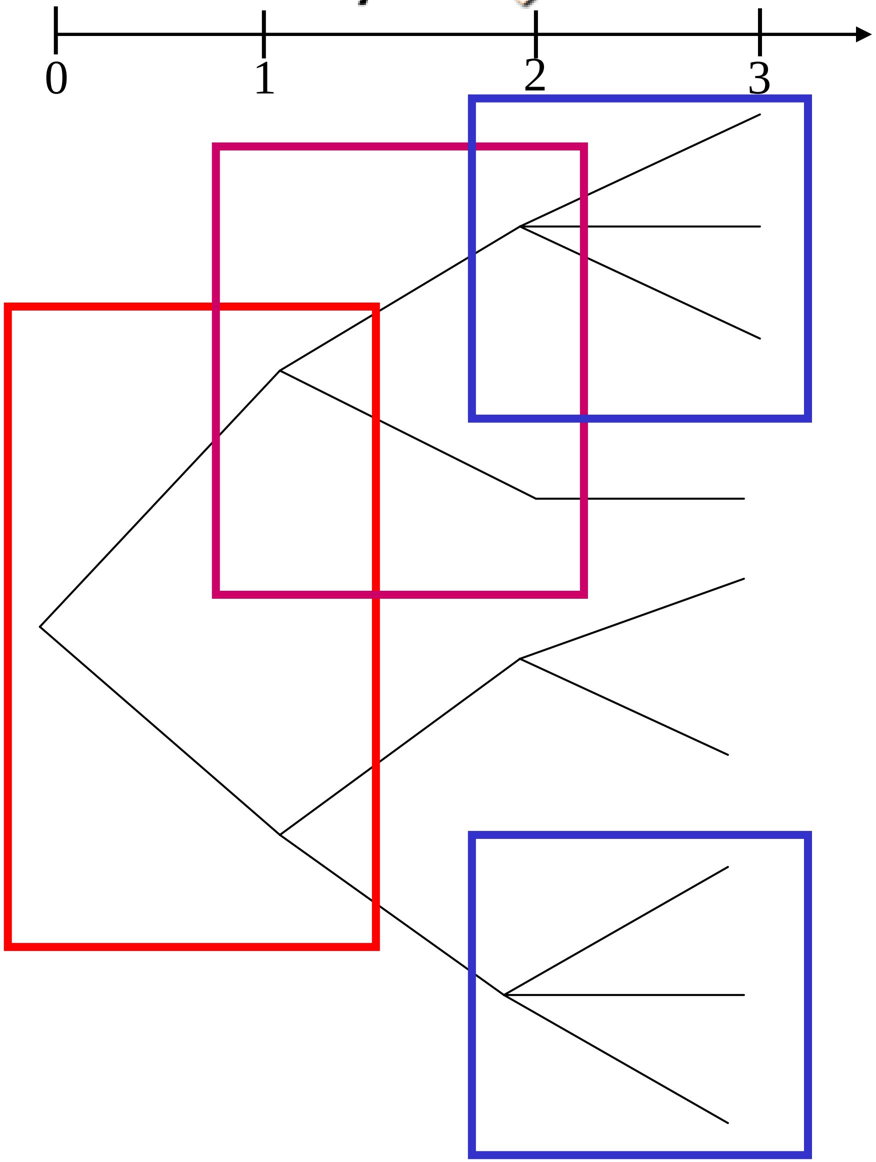
Lecture 07: Multi-period Model

Prof. Markus K. Brunnermeier

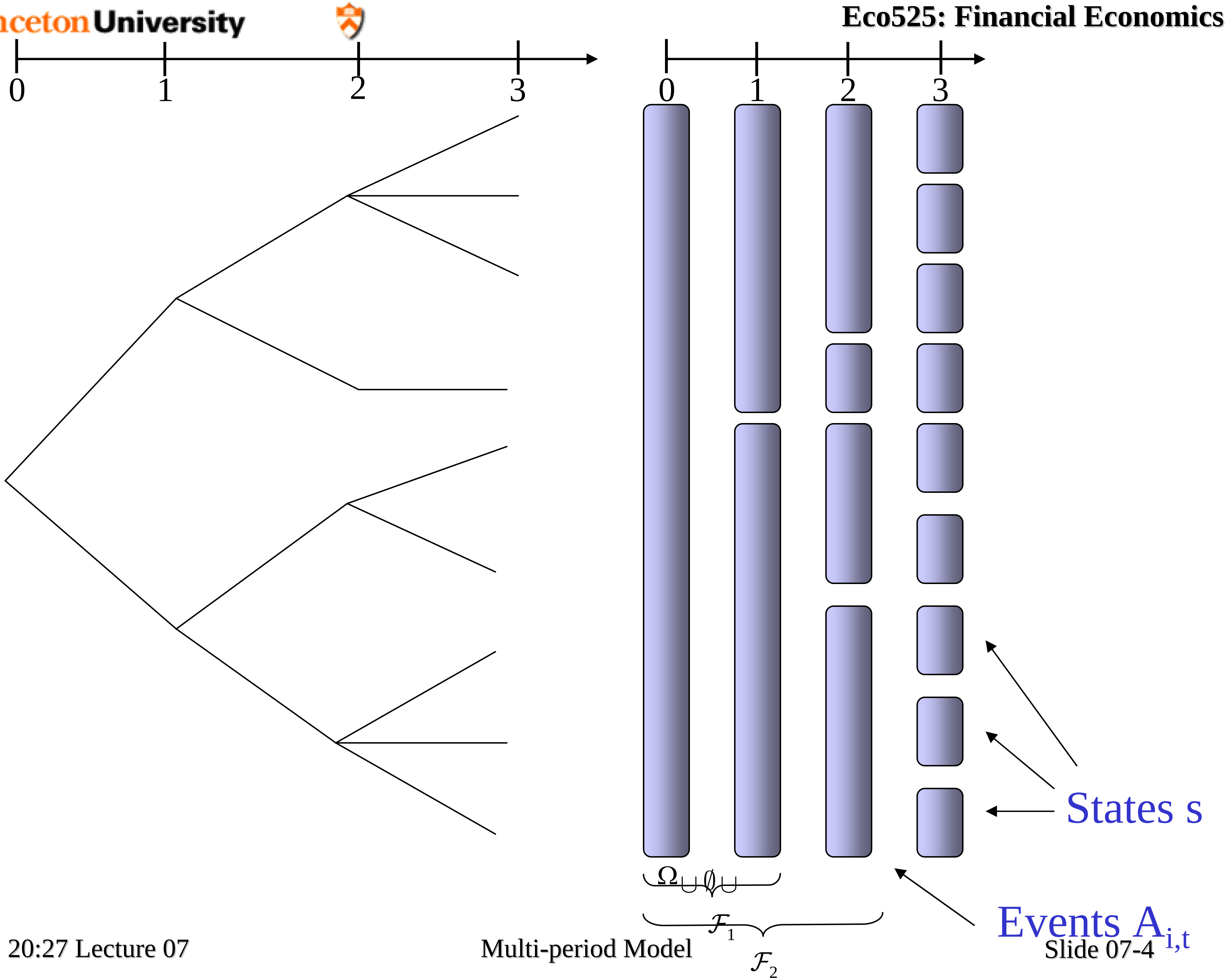
Introduction

- accommodate multiple and even infinitely many periods.
- several issues:
  - how to define assets in an multi period model,
  - how to model intertemporal preferences,
  - what market completeness means in this environment,
  - how the infinite horizon may the sensible definition of a budget constraint (Ponzi schemes),
  - and how the infinite horizon may affect pricing (bubbles).
- *This section is mostly based on Lengwiler (2004)*



many one period models

how to model information?


0
1
2
3
0
1
2
3
Ω
$\cup$
$\emptyset$
$\cup$
$\mathcal{F}_1$
$\mathcal{F}_2$
States s
Events $A_{i,t}$

from static to dynamic…

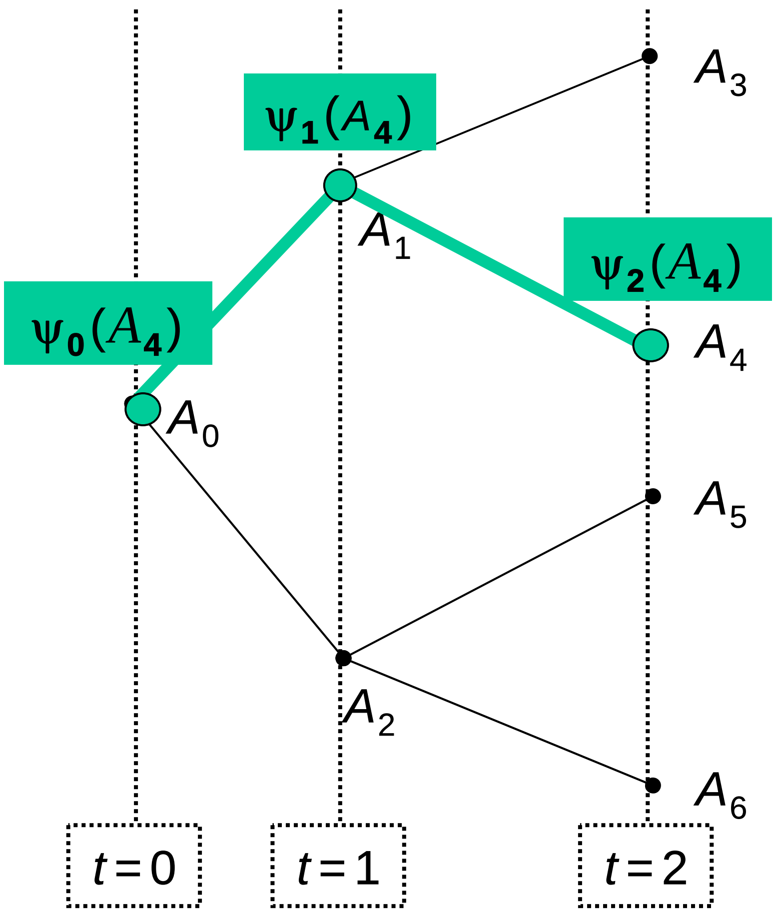
asset holdings	Dynamic strategy (adapted process)
asset payoff x	Next period's payoff $x_{t+1} + p_{t+1}$ Payoff of a strategy
span of assets	Marketed subspace of strategies
Market completeness	a) Static completeness (Debreu)
	b) Dynamic completeness (Arrow)
No arbitrage w.r.t. holdings	No arbitrage w.r.t strategies
State prices q(s)	Event prices $q_t(A_t(s))$

…from static to dynamic

State prices q(s)	Event prices $q_t(A(s))$
Risk free rate $\bar{r}$	Risk free rate $\bar{r}_t$ varies over time
	Discount factor from *t* to *0* $\rho_t(s)$
Risk neutral prob. $\pi^*(s) = q(s) \quad \bar{r}$	Risk neutral prob. $\pi^*(A_t(s)) = q_t(A_t(s)) / \rho_t\ (A_t)$
Pricing kernel $p^j = E[k_q\ x^j]$ $1 = E[k_q]\ \bar{r}$	Pricing kernel $k_t\ p_t^j = E_t[k_{t+1}(p^j_{t+1} + x^j_{t+1})]$ $k_t = \bar{r}_{t+1}\ E_t[k_{t+1}]$

