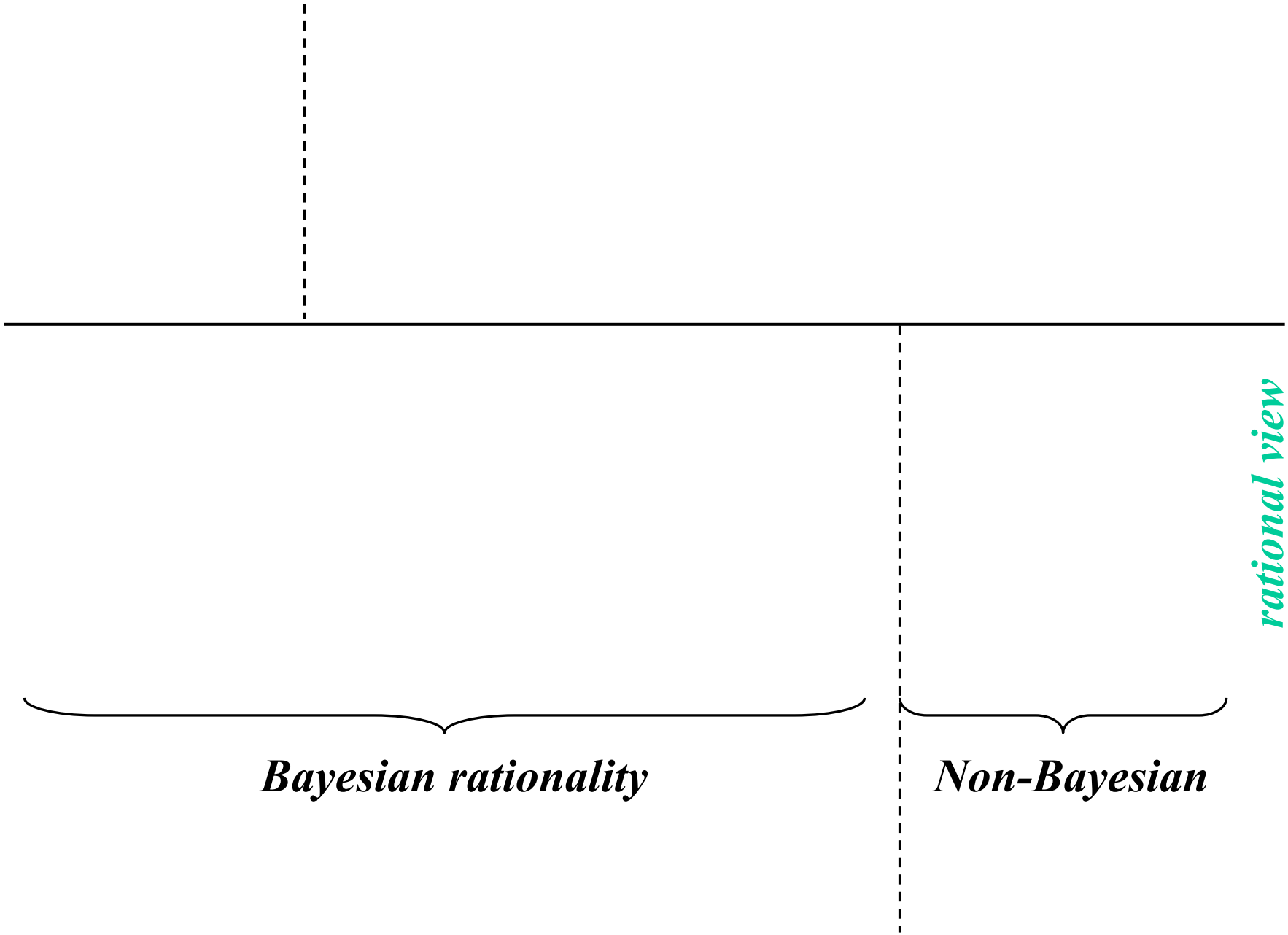
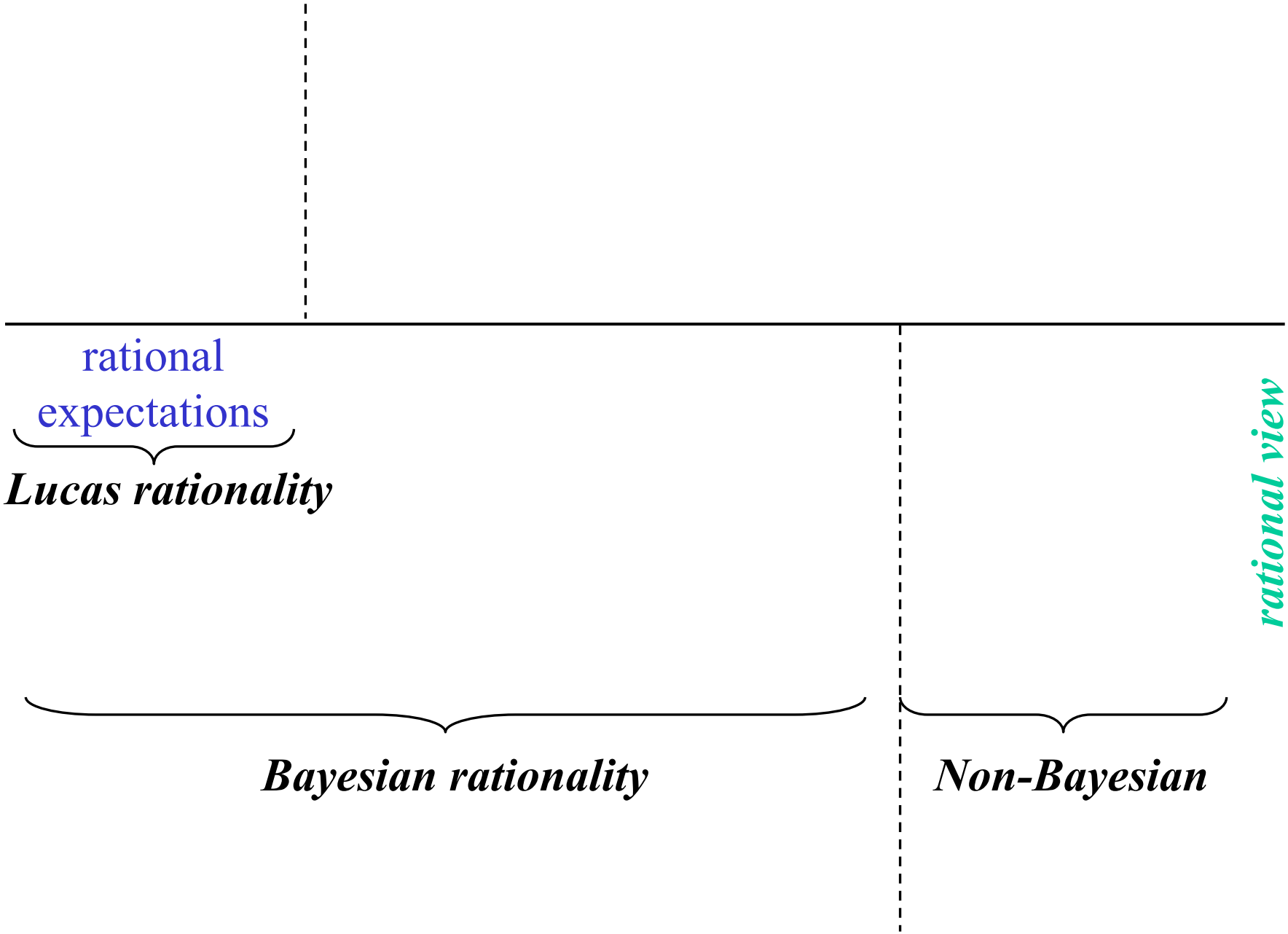
Optimal Expectations

