Predatory Short Selling*

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PRELIMINARY

Abstract

Financial institutions may be vulnerable to predatory short selling. When the stock of a financial institution is shorted aggressively, this can force the institution to liquidate long-term investments at fire sale prices to satisfy regulatory capital constraints. Predatory short selling can emerge in equilibrium when a financial institution is (i) close to its capital constraint (the vulnerability region) or (ii) violates its capital constraint even in the absence of short selling (the constrained region). The model provides a potential justification for temporary restrictions of short selling for vulnerable institutions. It also has implications regarding public disclosure of short positions and the design of regulatory constraints.

Keywords: Short selling, predatory trading, market manipulation, leverage constraints.

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The current credit crisis has led to a resurfacing of the discussion about short selling. As financial stocks fell sharply in the spring and summer of 2008, a number of banks, most notably Lehman Brothers and Morgan Stanley, blamed short sellers for their woes. In response, the SEC and a number of international financial regulators took measures against short selling; most significantly, some imposed temporary restrictions on the short selling of financial stocks, some even on short selling in general. The consensus view among economists, however, is that nothing is wrong with short selling. In fact, most would argue that short selling is a valuable activity—short sellers help enforce the law of one price, facilitate price discovery, and enhance liquidity. Moreover, short sale restrictions may lead to overvaluation and bubbles, and the ability to take short positions provides a valuable hedging tool to investors.[1] In the light of these findings, is there any economic justification to impose restrictions on short selling?

In this paper, we present a model of predatory short selling. We show that even though short selling activity is beneficial during 'normal times', at times of stress short sellers can destabilize a financial institution. Short selling in our model can become profitable as a form of trade-based manipulation: Through their trading, short sellers can trigger inefficient unwinding by a financial institution, to an extent that the value destruction from the fire sale makes the price decline triggered by a short seller self-fulfilling. The key mechanism that allows trade-based manipulation by short sellers is that financial institutions are subject to leverage constraints. Thus, when short sellers temporarily depress the stock price of a financial institution, this may cause financial institution to violate its leverage constraint and thus forces the institution to sell assets in order to repay debt. When long-term assets have to be unwound at a discount, this can make the short position of the short seller profitable.

Our model implies that financial institutions are vulnerable to attacks from predatory

[1] Diamond and Verrecchia (1987) show theoretically that a market with short-sale constraints incorporates information more slowly than a market in which short sales are not restricted. Empirical evidence for this finding can be found it Aitken, Frino, McCorry, and Swan (1998) and Danielsen and Sorescu (2001). For more detail on how short-sales constraints can lead to overvaluation, speculative trading and bubbles, see Miller (1977), Harrison and Kreps (1978), Scheinkman and Xiong (2003), and Hong and Stein (2003). Short positions are important hedging tools in a number of trading strategies, e.g. hedging options, convertible bonds, or market risk in long-short strategies.

short sellers when their balance sheets are weak. Predatory short selling occurs when a financial institution is either (i) close to its leverage constraint (*vulnerability region*) or (ii) violates the leverage constraint even in the absence of short selling (*constrained region*). In the vulnerability region there are two stable equilibria. In one equilibrium, no predatory short selling occurs. In that case the financial institution does not violate its constraint and can hold its long-term investments until maturity. In the second equilibrium, however, predatory short selling drives the financial institution into its constraint, causing a complete unwinding of its long-term asset holdings. In the constrained region there is a unique predatory equilibrium in which the financial institution unwinds all its asset holdings.

Comparing a regime with short sellers to one with short-sale restrictions shows that during 'normal times', when financial institutions are well capitalized, the equilibrium stock price and the investment policy of the financial institution is not affected by the presence of short sellers. In fact, this is the region in which short sellers exclusively fulfill their useful roles of providing liquidity and preventing overvaluation and bubbles. This suggests that restricting short selling during normal times is likely to have undesired negative consequences without providing benefits for the functioning of markets. This changes in the vulnerability region and in the constrained region. Here short sellers can cause the inefficient unwinding of the financial institutions' long-term investment. This is particularly striking in the vulnerability region, where short sellers can cause a complete unwinding even though the financial institution would have satisfied its leverage constraint in the absence of short sellers. While in the constrained region the financial institution needs to unwind part of its investments also in the absence of short sellers, also here predatory short sellers exacerbate the situation, forcing a complete rather than partial unwinding by the financial institution. The model thus provides a potential justification for temporary short sale restrictions for financial institutions at times when their balance sheets are weak.

The model has a number of additional implications for the regulatory response to short selling. First, since in the vulnerability region there are multiple equilibria, coordination among short sellers is important to drive down a financial institution. This means that full and timely disclosure of all short positions, which has been advocated to make the actions of short sellers more transparent, may in fact make it *easier* for short sellers to prey on vulnerable companies. Second, to the extent that short sellers force financial institutions to unwind as a response to violations of capital requirements, it may make sense to calculate constraints using averages that are taken over a period of time. That way, temporary price dislocations caused by short sellers are less likely to force financial institutions to unwind their long-term investments in a fire sale.

The paper is related to the theoretical literatures on short selling, market manipulation, and predatory trading. Goldstein and Gümbel (2008) provide an asymmetric information model, in which a feedback loop to real investment decisions allows a short seller to make a profit even in the absence of fundamental information. While their paper focuses on the effect short sellers can have on future real investment decisions, we focus on the effect they can have on financial institutions by forcing the unwinding of existing long-term investments. Allen and Gale (1992) provide a model in which a non-informed trader can make a profit if investors think the manipulator may be an informed trader. They label this type of manipulation 'trade-based manipulation', contrasting it with 'action-based' or 'information-based' manipulation.[2] Brunnermeier and Pedersen (2005) provide a model in which a predatory trader can exploit another trader's need to unwind. Our setup is related to their paper since, as in their setup, predatory traders (in this paper: short sellers) exploit the financial vulnerability of a financial institution. While in their paper the vulnerability stems from the need to unwind a position, in our paper the vulnerability results from the leverage restriction on the financial institution.

The remainder of the paper is structured as as follows. Section 1 gives a brief summary of regulatory response to short selling. Section 2 presents the model. Section 3 provides a discussion of the model's policy implications and empirical predictions. Section 4 concludes.

[2] Other papers that consider manipulative trading strategies include Allen and Gorton (1992), Benabou and Laroque (1992), Kumar and Seppi (1992), Gerard and Nanda (1993), Chakraborty and Yilmaz (2004), and Brunnermeier (2005).

1 Summary of regulatory response to short selling

As a result of the financial market turmoil in 2008, the SEC and a number of international financial market regulators put in effect a number of new rules regarding short selling. In July the SEC issued an emergency order banning so-called "naked" short selling[3] in the securities of Fannie Mae, Freddie Mac, and primary dealers at commercial and investment banks. In total 18 stocks were included in the ban, which took effect on Monday July 21 and was in effect until August 12.

