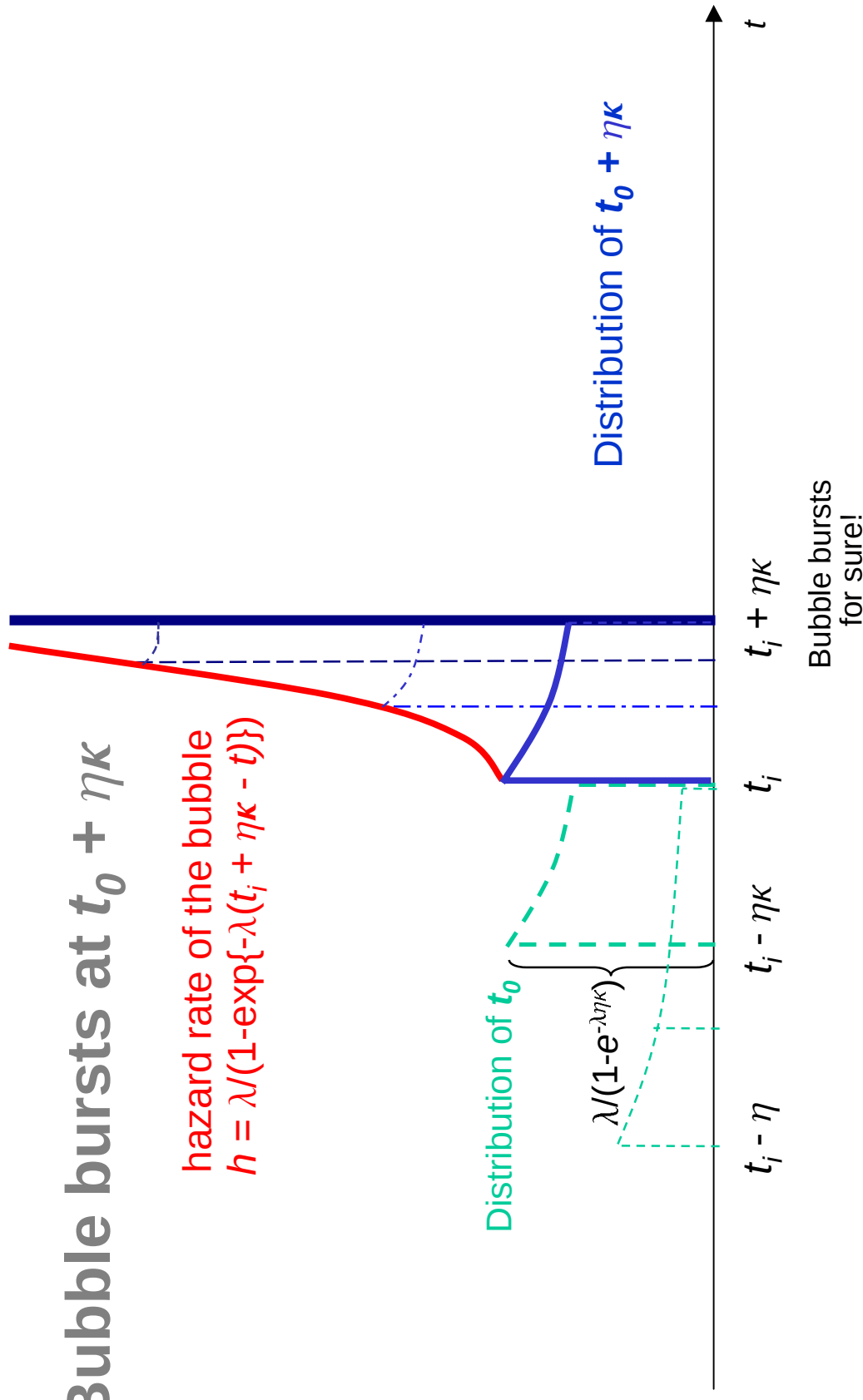


If  $t_0 < t_i - \eta\kappa$ , the bubble would have burst already.