Markus K. Brunnermeier and Jonathan A. Parker
Princeton University

2003


rational view
Bayesian rationality
Non-Bayesian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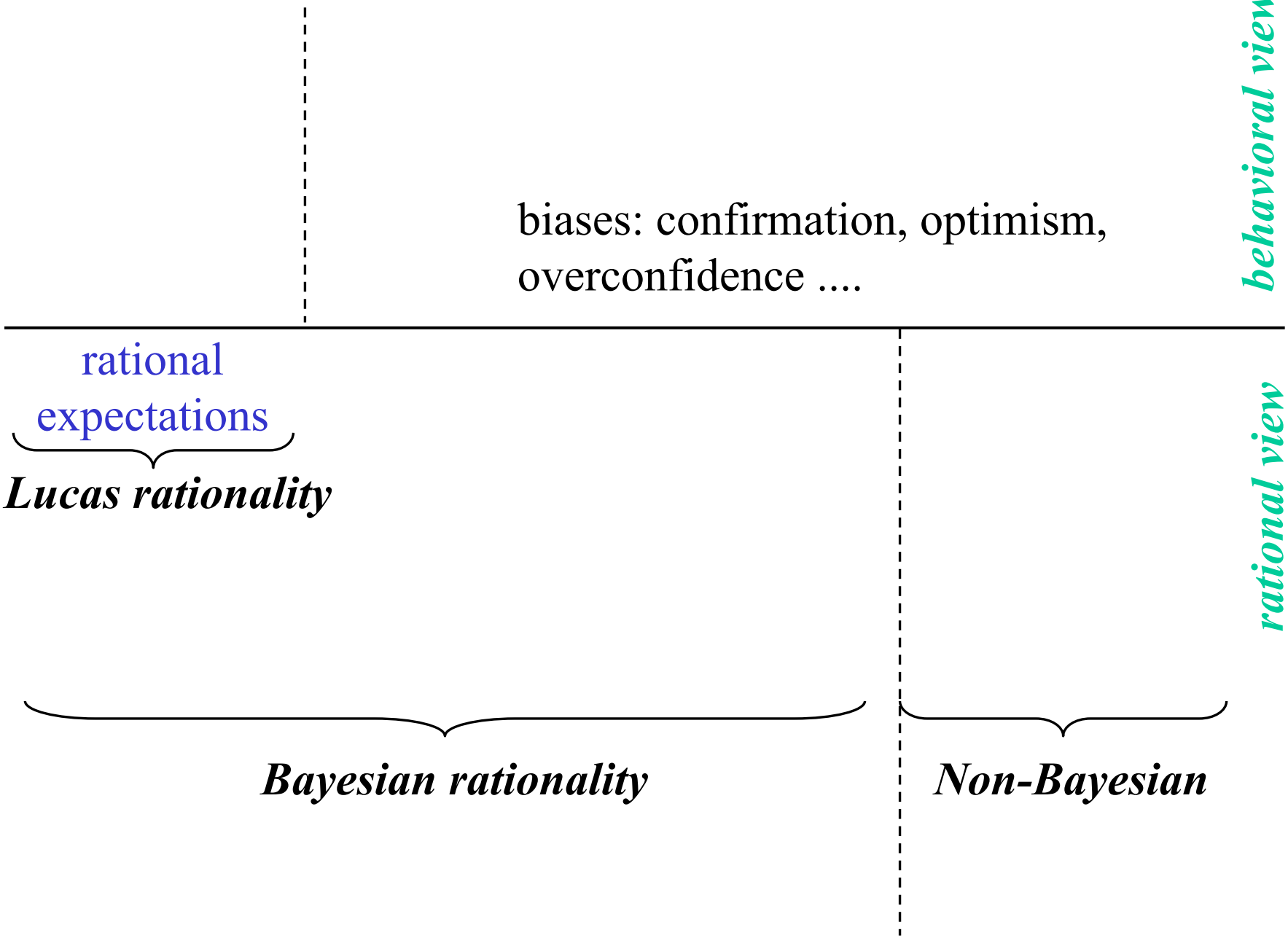
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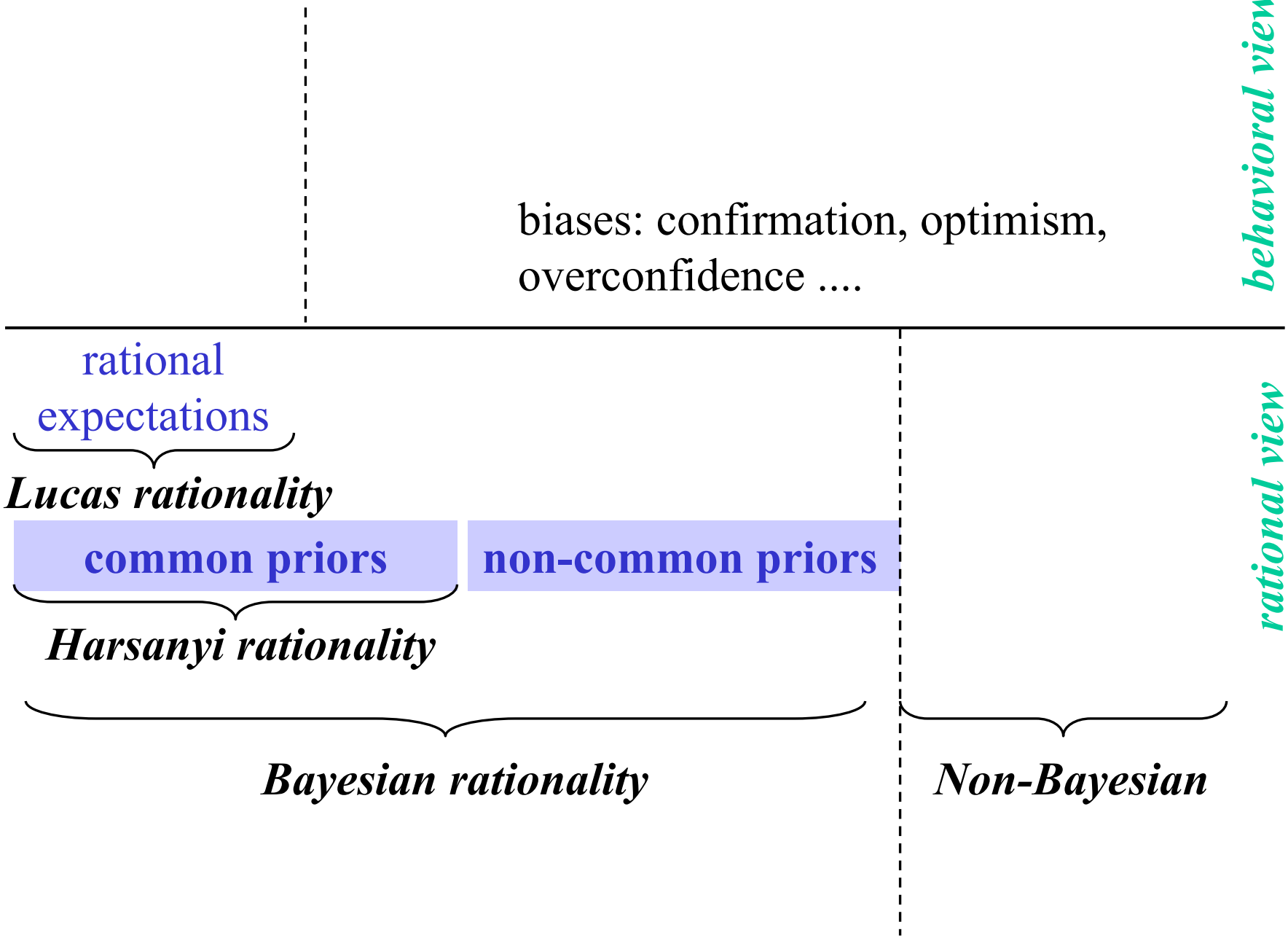
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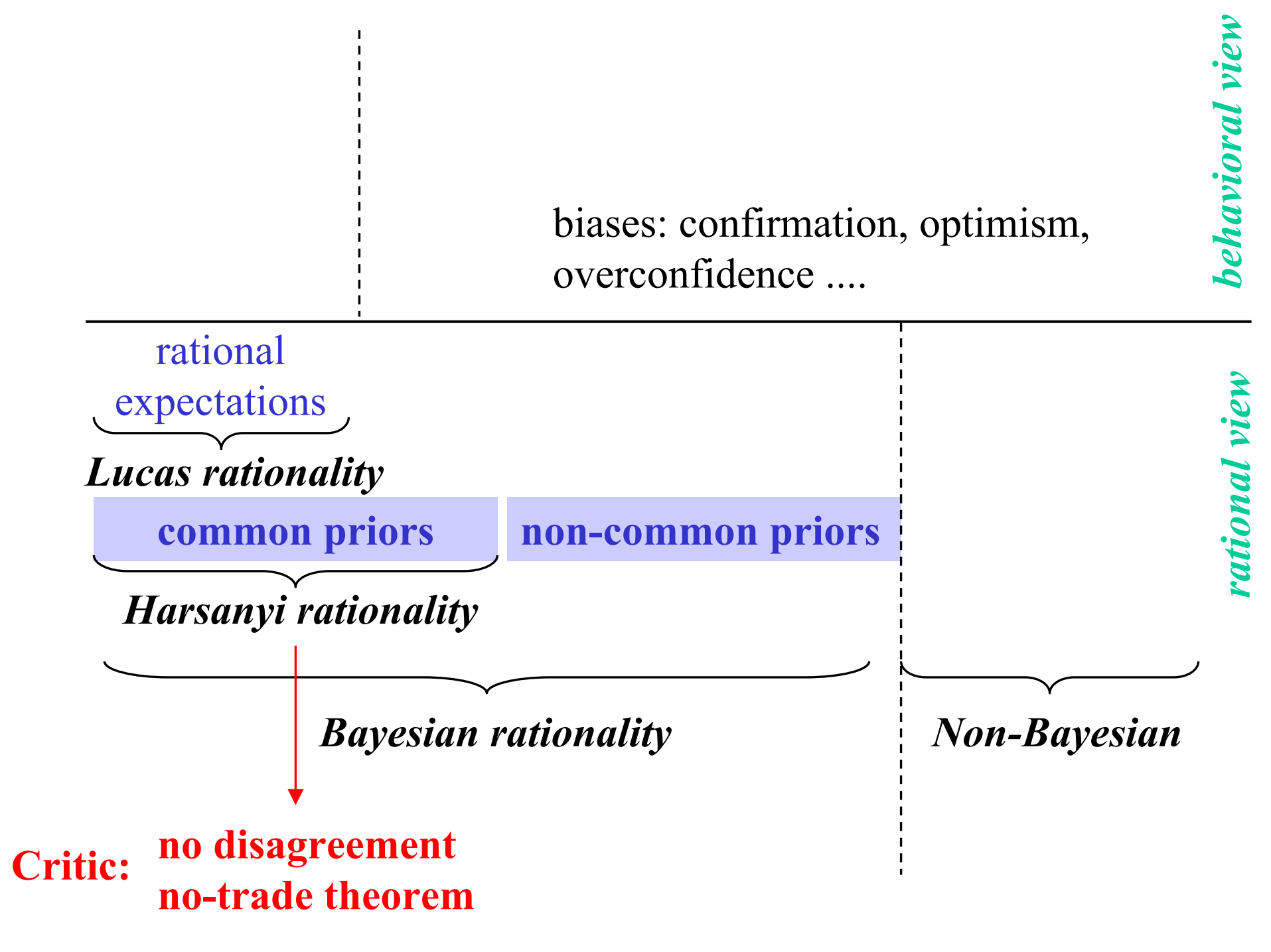
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no disagreement
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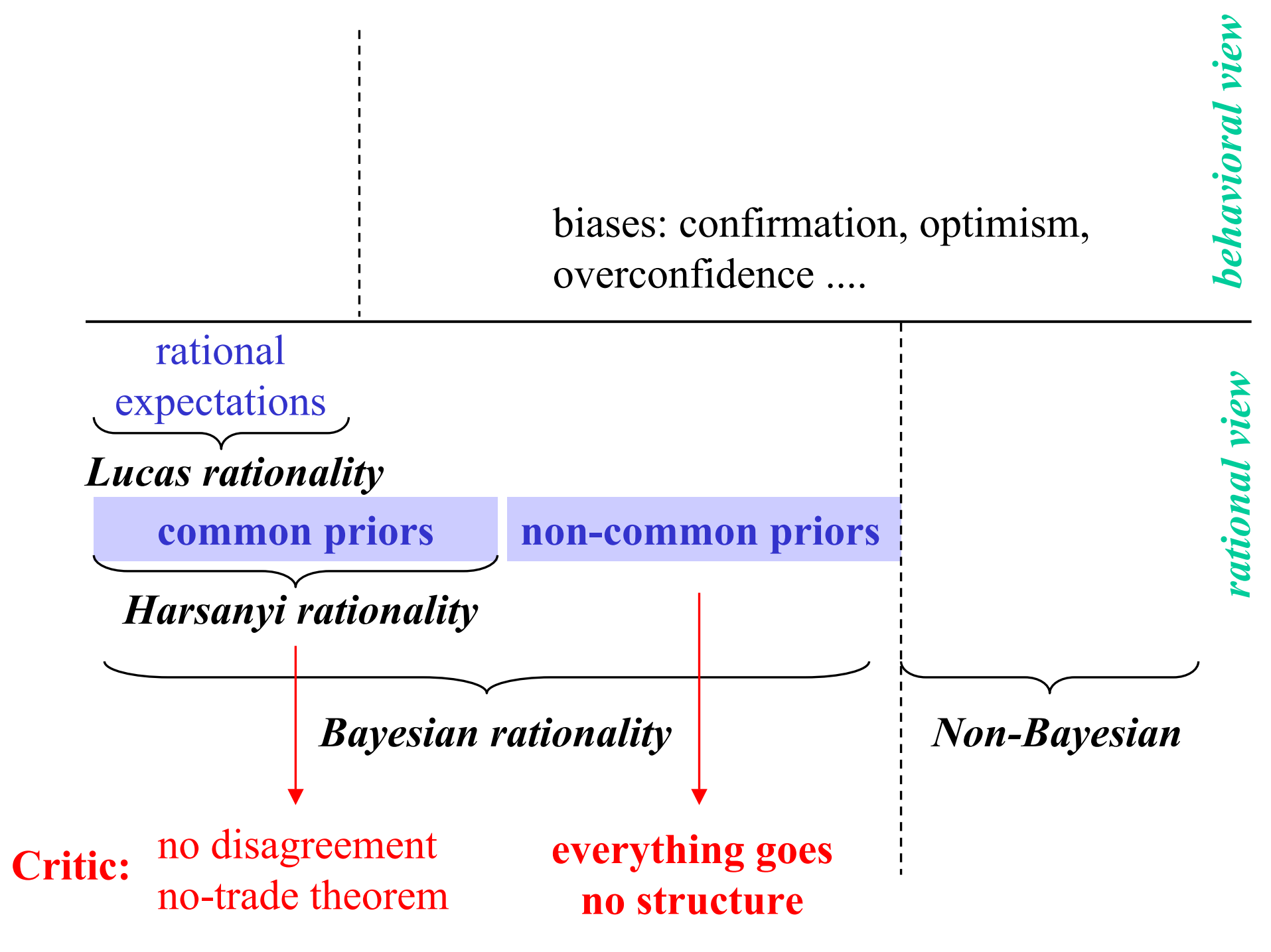
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everything goes
no structure