On September 19 2008, the SEC banned *all* short selling of stocks of financial companies. This much broader ban initially included a total of 799 firms, and more firms were added to this list over time. In a statement regarding the ban, SEC Chairman Christopher Cox said, "The Commission is committed to using every weapon in its arsenal to combat market manipulation that threatens investors and capital markets. The emergency order temporarily banning short selling of financial stocks will restore equilibrium to markets. This action, which would not be necessary in a well-functioning market, is temporary in nature and part of the comprehensive set of steps being taken by the Federal Reserve, the Treasury, and the Congress." This broad ban of all short selling in financial institutions was initially set to expire on October 2, but was extended until Wednesday October 9, i.e. three days after the emergency legislation (the bailout package) was passed.

In addition to measures taken by the SEC, a number of international financial regulators also acted in response to short selling. On September 21 2008, Australia temporarily banned all forms of short selling, with only market makers in options markets allowed to take covered short positions to hedge. In Great Britain, the FSA enacted a moratorium on short selling of 29 financial institutions from September 18 2008 until January 16 2009. Also Germany, Ireland, Switzerland and Canada banned short selling of some financial stocks, while France, the Netherlands and Belgium banned naked short selling of financial companies.

Of course, measures against short selling are not new to this crisis. In response to the

[3]In a naked short-sale transaction, the short seller does not borrow the share before entering the short position.

market crash of 1929, the SEC enacted the uptick rule, which restricts traders from selling short on a downtick. In 1940, legislation was passed that banned mutual funds from short selling. Both of these restriction were in effect until 2007. Going back even further in time, the UK banned short selling in the 1630s in response to the Dutch tulip mania.

2 Model

We consider a simple model with three periods, $t = 0, 1, 2$. At $t = 0$, a financial institution has invested in X units of a long-term asset. The financial institution has also taken out debt with face value D_0. Debt is due to be paid off at $t = 2$, but it can be paid back at $t = 1$ if the financial institution needs to reduce its leverage. We take both the initial position in the risky asset as well as the initial debt outstanding as given. Most of our analysis will focus on $t = 1$. We assume that at $t = 1$ the expectations is that the long-term asset will pay off R at $t = 2$. The long-term asset can be liquidated at $t = 1$, but early liquidation is subject to a discount; the liquidation value at $t = 1$ is given by δR, where $\delta < 1$. This means that early liquidation is inefficient. For simplicity we assume that the financial institution holds no cash, but the model could be straightforwardly extended to allow for cash holdings. Since none of the main finding change we focus on the simple case without cash.

Leverage Constraint. Key to the model is that the financial institution is subject to a capital constraint. In the model, this constraint takes the form of a leverage constraint, i.e. debt as a fraction of debt plus equity cannot exceed a critical amount $\gamma \in [0, 1]$, i.e.

$$\frac{D}{D+E} \leq \gamma. \tag{1}$$

The leverage constraint (1) captures regulatory capital requirements that financial institutions have to comply with. In particular, it captures the fact that capital constraints require financial institutions to reduce leverage in response to drops in market valuation. We assume that this constraint is calculated by directly using the market value of the financial institution's equity, which means that a drop in the financial institution's market valuation directly translates into an increase in leverage. Alternatively one could also assume that it is the market valuation of the assets on the financial institution's balance sheet that are used to determine its leverage constraint. This would result in a similar model, with the added complication that short sellers would short both the financial institution's equity and the assets the financial institution holds on its balance sheet (or an index that affects the valuation on the financial institution's balance sheet, e.g. the ABX).[4] What is important for the model is that—independent of its particular form—the leverage constraint implies that when equity falls below a certain value, the financial institution has to unwind some of the long-term asset and repay debt. As we will show, in certain circumstances this can make the financial institution vulnerable to short sellers in the equity market.

Equity Market. At $t = 1$ equity of the financial institution is traded in a financial market. This financial market has two types of investors, long-term investors and short sellers. The long-term investors (we assume that there is a mass-one continuum of them) offer demand schedules to the short sellers. This means that they form a residual demand curve that short sellers can sell into. Upon observing the demand schedules offered by the long-term investors, the short sellers decide whether to take a position in the stock. While the focus in this paper is on short selling, one may more generally think of the short sellers as arbitrageurs who can take both long and short positions. Short sellers are assumed to be competitive, i.e. they take prices as given and make zero profits in equilibrium.

We focus on the interaction in the equity market at the intermediate period, $t = 1$. At time $t = 1$, the two types of players, long-term investors and short sellers, interact in the following way. Long-term investors choose the slope and intercept of a demand schedule that they offer to the short sellers. We denote the intercept by $\overline{P}$ and the slope by λ. Formally the long-term investors' action space is thus given by the pair $(\overline{P}, \lambda) \in \mathbb{R} \times \mathbb{R}^{+}$. Note that by assumption $\lambda > 0$, i.e. the residual demand curve for the stock is downward sloping.

[4] Alternatively, banks may be forced to keep their leverage below a certain level because they know that otherwise they will be regarded as too risky by investors, or because drops in their market value of equity lead to ratings downgrades, which force the bank to unwind assets to comply with collateral requirements.

However, as we will argue below, the slope of the demand curve can be arbitrarily small.[5] Upon observing the demand schedules offered to them, the short sellers decide how much of the stock they want to sell short. Their action space is thus the size of their short position, $S \in \mathbb{R}$. Negative S denotes a long position. Putting this together, we can write price of the financial institution's equity at $t = 1$ as

$$\widetilde{P} = \overline{P} - \lambda S. \tag{2}$$

Equilibrium. The equilibrium amount of short selling will be determined by a zero profit condition for both long-term investors and short sellers. This means that the equity market price at $t = 1$, $\widetilde{P} = \overline{P} - \lambda S$ must be a rational prediction of the value of equity at $t = 2$, when the long-term investment pays off and equity investors receive their payoff. The equilibrium condition is thus that

$$\widetilde{P} = P, \tag{3}$$

where P is the payout to equity holders at $t = 2$.

2.1 Benchmark Case without Leverage Constraint

We first solve a benchmark model without the leverage constraint. As we will see, in this setup the short sellers may serve a role in ensuring that the financial institution's equity is fairly priced, but the short sellers' actions do not have any influence on the fundamental value of the financial institution. This means that in the absence of the leverage constraint, predatory short selling cannot occur in equilibrium.

Lemma 1 *When financial institutions are not subject to the leverage constraint, predatory short selling does not occur in equilibrium.*

[5] There are a number of ways to justify a downward sloping demand curve. For example, our assumption may capture in reduced form that long-term investors are risk averse and need to be compensated for risk that hey hold in equilibrium. The downward sloping demand curve may also the be the result of information asymmetries, as in Kyle (1985), that are not modeled explicitly here.

