### **Research Memorandum**

From: James H. Stock and Mark W. Watson Date: September 19, 2010 Subj: Distribution of quarterly values of GDP/GDI across months within the quarter

### **Background**

For many purposes it is useful to have estimates of the monthly values of GDP and GDI. The BEA constructs quarterly values of GDP/GDI (both real and nominal), but not monthly values. However, several of the components of GDP/GDI are available monthly (for example, personal consumption expenditures (a GDP component) and compensation of employees (a GDI component)), and even when a monthly component is not available, there is often another variable or set of related variables that is available monthly (for example, total private residential construction, a monthly series published by the Census Bureau, is related to the residential investment component of GDP).

In this memorandum we describe how we have used monthly series to construct monthly estimates of GDP and GDI.<sup>1</sup>

## Notation and general method

Our general method is to carry out distribution for components of nominal GDP and GDI. The sum of the monthly estimates of the nominal components is the monthly estimate of nominal GDP and GDI. Monthly estimates of the GDP deflator are also computed, and these estimates are used to deflate the monthly nominal values of GDP/GDI to produce monthly estimates of real GDP/GDI.

Distribution of the quarterly values of a specific component of GDP/GDI is constructed as follows. Let upper case variables denote the quarterly series/time index ( $Q_T$  denotes the quarterly value of variable Q in quarter T), and lower case denote monthly series/time index ( $q_t$  denotes the monthly value of variable q in month t.) Quarterly values are related to monthly values by the identity

$$Q_T = (q_{3T} + q_{3T-1} + q_{3T-2})/3$$

<sup>&</sup>lt;sup>1</sup> GDP and GDI are "flow" variables, and the BEA's quarterly values correspond to value of production/income over the quarter. The problem of allocating a quarterly flow to months within the quarter is referred to as "distribution" (that is, distributing the total quarterly over the constituent months). A related problem is "interpolation" which estimates monthly values of stock variables (such as the size of the labor force) from quarterly values. See Harvey, A.C. (1989), *Forecasting, Structural Time Series Models and the Kalman Filter*, Cambridge University Press, for a discussion of the related problems of distribution, interpolation, and temporal aggregation.

where we note that all of the variables (monthly and quarterly) are expressed at annual rates.

For the variables that we consider,  $Q_T$  and  $q_t$  are trending variables. Let  $S_T$  and  $s_t$  denote trends for the series (details discussed below). Let  $\tilde{Q}_T = Q_T / S_T$  and  $\tilde{q}_t = q_t / s_t$  which are approximately I(0) variables. This yields the identity:

$$\tilde{Q}_{T} = \frac{1}{3S_{T}} \begin{bmatrix} s_{3T} & s_{3T-1} & s_{3T-2} \end{bmatrix} \begin{bmatrix} \tilde{q}_{3T} \\ \tilde{q}_{3T-1} \\ \tilde{q}_{3T-2} \end{bmatrix}$$

Let  $x_t$  denote a set of monthly indicators. We model  $\tilde{q}_t$  as

$$\tilde{q}_t = \beta_0 + x_t'\beta + u_t$$
, where  $u_t = \rho u_{t-1} + \varepsilon_t$ .

Conditional on the parameters,  $\beta$ ,  $\rho$ , and  $\sigma_{\varepsilon}$ , trends  $S_T$  and  $s_t$ , and quarterly data  $Q_T$ , these two equations serve as the "measurement" and "transition" equations for a linear state-space model, and  $q_t$  can be estimated by the Kalman smoother.

The trend estimates  $S_T$  and  $s_t$  were computed by fitting a smooth curve (a cubic spline) to  $Q_T$ .

The parameters  $\beta$ ,  $\rho$ , and  $\sigma_{\varepsilon}$  are estimated by Gaussian maximum likelihood using the Q and x data.

Each component uses a different set of *x* variables, which are documented in the table below. In several instances, data availability dictates that the components of  $x_t$  change during the sample period. When a change occurs,  $\beta_0$ ,  $\rho$  and  $\sigma_{\varepsilon}$  are allow to change as well.

The same method was used to interpolate the implicit GDP deflator to compute a monthly price index.