Optimal Expectations

Our Goal: Provide structural model of subjective beliefs

- ◇ What is the direction of belief distortion?
- ◇ When are belief distortions large?
- ◇ Provide common framework for different biases

Three Main Elements

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3. Optimal beliefs balance these forces

- ◇ Beliefs maximize lifetime well-being

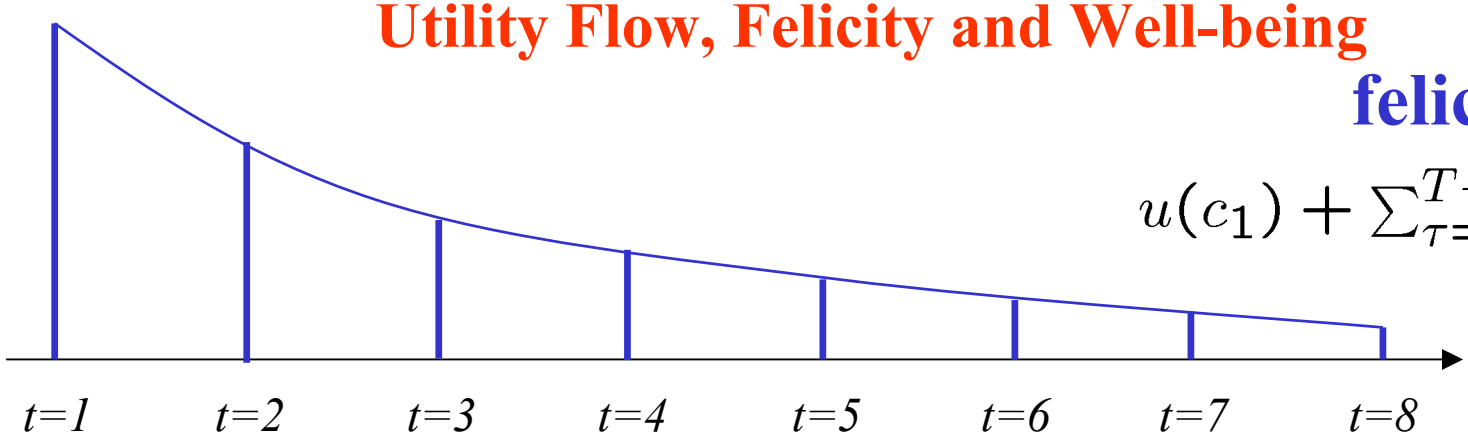
Outline

1.) The General Framework

2.) Applications and Empirical Implications

3.) Conclusion

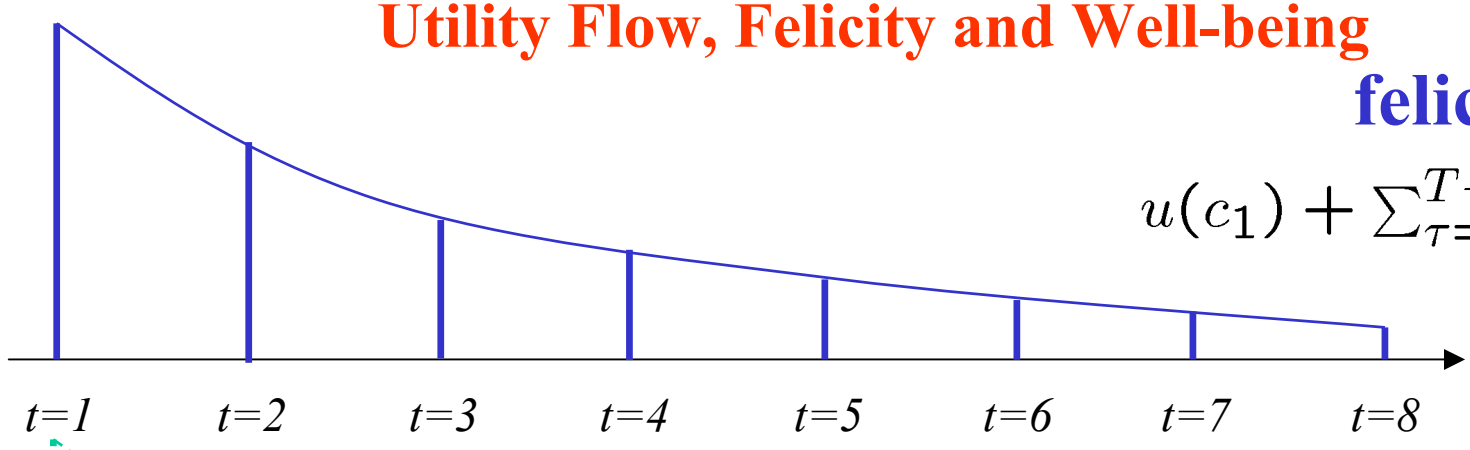
Utility Flow, Felicity and Well-being



felicity at $t = 1$

$$u(c_1) + \sum_{\tau=1}^{T-t} \beta^t u(c_{1+\tau})$$

Utility Flow, Felicity and Well-being



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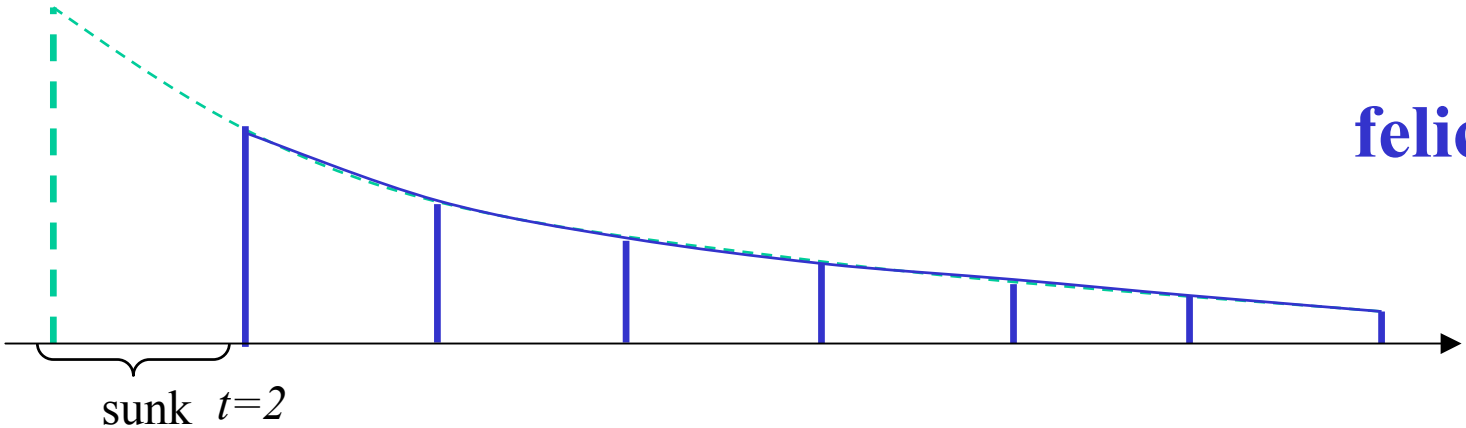
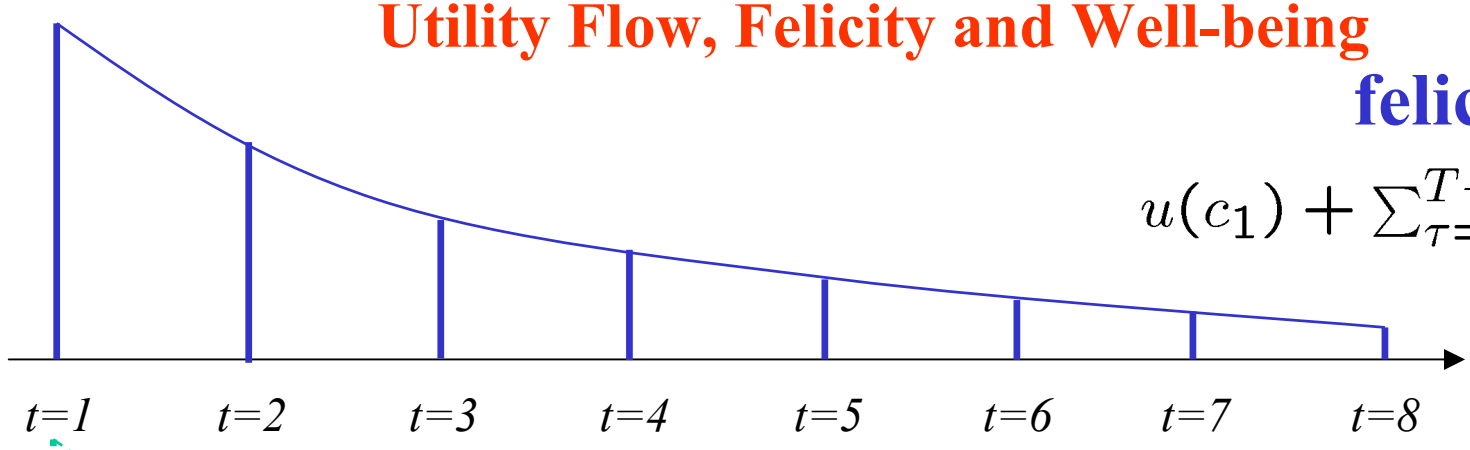
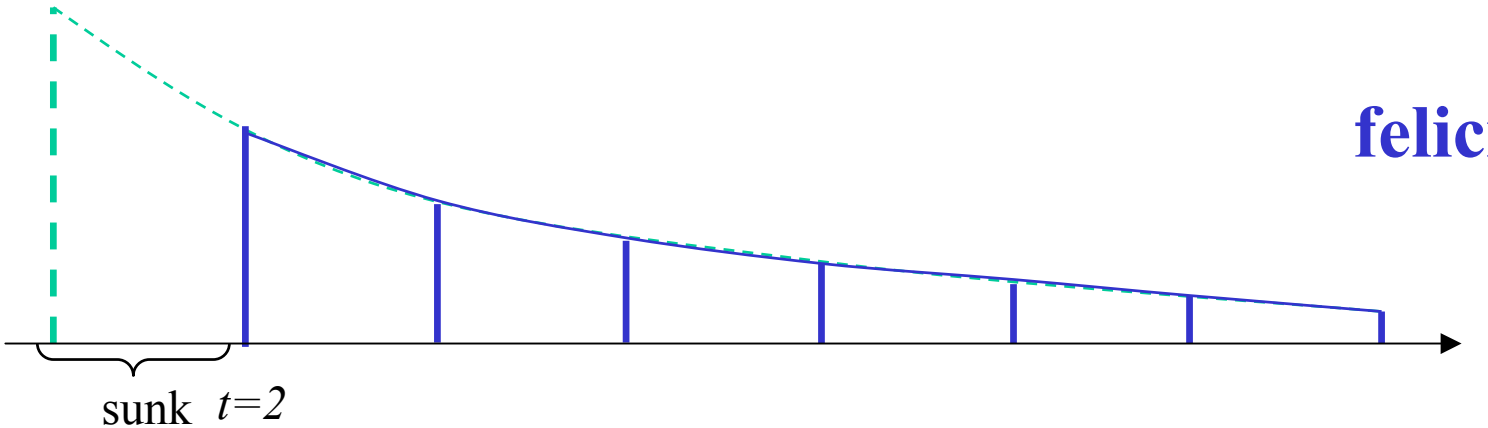
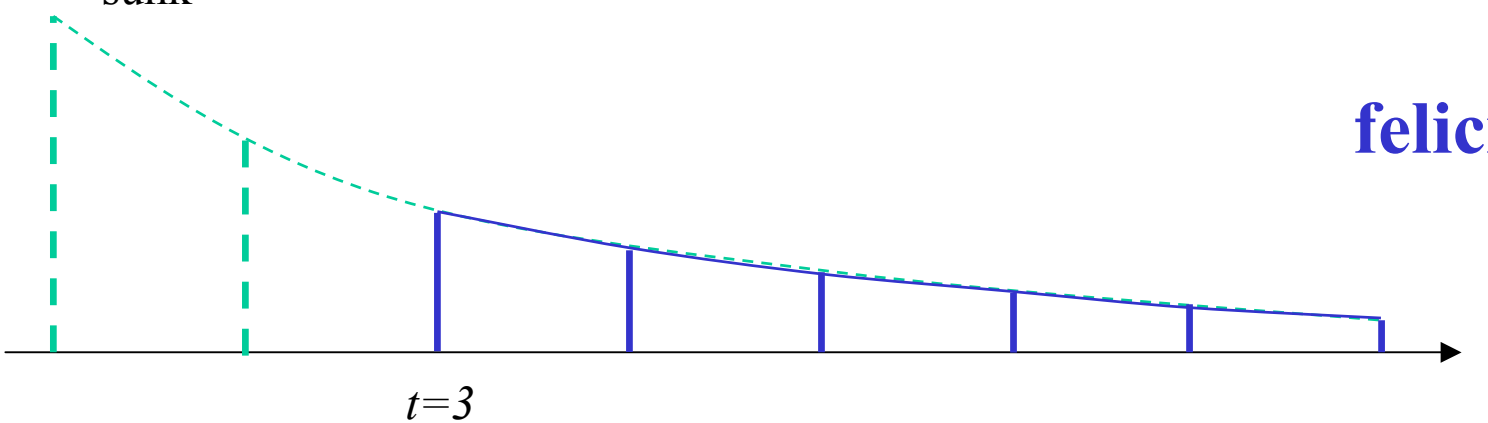


felicity at $t = 2$

Utility Flow, Felicity and Well-being







a

Lifetime well-being S

1. The General Framework

Felicity at t:

$$+ \quad u\left(c_t\right)$$

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Felicity at t: $+ V_t$