Proof. To see this, consider what happens when the equity market opens at $t = 1$. The fundamental value of the financial institutions equity is given by $XR - D_0$. This is the payoff that equity holders will receive at $t = 2$, after creditors have been paid off. This means that the equilibrium condition (3) can be rewritten as $\overline{P} - \lambda S = XR - D_0$. ■

Lemma 1 implies that when the long-term investors offer a demand schedule to the short sellers at $t = 1$, competition among short sellers will always ensure that the final price is equal to fundamental value. This is illustrated in figure 1. The left panel shows the case in which the intercept chosen by long-term investors is larger than the fundamental value of the equity, i.e. $\overline{P} > XR - D_0$. In that case short sellers will take a short position $S > 0$, and since competition ensures that short sellers will make zero profit in equilibrium, the equilibrium short position S will be such the final price will be equal to fundamental, i.e. $\widetilde{P} = XR - D_0$. The right panel shows the opposite case, in which the intercept chosen by the long-term investors is below fundamental value. In that case short sellers end up taking a long position to ensure that the equity if fairly priced. Of course, when the intercept chosen by the is equal to fundamental value, i.e. $\overline{P} = XR - D_0$, short sellers do not take a position in equilibrium, i.e. $S = 0$. This is depicted in figure 2.

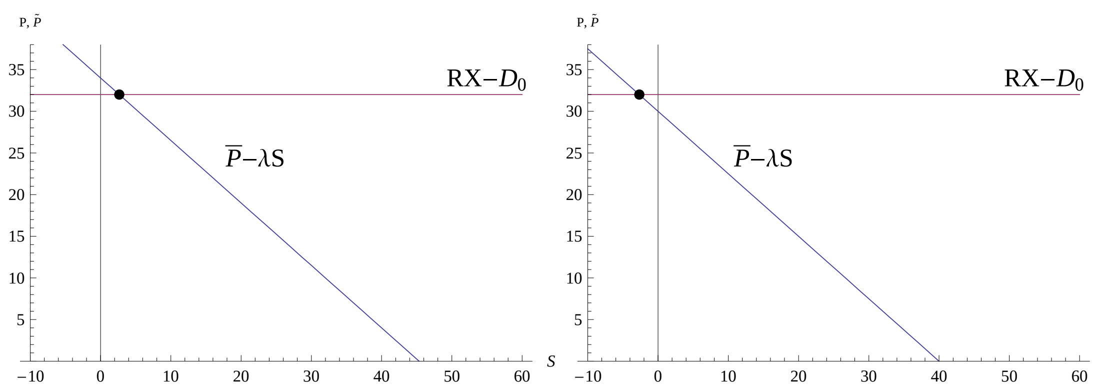

Figure 1: **Impact of short selling without leverage constraint.** In the left panel, the intercept chosen by long-term investors is larger than the fundamental value of equity, $\overline{P} > XR - D_0$. In that case short seller take a short position ensuring that $\widetilde{P} = \overline{P} - \lambda S = XR - D_0$. In the right panel, the intercept is less than fundamental value, such that the short sellers drive price up by taking a long position. The parameters in this example are: $X = 10, R = 10, D = 68, \lambda = 0.75$.

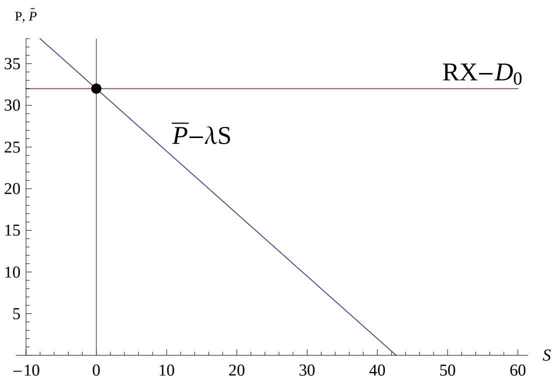

Figure 2: **Impact of short selling without leverage constraint.** When the intercept chosen by long-term investors is equal to fundamental value, i.e. $\overline{P} = XR - D_0$, short sellers never take a position in the absence of leverage constraints. Remaining parameters: $X = 10, R = 10, D = 68, \lambda = 0.75$.

For the rest of the paper we will focus on this third case, in which the intercept chosen by long-term investors reflects the fundamental value of equity in the absence of predatory short selling, i.e. $\overline{P} = XR - D_0$. Focusing on this case means that in the absence of leverage constraints, short sellers would never take a position. It also means that that the only form of short selling that can occur in equilibrium is predatory short selling, i.e. short selling that exploits the constraint of the financial institution. We will show in the next section how the leverage constraint can make a predatory short sale profitable, and under what condition predatory short selling equilibria exist. As we will also show below, focusing on $\overline{P} = XR - D_0$ is without loss of generality, the analysis generalizes to $\overline{P} \neq XR - D_0$.

2.2 Introducing the Leverage Constraint

We now introduce leverage constraint. Recall the the leverage constraint (1) requires the financial institution to keep leverage, i.e. debt as a fraction of debt plus equity, below a critical level, $\frac{D}{D+E} \leq \gamma$. When the leverage constraint is violated at $t = 1$, the financial institution must thus unwind some of the long-term asset and repay debt, such that the constraint is satisfied.

Denote the number of units of the long-term asset the financial institution has to sell at $t = 1$ by $\Delta X(S)$. If the financial institution has to unwind $\Delta X(S)$ units of the long-term asset at $t = 1$ to repay debt, this leads to an equity payout at time $t = 2$ of

$$P = \max\left[XR - D_0 - (1-\delta)R\Delta X(S), 0\right] \tag{4}$$

The reduction in equity value from unwinding, $(1-\delta)R\Delta X(S)$, stems from the fact that the long-term asset can only be sold at a discount. Using equation (4) we can thus rewrite the equilibrium conditions (3) as

$$\overline{P} - \lambda S = \max\left[XR - D_0 - (1-\delta)R\Delta X(S), 0\right]. \tag{5}$$

How much does the financial institution have to unwind? In order to find potential equilibria, we need to determine how much the financial institution needs to unwind at $t = 1$. To determine the value of $\Delta X(S)$, first notice that when the leverage constraint is not violated at time $t = 1$, the financial institution does not have to unwind any of its long-term investments. In that case, clearly $\Delta X(S) = 0$.

