Appendix E: Markov Models[1]

A Markov model describes the movements of members of a population through a set of states. A simple Markov model requires only two sets of information:

- the distribution of the population among the defined states at the initiation of the model; and
- for each state, the probability that any individual in that state will move into a different state (transition probabilities). In a simple Markov model, these probabilities remain the same with each iteration of the model. In the Markov model used in chapter 3, some probabilities change with successive iterations (e.g., the probability that women will die increases as the cohort grows older).

The transition probabilities can be portrayed in a matrix. For example, imagine a Markov model in which two states exist: healthy and cancer. Assume that in any given period, the chance of a healthy person getting cancer is 10 percent, and the chance of remaining healthy is 90 percent. For people with cancer, the chance of being cured (becoming healthy) during this period is 50 percent, and the probability of continuing to have cancer is also 50 percent. This situation is summarized in the following matrix:

		Period 2	
		Healthy	Cancer
Period	Healthy	0.9	0.1
	Cancer	.	0.5

Now, assume that there are 100 people in the population, and that initially (at the beginning of the model) 90 are healthy and 10 have cancer. Applying the above probabilities, the distribution of the population after one iteration of the model--i. e., in period 2--would be:

<u>Healthy:</u> (90 x 0.9) + (lo x 0.5) =
81 + 5 = 86 people

<u>Cancer:</u> (90 x 0.1) + (lo x 0.5) =
9 + 5 = 14 people

In this simple example, applying the transition probabilities successively to the population distribution from each previous iteration of the model, the population soon reaches a stable distribution:

Percent of population that:	Period 1	2	3	4	5
Is healthy	90	86	84	84	84
Has cancer	10	14	16	16	16

(In the model used in chapter 3, the population is artificially required to be stable by imposing the requirement that all women still alive at age 109 die in that iteration of the model (before reaching age 110).)

Markov models rely on certain assumptions that affect their application. Most important for the model applied in chapter 3 is that the probability that an individual will move into a different state depends only on her present state. A simple Markov model has no memory; a person's chance of having cancer in the next period in the example above depends only on his or her health in the current period. Also, the model can be applied only where there are mutually exclusive and exhaustive categories; an individual cannot be represented in two categories at once.

The conditions of Pap smear screening for elderly women are generally amenable to simulation with a Markov model. In the model used in the cost-effectiveness analysis (ch. 3), states of health are defined to be mutually exclusive and exhaustive as they relate to cervical cancer. The initial population

1 The information in this appendix is drawn from E. Stokey and R. Zeckhauser, <u>A primer for policy Analysis</u> (New York: W.W. Norton & Co., 1978).

distribution for the model is the existing prevalence of disease; the transition probabilities are mortality rates and rates for identification, progression, regression, and cure of disease. The purpose of the model as applied here is not to define a stable population distribution, but to track the costs and cases of cervical cancer through the lifetime of the defined cohort of women.

The most troublesome aspect of Markov modeling for cervical cancer screening is the assumption that the transition probabilities are the same for all individuals in each state. Some of the implications of this underlying assumption are discussed in the text.