$$\underbrace{\qquad}_{} + u(c_t) + \underbrace{\hat{E}_t \sum_{\tau=1}^{T-t} \beta^\tau u\left(c_{t+\tau}\right)}_{\text{'expected' utility} =}$$

$$\underbrace{\beta \hat{E}_t \left[V_{t+1}\left(x_{t+1}; \underline{s}_{t+1}\right)\right]}_{=V_t(x_t;\underline{s}_t)}$$

V_t = 'expected' utility from **current and future** consumption

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Felicity at t: $M_t + V_t$

$$\underbrace{\sum_{r=1}^{t-1} \delta^{t-r} u\left(c_{t-r}\right)}_{\text{'memory' utility} = \mathsf{M}_t\left(\underline{\mathsf{c}}_{t-1}\right)} + u\left(c_t\right) + \underbrace{\hat{E}_t \sum_{\tau=1}^{T-t} \beta^{\tau} u\left(c_{t+\tau}\right)}_{\text{'expected' utility} = \underbrace{\beta \hat{E}_t\left[V_{t+1}\left(x_{t+1}; \underline{\mathsf{s}}_{t+1}\right)\right]}_{=V_t\left(x_t; \underline{\mathsf{s}}_t\right)}}$$

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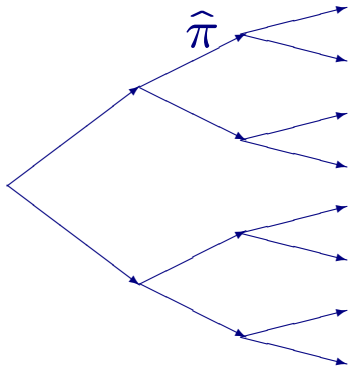
V_t = 'expected' utility from **current and future** consumption
M_t = 'memory' utility from **past** consumption

Stage 2: At each t choose c_t to maximize $V_t + M_t$ given subjective beliefs $\widehat{\pi}\left(s_t | \underline{\mathsf{s}}_{t-1}\right)$, state, x_t, and resource constraints.