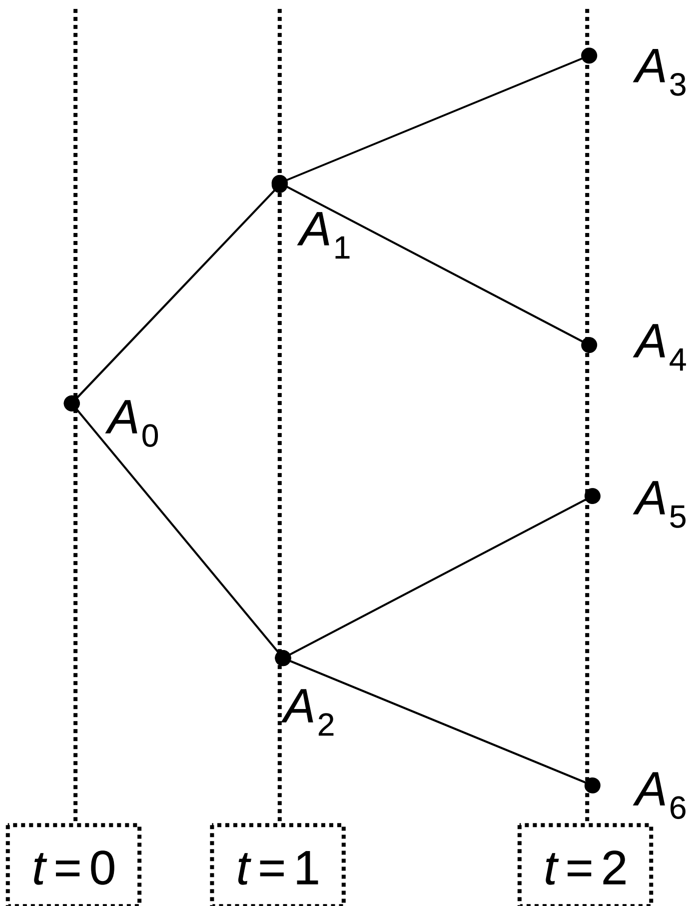
Assets in many periods

Multiple period uncertainty



- We recall the event tree that captures the gradual resolution of uncertainty.
- This tree has 7 events (A_0 to A_6). (Lengwiler uses e_0 to e_6)
- 3 time periods (0 to 2).
- If A is some event, we denote the period it belongs to as $\tau(A)$.
- So for instance, $\tau(A_2)=1$, $\tau(A_4)=2$.
- We denote a path with ψ as follows...

Multiple period uncertainty



- Last period events have prob., π_3 - π_6.
- The earlier events also have probabilities.
- To be consistent, the probability of an event is equal to the sum of the probabilities of its successor events.
- So for instance, $\pi_1 = \pi_3 + \pi_4$.

Multiple period assets

A typical multiple period asset is a coupon bond:

$$r_A := \begin{cases} \text{coupon} & \text{if } 0 < \tau(A) < t^*, \\ 1 + \text{coupon} & \text{if } \tau(A) = t^*, \\ 0 & \text{if } \tau(A) > t^*. \end{cases}$$

The **coupon bond** pays the coupon in each period & pays the coupon plus the principal at maturity t^*.

A **consol** is a coupon bond with $t^* = \infty$; it pays a coupon *forever*.

A discount bond (or **zero-coupon bond**) finite maturity bond with no coupon. It just pays 1 at expiration, and nothing otherwise.

Multiple period assets

- create **STRIPS** by extracting only those payments that occur in a particular period.
  - STRIPS are the same as discount bonds.
- More generally, arbitrary assets (not just bonds) could be striped.

Time preferences with many periods

Time preference

$$u(y^0) + \sum_t \delta(t)\, \mathrm{E}\{\mathrm{u}(y^t)\}$$

- Discount factor $\delta(t)$ number between 0 and 1
- Assume $\delta(t)>\delta(t+1)$ for all t.
- Suppose you are in period 0 and you make a plan of your present and future consumption: $y^0, y^1, y^2, \ldots$
- The relation between consecutive consumption will depend on the interpersonal rate of substitution, which is $\delta(t)$.
- Time consistency $\delta(t) = \delta^{\,t}$ **(exponential discounting)**

Pricing in a *static dynamic model*

A *static* dynamic model

- We consider pricing in a model that contains many periods (possibly infinitely many)…
- …and we assume that information is gradually revealed (this is the *dynamic* part)…
- …but we also assume that **all assets are only traded "at the beginning of time"** (this is the *static* part).
- There is dynamics in the model because there is time, but the *decision making is completely static*.

Maximization over many periods

- vNM exponential utility representative agent

$$\max\{\sum_{t=0} \delta^t \, \mathrm{E}\{\mathrm{u}(y^t)\} \mid y\text{-}w \in \mathcal{B}(p)\}$$

- If all Arrow securities (conditional on each event) are traded, we can express the first-order conditions as,

$$u'(y^0) = \lambda, \quad \delta^{\tau(A)}\pi_A u'(w^A) = \lambda q_A.$$

Multi-period SDF

- The equilibrium SDF is computed in the same fashion as in the static model we saw before

$$\frac{q_A}{\pi_A} = \delta^{\tau(A)} \frac{u'(w^A)}{u'(w^0)}$$

$$= \underbrace{\left(\delta \frac{u'(w^{\psi_1(A)})}{u'(w^0)}\right)}_{M_{\psi_1(A)}} \cdot \times \underbrace{\left(\delta \frac{u'(w^{\psi_2(A)})}{u'(w^{\psi_1(A)})}\right)}_{M_{\psi_2(A)}} \cdots \times \cdots \times \underbrace{\left(\delta \frac{u'(w^A)}{u'(w^{\psi_{\tau(A)-1}(A)})}\right)}_{M_{\psi_{\tau(A)-1}(A)}} =: \mathbf{M}_A$$

- We call M_A the "one-period ahead" SDF and $\mathbf{M}_A$ the multi-period SDF (“state-price density”).

