Interphase L-C Resonance and Stability Analysis of Series-Capacitor Buck Converters

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Abstract—Hybrid switched-capacitor power converters, such as the series-capacitor buck (SCB) converter, have intrinsic L-C resonant dynamics that might influence its control stability and transient response. This letter presents a systematic approach to analyzing this intrinsic resonant behavior, which can be classified into output L-C \( R_C \) resonance and interphase L-C \( R_C \) resonance based on common-mode and differential-mode decomposition. The impacts of coupled inductors on the resonant amplitude, frequency, and settling time during a step line transient are analyzed. The influence of intrinsic resonance on control stability is clarified, providing guidance for controller design. A two-phase SCB prototype was built and tested with discrete and coupled inductors under different operating conditions. All analysis is verified by simulation or experimental results.

Index Terms—series-capacitor buck, averaged model, multiphase converter, L-C resonance, coupled inductor, control

I. INTRODUCTION

HYBRID switched-capacitor (SC) converters have been proved attractive for many applications [1]–[6]. They leverage capacitors to achieve high power density and inductors to realize useful system functions such as soft charging and voltage regulation. In a hybrid-switched-capacitor converter, many capacitors and inductors may form a multi-resonant system with sophisticated dynamics. This characteristic could significantly impact the controller design of hybrid SC converters, which hasn’t been systematically investigated in literature. Using the series-capacitor buck (SCB) converter [5]–[7] as an example, this letter presents a systematic approach to analyzing the intrinsic L-C resonance of hybrid SC converters and its impacts on control dynamics.

A SCB converter operates similar to a multiphase buck converter with inputs in series and outputs in parallel [5]. The reduced equivalent input voltage leads to 1) extended duty ratio for high voltage conversion ratio, 2) reduced switch voltage stress and switching loss, 3) reduced inductor core loss, and 4) reduced current ripple. An inherent mutual balancing mechanism also exists, which can achieve automatic capacitor voltage balancing and inductor current sharing [8]. Previous large-/small-signal models of the SCB converter [9]–[11] only explain the overall converter dynamics, and interphase dynamics were not investigated. [8] unveiled the interphase L-C resonance whose damping ratio is proportional to the conduction-path resistance. However, a well-designed high efficiency converter with low conduction loss might result in an underdamped system with long settling time and large resonant amplitude. Models and design methods for describing and mitigating the interphase L-C resonance are still needed.

This letter systematically analyzes the interphase L-C resonance in SCB converters and provides comprehensive guidelines for controller design. The impact of coupled inductors on SCB interphase resonance is clarified. All theoretical analysis are verified by simulation and experimental results.

II. MODELING AND ANALYSIS OF INTRINSIC RESONANCE

This section analyzes the intrinsic L-C resonance by decomposing disturbance and its response into common-mode and differential-mode dynamics, streamlining the underlying mechanisms of SCB converter L-C resonant behaviors. Fig. 1 shows the circuit topology and key operation waveforms of a two-phase SCB converter. The analysis below starts with using discrete inductors, and the impacts of coupled inductors are discussed in Section III. In the SCB topology, the blocking capacitor \( C_B \) functions as a dc voltage source with \( v_{in}/2 \) across it. Switch node voltages step between 0 and \( v_{in}/2 \), doubling the duty ratio compared to a regular buck. Two phases are typically interleaved, and \( C_B \) is charged and discharged by the inductor currents of the two phases alternatively as their high-side switches \( (S_1\sim S_{2H}) \) turn on. Conduction-path resistances (including switch \( R_{ds} \), capacitor ESR, inductor winding resistance, etc.) are lumped into an effective resistance \( R_C \) in series with \( C_B \). The large-signal average model of the two-phase SCB converter is described in Fig. 2 together with the modeling equations and their equivalent circuits.

![Fig. 1. Circuit topology and operation waveforms of an example two-phase series-capacitor buck converter with discrete inductors. The maximum switch voltage stress is labeled in red. Coupled inductors can be utilized to replace the discrete ones, and phase number can be extended by stacking more series-capacitor buck cells [10], [11], as indicated by the grey lines and grey dots.](image-url)
The load transient dynamics of a SCB converter is similar to a multiphase buck converter and has been discussed in [11]. However, the line transient dynamics and their impacts on flying capacitor voltage and current sharing have not been systematically explored. The input voltage step change of a line transient results in blocking capacitor voltage variation and causes ringing and long settling time, which are the main focuses of this letter. Similar analysis methods can be applied to describe the responses to other perturbations, such as duty ratio change, unbalanced initial conditions, load transients, etc.