What happens when the constraint is violated at $t = 1$? When the equity value at $t = 1$, including the temporary price effects of short selling is such that the constraint is violated, i.e.

$$\frac{D_0}{D_0 + \overline{P} - \lambda S} > \gamma,$$

the financial institution has to sell $\Delta X(S)$ units of the long-term asset and repay debt in order to satisfy the constraint. The amount the financial institution has to unwind is determined by the following condition:

$$\frac{D_0 - \Delta X(S)\delta R}{D_0 - \Delta X(S)\delta R + \overline{P} - \lambda S} = \gamma, \tag{6}$$

which can be solved for $\Delta X(S)$. Combining the resulting expression with the facts that no liquidation is necessary when the constraint is satisfied, and that the maximum amount that can be liquidated is X, yields the following result:

Lemma 2 *The amount the financial institution needs to liquidate in the presence of the leverage constraint* (1) *and in the presence of short sellers that take an aggregate short position* S *is given by*

$$\Delta X(S) = \begin{cases} 0 & \textit{if } \frac{D_0}{D_0+\overline{P}-\lambda S} \leq \gamma \\ \min\left[\frac{(1-\gamma)D_0-\gamma(\overline{P}-\lambda S)}{R(\delta-\gamma\delta)}, X\right] & \textit{if } \frac{D_0}{D_0+\overline{P}-\lambda S} > \gamma \end{cases}. \tag{7}$$

Proof. In the case that the constraint is violated, the result follows directly from solving (6) for $\Delta X(S)$. Combining this with the fact that no liquidation occurs when the constraint is satisfied, and that the maximum amount that can be liquidated is X, yields the result. ■

We can rewrite equation (7) to show that the amount that needs to be liquidated in the case that the constraint is violated has two parts, $\Delta X(S) = X\frac{\frac{D_0}{XR}-\gamma}{\delta-\gamma\delta} + \gamma\frac{XR-(D_0+\overline{P}-\lambda S)}{R(\delta-\gamma\delta)}$. The first part, $X\frac{\frac{D_0}{XR}-\gamma}{\delta-\gamma\delta}$, is the amount the financial institution would have to unwind in the absence of predatory short sellers and with their equity priced at $XR - D_0$. Intuitively, this part depends on how far the financial institution is away from the constraint, i.e. $\frac{D_0}{XR} - \gamma$. The second part of the expression, $\gamma\frac{XR-(D_0+\overline{P}-\lambda S}{R(\delta-\gamma\delta)}$ depends on the short sellers' action. A larger short position forces the financial institution to unwind more to satisfy the constraint. Naturally, the incremental increase in ΔX as a result of more short selling is larger, the more short sales drive down the financial institution's share price, i.e. the larger the slope of the demand curve, λ.

Figure 3 illustrates what happens once we introduce the leverage constraint. When the financial institution is forced to unwind some of the long-term asset, the fundamental value of its equity drops. This is because the long-term asset has to be sold at a discount to fundamental value; when sold at $t = 1$ it yields δR rather than R. This means that in contrast to the case without the leverage constraint, when the short sellers take a large enough short position, the fundamental value of the financial institution drops as a result of this forced unwinding. Thus, rather than being a straight line as in figures 1 and 2, the

$t = 2$ fundamental value of the financial institution's equity now has a kink. To facilitate comparison with the benchmark case, the dotted line indicates the fundamental $t = 2$ value of equity in the absence of the leverage constraint.

Recall the equilibrium condition, $\widetilde{P} = P$. This condition implies that potential equilibria are intersections of the two lines in figure 3, i.e. points where the price in the equity market at $t = 1$ rationally reflects the fundamental value of the equity of the financial institution at $t = 2$. Turn first to the left panel of figure 3. In the illustration, long-term investors choose $\overline{P} = XR - D_0$.[6] Since the two lines only intersect once, the only equilibrium remains the one without short selling. Even though the short sellers can drive down the fundamental value of the financial institution by forcing it to liquidate some of its long-term investments, they cannot do so profitably. The losses they incur by driving down the price of the financial institution at $t = 1$ outweigh the gains to short sellers from value destruction. This is the case since the liquidation value of the long-term asset, which is determined by δ, is sufficiently large. Thus the value destruction on response to a violation of the leverage constrained is small, such that a predatory short sale is not profitable. The unique equilibrium is the one where $P = XR - D_0$. When $\overline{P} = XR - D_0$ this implies that the equilibrium amount of short selling is $S = 0$.

The right panel of figure 3, on the other hand, shows what happens when δ is sufficiently small. In this case, in addition to the equilibrium without short selling, two further equilibria emerge, as indicated by the two additional black dots in the figure. Both of these two additional equilibria involve predatory short selling, i.e. the decrease in the financial institution's equity value at $t = 1$ causes unwinding of long-term asset holdings that allow short sellers to break even. As is usually the case, the middle equilibrium is not stable, i.e. a small perturbation would lead to migration to either of the two other (stable) equilibria.

Equilibrium prices are independent of λ and $\overline{P}$. An important thing to notice is that as long as short sellers are not restricted in the number of shares they can short,

[6]As we show below, this assumption is not crucial. We focus on this case since it allows us to focus exclusively on predatory short selling.

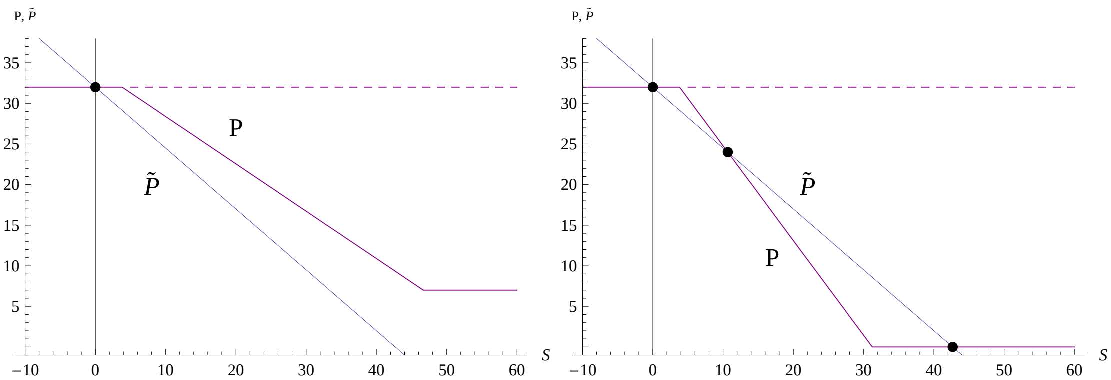

Figure 3: **Introducing the leverage constraint.** When the leverage constraint is introduced, a sufficiently large short position forces the financial institution to unwind. In the left panel the loss to the financial institution from unwinding the long asset is not large enough to make the short sale profitable ($\delta = 0.75$). The only equilibrium is the one in which no short selling occurs. In the right panel, on the other hand, we see that when the losses from liquidating the long-term asset are large enough two equilibria emerge with positive amounts of short selling emerge in addition to the no-short-selling equilibrium ($\delta = 0.6$). The middle equilibrium is unstable. The remaining parameters are: $X = 10, R = 10, D = 68, \lambda = 0.75$

the equilibrium prices are independent of the particular $\overline{P}$ and λ chosen by the long-term investors. This means that while there are many equilibria with involving different combinations of $\overline{P}$, λ and S, the equilibria are isomorphic in terms of equilibrium prices. This means that while focusing on $\overline{P} = XR - D_0$ allows us to focus exclusively on predatory short selling, equilibrium prices and the existence of predatory short selling equilibria do not depend on this assumption.