<b>Components of Nominal GDP</b>
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Q	Sample Period	x Notes		
Personal Consumption Expenditures	Full Sample	Personal Consumption Expenditures	Monthly PCE averages to Quarterly PCE. No interpolation needed	
Investment: NonResidential Structures	1959:1- 1992:12	CONP (Citibase); CONSTRUCT.PUT IN PLACE: TOTAL PRIVATE (MIL\$,SAAR) CONFR (Citibase): CONSTRUCT.PUT IN PLACE: PRIV RESIDENTIAL BLDGS (MIL\$,SAAR) x = CONP - CONFR	Citibase Series	
	1993:1 – End	PRIV: Total Private Construction RES: Total Private Residential Contruction. x = PRIV – RES	Series from : RES from Census C30 Reports: http://www.census.gov/const/www/c30index.html	
Investment: Equipment and Software	1959:1 – 1967:12 1968:1 – 1991:12	ATCGVS: Manufacturers Shipments: Total Capital Goods ANDEVS: Manufacturers Shipments: Nondefense Capital Goods	Data are from the Census M3 releases. Historical time series available at <a href="http://www.census.gov/manufacturing/m3/historical_data/index.html">http://www.census.gov/manufacturing/m3/historical_data/index.html</a> SIC prior to 1992 NAICS 1992:1-2009:12	
	1992:1 – End	AITIVS: Manufacturers Shipments: Information Technology Industries ANDEVS and AITIVS		
Investment: Residential	1959:1- 1992:12	CONFR (Citibase): CONSTRUCT.PUT IN PLACE: PRIV RESIDENTIAL BLDGS	Citibase Series	
Structures	1993:1 – End	RES: Total Private Residential Contruction.	From Census C30 Reports: http://www.census.gov/const/www/c30index.html	
Investment: Change in Private Inventories	1959:1- 1966:12	IVMT (Citibase): MFG & TRADE INVENTORIES: TOTAL BIL\$,EOM,SA)) $x = \Delta$ IVMT	Citibase Release Nov. 1998	
	1967:1- End	INVT_CHANGE: NIPA Underlying Detail Tables 5.6.5AM1 and 5.6.5BM1 . Change in Private Inventories by Industry	NIPA Underlying Detail Table. Change in Private Inventories by Industry	

Exports	1959:1 -	FSE602 EXPORTS EXCLUD.MILITARY AID	(Pre-1998 Citibase, from Bernanke-Gertler-Watson BPEA backup files):
	1904.12	SIII MENTS(MIL\$,SA)(BC1002) ESE602 EXPORTS EXCLUD MILITARY AID	(Pre. 1008 Citibase, from Bernanke Gertler Watson BPEA backup files)
	1901.12	SHIPMENTS(MIL\$ SA)(BC1602)	(110-1998 Chibase, nom Demarke-Oenter-watson Di EA backup mes).
	1771.12	SIII WENTS(WIE, SA)(Deno2)	
		FTE71 :U.S.MDSE EXPORTS:	
		NONELECTRICAL MACHINERY (MIL\$,SA)	
		(BCI Series number xxx)	
		FTEF · U S MDSE (EXPORTS)	
		AGRICULTURAL PRODUCTS (MILS SA)	
		(BCI Series number xxx)	
	1992:1-	EXPORTS: Exports of goods and services, BOP	Available at
	2009:12	Basis (SA) Millions of \$s.	http://www.bea.gov/international/index.htm
Imports	1959:1 –	FSM612: GENERAL IMPORTS (MIL\$,	(Pre-1998 Citibase, from Bernanke-Gertler-Watson BPEA backup files):
	1964:12	S.A.)(BCI-612)	
	1965:1-	FSM612: GENERAL IMPORTS (MIL\$, S.A.)	(Pre-1998 Citibase, from Bernanke-Gertler-Watson BPEA backup files):
	1991:12	(BCI-612)	
		ETM222 · U S MDSE IMDODTS·	
		PETROLEUM & PETROLEUM PRODUCTS	
		(MIL\$ SA) (BCI Series number xxx)	
		FTM732: U.S.MDSE IMPORTS:	
		AUTOMOBILES & PARTS (MIL\$,SA) (BCI	
		Series number xxx)	
	1002.1	IMPORTS: Imports of goods and services DOD	Available et
	1992.1- End	Basis (SA) Millions of \$s	http://www.bea.gov/international/index.htm
Government	1959:1-	WAGES G: Wage and Salary Disbursements by	
	1967:12	Industry, Government, NIPA Tables 2.7A and	
		2.7B	
		CONQ (Citibase): CONSTRUCT.PUT IN	
	10(0.1	PLACE: TOTAL PUBLIC, (MIL.\$,SAAR)	AMTNO/O and AMVDVO from from the Origina M2 subserve Historical
	1968:1 - 1001.12	WAGES_G: Wage and Salary Disbursements by Industry Covernment NIDA Tables 2.7A and	AMTMVS and AMADVS from from the Census M3 releases. Historical
	1991.12	2 7B	http://www.census.gov/manufacturing/m3/historical_data/index_html
		2.70	
		CONQ (Citibase): CONSTRUCT.PUT IN	