Assume $d_1 = d_2 = D$, $L_1 = L_2 = L$. The input voltage perturbation $v_{i_n}$ can be decomposed into common mode $\{\pm \frac{v_{in}}{\sqrt{2}}, \pm \frac{v_{in}}{\sqrt{2}}\}$ and differential mode $\{\pm \frac{v_{in}}{\sqrt{2}}, \pm \frac{v_{in}}{\sqrt{2}}\}$ for the two phases, as illustrated in Fig. 3. The common-mode perturbations drive the two phases to change in the same way, while the differential-mode perturbations cause opposite variations on the two phases. The resulting differential inductor currents $\pm \frac{\Delta i_L}{2}$ are cancelled at the output, so the common-mode current response is $+\frac{i_{L1}}{2}$ for each inductor. The overall current response of each inductor is:

$$i_{L1} = \frac{1}{2}i_0 + \frac{1}{2}\Delta i_L,$$

$$i_{L2} = \frac{1}{2}i_0 - \frac{1}{2}\Delta i_L. \quad (1)$$

Apply state-space-averaging, the $v_{i_n}$-to-$\tilde{v}_o$ transfer function is:

$$G_{v_{i_n}to\tilde{v}_o} = \frac{\tilde{v}_o}{v_{i_n}} = \frac{D}{2R_o + DR_C} \cdot \frac{\frac{s}{\omega_{nop}} + 1}{\frac{s}{\omega_{nip}} + 1}, \quad (2)$$

$$\omega_{nip} = \sqrt{\frac{2R_o + DR_C}{R_oC_oL}}; \quad Q_{op} = \sqrt{\frac{2R_oC_oL}{L + RC_oC_oD}}; \quad \omega_z = \frac{1}{R_oC_o}. \quad (3)$$

Accordingly, the line transient $v_{i_n}$-to-$\tilde{v}_o$ transfer function is:

$$G_{v_{i_n}to\tilde{v}_o} = \frac{\tilde{v}_o}{v_{i_n}} = G_{v_{i_n}to\tilde{v}_o} \cdot Z_o, \quad Z_o = \frac{R_o}{R_oC_o s + 1}. \quad (4)$$

Similarly, the $v_{i_n}$-to-$\Delta i_L$ transfer function is:

$$G_{v_{i_n}to\Delta i_L} = \frac{\Delta i_L}{v_{i_n}} = C_B \cdot \frac{s}{\frac{s}{\omega_{nip}} + 1}, \quad (5)$$

It can be seen from (2) and (5) that there exist two types of intrinsic L-C resonances in SCB converter dynamic responses:

1) Output $L-C_o$ resonance with $\omega_{nop} & Q_{op}$: higher $R_o$ leads to a higher $Q_{op}$ and lower damping ratio.

2) Interphase $L-C_B$ resonance with $\omega_{nip} & Q_{ip}$: higher $R_C$ results in a lower $Q_{ip}$ and higher damping ratio.

Dynamic responses of common-mode variables (e.g., $\tilde{i}_o$ and $\tilde{v}_o$) will only see the output $L-C_o$ resonant pole and their associated transfer functions (e.g., $G_{v_{i_n}to\tilde{v}_o}$, $G_{v_{i_n}to\tilde{i}_o}$, and $G_{d_{i_{L1}}to\tilde{v}_o}$) are the same as of a regular two-phase buck. Contrarily, responses of differential-mode variables (e.g., $\Delta i_L$ and $\tilde{v}_C$) will only see the interphase $L-C_B$ resonant pole, whose transfer functions (e.g., $G_{v_{i_n}to\Delta i_L}$ and $G_{d_{i_{L1}}to\Delta i_L}$) are different from the buck converter. According to (1), responses of $i_{L1}/i_{L2}$ contain both common-mode and differential-mode dynamics and will see both output $L-C_o$ and interphase $L-C_B$ resonant poles. The $v_{i_n}$-to-$i_{L1}/i_{L2}$ transfer functions can be obtained by combining $G_{v_{i_n}to\tilde{v}_o}$ and $G_{v_{i_n}to\Delta i_L}$. Same analysis approach and conclusions also apply to SCB converters with higher number of phases.