# Conj. 1 (ctd.): Immediate attack

⇒ Bubble bursts at  $t_0 + \eta\kappa$

hazard rate of the bubble  
 $h = \lambda(1 - \exp\{-\lambda(t_i + \eta\kappa - t)\})$



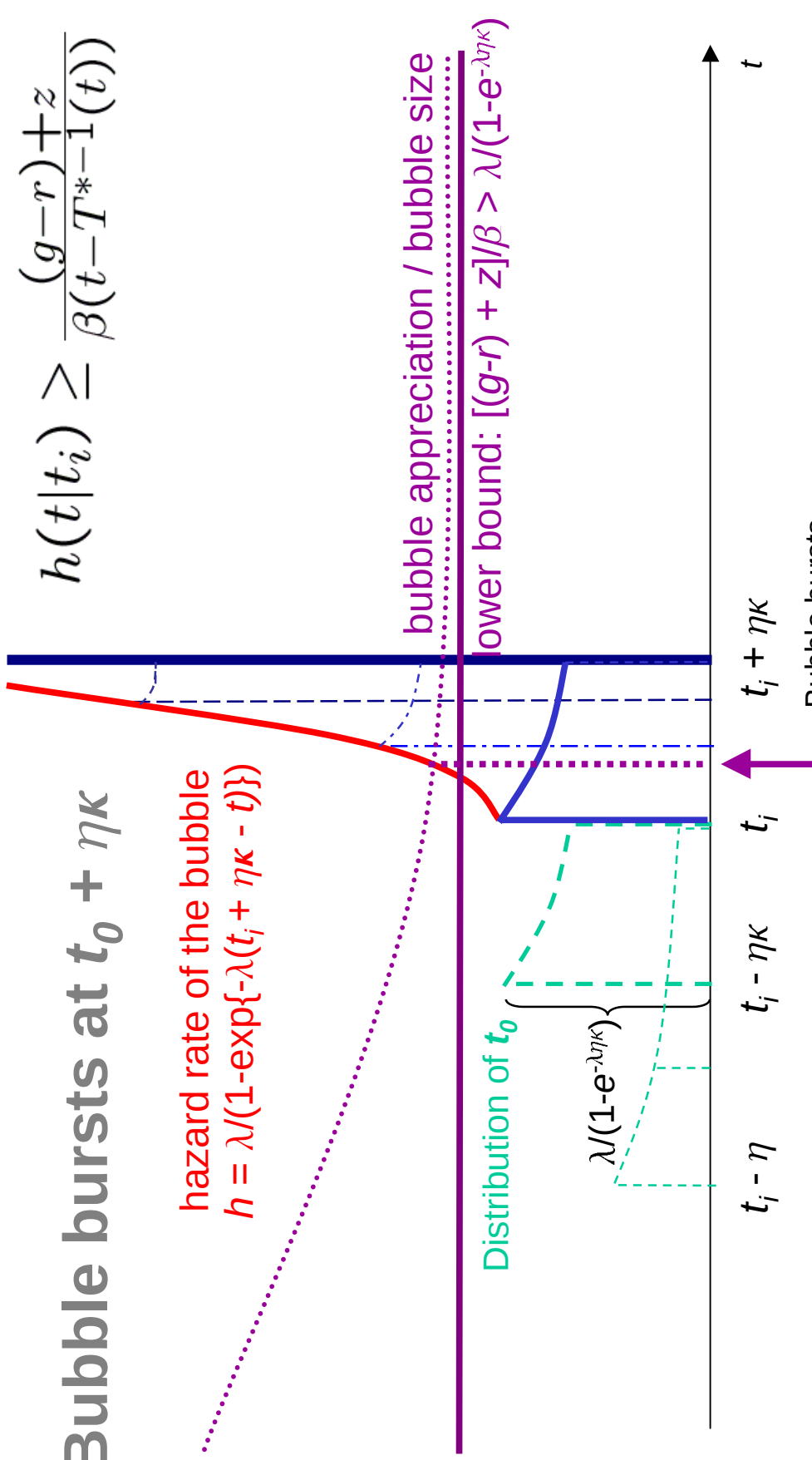
# Conj. 1 (ctd.): Immediate attack

⇒ Bubble bursts at  $t_0 + \eta\kappa$

Recall the sell out condition:

$$h(t|t_i) \geq \frac{(g-r)+z}{\beta(t-T^*-1(t))}$$

hazard rate of the bubble  
 $h = \lambda/(1-\exp\{-\lambda(t_i + \eta\kappa - t)\})$



Bubble bursts for sure!