Stage 1: At $t = 0$ assign optimal beliefs $\widehat{\pi}^{OE}\left(s_t|\underline{s}_{t-1}\right)$
(conditional probabilities to each branch of event tree)



that maximize

Lifetime well-being: $\mathcal{W} = E\left[\sum_{t=1}^{T} \beta^t \left(M_t + V_t\right)\right]$

Two-period example with consumption at $t = 2$

	$t = 1$	$t = 2$
t=1-self's felicity		$\beta\hat{E}[u(c_2)]$
t=2-self's felicity		$E[u(c_2)]$

Well-being: $\mathcal{W} = \beta\hat{E}[u(c_2)] + \beta E[u(c_2)]$

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A. It is optimal

"as if" interpretation

Scientific method

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Happiness may lead to better health or marriage prospects (Taylor and Brown (1988))

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C. Parents choose

Parents have the objective of optimal expectations

4. Applications

4a.) Portfolio choice

$\Rightarrow$ preference for **skewed** returns

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$\Rightarrow$ **"unexpected"** decline in consumption profile

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4d.) Optimal timing of a single task

$\Rightarrow$ Planning Fallacy, procrastination, context effect

4a. Portfolio choice

Setup:

1. Two period problem:
   invest in period 1, consume in period 2

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   S states, $\pi_s > 0$ for $s = 1$ to S, $Z_s < Z_{s+1}$, $Z_1 < 0 < Z_S$

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4. $c \geq 0$ in all states

Stage 2: Agent $\max_w \beta \sum_{s=1}^{S} \widehat{\pi}_s u\left(R + wZ_s\right)$

$$\text{FOC:} \quad 0 = \sum_{s=1}^{S} \widehat{\pi}_s u'\left(R + wZ_s\right) Z_s \qquad \Rightarrow w^*(\widehat{\pi})$$

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Stage 1: Choose $\widehat{\pi}_s$ to maximize lifetime well-being

$$\underbrace{\beta \sum_{s=1}^{S} \widehat{\pi}_s u\left(R + w^* Z_s\right)}_{\text{'expected' utility at } t=1} + \underbrace{\beta \sum_{s=1}^{S} \pi_s u\left(R + w^* Z_s\right)}_{\text{utility flow at } t=2}$$

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$$\text{FOC:} \quad \underbrace{\beta\left(u_S - u_{s'}\right)}_{\text{marginal 'expected' utility}} = \underbrace{\beta \sum_{s=1}^{S} \pi_s u'\left(R + w^* Z_s\right) Z_s \frac{dw^*}{d\widehat{\pi}_{s'}}}_{\text{marginal cost of distortion}}$$

Proposition Excess risk taking due to optimism

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(i) Agents are optimistic about states with high portfolio

(ii) Agents go even more long (short) than agent with RE **or** even in the opposite direction
if $E[Z] > 0$, then $w^{RE} > 0$, and $w^* > w^{RE}$ or $w^* < 0$;
if $E[Z] < 0$, then $w^{RE} < 0$, then $w^* < w^{RE}$ or $w^* > 0$;

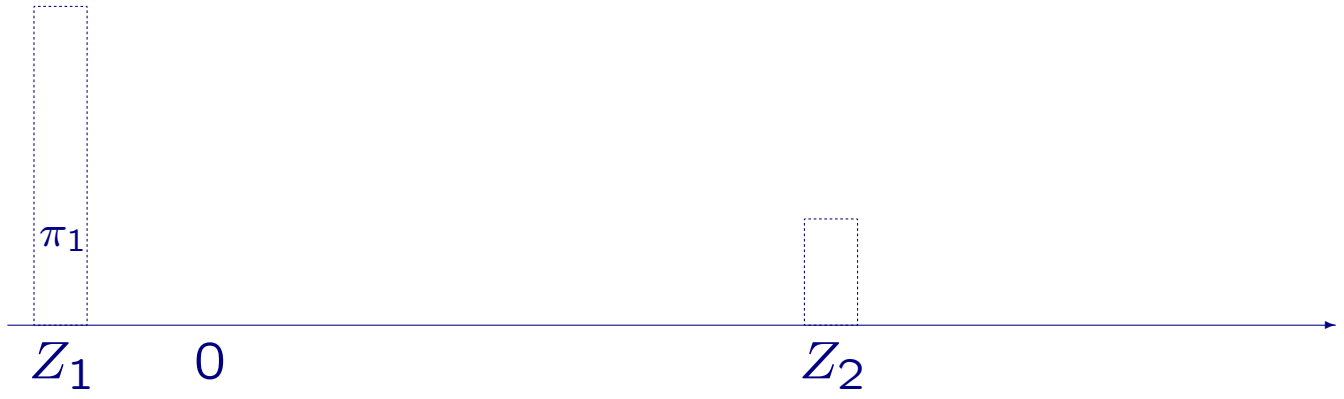
When Do agents buy asset with $E[Z] < 0$?

Empirical Phenomena: Preference for Skewness

- ◇ Horse race long shots: Golec and Tamarkin (1998)
- ◇ Lottery demand: Garrett and Sobel (1999)
- ◇ Security design: LYONs, EPNs, ELNs, Swedish lottery bonds

Setup:

- ◇ 2 states with payoffs: $Z_1 < 0 < Z_2$,
- ◇ hold mean $E[Z] < 0$ and variance $Var[Z]$ fixed
- ◇ the higher π_1, the more skewed (like lottery ticket)



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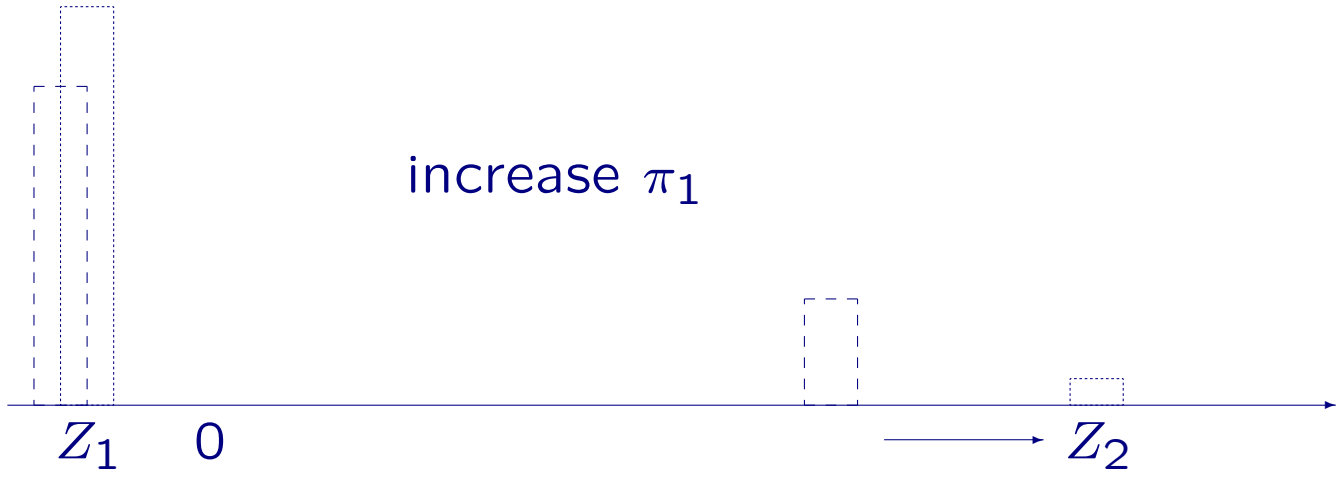

Proposition

There exists a $\underline{\pi}$ such that for all $\pi_1 > \underline{\pi}$ (i.e. if returns are sufficiently skewed), OE agent with an unbounded utility function goes long an asset even though its mean payoff is negative.

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Remarks:

- ◇ there is not much room to distort beliefs.
- ◇ shorting becomes very risky.