Lemma 3 *When short sellers are unconstrained in the size of the short position they take, the equilibrium prices and the amount that has to be unwound by the financial institution is independent of λ and $\overline{P}$.*

Proof. This result comes from the fact that, in equilibrium, a change in either $\overline{P}$ or λ will be exactly offset by a corresponding change in the equilibrium level of the short position S, such that the equilibrium condition $\widetilde{P} = P$ is satisfied. Equilibrium prices and the equilibrium amount that has to be unwound by the financial institution thus remain unaffected. ■

This independence result is illustrated in figure 4. Lemma 3 is convenient since it allows us to classify equilibria by just looking at equilibrium prices and the amount of the long-term asset that has to be unwound by the financial institution.

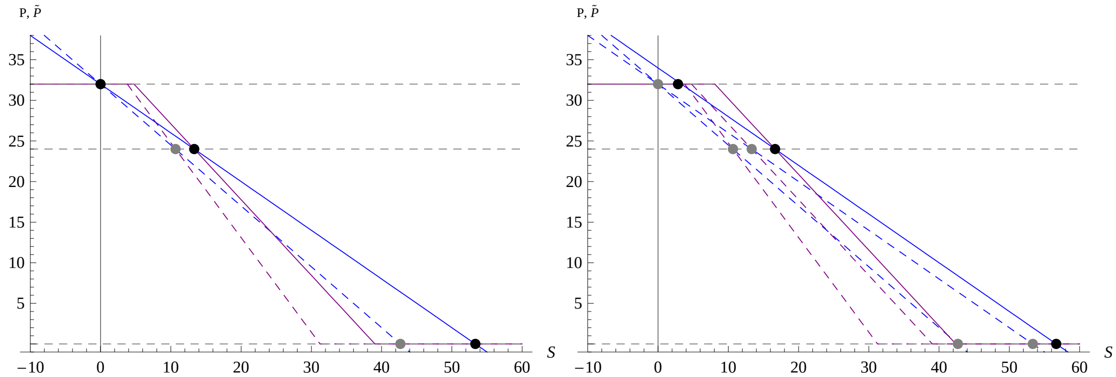

Figure 4: **Equilibrium prices are independent of $\overline{P}$ and λ.** The left panel shows that when λ is decreased from 0.75 (dashed line) to 0.6 (solid line), the equilibrium amount of short selling changes, but equilibrium prices remain the same. The right panel shows that when in addition also the intercept $\overline{P}$ is increased from 32 to 34, the equilibrium prices again remain unchanged. The remaining parameters are $R = 10, X = 10, \delta = 0.6, \gamma = 0.7, D = 68$.

Overview of Equilibria. We are now in a position to summarize the classes of equilibria in the equity market at $t = 1$. In the proposition, we focus on the case where $\delta < \gamma$, which is the case depicted in the right panel of figure 3.[7]

Proposition 1 *In the presence of the leverage constraint, when $\delta < \gamma$ we distinguish three regions.*

1. *Well-capitalized region: When financial institutions are well capitalized, $R > \frac{D_0}{\delta X}$, there is a unique equilibrium in which the financial institution does not have to unwind any of its long-term holdings. No predatory short selling can occur, $\Delta X(S) = 0$, and $P = XR - D_0$.*

2. *Vulnerability region: When $\frac{D_0}{\gamma X} \leq R < \frac{D_0}{\delta X}$, there are two stable equilibria and one unstable equilibrium.*

   (a) *In one stable equilibrium the financial institution does not unwind any of its long-term holdings, $\Delta X(S) = 0$, and $P = XR - D_0$.*

[7] For the cases when $\delta \geq \gamma$, see the appendix.

(b) In the other stable equilibrium the financial institution is forced to unwind its entire holdings of the long-term asset, i.e. $\Delta X(S) = X$ *and* $P = 0$*.*

(c) In the unstable equilibrium the financial institution has to unwind part of its equity holdings, $\Delta X(S) = X\frac{\gamma - \frac{D_0}{XR}}{\gamma - \delta}$ *and* $P = \frac{1-\gamma}{\gamma-\delta}(D_0 - \delta XR)$

3. *Constrained region: When* $R < \frac{D_0}{\gamma X}$*, there is a unique equilibrium in which the financial institution unwinds its entire holdings of the long-term asset,* $\Delta X = X$ *and* $P = 0$*.*

Proof. See appendix. ■

Figure 5 illustrates the equilibria as a function of R, the $t = 1$ expected payoff of the long-term asset. As Proposition 1 points out, there are three regions of interest. First, when R is sufficiently high, short sellers cannot profitably cause the financial institution to unwind. The only equilibrium is the one in which the financial institution holds its long-term investments until maturity. Expecting this, the equity value at $t = 1$ is given by $XR - D_0$. We can think of this region as characterizing the state in which financial institutions are healthy and well-capitalized. When this is the case, short sellers play no role, except potentially fulfilling the beneficial function of correcting the equity value of the financial institution when the long-term investors (for some reason) offer an intercept that deviates from fundamental value. Importantly, this means that when financial institutions are healthy, we should expect short sellers to perform the beneficial tasks of price discovery and liquidity provision. We should not, however, be concerned about predatory behavior by short sellers in this region.

However, when R drops sufficiently there is a second region with multiple equilibria. In this region, the leverage constraint is satisfied if short sellers do not take any predatory short positions that cause unwinding of the long-term asset. Thus, there is still an equilibrium without short selling and without unwinding by the financial institution. However, there are two equilibria in which short sellers take positions and force the financial institution to unwind by driving it into the constraint. This means that in this region the financial institution is vulnerable to short sellers, even though the leverage constraint is not binding. We thus refer to this region of multiple equilibria as the *vulnerability region*. As is usually the case, the middle equilibrium is unstable. One way to select between the remaining two equilibria is to take the limit from a setting where short sellers have market power. In that case, only the stable equilibrium with short selling remains an equilibrium. The reason is that as long as short sellers have a little bit of market power (i.e. they know they can move the price) they will have a profitable deviation when starting from the equilibrium without short selling. The reason is that any point in the region between the unstable equilibrium and the stable short selling equilibrium give positive profits to short sellers (since in that region $P < \tilde{P}$), such that each individual short seller has a profitable deviation when starting in the non-short selling equilibrium. Taking the limit to the competitive case, the only equilibrium is thus the stable equilibrium with short selling. We thus say that in the vulnerability region, the equilibrium without short selling is not robust to strategic interaction.

Finally, there is a third region in which there is a unique equilibrium in which the short sellers are active. This is the case when R is sufficiently low such that the leverage constraint is violated even in the absence of short selling. We thus refer to this region as the *constrained region*. In this region, short sellers exploit the financial weakness of the financial institution, driving down the stock price such that the financial institution has to unwind its entire holdings of the risky asset, rather than just part of it, which would be the case in the absence of short sellers. In the constrained region, the equity value of the financial institution in the presence of shot sellers is given by $P = 0$.