For a general $M$-phase SCB converter, transfer functions can be calculated based on state-space modeling as summarized in the Appendix. A more intuitive way of deriving the response to a perturbation is by superposing the responses to its common-mode and differential-mode components. Fig. 4a plots the small-signal average model of an $M$-phase SCB
converter and its equivalent circuits seen by common-mode and differential-mode perturbations individually. Denote the effective perturbations applied to each phase as \( \tilde{v}_1 \sim v_{1M} \), which can refer to input voltage or converted duty ratio perturbations (e.g., \( v_1 = \tilde{v}_{in} \), \( v_2 \sim M = 0 \) for the input voltage perturbation). As shown in Fig. 4a, the common-mode perturbation component \( v_{cm} = (\tilde{v}_{in} + v_{2} M) \) is effectively applied to an output \( L-C_B \) network with the resonant frequency of \( \omega_{inop} = \sqrt{\frac{1}{L C_B}} \). Same for the output load transient that contains only common-mode perturbation component.

As for differential-mode perturbations, the incurred variations are canceled at the output, so the output terminals are effectively shorted in the small-signal average model, resulting in an equivalent \( M \)-level \( L - C_B \) ladder network. Transfer function for this \( L-C \) ladder circuit can be determined by using DFFz triangles [12], which contains up to \( M-1 \) resonant poles (assume \( C_{B,k} \) are identical):

\[
\omega_{inp,k} = \frac{2D}{\sqrt{L C_B}} \sin \left( \frac{k \pi}{2M} \right), \quad k = 1 \sim M - 1. \tag{7}
\]

Lumping the common-mode and the differential-mode dynamic responses yields the overall response. Fig. 4b shows the \( v_{in} \)-to-\( i_L \) transfer functions of an example 3-phase SCB converter, which has two interphase \( L-C_B \) resonant poles and one output \( L-C_o \) resonant pole, as expected.

III. IMPACTS OF COUPLED INDUCTORS

Coupled inductors that exhibit different inductances to common-mode and differential-mode excitations can improve inductor current sharing and capacitor voltage balancing for multiphase hybrid switched-capacitor-magnetic topologies [13], [14]. This section discusses the impacts of coupled inductors on intrinsic resonance of the SCB converter. Table I lists the parameters of an example two-phase SCB for all calculations and simulations in Sections III and IV, unless otherwise specified. Fig. 5 shows the large-signal average model with coupled inductors. \( L_S \) and \( L_M \) are self and mutual inductances in the inductance matrix, and \( L_k \) and \( \beta \) are effective leakage inductance and coupling coefficient as defined in [14], [15]. A higher \( \beta \) indicates higher coupling coefficient for the coupled inductor. When using coupled inductors, transfer functions \( G_{v_{in},i_L} \), \( G_{v_{in},v_o} \), and \( G_{v_{in},\Delta i_L} \) have the same expressions as in Eqs. (2) – (6) except that the \( \omega_n \) and \( Q \) are changed to:

\[
\omega_{inop} = \sqrt{\frac{2R_o + DR_C}{R_o C_o L_k}}, \quad Q_{op} = \sqrt{\frac{R_C L_k}{L_k + R_C R_o C_o D}}.
\]

\[
\omega_{inp} = D \sqrt{\frac{2}{(1+\beta)L_k C_B}}, \quad Q_{ip} = \frac{1}{R_C} \sqrt{\frac{2(1+\beta)L_k}{C_B}}.
\]
Accordingly, common-mode or output dynamics will see a small inductance $L_k$, while the differential-mode or interphase dynamics will see a large inductance $(1 + \beta)L_k$. If $L_k$ is fixed (i.e., under the same transient speed), $\beta$ will only influence differential-mode dynamics. As shown in Fig. 6, a larger coefficient $\beta$ results in a lower interphase resonant frequency $\omega_{nip}$ and a higher quality factor $Q_{iop}$, but the gain at resonance remains unchanged as $\frac{1}{Q_{iop}}$. When $\beta$ increases, higher $Q$ with narrower high-gain bandwidth may benefit the line transient response, since a $v_{in}$ step change contains multiple frequency components. In frequency domain, a $v_{in}$ step change is $v_{in} = \frac{U}{s}$ ($U$ is the step amplitude), and the $\Delta i_L$ response is $\Delta i_L = G_{v_{in}i_L} \cdot \frac{U}{s}$. Accordingly, its time domain response is:

$$\Delta i_L(t) = L^{-1}\left\{G_{v_{in}i_L} \cdot \frac{U}{s}\right\} = A \cdot e^{-\sigma t} \sin(\omega_d t), \quad \text{(10)}$$

$$A = 2U \sqrt{\frac{C_B}{8(1+\beta)L_k-R_C^2C_B}} \cdot \frac{D}{(1+\beta)L_k}, \quad \text{(11)}$$

$$\omega_d = \frac{D}{2(1+\beta)L_k} \sqrt{\frac{8(1+\beta)L_k-R_C^2C_B}{C_B}}.$$