The fundamental pricing formula

- To price an arbitrary asset x, portfolio of STRIPed cash flows, $x^j = x_1^j + x_2^j + \cdots + x_\infty^j$, where x_t^j denotes the cash-flows in period t.
- The price of asset x^j is simply the sum of the prices of its STRIPed payoffs, so

$$\boxed{p_j = \sum_t E\{\mathbf{M}_t x_t^j\}}$$

- This is the ***fundamental pricing formula***.
- Note that $\mathbf{M}_t = \delta^t$ if the repr agent is risk neutral. The fundamental pricing formula then just reduces to the present value of expected dividends, $p_j = \sum \delta^t E\{x_t^j\}$.

Dynamic completion

Dynamic trading

- In the "static dynamic" model we assumed that there were many periods and information was gradually revealed (this is the *dynamic* part)…
- …but all assets are traded "at the beginning of time" (this is the *static* part).
- Now consequences of re-opening financial markets. ***Assets can be traded at each instant.***
- This has deep implications.
  - allows us to reduce the number of assets available at each instant through dynamic completion.
  - It opens up some nasty possibilities (Ponzi schemes and bubbles),

Completion with short-lived assets

- If the horizon is infinite, the number of events is also infinite. Does that imply that we need an infinite number of assets to make the market complete?
- Do we need assets with all possible times to maturity and events to have a complete market?
- No. Dynamic completion.
  Arrow (1953) and Guesnerie and Jaffray (1974)

Completion with short-lived assets

- Asset is `short lived´ if it pays out only in the period immediately after the asset is issued.
- Suppose for each event A and each successor event A' there is an asset that pays in A' and nothing otherwise.
- It is possible to achieve arbitrary transfers between all events in the event tree by trading only these short-lived assets.
- This is straightforward if there is no uncertainty.

Completion with short-lived assets

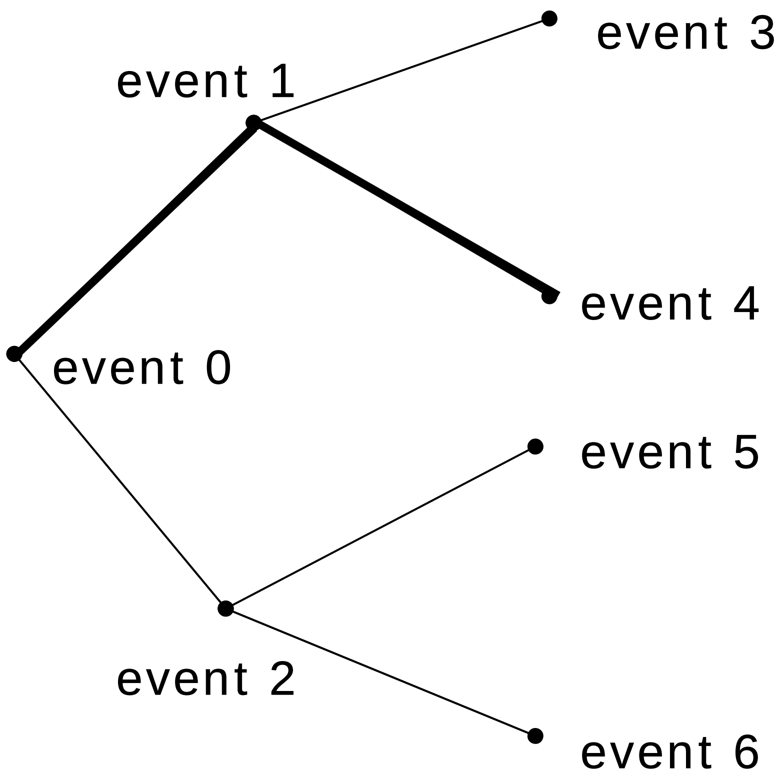
- Without uncertainty, and T periods (T can be infinite), there are T one period assets, from period 0 to period 1, from period 1 to 2, etc.
- Let p_t be the price of the bond that begins in period t-1 and matures in period t.
- For the market to be complete we need to be able to transfer wealth between any two periods, not just between consecutive periods.
- This can be achieved with a *trading strategy*.

Completion with short-lived assets

- *Example:* Suppose we want to transfer wealth from period 1 to period 3.
- In period 1 we cannot buy a bond that matures in period 3, because such a bond is not traded then.
- Instead buy a bond that matures in period 2, for price p_2.
- In period 2, use the payoff of the period-2 bond to buy period-3 bonds.
- In period 3, collect the payoff.
- The result is a transfer of wealth from period 1 to period 3. The price, as of period 1, for one unit of purchasing power in period 3, is $p_2 p_3$.

Completion with short-lived assets

With **uncertainty** the process is only slightly more complicated. It is easily understood with a graph.



- Let p_A be the price of the asset that pays one unit in event A. This asset is traded only in the event immediately preceding A.
- We want to transfer wealth from event 0 to event 4.
- Go backwards: in event 1, buy one event 4 asset for a price p_4.
- In event 0, buy p_4 event 1 assets.
- The cost of this today is $p_1 p_4$. The payoff is one unit in event 4 and nothing otherwise.