4b. General Equilibrium

Empirical Phenomena:

- ◇ betting & gambling
- ◇ high trading volume (stock and FX market)
  $\Leftarrow$ **endogenous** heterogenous prior beliefs
- ◇ home bias puzzle
- ◇ 'over-investment' in employer's stock

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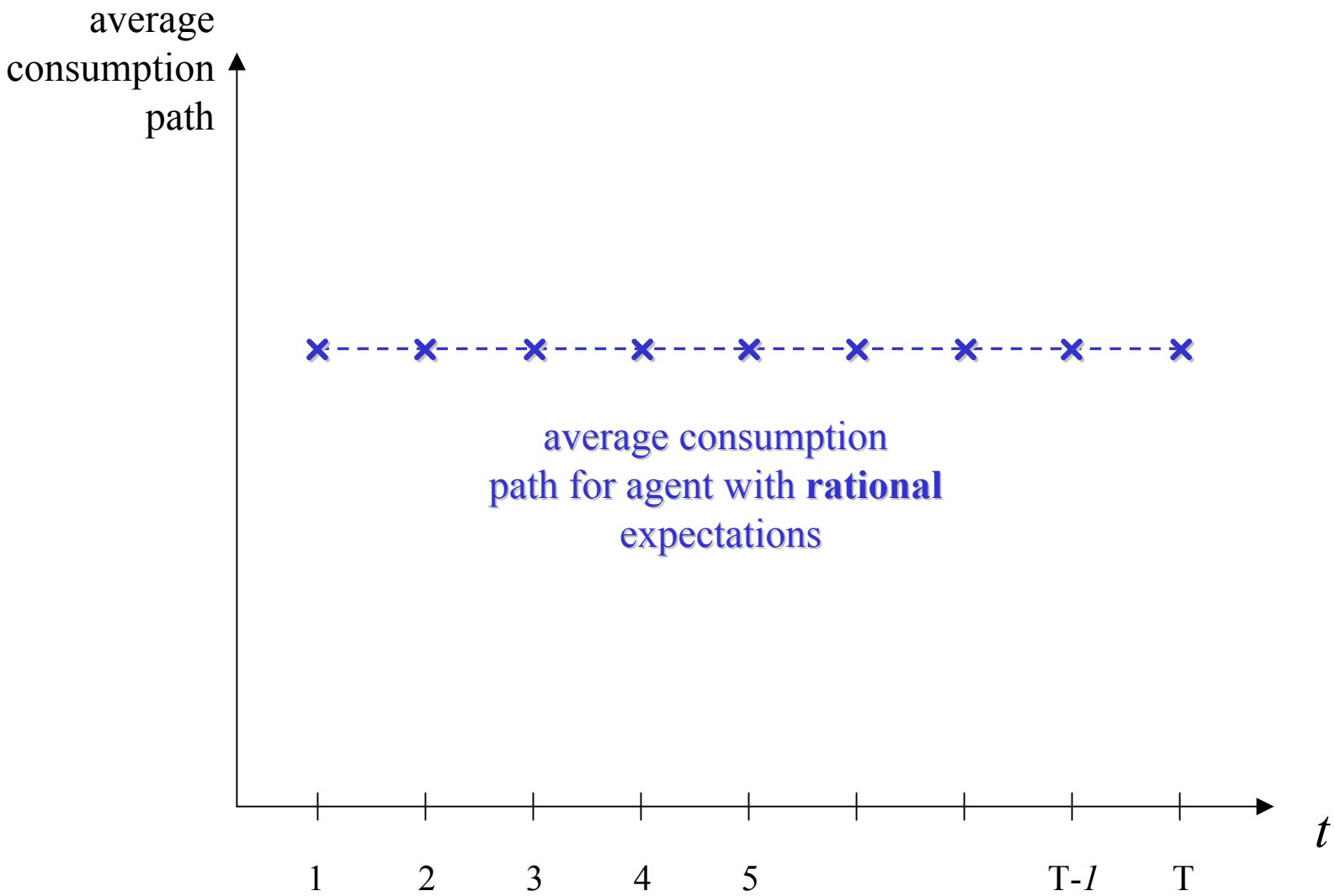
Proposition

(iii) **Heterogeneous prior beliefs**
In any equilibrium, each agent bets on a different state
i believes in "heads": $\hat{\pi}_1^i > \pi_1$, $\hat{\pi}_2^i < \pi_2$, $w^i < 0$, $c_1^i > c_2^i$, and
$-i$ believes in "tails": $\hat{\pi}_2^{-i} > \pi_2$, $\hat{\pi}_1^{-i} < \pi_1$, $w^{-i} > 0$, $c_2^{-i} > c_1^{-i}$

4c. Consumption and Saving

Empirical Phenomena:

- ◇ households **expect** upward sloping consumption profile (Barsky et al. 1997)
- ◇ **actual** average consumption growth is non-positive and profiles are concave (Gourinchas & Parker (2002))


