FIN 501 Problem Set 3
Midterm Exam 2004

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October 24, 2005, 9.30-10.45

Please answer all questions. The total number of points is 100 and you have 75 minutes. It is crucial that you explain how you arrive at your answers. If you cannot show something mathematically, try to give some intuition. Graphical arguments are also helpful. Good luck.

Problem 1 (10 points) At the horse races on Saturday afternoon Gavin Jones studies the racing form and concludes that he horse "No Arbitrage" has a 25 % chance to win and is posted at 4 to 1 odds. (For every dollar Gavin bets, he receives $5 if the horse wins and nothing if it loses.) He can either bet on this horse or keep his money in his pocket. Gavin decides that he has a square-root utility for money.

a) (6 points) What fraction of his money should Gavin bet on "No Arbitrage"?

b) (4 points) What is the implied winning payoff (in terms of utility) of a $1 bet against "No Arbitrage"?

Problem 2 (30 points) Consider an economy with one risky asset and one risk-free asset. Suppose that the return of the risky asset can either exceed or fall short of the risk-free rate, but that on average it is strictly greater than the risk-free rate.

a) (12 points) Show that a risk-averse investor will always invest a strictly positive amount in the risky asset.

What if there are two risky assets with gross returns given by the table below? Answer the question in light of the setting where we set the risk-free rate to zero ($R^f = 1$) and both states are equally likely.

	state 1	state 2
risky asset 1	1/2	2
risky asset 2	3	3/4

b) (6 points) How much would you invest in asset 1, asset 2 and the risk-free asset (a characterization of the solution is sufficient)?

Now, assume instead that risky assets' payoffs are normally distributed with mean $\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$ and (co)variance matrix $\begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_1^2 \end{pmatrix}$ and your utility function is $u(W) = -e^{-\gamma W}$. (Hint: Recall the certainty equivalent of this utility function.)

c) (8 points) How much would you invest in each asset?

Problem 3 (24 points) Assume that the economy consists of two assets with payoffs and prices given by

$$X = \begin{pmatrix} 1 & 2 \\ 2 & a \end{pmatrix}, \; p = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

In X, the rows corresponds to different assets and the columns to the payoffs in the two states. The state probabilities are given by

$$\pi = \begin{pmatrix} 0.5 & 0.5 \end{pmatrix}.$$

a) (6 points) For what values of a does the law of one price (LOOP) hold? For what values of a is the market complete?

b) (6 points) For what values of a is the market arbitrage free?

c) (6 points) Assume that $a = 8$. What is the risk free rate?

d) (6 points) Calculate the risk neutral probability when $a = 8$.

Problem 4 (18 points) Consider a frictionless two-period economy where the basis payoffs are given by

$$X = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{pmatrix}$$

and the vector of probabilities is $\pi = [0.2, 0.6, 0.2]'$. Let consumption growth in each state of nature be given by the vector $y = [1.6, 1, 1.8]$. Assume that the LOOP holds. Suppose further that prices are set by a representative investor whose utility in each period is given by

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

and her total utility is $U(c_0, c_1) = u(c_0) + \beta E[u(c_1)]$.

a) (2 points) Are markets complete in this economy? Explain your answer.

b) (6 points) Compute the prices for two payoffs (two assets with X-payoff) for the case where $\beta = \gamma = 1$. [Hint: Note that for $\gamma = 1$, $u(c) = \ln c$.]

c) (6 points) For the prices computed in question b) find a vector of state prices. Can you find more than one?

d) (4 points) Compute the risk-free rate for the prices computed in question b).

Problem 5 (18 points) Consider different payoffs of assets to study the concept of stochastic dominance.

a) (5 points) Provide an example in which payoff of asset D first order stochastically dominates E, but asset D does NOT state-by-state dominate asset E.

b) (5 points) Find and carefully describe a pair of assets A and B with the property that A Second Order Stochastically Dominates (SOSD) B but not A First Order Stochastically Dominates (FOSD) B.

c) (8 points) If A SOSD B and B SOSD C, is it neccessarily true that A SOSD C? Provide a proof or a counterexample.