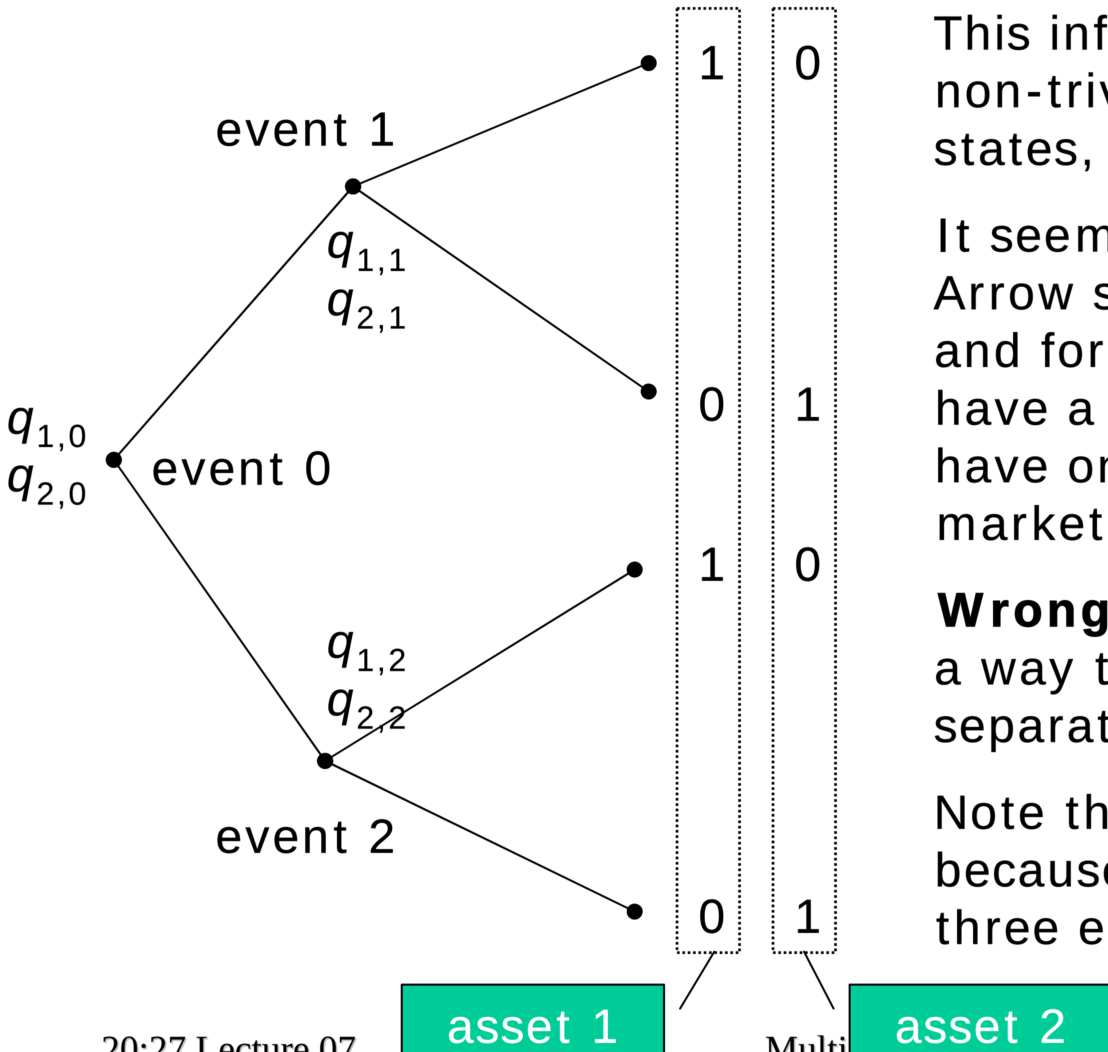
Completion with long-lived assets

- dynamic completion with long-lived assets, Kreps (1982)
- T-period model without uncertainty ($T < \infty$).
- assume there is a single asset:
  - a discount bond maturing in T.
  - bond can be purchased and sold in each period, for price p_t, t=1,…,T.

Completion with a long maturity bond

- So there are T prices (not simultaneously, but sequentially).
- Purchasing power can be transferred from period t to period $t' > t$ by purchasing the bond in period t and selling it in period t'.

A simple information tree



This information tree has three non-trivial events plus four final states, so seven events altogether.

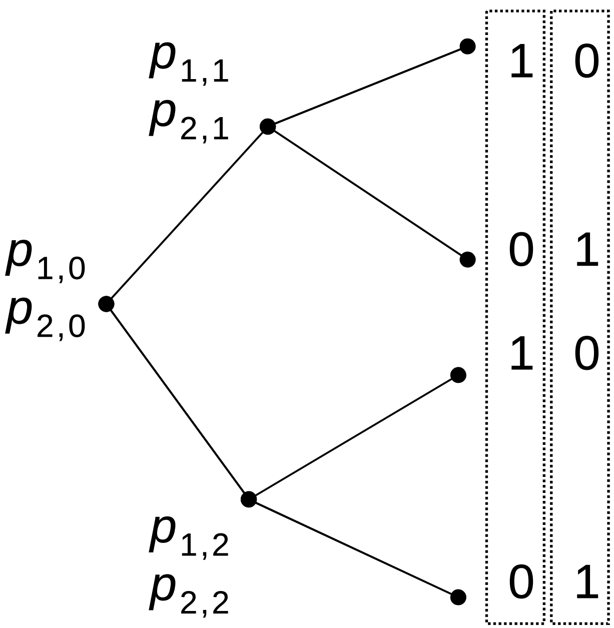
It seems as if we would need six Arrow securities (for events 1 and 2 and for the four final states) to have a complete market. Yet we have only two assets. So the market cannot be complete, right?

Wrong! Dynamic trading provides a way to fully insure each event separately.

Note that there are six prices because each asset is traded in three events.

One-period holding

- Call "asset [*j*,*A*]" the cash flow of asset *j* that is purchased in event *A* and is sold one period later.
- How many such assets exist? What are their cash flows?



There are six such assets: [1,0], [1,1], [1,2], [2,0], [2,1], [2,2].

(Note that this is potentially sufficient to span the complete space.)

