Logit, Probit, and Multinomial Logit models in R

(v. 3.6)

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If outcome or dependent variable is binary and in the form 0/1, then use logit or probit models.
Some examples are:

Did you vote in the last election?	Do you prefer to use public transportation or to drive a car?
0 'No'	
1 'Yes'	0 'Prefer to drive'
	1 'Prefer public transport'

If outcome or dependent variable is categorical but are ordered (i.e. low to high), then use ordered logit or ordered probit models. Some examples are:

Do you agree or disagree with the President?	What is your socioeconomic status?
1 'Disagree'	1 'Low'
2 'Neutral'	2 'Middle'
3 'Agree'	3 'High'

If outcome or dependent variable is categorical without any particular order, then use multinomial logit. Some examples are:

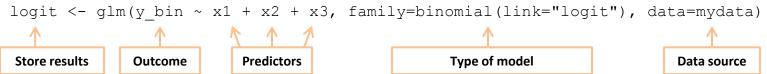
If elections were held today, for which party	What do you like to do on the weekends?
would you vote?	
	1 'Rest'
1 'Democrats'	2 'Go to movies'
2 'Independent'	3 'Exercise'
3 'Republicans'	TR 2

Logit model

Getting sample data

```
library(foreign)
mydata <- read.dta("https://www.princeton.edu/~otorres/Panel101.dta")</pre>
```

Running a logit model



summary(logit)

```
Call: qlm(formula = y bin \sim x1 + x2 + x3, family = binomial(link = "logit"),
```

data = mydata)

Deviance Residuals:

Min 1Q Median 3Q Max -2.0277 0.2347 0.5542 0.7016 1.0839

Coefficients:

x3	0.7512	0.4548	1.652	0.0986	•
x2	0.3665	0.3082	1.189	0.2343	
x1	0.8618	0.7840	1.099	0.2717	
(Intercept)	0.4262	0.6390	0.667	0.5048	
	Estimate	Sta. Error	z value	Pr(> z)	

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 70.056 on 69 degrees of freedom Residual deviance: 65.512 on 66 degrees of freedom

AIC: 73.512

The Pr(>|z|) column shows the two-tailed p-values testing the null hypothesis that the coefficient is equal to zero (i.e. no significant effect). The usual value is 0.05, by this measure none of the coefficients have a significant effect on the log-odds ratio of the dependent variable. The coefficient for x3 is significant at 10% (<0.10).

The **z value** also tests the null that the coefficient is equal to zero. For a 5% significance, the z-value should fall outside the ±1.96.

The **Estimate** column shows the coefficients in log-odds form. When $\times 3$ increase by one unit, the expected change in the log odds is 0.7512. What you get from this column is whether the effect of the predictors is positive or negative. See next page for an extended explanation.

Logit model

The stargazer() function from the package -stargazer allows a publication quality of the logit model.

The model will be saved in the working directory under the name 'logit.htm' which you can open with Word or any other word processor.

library(stargazer)
stargazer(logit, type="html", out="logit.htm")

	$Dependent\ variable:$
	y_bin
x1	0.862
	(0.784)
x2	0.367
	(0.308)
x3	0.751*
	(0.455)
Constant	0.426
	(0.639)
Observations	70
Log Likelihood	-32.756
Akaike Inf. Crit.	73.512
Note:	*p**p***p<0.01

NOTE: Use the option type = "text" if you want to see the results directly in the RStudio console.

Logit model: odds ratio

Odds ratio interpretation (OR): Based on the output below, when x3 increases by one unit, the odds of y = 1 increase by 112% -(2.12-1)*100-. Or, the odds of y = 1 are 2.12 times higher when x3 increases by one unit (keeping all other predictors constant). To get the odds ratio, you need explonentiate the logit coefficient.