average
consumption
path
average consumption
path for agent with **rational**
expectations
t
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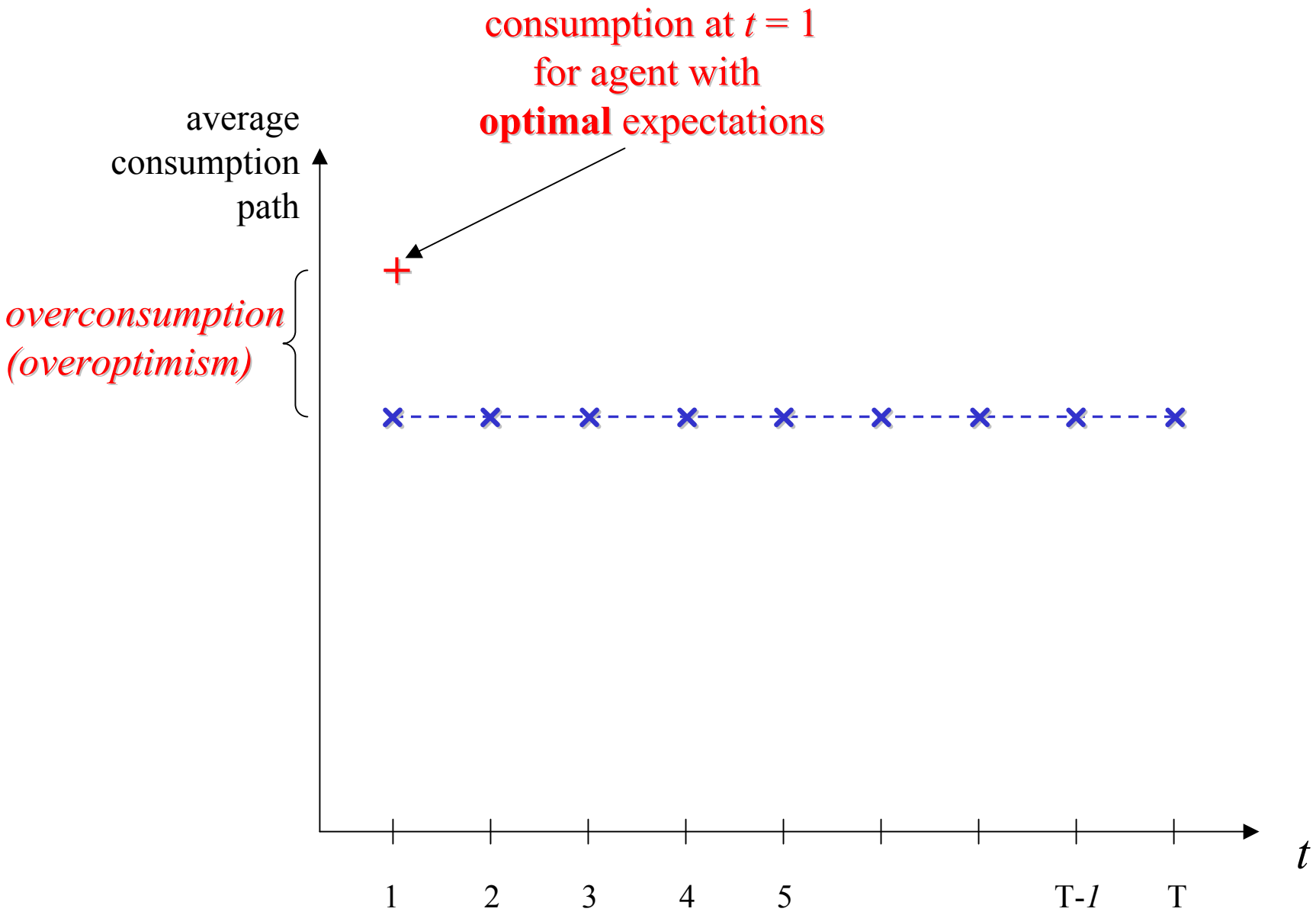
consumption at $t = 1$
for agent with
optimal expectations
average
consumption
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overconsumption
(overoptimism)
t
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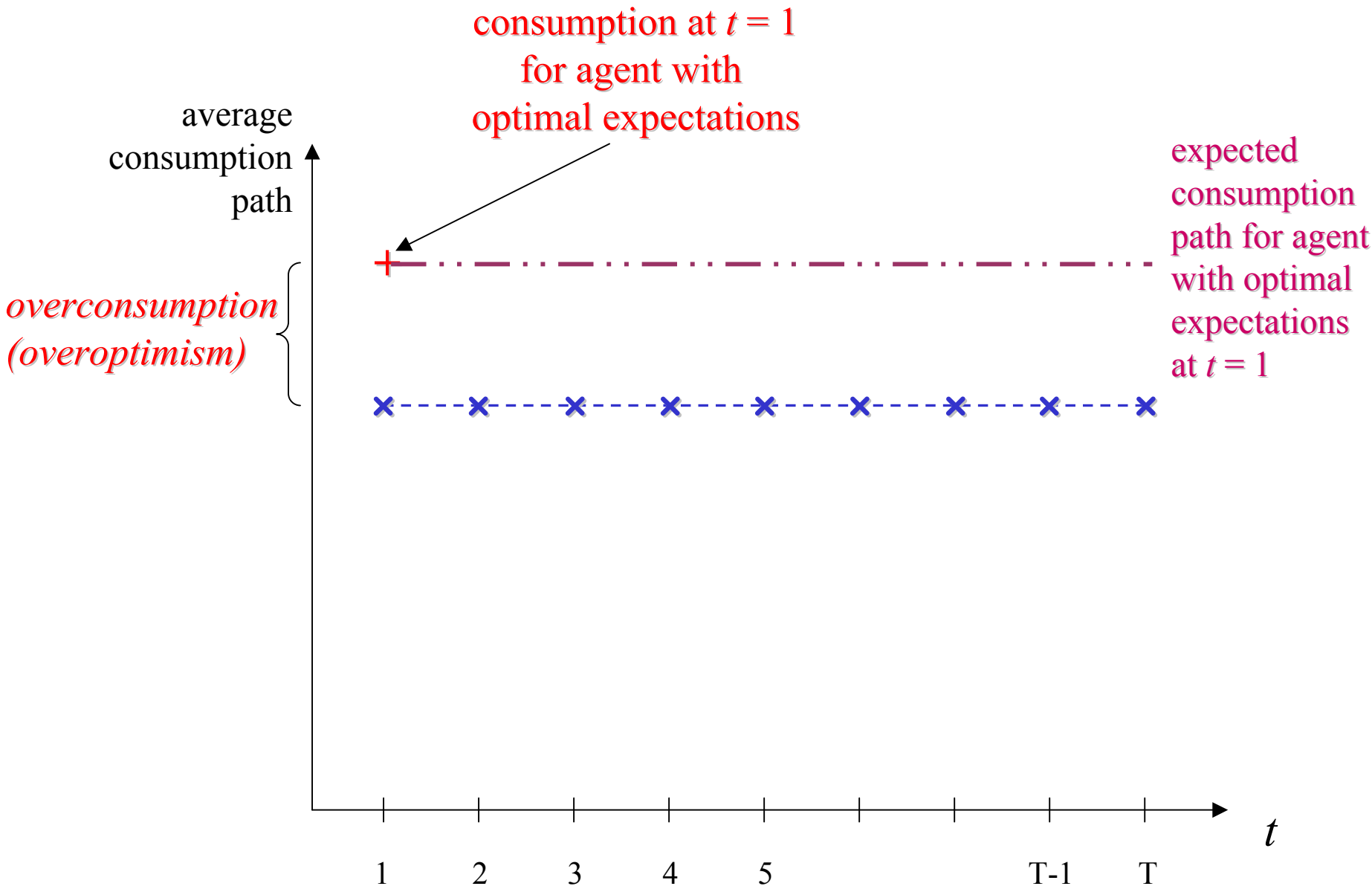
consumption at $t = 1$
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optimal expectations
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path for agent
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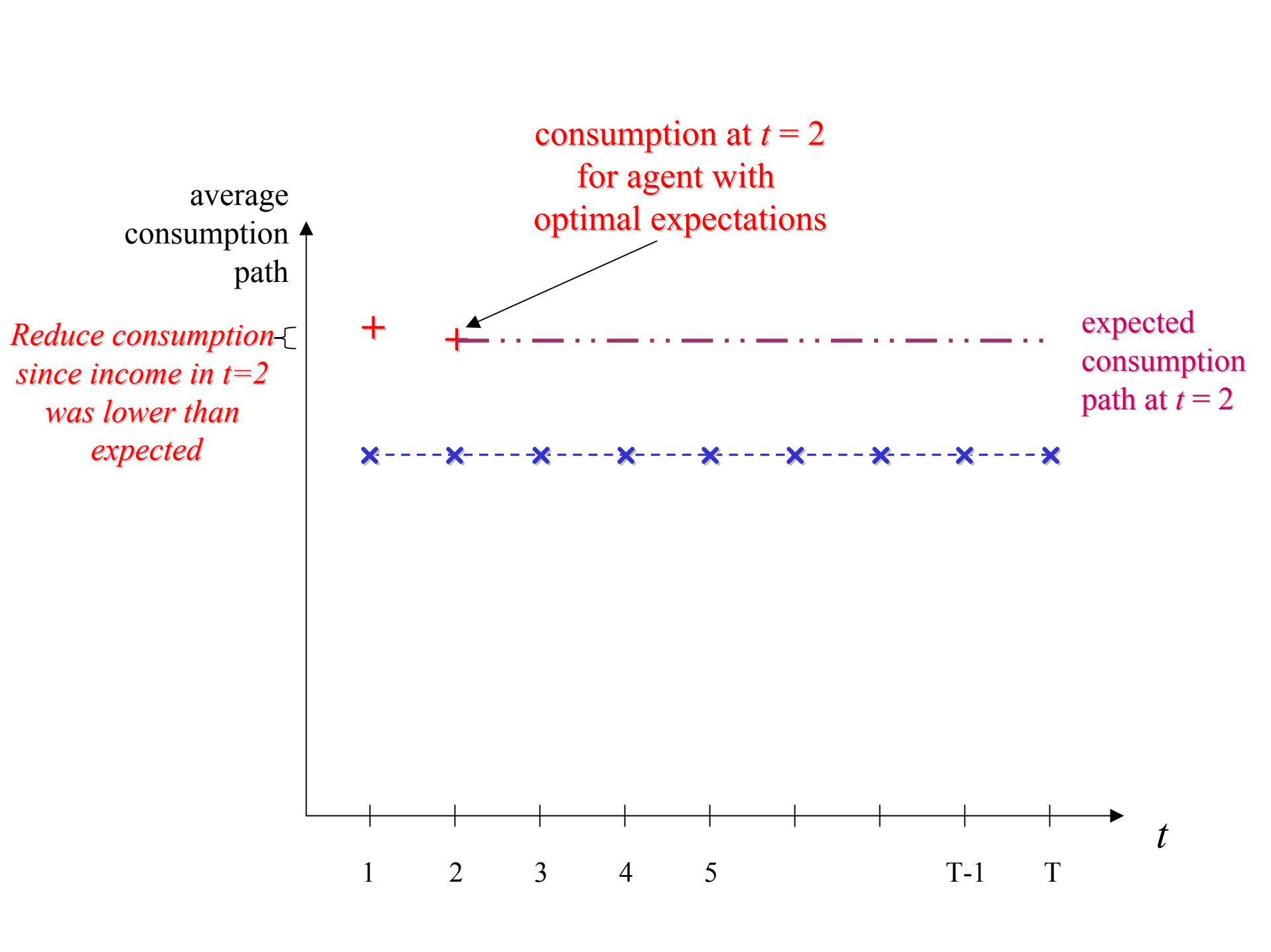
consumption at $t = 2$
for agent with
optimal expectations
average
consumption
path
Reduce consumption since income in t=2 was lower than expected
expected
consumption
path at $t = 2$
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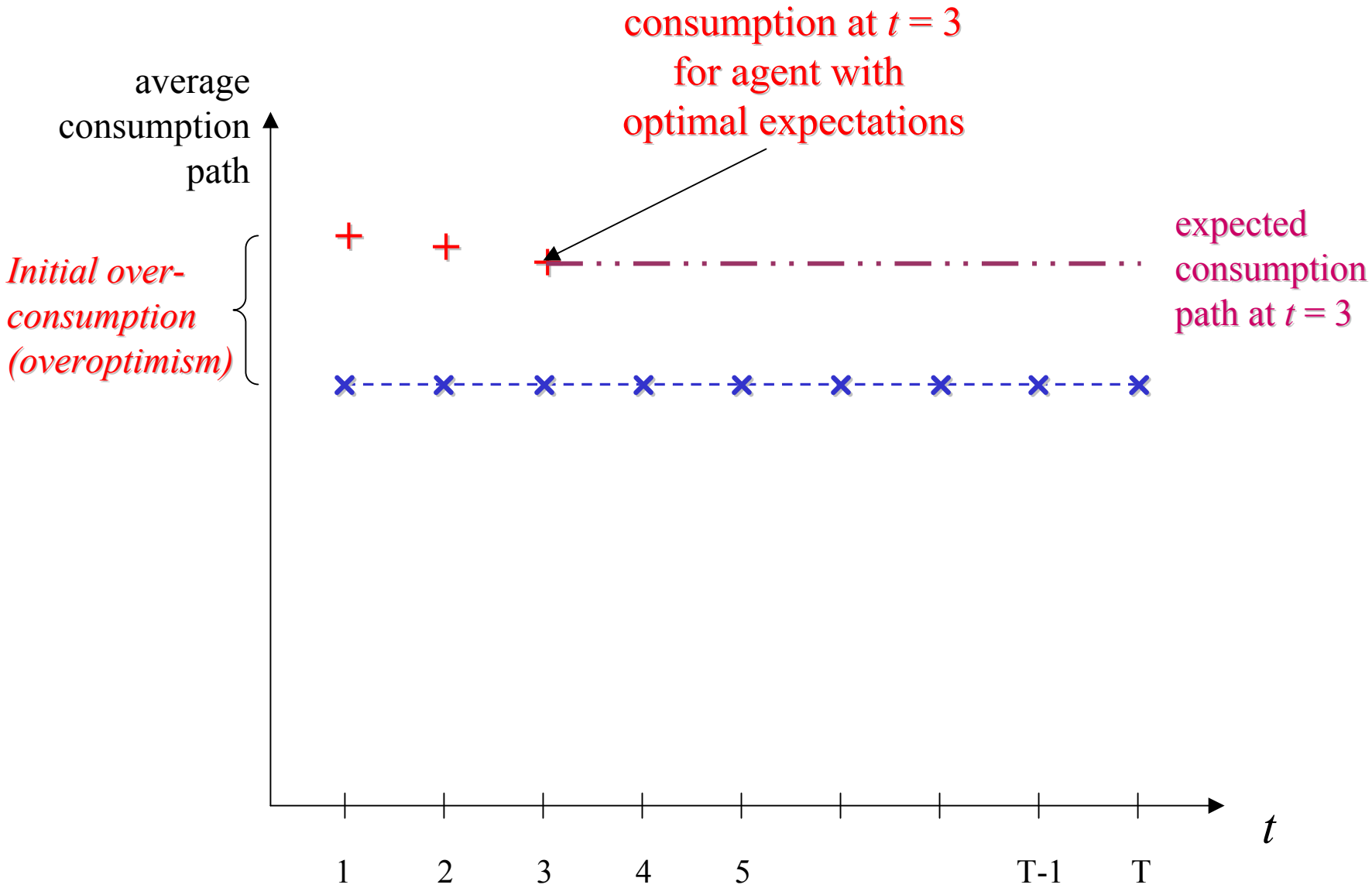
consumption at $t = 3$
for agent with
optimal expectations
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path
Initial over-
consumption
(overoptimism)
expected
consumption
path at $t = 3$
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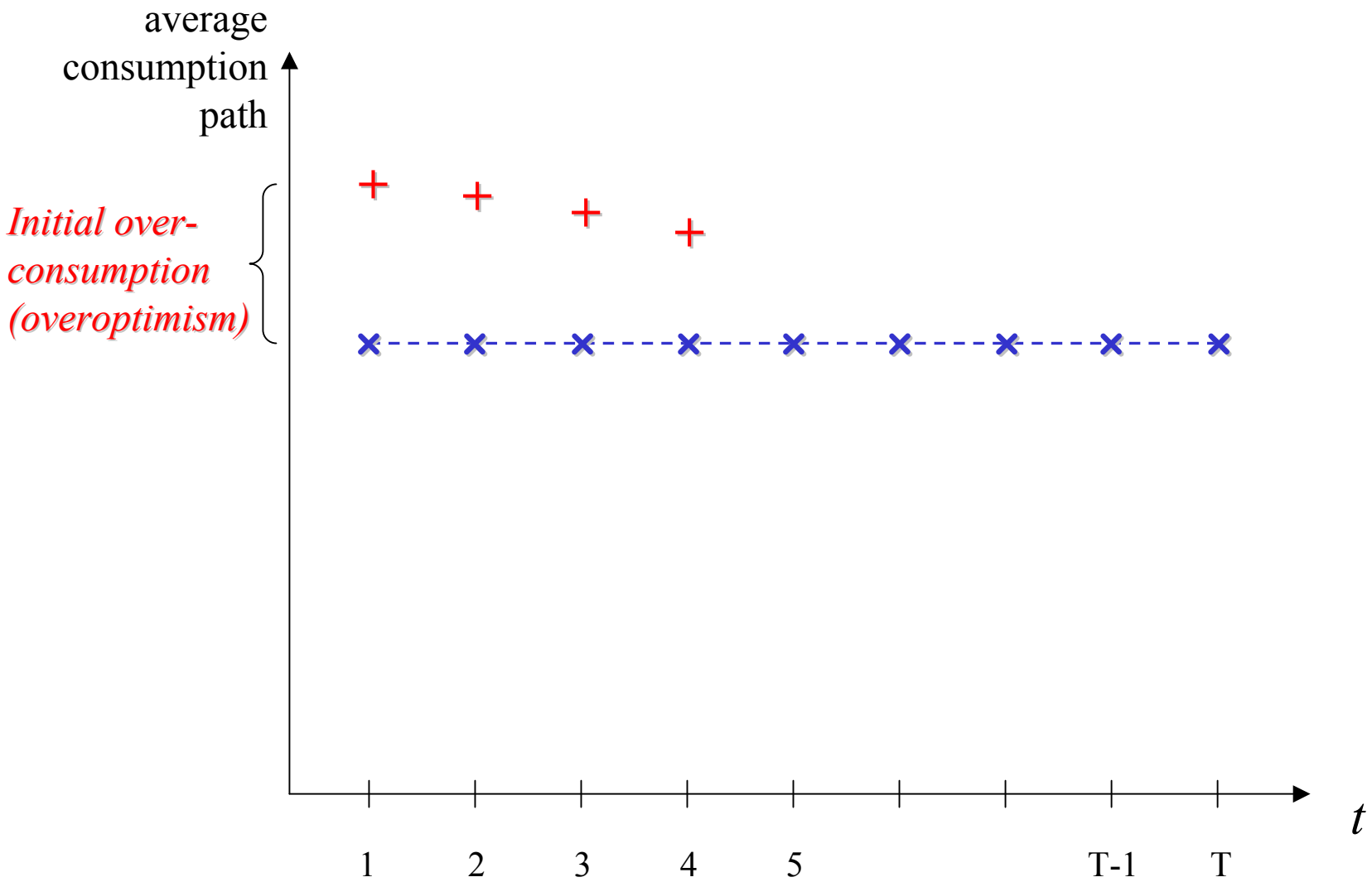