⇒ “delayed attack is optimal”  
 no “immediate attack” equilibrium!

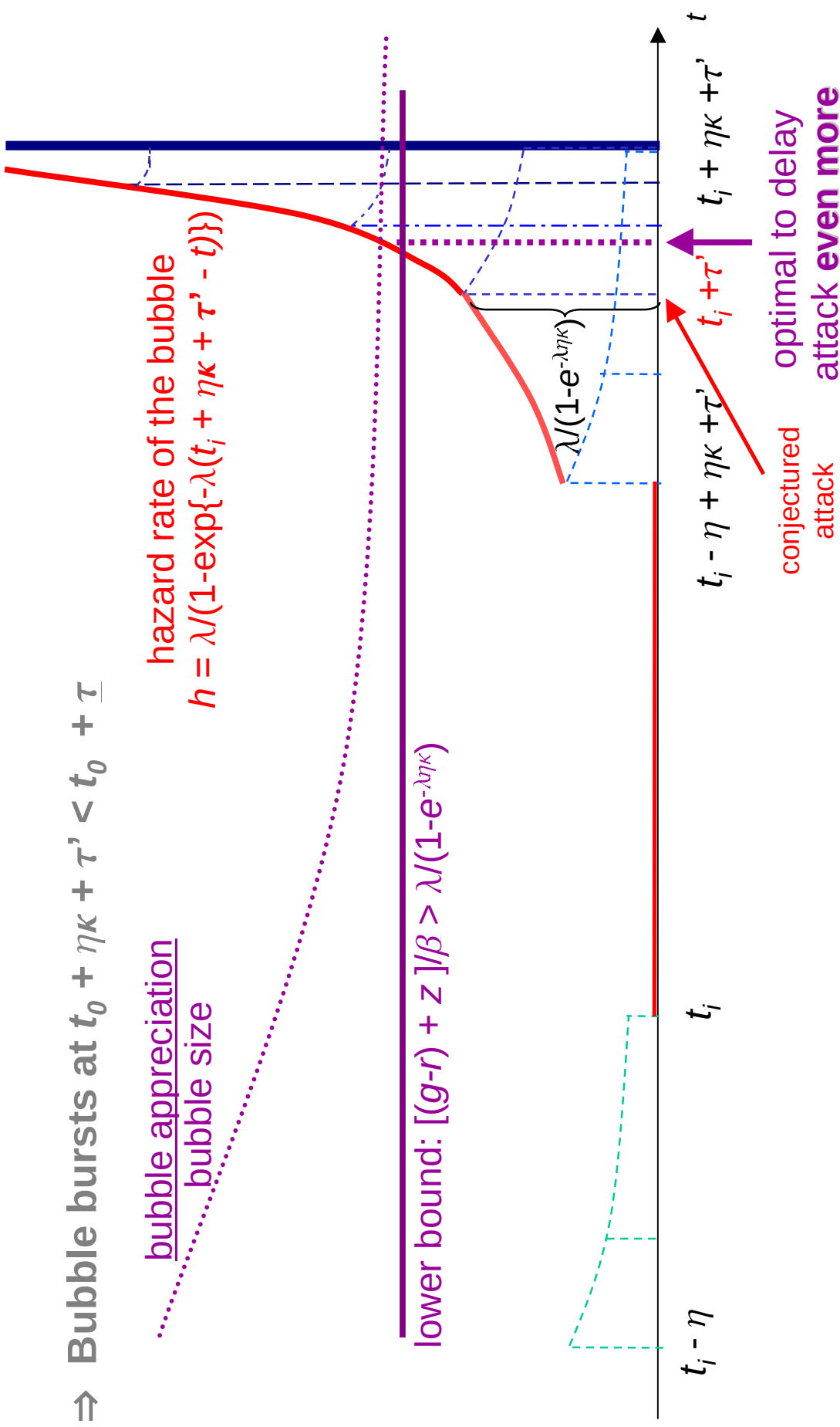
## Conj. 2: Delayed attack by arbitrary $\tau'$