Fig. 7 shows the simulated and calculated responses of $\Delta i_L$ to an input voltage step change, in which the calculated results match well with the simulated ones, validating the analysis. The 2% settling time of $\Delta i_L$ envelop is $t_s = \frac{\pi}{\omega_d}$. Fig. 7 also indicates that using coupled inductors can effectively suppress the amplitude of interphase resonance for SCB converters with the tradeoff of increased settling time. This feature fundamentally comes from larger effective inductance $(1 + \beta)L_k$ for differential-mode (i.e., interphase) dynamics. The relationship between the resonant $\Delta i_L$ amplitudes and the 2% settling time ($t_s$) during a unit $v_{in}$ step change are plotted in Fig. 8, which implies: (1) larger $C_B$ leads to smaller capacitor voltage ripple but results in larger resonant current amplitude; (2) as $t_s$ increases, the resonant current amplitude decreases.

IV. INFLUENCE ON CONTROL STABILITY

This section explains the impacts of intrinsic resonance on control stability when the SCB converter is controlled in voltage-mode. A typical multiphase PWM voltage-mode controller generates identical duty ratio command for each phase by sensing $\dot{v}_o$. Fig. 10 plots its block diagram. $H_s$ and $A_c$ are transfer functions for sampling and compensation networks respectively. As implied by the equivalent circuit model in Fig. 2, the identical duty commands will cause common-mode variations (e.g., $\dot{v}_o$), but will not incur differential-mode variations (e.g., $\Delta i_L$ and $\dot{v}_c$). Substituting $d_1 = d_2 = D + \dot{d}$ into the average model, the $\dot{d}$-to-$\dot{v}_o$ transfer function is:

$$G_{d_{io}} = \frac{\dot{v}_o}{\dot{d}} = V_{in} - L_o R_C \cdot \frac{\frac{\sigma^2}{\omega_{nop}^2} + 1}{\frac{\omega_z^2}{\omega_{nop}^2} + \frac{\omega_{z}^2}{\omega_{nop}^2} + 1}, \quad \text{(12)}$$

where $\omega_z$, $\omega_{nop}$, and $Q_{op}$ are for output $L-C_o-R_o$ network and are the same as in (3) or (8). $G_{d_{vo}} = G_{d_{io}} \cdot Z_o$. Since the sensed $\dot{v}_o$ is also a common-mode variable, the overall feedback loop only senses and affects common-mode dynamics; it will not be influenced by or have impacts on differential-mode dynamics. Consequently, interphase $L-C_B$ resonance doesn’t affect control stability; stable loop design only needs to consider the output $L-C_o$ resonance, which is the same as a multiphase buck. Similar conclusions can be drawn for other control methods that sense the common-mode dynamics and generate identical commands for all the phases.

Fig. 9 shows the simulated open loop and closed loop transfer functions for an example SCB converter with a typical voltage-mode controller. Denote the loop gain as $T = Z_o \cdot H_s \cdot A_c \cdot G_{d_{io}}$. Smaller $L_k$ will result in larger $\omega_{nip}$ in the $G_{d_{io}}$, allowing to design higher loop-gain bandwidth to achieve faster transient speed. Responses of $\dot{v}_c$ and $\dot{v}_o$ are involved in the loop, so the closed loop gains of $G_{v_{in}i_o}$ and $G_{v_{in}v_o}$ are greatly suppressed; responses of $\Delta i_L$ is not affected.
Fig. 11. Simulated voltage and current responses to a line transient \( (V_{in} = 12\text{~V} \rightarrow 14\text{~V} \rightarrow 12\text{~V}) \) in the case of (a) open loop and (b) closed loop. \( (\beta = 0) \)

Fig. 12. Bode plots of \( G_{\Delta d \Delta i_L} \) under different load conditions.

