Introduction to Econometrics (4th Edition)

by

James H. Stock and Mark W. Watson

Solutions to Odd-Numbered End-of-Chapter Exercises: Chapter 5

(This version September 14, 2018)

5.1 (a) The 95% confidence interval for β_1 is $\{-5.82 \pm 1.96 \times 2.21\}$, that is

$-10.152 \le \beta_1 \le -1.4884.$

(b) Calculate the *t*-statistic:

$$t^{act} = \frac{\hat{\beta}_1 - 0}{\text{SE}(\hat{\beta}_1)} = \frac{-5.82}{2.21} = -2.6335.$$

The *p*-value for the test $H_0: \beta_1 = 0$ vs. $H_1: \beta_1 \ne 0$ is

$$p\text{-value} = 2\Phi(-|t^{act}|) = 2\Phi(-2.6335) = 2 \times 0.0042 = 0.0084.$$

The *p*-value is less than 0.01, so we can reject the null hypothesis at the 5% significance level, and also at the 1% significance level.

(c) The *t*-statistic is

$$t^{act} = \frac{\hat{\beta}_1 - (-5.6)}{\text{SE}(\hat{\beta}_1)} = \frac{0.22}{2.21} = 0.10$$

The *p*-value for the test $H_0: \beta_1 = -5.6$ vs. $H_1: \beta_1 \ne -5.6$ is

$$p\text{-value} = 2\Phi(-|t^{act}|) = 2\Phi(-0.10) = 0.92$$

The *p*-value is larger than 0.10, so we cannot reject the null hypothesis at the 10%, 5% or 1% significance level. Because $\beta_1 = -5.6$ is not rejected at the 5% level, this value is contained in the 95% confidence interval.

(d) The 99% confidence interval for β_0 is $\{520.4 \pm 2.58 \times 20.4\}$, that is,

$467.7 \le \beta_0 \le 573.0.$

5.3. The 99% confidence interval is $1.5 \times \{3.94 \pm 2.58 \times 0.31)$ or

$$4.71 \text{ lbs} \leq \text{WeightGain} \leq 7.11 \text{ lbs}.$$

5.5 (a) The estimated gain from being in a small class is 13.9 points. This is equal to approximately 1/5 of the standard deviation in test scores, a moderate increase.

(b) The *t*-statistic is $t^{act} = \frac{13.9}{2.5} = 5.56,$ which has a *p*-value of 0.00. Thus the null hypothesis is rejected at the 5% (and 1%) level.

(c) $13.9 \pm 2.58 \times 2.5 = 13.9 \pm 6.45.$

(d) Yes. Students were randomly assigned to small or regular classes, so that *SmallClass* is independent of characteristics of the student, including those affecting testscores, that is u. Thus $E(u_i \mid ClassSize_i) = 0$.

5.7. (a) The t-statistic is $\frac{3.2}{1.5} = 2.13$ with a p-value of 0.03; since the p-value is less than 0.05, the null hypothesis is rejected at the 5% level.

(b) $3.2 \pm 1.96 \times 1.5 = 3.2 \pm 2.94$

(c) Yes. If Y and X are independent, then $\beta_1 = 0$; but the p-value in (a) was 0.03. This means that only in 3% of all samples, the absolute value of t-statistic would be 2.13 (the value actually observed in this sample) or larger.

(d) β_1 would be rejected at the 5% level in 5% of the samples; 95% of the confidence intervals would contain the value $\beta_1 = 0$.

5.9. (a) $\bar{\beta} = \frac{\frac{1}{n}(Y_1 + Y_2 + \cdots + Y_n)}{\bar{X}}$ so that it is linear function of $Y_1, Y_2, \ldots, Y_n$.

(b) $E(Y_i|X_1, \ldots, X_n) = \beta_1 X_i$, thus

$$E(\bar{\beta}|X_1, \ldots, X_n) = E\frac{1}{\bar{X}}\frac{1}{n}(Y_1 + Y_2 + \cdots + Y_n)|X_1, \ldots, X_n)$$
$$= \frac{1}{\bar{X}}\frac{1}{n}\beta_1(X_1 + \cdots + X_n) = \beta_1$$

5.11. Using the results from 5.10, $\hat{\beta}_0 = \overline{Y}_m$ and $\hat{\beta}_1 = \overline{Y}_w - \overline{Y}_m$. From Chapter 3, $\text{SE}(\overline{Y}_m) = \frac{S_m}{\sqrt{n_m}}$ and $\text{SE}(\overline{Y}_w - \overline{Y}_m) = \sqrt{\frac{s_m^2}{n_m} + \frac{s_w^2}{n_w}}$.

Plugging in the numbers $\hat{\beta}_0 = 523.1$ and $\text{SE}(\hat{\beta}_0) = 6.22$; $\hat{\beta}_1 = -38.0$

and $\text{SE}(\hat{\beta}_1) = 7.65$.

5.13. (a) Yes, this follows from the assumptions in KC 4.3.

(b) Yes, this follows from the assumptions in KC 4.3 and conditional homoskedasticity

(c) They would be unchanged for the reasons specified in the answers to those questions.

(d) (a) is unchanged; (b) is no longer true as the errors are not conditionally homoskesdastic.

5.15. Because the samples are independent, $\hat{\beta}_{m,1}$ and $\hat{\beta}_{w,1}$ are independent. Thus $\text{var}(\hat{\beta}_{m,1} - \hat{\beta}_{w,1}) = \text{var}(\hat{\beta}_{m,1}) + \text{var}(\hat{\beta}_{w,1})$. $\text{Var}(\hat{\beta}_{m,1})$

is consistently estimated as $[SE(\hat{\beta}_{m,1})]^2$ and $\text{Var}(\hat{\beta}_{w,1})$ is consistently estimated as $[SE(\hat{\beta}_{w,1})]^2$, so that $\text{var}(\hat{\beta}_{m,1} - \hat{\beta}_{w,1})$ is consistently estimated by $[SE(\hat{\beta}_{m,1})]^2 + [SE(\hat{\beta}_{w,1})]^2$, and the result follows by noting the SE is the square root of the estimated variance.