⇒ Bubble bursts at  $t_0 + \eta_K + \tau' < t_0 + \tau$

bubble appreciation  
bubble size

hazard rate of the bubble  
 $h = \lambda(1 - \exp\{-\lambda(t_i + \eta_K + \tau' - t)\})$

lower bound:  $[(g-r) + z]/\beta > \lambda(1 - e^{-\lambda\eta_K})$



⇒ attack is never successful

⇒ bubble bursts for exogenous reasons at  $t_0 + \tau$

# Endogenous crashes

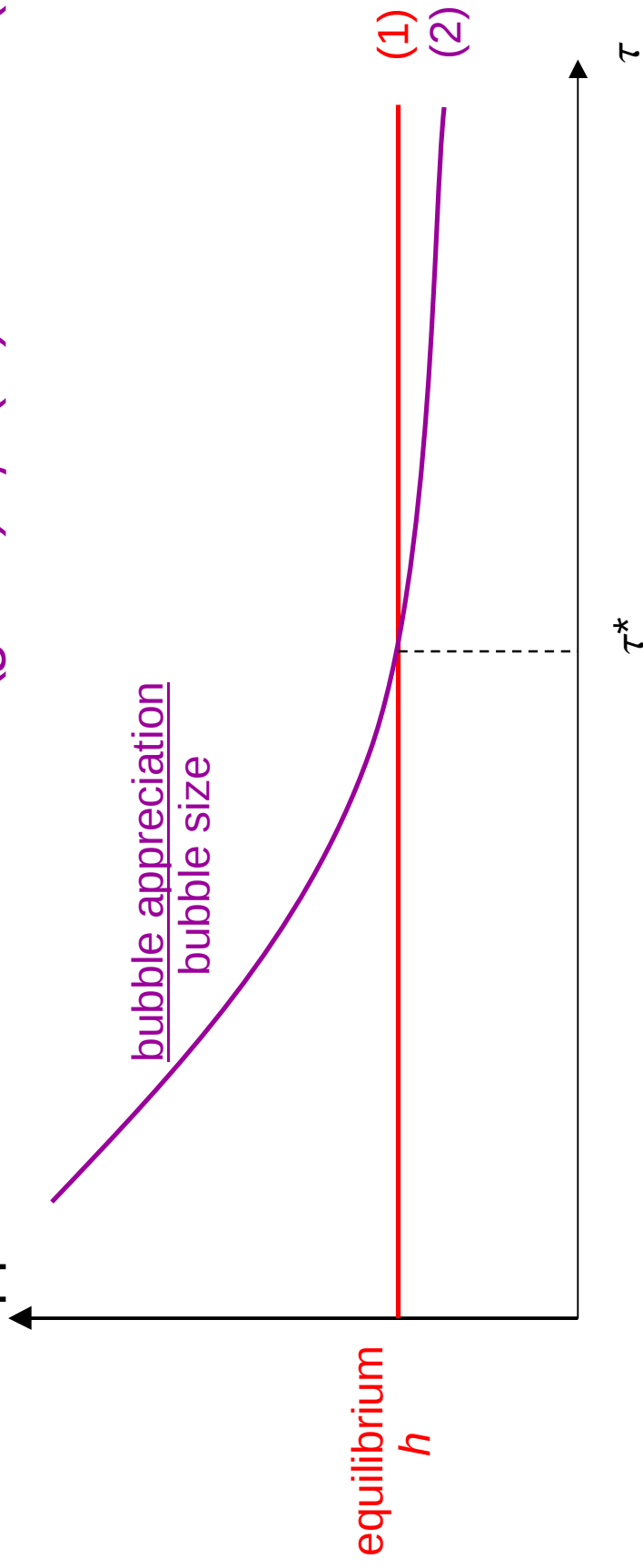
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- **Proposition 3:** Suppose  $\frac{\lambda}{1-e^{-\lambda\eta\kappa}} > \frac{g-r}{\beta}$ .
  - ▶ ‘**unique**’ trading equilibrium.
  - ▶ traders begin attacking after a delay of  $\tau^*$  periods.
  - ▶ bubble ***bursts*** due to endogenous selling pressure at a size of  $p_t$  times