The effect of banning short selling. We are now in a position the compare outcomes between regimes in which short selling is allowed versus regimes in which short selling is banned. Of course, in the case in which short selling is banned, the financial institution only needs to unwind in period 1 when the leverage constraint is violated *absent* any short selling. Proposition 2 compares the two regimes.

Proposition 2 *The effect of banning short selling on equilibrium prices and the quantity of the long-term investment unwound by the financial institution depends on the equilibrium region:*

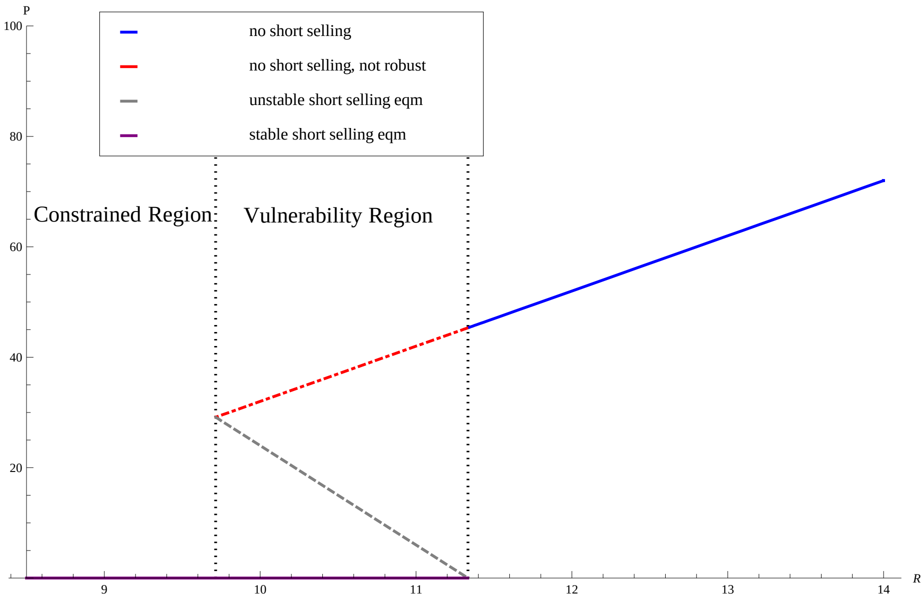

Figure 5: **Overview of Equilibria.** This plot shows the equilibrium equity value of the financial institution as a function of R. For high values of R, there is a unique equilibrium without predatory short selling. Once R drops sufficiently low, there is a region with multiple equilibria (when $\gamma > \delta$). In this *vulnerability region* predatory short selling can emerge. The middle equilibrium is unstable. When R is so low that the leverage constraint binds in the absence of short selling, there is a unique equilibrium with short selling. This is the *constrained region*. The parameter values in this graph are $X = 10, \delta = 0.6, \gamma = 0.7, D = 68$.

1. *When the financial institution is well capitalized, equilibrium prices and the amount the financial institution needs to liquidate coincide. In both cases* $P = XR - D_0$ *and* $\Delta X = 0$.

2. *In the vulnerability region, without short selling, no unwinding takes place and* $P = XR - D_0$. *When short sellers are present, on the other hand, in the robust equilibrium the financial institution unwinds its entire position and* $P = 0$.

3. *In the constrained region, the financial institution unwinds part of the long-term asset holdings when short selling is restricted,* $\Delta X(0) = \frac{\frac{D_0}{XR} - \gamma}{\delta - \gamma\delta}$ *and* $P = XR - D_0 - (1 - \delta)R\frac{\frac{D_0}{XR} - \gamma}{\delta - \gamma\delta}$. *In the presence of short sellers, the financial institution unwinds its entire holdings and* $P = 0$.

Proof. See appendix. ■

This is illustrated in figure 6. There are two important differences between the short selling and no-short selling regimes. First, when short sales are restricted, there is no vulnerability region. This means that the financial institution only has to unwind the long-term asset when the leverage constraint is violated in absence of temporary price movements caused by short sellers. Second, even when the constraint is violated in the absence of short selling, the amount the financial institution has to unwind is (weakly) smaller than in the case with short selling. This is the case since when the constraint is violated, short sellers always force the financial institution to unwind its entire portfolio. When no short sellers are present, on the other hand, the financial institution can in general satisfy the leverage constraint by selling only part of its asset holdings, except when R drops so low that the financial institution enters the 'death spiral', and has to unwind all its holdings even when no short sellers are present. In the figure, this is the point where no short selling curve meets the x-axis.

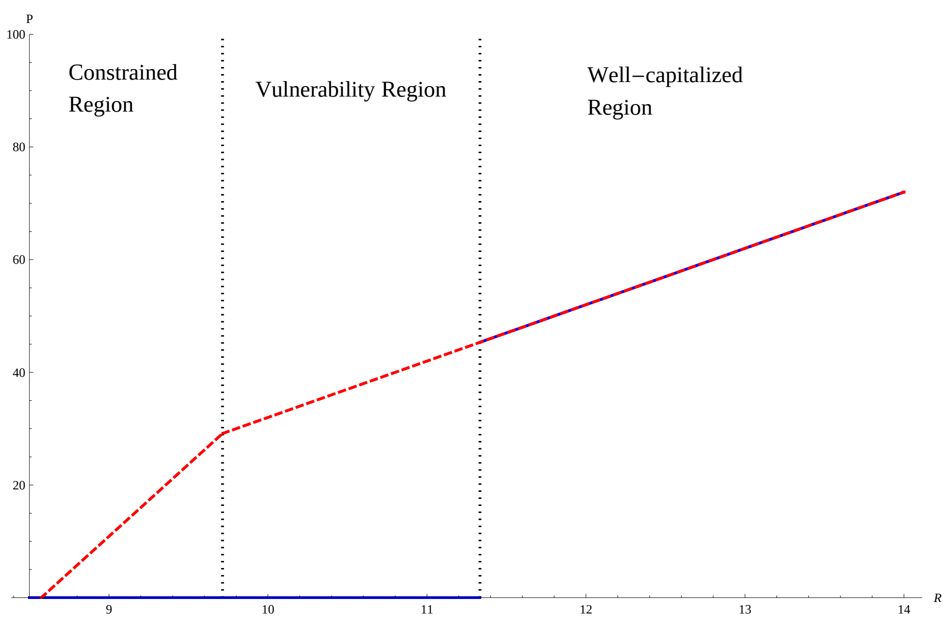

Figure 6: **The effect of banning short selling.** This figure compares equilibria with and without short sellers, focusing on robust, stable equilibria. When short selling is allowed, there is a discontinuous drop in price as the financial institution enters the vulnerability region. Once in the vulnerability region, short sellers force the financial institution to unwind its entire portfolio and $P = 0$. When short selling is not allowed, the financial institution does not have to unwind unless R drops far enough, even without short sellers present. Moreover, even in the region where the constraint is binding, the financial institution does not have to unwind its entire portfolio, except when its equity value truly hits zero. The parameter values in this graph are $X = 10, \delta = 0.6, \gamma = 0.7, D = 68$.