"Asset [1,1]" costs $p_{1,1}$ and pays out 1 in the first final state and zero in all other events.

"Asset [1,0]" costs $p_{1,0}$ and pays out $p_{1,1}$ in event 1, $p_{1,2}$ in event 2, and zero in all the final states.

The extended return matrix

The trading strategies [1,0] ... [2,2] give rise to a new 6x6 return matrix.

asset	[1,0]	[2,0]	[1,1]	[2,1]	[1,2]	[2,2]
event 0	$-p_{1,0}$	$-p_{2,0}$	0	0	0	0
event 1	$p_{1,1}$	$p_{2,1}$	$-p_{1,1}$	$-p_{2,1}$	0	0
event 2	$p_{1,2}$	$p_{2,2}$	0	0	$-p_{1,2}$	$-p_{2,2}$
state 1	0	0	1	0	0	0
state 3	0	0	0	1	0	0
state 3	0	0	0	0	1	0
state 4	0	0	0	0	0	1

This matrix is regular (and hence the market complete) if the grey submatrix is regular (= of rank 2).

The extended return matrix

- Is the gray submatrix regular?
- Components of submatrix are prices of the two assets, conditional on period 1 events.
- There are cases in which (p_{11}, p_{21}) and (p_{12}, p_{22}) are collinear in equilibrium.
- If per capita endowment is the same in event 1 and 2, in state 1 and 3, and in state 2 and 4, respectively, and if the probability of reaching state 1 after event 1 is the same as the probability of reaching state 3 after event 2 → submatrix is singular (only of rank 1).
- But then events 1 and 2 are effectively identical, and we may collapse them into a single event.

The extended return matrix

- A *random* square matrix is regular. So outside of special cases, the gray submatrix is regular ("almost surely").
- The 2x2 submatrix may still be singular *by accident*.
- In that case it can be made regular again by applying a small perturbation of the returns of the long-lived assets, by perturbing aggregate endowment, the probabilities, or the utility function.
- *Generically*, the market is dynamically complete.

How many assets to complete market?

- *branching number* = The maximum number of branches fanning out from any event in the uncertainty tree.
- This is also the number of assets necessary to achieve dynamic completion.
- Generalization by Duffie and Huang (1985): continuous time → continuity of events → but a small number of assets is sufficient.
- The large power of the event space is matched by continuously trading few assets, thereby generating a continuity of trading strategies and of prices.

Example: Black-Scholes formula

- Cox, Ross, Rubinstein binominal tree model of B-S
- Stock price goes up or down (follows binominal tree) interest rate is constant
- Market is dynamically complete with 2 assets
  - Stock
  - Risk-free asset (bond)
- Replicate payoff of a call option with (dynamic Δ hedging)
- (later more)

Ponzi schemes

Ponzi schemes: infinite horizon max. problem

- Infinite horizon allows agents to borrow an arbitrarily large amount without effectively ever repaying, by rolling over debt forever.
  - Ponzi scheme - allows infinite consumption.
- Consider an infinite horizon model, no uncertainty, and a complete set of short-lived bonds.

  [z^t is the amount of bonds maturing in period t in the portfolio, β_t is the price of this bond as of period t-1]

$$\max\left\{\sum_{t=0}^{\infty}\delta^t u(y^t)\;\middle|\;\begin{array}{l} y^0 - w^0 \le -\beta_1 z^1 \\ y^t - w^t \le z^t - \beta_{t+1} z^{t+1} \quad \text{for } t>0 \end{array}\right\}.$$

Ponzi schemes: rolling over debt forever

- The following consumption path is possible:

  $$y^t = w^t + 1 \text{ for all } t.$$

- Note that agent consumes *more than his endowment in each period*, ***forever***.
- This can be financed with *ever increasing debt*:

  $$z^1 = -1/\beta_1,\ z^2 = (-1+z^1)/\beta_2,\ z^3 = (-1+z^2)/\beta_3 \ldots$$

- Ponzi schemes can never be part of an equilibrium. In fact, such a scheme even destroys the existence of a utility maximum because the choice set of an agent is unbounded above. We need an additional constraint.

Ponzi schemes: transversality

- The constraint that is typically imposed on top of the budget constraint is the *transversality condition*,

$$\lim_{t\to\infty} \beta_t \, z^t \geq 0.$$

- This constraint implies that the value of debt cannot diverge to infinity.
  - More precisely, it requires that all debt must be redeemed eventually (i.e. in the limit).

Bubbles

The price of a consol

- In an infinite dynamic model, in which assets are traded repeatedly, there are additional solutions besides the "fundamental pricing formula."
  - ➢New solutions have “bubble component”.
  - ➢No market clearing at infinity.
- Consider
  - ➢model without uncertainty
  - ➢Consol bond delivering $1 in each period forever

The price of a consol

- According to the static-dynamic model (the fundamental pricing formula), the price of the consol is $p = \sum_t \mathbf{M}_t$.

$$\mathbf{M}_t := M_1 \times M_2 \times \cdots \times M_t$$

- Consider re-opening markets now. The price at time 0 is just the sum of all Arrow prices, so
- $p_0 = \sum_{t=1}^{\infty} q_t$.
- q_t is the marginal rate of substitution between consumption in period t and in period 0,
- $q_t = \delta^t \left(u'(w^t)/u'(w^0)\right)$, so $p_0 = \sum_{t=1}^{\infty} \delta^t \frac{u'(w^t)}{u'(w^0)}$.