Fig. 13. Bode plots of \( G_{\Delta \nu_c \Delta i_L} \) under different load conditions.

by the loop, so the closed loop gain of \( G_{\nu_c \Delta i_L} \) is unchanged:

\[
(G_{\nu_c \Delta i_L})^{CL} = \frac{G_{\nu_c \nu_o}^{CL}}{1 + T}, \quad (G_{\nu_c \nu_o})^{CL} = \frac{G_{\nu_c \nu_o}}{1 + T},
\]

Equation (13) indicates that the voltage-mode control loop can restrain the output variation, but it cannot suppress the interphase resonance. Therefore, as shown in Fig. 11, while the output voltage and current are effectively controlled to maintain stable against a line transient, the resonance of \( \nu_c \) and \( \Delta i_L \) are still left underdamped with high resonance amplitude and long settling time.

Actively controlling \( \Delta i_L \) resonance with unequal duty ratios will face more complicate \( \Delta d \)-to-\( \Delta i_L \) dynamics than that of the multiphase buck. Substituting \( d_1 = D + \frac{1}{2} \Delta \tilde{d} \) and \( d_2 = D - \frac{1}{2} \Delta \tilde{d} \) into the average model, the \( \Delta d \)-to-\( \Delta i_L \) transfer function can be obtained:

\[
G_{\Delta d \Delta i_L} = \frac{\Delta \tilde{i}_L}{\Delta d} = \frac{I_o}{2D} \cdot \frac{1 - \frac{s}{\omega_{nip}} - \frac{s^2}{\omega_{nip}^2} + \frac{s}{Q_{ip} \omega_{nip}} + 1}{1 + \frac{s}{\omega_{rhp}}}, \tag{14}
\]

where \( \omega_{nip} \) and \( Q_{ip} \) are the same as in (6) or (9), and the right-half-plane zero is \( \omega_{rhp} = \frac{2I_o D}{V_{in} (1 - \beta) L_C} \). Fig. 12 plots the bode plots of \( G_{\Delta d \Delta i_L} \) under different load conditions. The right-half-plane zero together with the interphase resonant poles results in a 270° phase reduction. As \( I_o \) decreases, both \( \omega_{rhp} \) and the dc gain will reduce towards zero. The dc gain might even flip the sign due to nonlinear factors at very light load. All these issues could bring challenges to the active control of \( \Delta i_L \) and needs to be properly handled.

An alternative way of actively suppressing interphase resonance is to control \( \nu_c \) resonance. Similar to (14), the \( \Delta d \)-to-\( \nu_c \) transfer function can be derived as:

\[
G_{\Delta d \nu_c} = \frac{\nu_c}{\Delta d} = \frac{V_{in}}{4D} \cdot \frac{1 + \frac{s}{\omega_{niph}} - \frac{s^2}{\omega_{niph}^2} + \frac{s}{Q_{ip} \omega_{niph}} + 1}{1 + \frac{s}{\omega_{rhp}}}, \tag{15}
\]

where \( \omega_{niph} \) and \( Q_{ip} \) are the same as in (6) or (9), and \( \omega_{rhp} = \frac{D V_{in}}{I_o (1 - \beta) L_C} \). The dc gain of \( G_{\Delta d \nu_c} \) can describe the impacts of unequal duty ratios on the unbalanced value of \( C_B \) steady-state voltage, which could also be caused by resistance variation between phases [8], phase shift error [13], and source impedance [16]. Fig. 13 shows the bode plots of \( G_{\Delta d \nu_c} \), in which there is no right-half-plane zero and the maximum phase reduction is 180° under all load conditions, making it attractive to design a \( C_B \) voltage control loop for suppressing the interphase \( L-C_B \) resonance.

V. EXPERIMENTAL RESULTS

To verify the analysis, a two-phase SCB converter was built and tested with both discrete and coupled inductors. Fig. 14 shows the overall hardware prototype. An input connector board (Fig. 14b) for the line transient test is designed, which can provide an input voltage step change by switching in a second voltage source \( \Delta v \). Fig. 14c annotates the tested SCB converter. Key component parameters are tabulated in Table II. The prototype designed herein is to verify the theoretical analysis. Demonstrating high efficiency or power density is
out of the scope of this paper. Following experiments are performed as the SCB prototype converts 6-V to 0.5-V and switches at 300-kHz, unless otherwise specified.

Fig. 15 shows key operation waveforms of the SCB converter. The discrete design contains two standalone inductors, while the coupled case contains both standalone inductors and a toroidal coupled inductor. This setup ensures the discrete and the coupled cases have roughly the same leakage inductance (or transient $\frac{di}{dt}$). As shown in Fig. 15, the coupled inductor can greatly reduce the current ripple while maintaining roughly the same transient speed for both designs.