The figure shows that when financial institutions are well-capitalized, the equilibria with and without short selling coincide. Differences only occur once the financial institution enters the vulnerability region or the constrained region. The difference is most notable in the vulnerability region; once the financial institution is vulnerable, short seller can cause a large, discontinuous drop in the equity value of the financial institution, together with the inefficient unwinding of the entire position in the long-term asset. In this region, comparing the financial institution's equity value without short selling to the robust equilibrium[8] with short selling shows that short sellers reduce the equity value of the financial institution from $XR - D_0$ to 0. Total value of the financial institution is reduced by $(1 - \delta)XR$. Since short sellers do not make a profit to offset this loss in value, this is an inefficiency.

In the constrained region the financial institution has to unwind some of the long-term asset even in the absence of short seller. However, it can satisfy the leverage constraint by selling strictly less when no short sellers are present. This is because short seller prey on the weak financial institution by shorting its shares, forcing it to unwind even more of its long-term investments. In fact, in the presence of short sellers, the only equilibrium in the constrained region is the one in which the financial institution unwinds its entire portfolio and the equity value is 0.

Regular selling vs. short selling. The analysis above has focused on predatory short selling, rather than predatory selling. An important question is thus why predatory behavior works through short selling, but not through regular selling. The reason is simple: a regular seller cannot profit from this type of predatory behavior. Imagine a current owner of the stock considering to sell the stock as the financial institution enters the vulnerability region. Since the investor already owns the share, individually he has no incentive to drive down the price through selling the stock. In other words, what makes predation possible is buying back on the cheap after driving down the price. This means that in contrast to a short seller who can make positive profits once the financial institution enters the vulnerability region,

[8]We focus here on the robust equilibrium using the refinement explained above. Alternatively, without this refinement, there is price multiplicity in this region and the equilibrium price will depend on which equilibrium short sellers coordinate on.

an existing owner of the asset cannot make a profit and thus has no incentive to sell the share to induce unwinding by the financial institution.[9]

3 Discussion

While the simple model presented above does not provide a fully-fledged welfare analysis of short selling, the model generates a number of important insights that allow to devise a more differentiated approach to regulating short selling than we have seen in the past. Below, we briefly outline what we consider the main points regulators can take away from our analysis. We also briefly discuss empirical predictions of the model.

Vulnerability of financial institutions. On of the main implications of the model is that while short sellers are not able to prey on financial institutions when these are well-capitalized, in times of stress it is possible that financial institutions become vulnerable to predatory short sellers. The reason for this is that financial institutions face financing constraints that allow short sellers to capitalize on financial weakness by forcing an institution to unwind investments, leading to a reduction in fundamental value. This reduction in fundamental value, in turn, makes the short sale profitable.

In terms of regulating short selling, the model thus implies that while banning short selling during normal times is not desirable, it may make sense to restrict short selling temporarily when financial institutions balance sheets are weak. The reason is that when banks are well-capitalized (and predatory short selling does not occur in equilibrium) short sellers only carry out their beneficial roles of enforcing the law of one price, providing liquidity and incorporating information into prices. The model thus suggests that there is no justification for a *general ban* of short selling on the grounds of predatory behavior. However, the model does provide a potential justification for *temporary* restrictions of short selling. In particular, when financial institutions' balance sheets are weak and they enter the vulnerability region,

[9]The only way in which an equilibrium similar to the predatory short selling equilibrium can occur through regular selling is by coordination on a bad equilibrium: Regular owners may sell their shares because they sell everyone else to sell. This is comparable to the dominated equilibria in Diamond and Dybvig (1983) and Bernardo and Welch (2004).

predatory short selling can be destabilizing, leading to negative skewness and inefficient unwinding of long-term investments by the vulnerable institutions. While the paper does not provide a full welfare analysis, this result provides a potential justification for temporary short sales restrictions to curb predatory behavior.

Disclosure of short positions. Another important implication of the model is the role of multiplicity in the vulnerability region. Since there are two stable equilibria in this region, coordination among short sellers is crucial for which of the two equilibria we end up in. This has important implications for the disclosure of short positions. In particular, in addition to recent short-sale restrictions, a number of regulators have enacted tougher disclosure requirements for short positions. In the US, the SEC now requires short sellers to publicly disclose their positions weekly. In the UK, the FSA implemented a rule that requires that investors disclose on each day any short positions in excess of 0.25% of the ordinary share capital of financial companies at the end of trading the previous day. However, the results from this model imply that this may, in fact, be counterproductive. In particular, requiring public disclosure of all short positions may in fact *facilitate* coordination among predatory short sellers. When short sellers are required to publicly disclose positions, it may thus be more likely that we end up in the predatory equilibrium when in the vulnerability region. This suggests that disclosure should either just be to the regulator, or should be made public only with sufficient time delay.

Constraints based on averages. The model also has implications for the design of regulatory constraints that financial institutions have to conform with. In particular, the ability of the short sellers to prey on financial institution crucially depends on their ability to move the financial institution's constraint by temporarily driving down the stock price. In the model, this is possible since the constraint is calculated using the current market price of equity, which means that temporary price dislocations caused by short sellers are immediately reflected in the financial institution's constraint. A constraint that uses an average of past prices taken over a certain period would weaken this link and make it harder for short sellers to affect the financial institution's constraint and cause unwinding.

Empirical predictions. The model predicts that large downward price movements can occur when a financial institution enters the vulnerability region or the constrained region. This means that in our model short selling increases negative skewness in equity prices. This is opposite to the prediction in Hong and Stein (2003), whose model predicts that *banning* short selling leads to negative skewness, and consistent with the empirical evidence in Bris, Goetzmann, and Zhu (2007), who find evidence that there is significantly less negative skewness in markets in which short selling is either not legal or not practiced. This paper additionally predicts that in the cross-section, negative skewness should be observed particularly for financial institutions with weak balance sheets, i.e. those that enter the vulnerability region or the constrained region.

4 Conclusion

This paper provides a simple model of predatory short selling. Predatory short selling occurs when short sellers exploit financial weakness or liquidity problems of a financial institution. In our model, predatory short sales occur in equilibrium since the equity price drop caused by the short sellers causes a leverage-constrained financial institution to unwind long-term asset holdings at a discount. Predatory short selling only occurs when a financial institution is either (i) close to its leverage constraint (*vulnerability region*) or (ii) violates the leverage constraint even in the absence of short selling (*constrained region*). In the vulnerability region there are two stable equilibria. In addition to an equilibrium without short selling, there is a predatory equilibrium in which short sellers cause a complete unwinding of the financial institution's long-term asset holdings. In the constrained region there is a unique predatory equilibrium in which the financial institution unwinds all its asset holdings. The model provides a potential justification for temporary short sale restrictions for financial institutions. Moreover, the model suggests that disclosure of short positions may facilitate coordination among predatory short sellers, and that constraints of financial institutions should be based on averages rather than snapshots.