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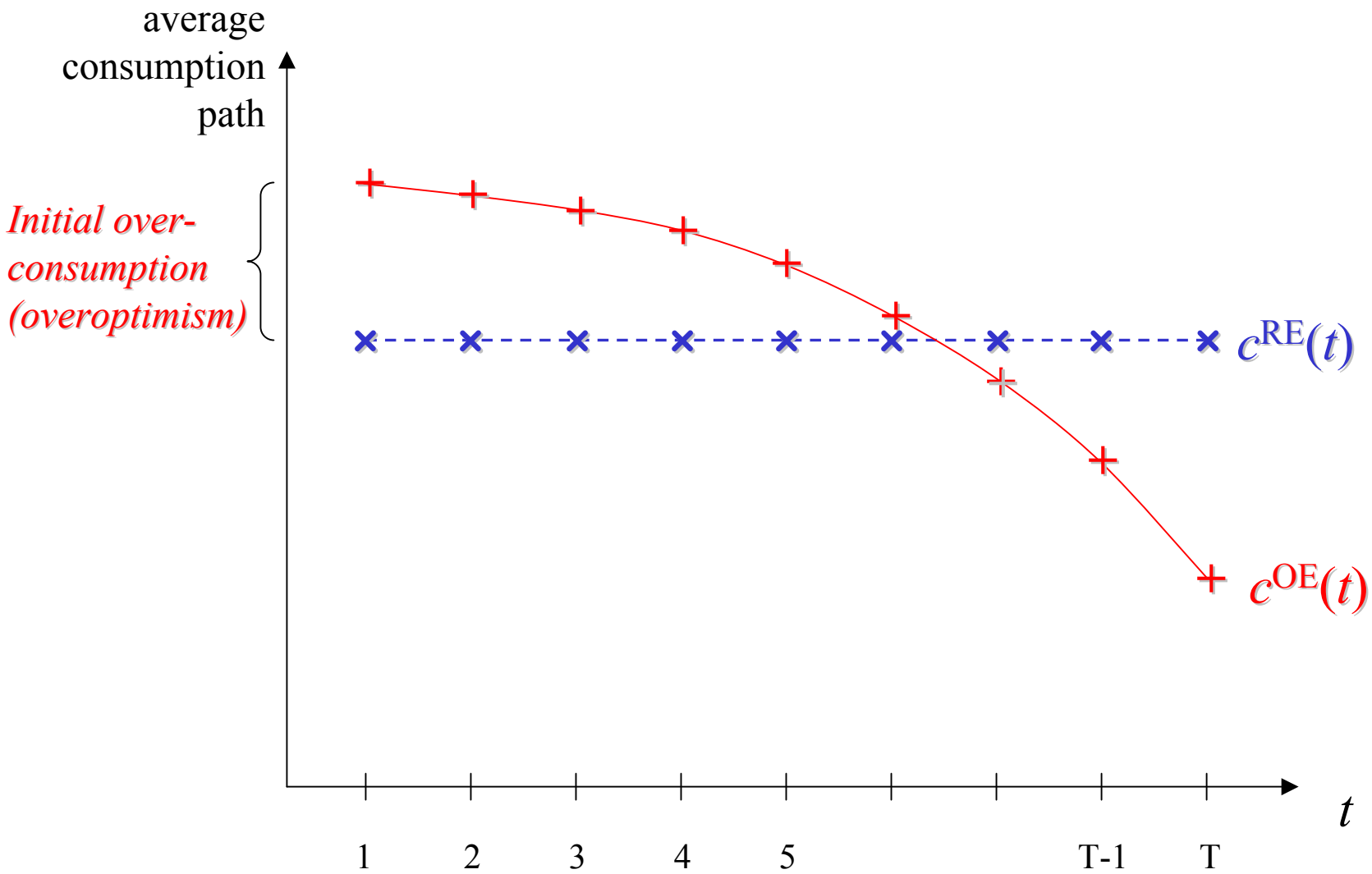
average
consumption
path
Initial over-
consumption
(overoptimism)
$c^{RE}(t)$
$c^{OE}(t)$
t
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T

4d. Optimal Timing of a Single Action

Empirical Phenomena:

- ◇ planing fallacy: underestimation of time to complete task
  - ◇ referee report
  - ◇ heavy briefcases for weekend
- ◇ additional options (even when not chosen) alters choice

Intuition:

- ◇ 'Optimal beliefs' underestimate how difficult it is to do a task tomorrow (relative to today)

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- ◇ Procrastination due to belief distortion and not preference distortion.

Conclusion

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1. Structural model of "priors"

- ◇ beliefs are most distorted, when decision errors are small
- ◇ endogenous heterogenous beliefs $\Rightarrow$ trade and speculation
- ◇ excess risk taking due to optimism
- ◇ preference for skewness
- ◇ realistic consumption profile

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2. Features of procrastination (due to belief distortions)

- ◇ intertemporal preference reversal, context effect