$$\beta^* \equiv \frac{1-e^{-\lambda\eta\kappa}}{\lambda}(g-r)$$

# Endogenous crashes - deriving $\tau^*$

- In equilibrium trader  $t_i = t_0 + \eta_K$  bursts the bubble.
- When she sells his shares her support of  $t_0$  is  $[t_i - \eta_K, t_i]$ , hence his hazard rate is  $h = \lambda(1 - \exp\{-\lambda\eta_K\})$  (1)
- The bubble bursts at  $t_i = t_0 + \eta_K + \tau^*$ , hence it bursts at a size of  $e^{gt} \beta^*(\tau^*)$  bubble appreciation/ size =  $(g - r + z) / \beta^*(\tau^*)$  (2)



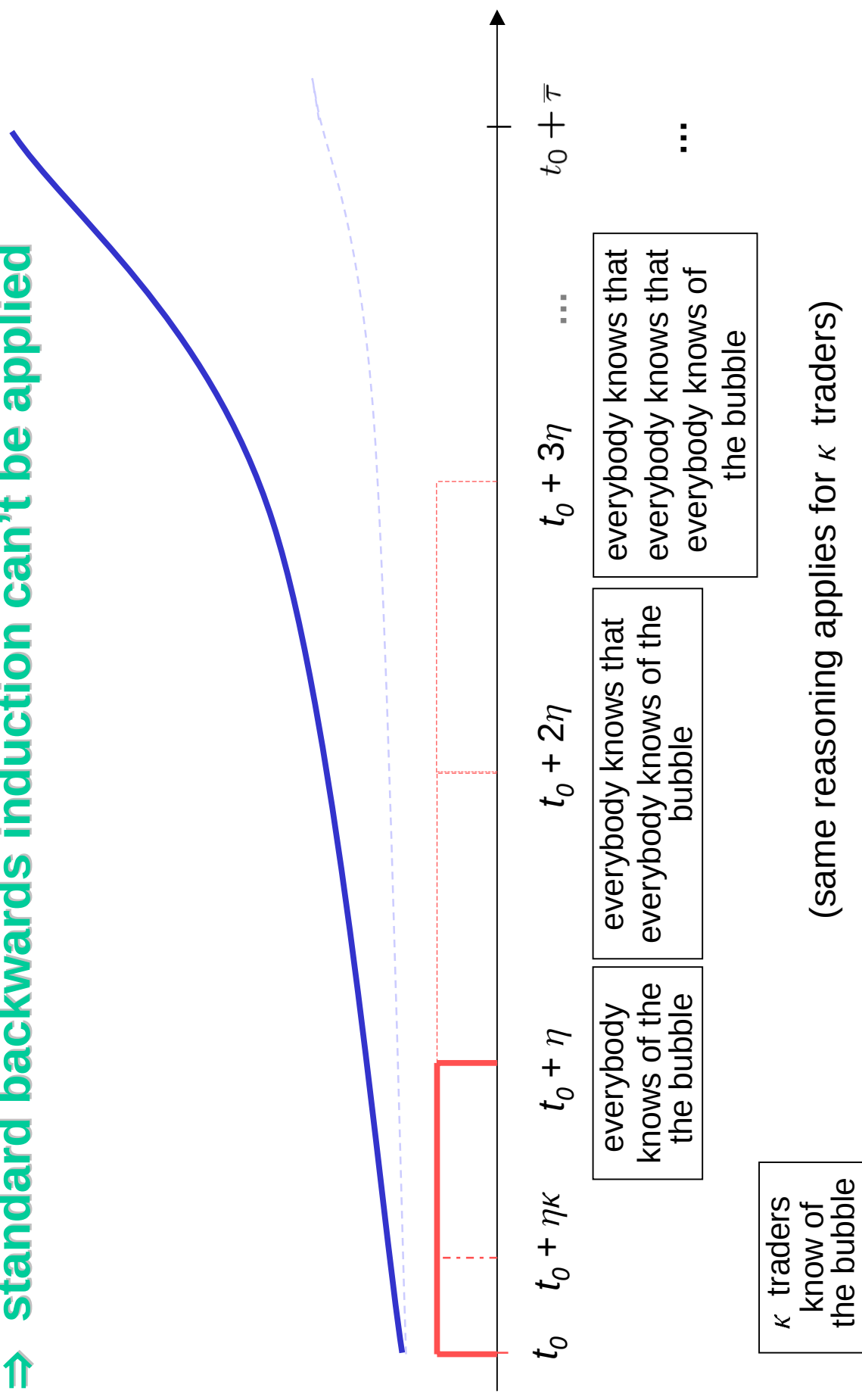
# Comparative statics

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- Role of information dispersion  $\lambda, \eta$ 
  - ▶ Prior distribution of  $t_0$   $F(t_0) = 1 - \exp\{-\lambda t_0\}$ 
    - the smaller  $\lambda$ , the larger  $\beta^*$ , the size of bubble
    - $\lambda \rightarrow \infty \Rightarrow t_0 = 0$ , no info dispersion  $\Rightarrow$  no bubble
    - $\lambda \rightarrow 0 \Rightarrow$  distributions  $\rightarrow$  uniform [size is  $\eta\kappa(g-r)$ ]
  - ▶ Dispersion of opinion  $\eta$ 
    - as  $\eta \uparrow$   $\Rightarrow$  bubble's life-span  $\uparrow$
    - for  $\eta > -\frac{1}{\lambda\kappa} \ln(1 - \lambda \frac{g-r}{\beta})$   $\Rightarrow$  exogenous crash
- Role of momentum traders  $\kappa \Rightarrow$  same as for  $\eta$

# Lack of common knowledge

⇒ **standard backwards induction can't be applied**



# Related theoretical literature

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- Asynchronized clocks
  - ▶ Halpern & Moses (1984) [computer science]
  - ▶ Morris (1995)
    - restricted strategy space: condition only on own clock  
no conditioning on calendar time, past payoffs, etc.
    - no competitive element ( $\kappa = 1$  - case only)
- Global Games  
