

# Fin 501: Asset Pricing

## Pricing Models and Derivatives

### Problem Set 1

Prof. Markus K. Brunnermeier

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Please put your solution in Martin Oehmke's mailbox in Fisher 001

#### 1.1 Problem

There are three states of the world in period 1 and two securities. The first security, the "bond", pays  $(1, 1, 1)$ , whereas the second security, the "stock", pays  $(0, 1, 1)$ .

(a) Calculate the set of all payoffs that can be achieved by portfolios containing the bond and the stock.

From now on assume that there is no arbitrage.

(b) What restrictions does the vector  $(p^1, p^2)$  of the prices of the bond and the stock satisfy?

(c) Find all state prices that are compatible with  $(p^1, p^2) = (3, 2)$ . What are the minimum and maximum prices that a security that pays  $(1, 1, 0)$  can sell for? What about the security  $(1, 0, 0)$ ?

(d) Identify all securities for which there is a unique price compatible with a given prices  $p^1$  for the bond and  $p^2$  for the stock. How does your answer compare with the answer to (a). Why?

(e) Suggest a security that if you could identify the price, in addition to observing the price of the bond and the stock, you could then determine the price of any other security.

#### 1.2 Problem

There are  $S$  states of the world in period 1 and two securities. The first one, the bond, pays 1 in each state of the world. The second one, the stock, pays a vector  $x \neq (1, \dots, 1) \in R^S$ . the price of the bond is  $p^1$  whereas the

price of the stock is  $p^2$ . A forward contract on the stock is an agreement to pay  $F$ , the *forward price* at 1, in exchange for a payment of  $x^s$  in each state  $s = 1, \dots, S$ . At time zero nothing is exchanged. Assume no arbitrage and determine  $F$ .

### 1.3 Problem

Suppose there are  $S$  possible states of the world in period 1 and that each state has a "true" probability of occurrence  $\eta^s > 0$ . ( $\sum_s \eta^s = 1$ ). For any vector  $y \in R^S$ , write  $E(y) = \sum_s \eta^s y^s$ . A state price density is a vector  $\mu \in R^S$  with  $\mu^s = \frac{q^s}{\eta^s}$ , where  $q$  is a state-price vector.

(a) Consider an asset with payoff  $(x^1, \dots, x^S)$ . Show that the price of the asset must equal  $E(z)$ , where  $z^s = x^s \mu^s$ . Give an interpretation to this result.

(b) The rate of return of an asset with price  $p > 0$  in state  $s$  is  $r^s = \frac{x^s}{p}$ . Let  $w^s = r^s \mu^s$ . Show that  $E(w) = 1$ , and interpret this result.

### 1.4 Problem

A stock today trades at a price  $S_0$ . Recall that a European call (put) with strike price  $K$  gives the owner the right, but not the obligation, to buy (sell) the stock at a given date at price  $K$ . An at-the-money European call (one with strike price  $S_0$ ) that can be exercised a year from now on this stock sells at price \$10. An at-the-money European put that can be exercised a year from now on the same stock sells for \$8. A riskless bond that pays \$100 a year from now trades for \$95. If there is no arbitrage, what is the price of the stock today?

### 1.5 Problem

There are two dates  $t = 0, 1$  and two states of the world  $\omega^1, \omega^2$ . The prevailing state is revealed at date 1. Three assets trade at date 0. The first pays one unit in each of the two states and trades at date zero at a price of 1. The second pays 4 units at  $\omega^1$  and 2 units at  $\omega^2$  and trades at 0 for 5/2. The third asset is the option to buy asset two (cum-dividend) at time 1 for 3 units. Assume there is no arbitrage and calculate the price for this asset.

## 1.6 Problem

There are three states of the world and three assets are traded. The payoff of the assets are  $x_1 = (1, 2, 5)$ ,  $x_2 = (1, 2, 2)$  and  $x_3 = (2, 4, 1)$ . Show whether or not markets are complete. Now imagine that instead of asset 3 a riskless asset is traded. Are markets complete?