5 Appendix

Proof of Proposition 1: First focus on the case when $\delta < \gamma$. Intuitively, when $\delta < \gamma$, the P curve is steeper than the $\widetilde{P}$ curve, as depicted in the right panel of figure 3. We will consider the three regions in turn. First, we compute the region in which no predatory short selling can occur. Short sellers cannot succeed in predatory short selling when, after forcing the financial institution to unwind the entire holdings of the long-term asset, the fundamental value at $t = 2$ exceeds the price to which the short sellers need to drive the price to force the unwinding. This is so because it means that short sellers have to buy back at a higher price than they receive when selling the stock short.

To check this condition, assume that short sellers choose a short position $\overline{S}$ such that the entire portfolio of the financial institution is liquidated, i.e. $\Delta X(\overline{S}) = X$. This requires that

$$\frac{(1-\gamma)D_0 - \gamma(\overline{P} - \lambda S)}{R(\delta - \gamma\delta)} = X \tag{8}$$

which yields

$$\overline{S} = \frac{\gamma\overline{P} + (1-\gamma)(\delta XR - D_0)}{\lambda\gamma} \tag{9}$$

As pointed out above, this cannot be profitable when

$$XR - D_0 - \lambda\overline{S} < \delta XR - D_0 \tag{10}$$

Using (9) to solve (10) for R yields

$$R > \frac{D_0}{\delta X}, \tag{11}$$

which is the expression in the proposition. This implies that the only equilibrium is the one without predatory short selling, in which $P = \tilde{P} = XR - D_0$.

Second, consider the region in which $\frac{D_0}{\gamma X} \leq R < \frac{D_0}{\delta X}$. Note that in this region the leverage constraint is not violated in the absence of predatory short selling, since $\frac{D}{XR} < \gamma$. This means that there is still an equilibrium in which no predatory short selling occurs and $\widetilde{P} = P = XR - D_0$. However, from (11), we know that there is now also an equilibrium in which short sellers force the financial institution to unwind its entire asset holdings. In this equilibrium, by the zero profit condition, we have

$$\tilde{P} = XR - D_0 - \lambda\overline{S} = \max\left[\delta XR - D_0, 0\right] = P. \tag{12}$$

Since we know that in this region $\delta XR - D_0 < 0$, we the equilibrium price must be $P = 0$. Both of these equilibria are stable. Moreover, there is a third, unstable equilibrium, in which only part of the financial institution's investment is unwound. Denote the amount of short selling in the unstable equilibrium by S^*. In this equilibrium, since $\Delta X(S^*) < X$, we must have

$$\overline{P} - \lambda S^* = XR - D_0 - (1-\delta)\Delta X(S^*). \tag{13}$$

Substituting in for $\Delta X(S^*)$ from equation (9) and solving for S^* yields

$$S^* = \frac{\overline{P}}{\lambda} + \frac{(1-\gamma)(\delta XR - D_0}{\lambda(\gamma - \delta)} \tag{14}$$

Using this expression for S^*, we can determine the price in the unstable equilibrium as

$$\widetilde{P} = P = \overline{P} - \lambda S^* = \frac{1-\gamma}{\gamma-\delta}(D - \delta XR). \tag{15}$$

Substituting in to (9) yields that the amount the financial institution has to unwind in the unstable equilibrium is $\Delta X(S^*) = X\frac{\gamma - \frac{D_0}{XR}}{\gamma - \delta}$, as stated in the proposition.

Third, consider the region in which the leverage constraint is violated even in the absence of predatory short selling, i.e $\frac{D}{XR} > \gamma$. In this region, the equilibrium must involve short selling to an extent that the entire portfolio of the financial institution is liquidated. The reason is the following. Assume that $\frac{D}{XR} > \gamma$. Absent short selling, the financial institution would unwind an amount $\Delta X(0)$ such that the constraint is satisfied. This reduces the value of the financial institution at $t = 2$ by $(1 - \delta)\Delta X(0)$. This however implies that absent short selling at $t = 1$, $\tilde{P} = XR - D_0 > XR - D_0 - (1-\delta)\Delta X(0) = P$. This means that short sellers at $t = 1$ have an incentive to take a short position. This, in turn, exacerbates the leverage constraint and forces the financial institution to unwind more. The only equilibrium is the one where $\Delta X = X$ and $\tilde{P} = P = 0$. Intuitively, this is the case since the two lines cannot intersect above the x-axis in this case; the P curve starts below the $\widetilde{P}$ curve, and is steeper, such that the only intersections is on the flat part that lies on the x-axis.

Now consider what happens when $\delta > \gamma$. Intuitively, when $\delta < \gamma$, the P curve is less steep than the $\widetilde{P}$ curve, as depicted in the left panel of figure 3. The first thing to note is that now the vulnerability region disappears, since $\frac{D_0}{\gamma X} > \frac{D_0}{\delta X}$. In this case, as long as $R > \frac{D_0}{\gamma X}$, the financial institution is well-capitalized and no predatory short selling occurs. As a result, unwinding only takes place when $R < \frac{D_0}{\gamma X}$. As before, when $R < \frac{D_0}{\gamma X}$, short sellers will be active. When $R > \frac{D_0}{\delta X}$, short sellers force a partial unwinding of the financial institution's position in the long-term asset. When $R < \frac{D_0}{\delta X}$, short sellers force a complete unwinding.

Finally, in the knife-edge case when $\gamma = \delta$, the P and $\widetilde{P}$ curves have the same slope. This means that when $R > \frac{D_0}{\gamma X}$ predatory short selling cannot occur, while when $R < \frac{D_0}{\gamma X}$, the financial institution is unforced to unwind all its holdings and $P = 0$. When $R = \frac{D_0}{\gamma X}$, the equilibrium price can lie on any point on the interval $[0, XR - D_0]$, as the two lines lie directly on top of each other.

Proof of Proposition 2: We focus on the more interesting case when $\delta < \gamma$. The price in the presence of short sellers follows from proposition 1. The equilibrium $t = 1$ prices when short selling is restricted are determined as follows. Assume that the long-term investors are rational, such that they correctly anticipate the $t = 2$ payoff $XR - D_0 - (1 - \delta)R\Delta X(0)$. As long as the leverage constraint is not violated, i.e. $\frac{D}{XR} < \gamma$, the financial institution does not have to liquidate, and $P = XR - D_0$. When $\frac{D}{XR} > \gamma$, the financial institution has to unwind $\Delta X(0) = X\frac{\frac{D_0}{XR} - \gamma}{\delta - \gamma\delta}$ units of the long-term asset, yielding an equilibrium price of $P = XR - D_0 - (1 - \delta)XR\frac{\frac{D_0}{XR} - \gamma}{\delta - \gamma\delta}$. When $\delta > \gamma$, the plot would look similar, but there would be no vulnerability region, and in part of the constrained region the financial institutions equity value would still be positive, since only a part of the long-term asset gets unwound.

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