(uniqueness of equilibrium in static games with strategic complementarities)
  - ▶ Carlson & van Damme (1994)
  - ▶ Morris & Shin (1998)
- Other timing games
  - ▶ war of attrition - preemption games (private values)
  - ▶ herding models with endogenous sequencing (observable actions)

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**synchronizing public events**

pre-scheduled news

unanticipated public events

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# Pre-scheduled public news

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- Pre-scheduled public events
  - ▶ news is unknown, but timing is fixed in advance.  
(macroeconomic news etc.)
  - ▶  $p_t = E_t[p_s]$  for all  $s > t$ .
  - ▶ *⇒ pre-scheduled news will only move price by its fundamental content, but not beyond.*
    - Why? It cannot serve as a synchronization device.
    - If it would, then the bubble would burst with strictly positive probability on this date. In this case arbitrageurs have incentive to attack slightly earlier  
(same as Friday 13th of July)

# Unanticipated public news

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- Unanticipated public events
  - ▶ pre-emption argument does not apply!
  - ▶ can serve as synchronization device.
  - ▶ there are millions of public events (weather, etc.)
  - ▶ viewing something as a public event is also a coordination problem in itself.
- ▶ Extended setting
  - focus on news with *no* informational content (sunspots).
  - public event occurs with Poisson arrival rate  $\theta$ .
  - Arbitrageurs who are aware of the bubble become increasingly worried about it over time.
    - » only traders who became aware of the mispricing more than  $\tau_e$  periods ago observe (look out for) public events.