The price of a consol at t=1

- At time 1, the price of the consol is the sum of the remaining Arrow securities,

$$q_1 = \sum_{t=2}^{\infty} q_t = \sum_{t=2}^{\infty} \delta^{t-1} \frac{u'(w^t)}{u'(w^1)}.$$

- This can be reformulated,

$$p_1 = \delta^{-1} \frac{u'(w^0)}{u'(w^1)} \sum_{t=2}^{\infty} \delta^t \frac{u'(w^t)}{u'(w^0)}.$$

- The second part (the sum from 2 to ∞) is almost equal to q_0,

$$\sum_{t=2}^{\infty} \delta^t \frac{u'(w^t)}{u'(w^0)} = \left(\sum_{t=1}^{\infty} \delta^t \frac{u'(w^t)}{u'(w^0)}\right) - \delta \frac{u'(w^1)}{u'(w^0)} \quad = p_0 - \delta \frac{u'(w^1)}{u'(w^0)}.$$

Solving forward

- More generally, we can express the price at time t+1 as a function of the price at time t,

$$p_{t+1} = \delta^{-1} \frac{u'(w^t)}{u'(w^{t+1})} \left(q_t - \delta \frac{u'(w^{t+1})}{u'(w^t)} \right), \quad \text{thus}$$

$$p_t = \delta \frac{u'(w^{t+1})}{u'(w^t)} + \delta \frac{u'(w^{t+1})}{u'(w^t)} p_{t+1} = M_{t+1} + M_{t+1} q_{t+1}.$$

- We can solve this forward by substituting the t+1 version of this equation into the t version, *ad infinitum*,

$$p_0 = M_1 + M_1 p_1 = M_1 + M_1 [M_2 + M_2 p_2] = \cdots$$

$$\boxed{\Rightarrow p_0 = \underbrace{\sum_{t=1}^{\infty} \mathbf{M}_t}_{\text{fundamental value}} + \underbrace{\lim_{T \to \infty} \mathbf{M}_T p_T}_{\text{bubble component}}.}$$

Money as a bubble

$$p_0 = \underbrace{\sum_{t=1}^{\infty} \mathbf{M}_t}_{\text{fundamental value}} + \underbrace{\lim_{T\to\infty} \mathbf{M}_T p_T}_{\text{bubble component}}.$$

- The fundamental value = price in the static-dynamic model.
- Repeated trading gives rise to the possibility of a bubble.
- Fiat money can be understood as an asset with no dividends. In the static-dynamic model, such an asset would have no value (the present value of zero is zero). But if there is a bubble on the price of fiat money, then it can have positive value (Bewley, 1980).
- In asset pricing theory, we often rule out bubbles simply by imposing $\lim_{T\to\infty} M_T p_T = 0$.

Martingales

Martingales

- Let $\boldsymbol{X}_1$ be a random variable and let x_1 be the realization of this random variable.
- Let $\boldsymbol{X}_2$ be another random variable and assume that the distribution of $\boldsymbol{X}_2$ depends on x_1.
- Let $\boldsymbol{X}_3$ be a third random variable and assume that the distribution of $\boldsymbol{X}_3$ depends on x_1, x_2.
- Such a sequence of random variables, $(\boldsymbol{X}_1,\boldsymbol{X}_2,\boldsymbol{X}_3,\ldots)$, is called a **stochastic process**.
- A stochastic process is a **martingale** if

$$\mathrm{E}[x_{t+1} \mid x_t, \ldots, x_1] = x_t.$$

Prices are martingales…

- Samuelson (1965) has argues that prices have to be martingales in equilibrium.
- one has to assume that
  1. no discounting
  2. no dividend payments (intermediate cash flows)
  3. representative agent is risk-neutral

1. With discounting:
   - ➢ Discounted price process should follow martingale $E[\delta p_{t+1} | p_t] = p_t$.

Reinvesting dividends

2. With dividend payments
   - because p_t depends on the **dividend** of the asset in period t+1, but p_{t+1} does not (these are ex-dividend prices).
   - Consider, value of a fund that keeps reinvesting the dividends follows a martingale LeRoy (1989).

Reinvesting dividends

- Consider a fund owning nothing but one units of asset j.
- The value of this fund at time 0 is $f_0 = p_0 = \mathrm{E}[\sum_{t=1}^{\infty} \delta^t x_t^j]$ $= \delta\, \mathrm{E}[x_1^j + \mathrm{p}_1]$. (if representative agent is risk-neutral)
- After receiving dividends x_1^j (which are state contingent) it buys more of asset j at the then current price p_1, so the fund then owns $1 + x_1^j/\mathrm{p}_1$ units of the asset.
- The *discounted* value of the fund is then

  $f_1 = \delta\, p_1 \,(1+x_1^j/p_1) = \delta\,(p_1+x_1^j) = p_0 = f_0$,

  so the discounted value of the fund is indeed a martingale.

…and with risk aversion?

- A similar statement is true if the representative agent is **risk averse**.
- The difference is that
  - we must discount with the risk-free interest rate, not with the discount factor,
  - we must use the risk-neutral probabilities (also called equivalent martingale measure for obvious reasons) instead of the objective probabilities.
- Just as in the 2-period model, we define the risk-neutral probabilities as

$$\pi^*_A = \pi_A \mathbf{M}_A / \rho_{\tau(A)},$$

- where $\rho_{\tau(A)}$ is the discount-factor from event A to 0.

(state dependent)

…and with risk aversion?

- The initial value of the fund is

  $f_0 = p_0 = \sum_{t=1}^{\infty} \mathrm{E}[\mathbf{M}_t\, x_t^{\,j}]$.
- Let us elaborate on this a bit,

  $f_0 = \mathrm{E}[\sum_{t=1}^{\infty} \mathbf{M}_t\, x_t^{\,j}] = \mathrm{E}\{M_1\, x_1^{\,j} + \sum_{t=2}^{\infty} \mathbf{M}_t\, \mathrm{x}_t^{\,j}]$

  $= \mathrm{E}[M_1\, (x_1^{\,j} + \sum_{t=2}^{\infty} \prod_{t'=2}^{t} M_{t'}\, x_t^{\,j}\,)]$

  $= \mathrm{E}[M_1\, (x_1^{\,j} + p_1)]$.
- <u>E*</u> = expectations under the risk-neutral distribution <u>π*</u>, this can be rewritten as

  $f_0 = \rho_1\, \underline{\mathrm{E}^*[}x_1^{\,j} + p_1] = \rho_1\, \underline{\mathrm{E}^*[}f_1]$.
- The properly discounted (ρ instead of δ) and properly expected (π* instead of π) value of the fund is indeed a martingale.