	PLACE: TOTAL PUBLIC, (MIL.\$,SAAR)	
	AMTMVS-AMXDVS, where	
	AMTMVS: Manufacturing and Trade Sales: Total Manufacturing (SIC 1968-1991)	
	AMXDVS: Manufacturing and Trade Sales: Manufacturing excluding Defense (SIC 1968-	
1002 1	1771) WACEG C W 101 D'1 (1	
1992:1 -	WAGES_G: Wage and Salary Disbursements by	CON_Gov from Census C30 Reports:
End	Industry, Government, NIPA Tables 2.7A and	http://www.census.gov/const/www/c30index.html
	2.7B	
		AMTMVS and AMXDVS from from the Census M3 releases. Historical
	CON Gov. Total Public Construction put in	time series available at
	place	http://www.census.gov/manufacturing/m3/historical_data/index_html
	place	
	AMTMVS-AMXDVS, where	
	AMTMVS: Manufacturing and Trade Sales: Total Manufacturing (NAICS 1992-2009)	
	AMXDVS: Manufacturing and Trade Sales: Manufacturing excluding Defense (NAICS 1992- 2009)	

Q	Sample	x	
	Period		
Employee	1959:1-	COMPE: Compensation of Employees from monthly Personal Income	
Compensation	End		2.6, line 2
Proprietors Income	1959:1-	PROP_INC: Proprietors Income from monthly Personal Income.	NIPA Table
	End		2.6, line 7
Rental Income	1959:1-	Rental Income (with CCA): from monthly Personal Income	NIPA Table
	End		2.6, line 10
		Note: Current version of Table 2.6 shows no variation in monthly rents within a quarter for dates prior to	
		January 1977. For the pre-1977 we have used series GMPREN from a previous version of CITIBASE.	
Net Interest	1959:1-	INTEREST: Personal Interest Income from monthly Personal Income	NIPA Table
	End		2.6, line 12
Corporate Profits	1959:1-	PROP_INC: Proprietors Income from monthly Personal Income.	NIPA Table
	End		2.6, line 7
Other GDI	1959:1-	COMPE: Compensation of Employees from monthly Personal Income	NIPA Table
	End		2.6, line 2
		PROP_INC: Proprietors Income from monthly Personal Income.	
			NIPA Table
			2.6, line 7

# **Components of Nominal GDI**

# **GDP Price Deflator**

Q	Sample Period	x	Notes
GDP Price Deflator	1959:1-End	PCED: PCE Price Deflator	NIPA Table 2.8.4, line 1

Replication files are available at http://www.princeton.edu/~mwatson/mgdp\_gdi.html:

The replication files consist of input data (Q and x) and computer programs.

Data: All Input Data are listed in DISTRIBUTE\_GDP\_GDI\_INPUT.XLS

Programs: (All programs use GAUSS (v10.0) and require the GAUSS program MAXLIK) DISTRIBUTE\_NOMINAL\_COMPONENTS.GSS: Distributes each of the monthly components listed above DISTRIBUTE\_GDP\_DEFLATOR.GSS: Distributes the GDP deflator DISTRIBUTE\_REAL\_GDP\_GDI.GSS: Uses output from the two programs about to compute monthly estimates of nominal and real values of GDP/GDI.