# Public events & Market rebounds

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## ■ Proposition 5: In ‘responsive equilibrium’

**Sell out** a) always at the time of a public event  $t_e$ ,

b) after  $t_i + \tau^{e*}$  (where  $\tau^{e*} < \tau^*$ ),

**except** after a failed attack at  $t_p$ , **re-enter** the market  
for  $t \in (t_e, t_e - \tau_e + \tau^{e*})$ .

## ■ Intuition for re-entering the market:

- ▶ for  $t_e < t_0 + \eta_K + \tau_e$  attack fails, agents learn  $t_0 > t_e - \tau_e - \eta_K$
- ▶ without public event, they would have learnt this only at  $t_e + \tau_e - \tau^{e*}$ .
  - the existence of bubble at  $t$  reveals that  $t_0 > t - \tau^{e*} - \eta_K$
  - that is, no additional information is revealed till  $t_e - \tau_e + \tau^{e*}$
  - density that bubble bursts for endogenous reasons is zero.

# Role of information

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- Only unanticipated public news can burst a bubble.
- News which is considered as important can be more important than real fundamental news.
- Fads and fashions in information.

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# Price cascades and rebounds

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- Price drop as a synchronization device (public event).
  - ▶ through psychological resistance line
  - ▶ by more than, say 5 %

## ■ Exogenous price drop

- ▶ after a price drop
  - if bubble is ripe
    - ⇒ bubble bursts and price drops further.
  - if bubble is not ripe yet
    - ⇒ price bounces back and the bubble is strengthened for some time.

# Price cascades and rebounds (ctd.)

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## ■ Proposition 6:

**Sell out** a) after a price drop if  $\tau_i \geq \tau_p(H_p)$

b) after  $t_i + \tau^{***}$  (where  $\tau^{***} < \tau^*$ ),

**re-enter** the market after a rebound at  $t_p$

for  $t \in (t_p, t_p - \tau_p + \tau^{***})$ .

- ▶ attack is costly, since price might jump back  
 $\Rightarrow$  only arbitrageurs who became aware of the bubble more than  $\tau_p$  periods ago attack the bubble.
- ▶ after a rebound, an endogenous crash can be temporarily ruled out and hence, arbitrageurs re-enter the market.
- ▶ Even sell out after another price drop is less likely.

# Conclusion

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## ■ Bubbles

- ▶ Dispersion of opinion among arbitrageurs causes a synchronization problem which makes coordinated price corrections difficult.
- ▶ Arbitrageurs time the market and ride the bubble.
- ▶  $\Rightarrow$  Bubbles persist

## ■ Crashes

- ▶ can be triggered by unanticipated news without any fundamental content, since
- ▶ it might serve as a synchronization device.

## ■ Rebound

- ▶ can occur after a failed attack, which temporarily strengthens the bubble.

# Formal analysis for symmetric strategies

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- Suppose endogenous selling pressure would burst bubble at  $t = t_0 + \hat{\tau}$ , where  $\hat{\tau} < \bar{\tau}$ 
  - ▶ for  $t < t_i - \theta + \hat{\tau}$   $h(t|\underline{t}_i) = 0$
  - ▶ for  $t \geq t_i - \theta + \hat{\tau}$   $h(t|\underline{t}_i) = \frac{\lambda}{1 - e^{-\lambda(t_i + \hat{\tau} - t)}}$
- (from sell out condition) sell shares at
 
$$t_i + \hat{\tau} + \ln\left[\frac{(g-r) - \lambda + \lambda e^{-(g-r)\hat{\tau}}}{g-r+c}\right] \left[\frac{1}{\lambda}\right]$$
- mass of arbitrageurs aware of the bubble  $\max\{\frac{1}{\theta}(t - t_0), 1\}$
- mass of arbitrageurs attacking (= selling pressure)
 
$$\max\left\{\frac{1}{\theta}(t - \hat{\tau} - \ln\left[\frac{(g-r) - \lambda + \lambda e^{-(g-r)\hat{\tau}}}{g-r+c}\right]) - t_0, 1\right\}$$
- ▶ for  $t = t_0 + \hat{\tau}$   $-\frac{1}{\theta\lambda} \ln\left[\frac{(g-r) - \lambda + \lambda e^{-(g-r)\hat{\tau}}}{g-r+c}\right] < \kappa$
- ▶ *contradiction!*

# Endogenous crashes

⇒ Bubble bursts at  $t_0 + \theta_K + \tau^*$

