Introduction to Econometrics (3rd Updated Edition)

by

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Solutions to Odd-Numbered End-of-Chapter Exercises: Chapter 12

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12.1. (a) The change in the regressor, \(\ln(P_{cigarettes}^{1995}) - \ln(P_{cigarettes}^{1985})\), from a $0.50 per pack increase in the retail price is \(\ln(8.00) - \ln(7.50) = 0.0645\). The expected percentage change in cigarette demand is \(-0.94 \times 0.0645 \times 100\% = -6.07\%\). The 95% confidence interval is \((-0.94 \pm 1.96 \times 0.21) \times 0.0645 \times 100\% = [-8.72\%, -3.41\%].\)

(b) With a 2% reduction in income, the expected percentage change in cigarette demand is \(0.53 \times (-0.02) \times 100\% = -1.06\%\).

(c) The regression in column (1) will not provide a reliable answer to the question in (b) when recessions last less than 1 year. The regression in column (1) studies the long-run price and income elasticity. Cigarettes are addictive. The response of demand to an income decrease will be smaller in the short run than in the long run.

(d) The instrumental variable would be too weak (irrelevant) if the \(F\)-statistic in column (1) was 3.7 instead of 33.7, and we cannot rely on the standard methods for statistical inference. Thus the regression would not provide a reliable answer to the question posed in (a).
12.3. (a) The estimator \( \hat{\sigma}_a^2 = \frac{1}{n-2} \sum_{i=1}^{n} (Y_i - \hat{\beta}_{0T}^{TSL} - \hat{\beta}_{1T}^{TSL} \hat{X}_i)^2 \) is not consistent. Write this as
\[
\hat{\sigma}_a^2 = \frac{1}{n-2} \sum_{i=1}^{n} (\hat{u}_i - \hat{\beta}_{1T}^{TSL} (\hat{X}_i - X_i))^2,
\]
where \( \hat{u}_i = Y_i - \hat{\beta}_{0T}^{TSL} - \hat{\beta}_{1T}^{TSL} X_i \). Replacing \( \hat{\beta}_{1T}^{TSL} \) with \( \beta_1 \), as suggested in the question, write this as
\[
\hat{\sigma}_a^2 \approx \frac{1}{n} \sum_{i=1}^{n} (u_i - \beta_1 (\hat{X}_i - X_i))^2 = \frac{1}{n} \sum_{i=1}^{n} u_i^2 + \frac{1}{n} \sum_{i=1}^{n} [\beta_1^2 (\hat{X}_i - X_i)^2 + 2u_1 \beta_1 (\hat{X}_i - X_i)].
\]
The first term on the right hand side of the equation converges to \( \sigma_u^2 \), but the second term converges to something that is non-zero. Thus \( \hat{\sigma}_a^2 \) is not consistent.

(b) The estimator \( \hat{\sigma}_b^2 = \frac{1}{n-2} \sum_{i=1}^{n} (Y_i - \hat{\beta}_{0T}^{TSL} - \hat{\beta}_{1T}^{TSL} X_i)^2 \) is consistent. Using the same notation as in (a), we can write \( \hat{\sigma}_b^2 \approx \frac{1}{n} \sum_{i=1}^{n} u_i^2 \), and this estimator converges in probability to \( \sigma_u^2 \).
12.5. (a) Instrument relevance. $Z_i$ does not enter the population regression for $X_i$.

(b) $Z$ is not a valid instrument. $\hat{X}$ will be perfectly collinear with $W$. (Alternatively, the first stage regression suffers from perfect multicollinearity.)

(c) $W$ is perfectly collinear with the constant term.

(d) $Z$ is not a valid instrument because it is correlated with the error term.
12.7. (a) Under the null hypothesis of instrument exogeneity, the $J$ statistic is distributed as a $\chi^2_1$ random variable, with a 1% critical value of 6.63. Thus the statistic is significant, and instrument exogeneity $E(u_i|Z_{1i}, Z_{2i}) = 0$ is rejected.

(b) The $J$ test suggests that $E(u_i|Z_{1i}, Z_{2i}) \neq 0$, but doesn’t provide evidence about whether the problem is with $Z_1$ or $Z_2$ or both.
12.9. (a) There are other factors that could affect both the choice to serve in the military and annual earnings. One example could be education, although this could be included in the regression as a control variable. Another variable is “ability” which is difficult to measure, and thus difficult to control for in the regression.

(b) The draft was determined by a national lottery so the choice of serving in the military was random. Because it was randomly selected, the lottery number is uncorrelated with individual characteristics that may affect earning and hence the instrument is exogenous. Because it affected the probability of serving in the military, the lottery number is relevant.