With measured $L_k$ and $C_B$, the interphase resonant frequency for the discrete design can be obtained as 9.26-kHz.

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With measured $L_k$ and $C_B$, the interphase resonant frequency for the discrete design can be obtained as 9.26-kHz.
VI. CONCLUSIONS
This letter presents a systematic approach of analyzing the intrinsic L-C resonant behavior in SCB converters. By decomposing disturbance and its response into common-mode and differential-mode dynamics, the intrinsic resonant behavior can be classified into output $L-C_o$ resonance and interphase $L-C_B$ resonance. Similar analysis approach can be extended to higher number of phases, enabling a more intuitive understanding of SCB converter transient and balancing behaviors. The impacts of coupled inductors is analyzed, indicating that higher coupling coefficient results in smaller resonant amplitude and lower resonant frequency with longer settling time. Comprehensive guidelines for designing a controller that cover both input-output dynamics and interphase resonance are provided. All analysis is verified by simulation or experimental results.

APPENDIX
STATE-SPACE MODEL OF AN M-PHASE SCB CONVERTER
An $M$-phase SCB converter contains $M$ inductors, $M - 1$ blocking capacitors, and one output capacitor, so there are $2M$ state variables. Select the state vector as $x = \begin{bmatrix} i_{L1}, i_{L2}, \ldots, i_{LM} & v_{C1}, v_{C2}, \ldots, v_{CM-1} & v_o \end{bmatrix}^T$, the input vector as $u = \begin{bmatrix} i_{in} \end{bmatrix}$, and the output vector as $y = \begin{bmatrix} i_o, v_o \end{bmatrix}^T$. Applying switching-cycle averaging, the state-space model can be obtained as:

$$\dot{x} = Ax + Bu,$$

$$y = Ex,$$

where the coefficient matrix $A$ is:

$$A = \begin{bmatrix} 0_{M \times M} & \frac{A_{12}}{-A_{21}} & \frac{A_{22}}{1} \end{bmatrix},$$

and the block matrices $A_{12}, A_{21},$ and $A_{22}$ are:

$$A_{12} = \begin{bmatrix} -D_{L1} & 0 & \cdots & 0 & \frac{1}{L_1} \\ \frac{D_{L2}}{L_2} & -D_{L2} & \cdots & \frac{1}{L_2} & \vdots \\ 0 & \cdots & \frac{D}{L_{M-1}} & -\frac{D}{L_{M-1}} & \frac{1}{L_{M-1}} \\ 0 & \cdots & 0 & \frac{1}{L_M} & \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} \frac{D}{C_{B1}} & \frac{D}{C_{B1}} & \cdots & 0 & 0 \\ 0 & \frac{D}{C_{B2}} & \cdots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & \frac{D}{C_{BM-1}} & -\frac{D}{C_{BM-1}} \\ \frac{1}{C_o} & \frac{1}{C_o} & \cdots & \frac{1}{C_o} & \end{bmatrix},$$

$$A_{22} = \begin{bmatrix} 0_{M-1 \times M-1} & 0_{M-1 \times 1} & 0_{1 \times M-1} & 0_{1 \times 1} \\ 0_{1 \times 2M-1} & 0_{M \times M} \end{bmatrix}.$$  

The coefficient matrices $B$ and $E$ are:

$$B = \begin{bmatrix} D_{L1} & 0_{1 \times 2M-1} \end{bmatrix}^T,$$

$$E = \begin{bmatrix} \frac{1}{L_1}, \frac{1}{L}, \ldots, \frac{1}{L} & 0_{M \times M} \\ 1, 1, \ldots, 1 & 0, 0, \ldots, 0 \\ 0, 0, \ldots, 0 & 1, 0, \ldots, 0, 1 \end{bmatrix}.$$  

Accordingly, the transfer functions can be derived as:

$$G_{v_{in}^{iL_k}} = \frac{\hat{i}_{L_k}}{v_{in}} = (E)_{row \ k} \cdot (s I - A)^{-1} B,$$

$$G_{v_{in}^{i_o}} = \frac{\hat{i}_o}{v_{in}} = (E)_{row \ M+1} \cdot (s I - A)^{-1} B,$$

$$G_{v_{in}^{v_o}} = \frac{\hat{v}_o}{v_{in}} = (E)_{row \ M+2} \cdot (s I - A)^{-1} B.$$  

REFERENCES