Models of the real interest rate

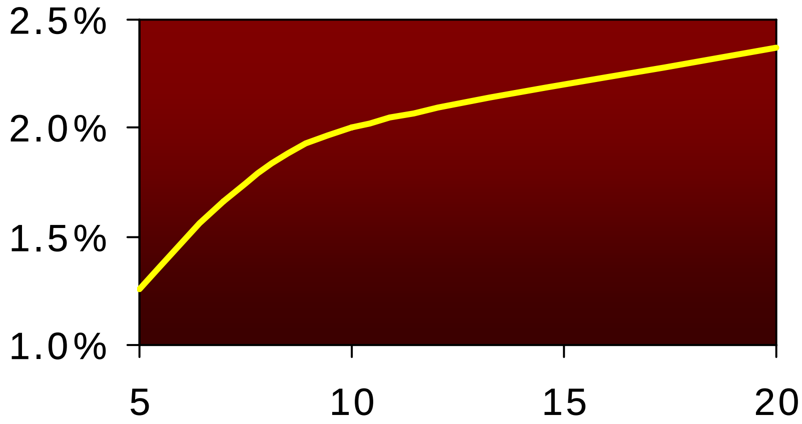
Term structure of real interest rates

- Bond prices carry all the information on intertemporal rates of substitution,
  - primarily affected by expectations, and
  - only indirectly by risk considerations.
- Collection of interest rates for different times to maturity is a meaningful predictor of future economic developments.
  - More optimistic expectations produce an upward-sloping term structure of interest rates.

Term structure

- The price of a risk-free discount bond which matures in period *t* is $\beta_t = E[\mathbf{M}_t]$
- The corresponding yield or interest rate is $r_t = (\beta_t)^{-t} = \delta^{-1} [\ E[u'(w^t)] / u'(w^0)\ ]^{-1/t}$.
- Collection of interest rates is the term structure, $(r_1, r_2, r_3, \ldots)$.
- Note that these are real interest rates (net of inflation), as are all prices and returns.

Term structure



- Here's an example of the term structure of real interest rates, measured with U.S. Treasury Inflation Protected Securities (TIPS), on August 2, 2004.
- **Source:** www.ustreas.gov/offices/domestic-finance/debt-management/interest-rate/real_yield-hist.html

Term structure

$r_t = \delta^{-1} \; \{E[u'(w^t)] / u'(w^0) \}^{-1/t}.$

- Let g_t be the (state dependent) growth rate per period between period t and period 0, so $(1+g_t)^t = w^t / w^0$.
- Assume further that the representative agent has CRRA utility and a first-order approximations yields $r_t \approx \gamma \, E\{g_t\} - \ln \delta$. (Homework!)
- The yield curve measures expected growth rates over different horizons.

Term structure

$$\xi_t \approx \gamma \, E\{g_t\} - \ln \delta$$

- Approximation ignores second-order effects of uncertainty
- …but we know that more uncertainty depresses interest rates if the representative agent is prudent.
- Thus, if long horizon uncertainty about the per capita growth rate is smaller than about short horizons (for instance if growth rates are mean reverting), then the term structure of interest rates will be upward sloping.

The expectations hypothesis

- *cross section of prices*:
  The term structure are bond prices *at a particular point in time*. This is a cross section of prices.
- *time series properties*:
  how do interest rates evolve as time goes by?
- Time series view is the relevant view for an investor how tries to decide what kind of bonds to invest into, or what kind of loan to take.
- enhance notation and write $r_{t,t'}$ to denote the return rate of a bond that begins in t and ends in $t+t'$.

The expectations hypothesis

- Only the first strategy is truly free of risk.
- The other two strategies are risky, since
  - ➢price of 3-year bond in period 2 is unknown today, &
  - ➢tomorrow's yield of a 1-year bond is not known today.
- **Term premia:**
  the possible premium that these risky strategies have over the risk-free strategy are called *term premia*. (special form of risk premium).

The expectations hypothesis

- Consider a *t*-period discount bond. The price of this bond

$$\beta_{0,t} = \mathrm{E}[\boldsymbol{M}_t] = \mathrm{E}[M_1 \cdots M_t].$$

  - one has to invest $\beta_{0,t}$ in t=0 in order to receive one consumption unit in period *t*.

- Alternatively, one could buy 1-period discount bonds and roll them over *t*-times. The investment that is necessary today to get one consumption unit (in expectation) in period t with this strategy is

$$\mathrm{E}[M_1] \cdots \mathrm{E}[M_t].$$

The expectations hypothesis

- Two strategies yield same expected return rate if and only if

$$E[M_1 \cdots M_t] = E[M_1] \cdots E[M_t],$$

which holds if M is serially uncorrelated.

  - In that case, there are no term premia — an assumption known as the expectations hypothesis.
  - Whenever Ms is serially correlated (for instance because the growth process is serially correlated), then expectations hypothesis may fail.

Conditional versus unconditional CAPM

- If β of each subperiod CAPM are time-independent, then
  conditional CAPM = unconditional CAPM
- If βs are time-varying they may co-vary with R_m and hence CAPM equation does not hold for unconditional expectations.
  - Additional co-variance terms have to be considered!

(Dynamic) Hedging Demand

- we will illustrate this concept when we talk about noise trader risk.