Estimating the odds ratio by hand

The **Estimate** column shows the coefficients in log-odds form. When x3 increase by one unit, the expected change in the log odds is 0.7512. Lets hold x1 and x2 constant at their means, and vary x3 with values 1, 2, and 3, to get the predicted log-odds given each of the three values of x3:

```
When x3 increases from 1 to 2, the log-odds increases:
r2-r1
0.7512115
When x3 increases from 2 to 3, the log-odds increases:
r3-r2
0.7512115
Which corresponds to the estimate for x3 above.
```

The odds ratio, is the exponentiation of the difference of the log-odds

```
> exp(r2-r1)
2.119566
```

Or, the ratio of the exponentiation of each of the log-odds.

```
> exp(r2)/exp(r1)
2.119566
```

OTR Which corresponds to the OR value for x3 above. 5

Logit model: odds ratios

Relative risk ratios allow an easier interpretation of the logit coefficients. They are the exponentiated value of the logit coefficients.

library(stargazer)

stargazer(logit, type="html", coef=list(logit.or), p.auto=FALSE, out="logitor.htm")

	Dependent variable:
	y_bin
x1	2.367
	(0.784)
x2	1.443
	(0.308)
x3	2.120*
	(0.455)
Constant	1.531
	(0.639)
Observations	70
Log Likelihood	-32.756
Akaike Inf. Crit.	73.512
Note:	*p**p***p<0.01

Keeping all other variables constant, when x1 increases one unit, it is 2.367 times more likely to be in the 1 category. In other words, the odds of being in the 1 category (as opposed to the 0 category) are 136% higher when x1 move one unit (2.36 - 1). The coefficient, however, is not significant.

NOTE: Use the option type = "text" if you want to see the results directly in the RStudio console.

The logit model can be written as (Gelman and Hill, 2007):

$$Pr(y_i = 1) = Logit^{-1}(X_i\beta)$$

In the example:

Estimating the probability at the mean point of each predictor can be done by inverting the logit model. Gelman and Hill provide a function for this (p. 81), also available in the R package -arm-

```
invlogit = function (x) \{1/(1+\exp(-x))\}

invlogit(coef(logit)[1]+

coef(logit)[2]*mean(mydata$x1)+

coef(logit)[3]*mean(mydata$x2)+

coef(logit)[4]*mean(mydata$x3))

Pr(y_i = 1) = 0.8328555
```

Adding categorical variable, the model would be:

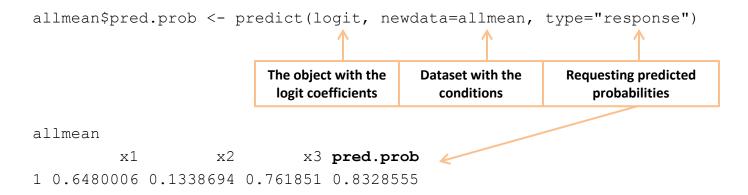
Estimating the probability when opinion = 'Agree'

```
invlogit = function (x) \{1/(1+\exp(-x))\}
Estimating the probability when opinion = 'Disagree'
                  invlogit(coef(logit.cat)[1]+
                           coef(logit.cat)[2]*mean(mydata$x1)+
                           coef(logit.cat)[3]*mean(mydata$x2)+
                           coef(logit.cat)[4]*mean(mydata$x3)+
                           coef(logit.cat)[6]*1)
                Pr(y_i = 1 | opinion = "Disagree") = 0.9077609
Estimating the probability when opinion = 'Strongly disagree'
               invlogit(coef(logit.cat)[1]+
                           coef(logit.cat)[2]*mean(mydata$x1)+
                           coef(logit.cat)[3]*mean(mydata$x2)+
                           coef(logit.cat)[4]*mean(mydata$x3)+
                           coef(logit.cat)[7]*1)
                Pr(y_i = 1 | opinion = "Strongly disagree") = 0.933931
Estimating the probability when opinion = 'Strongly agree'
                 invlogit(coef(logit.cat)[1]+
                           coef(logit.cat)[2]*mean(mydata$x1)+
                           coef(logit.cat)[3]*mean(mydata$x2)+
                           coef(logit.cat) [4] *mean(mydata$x3))
                Pr(y_i = 1 | opinion = "Strongly agree") = 0.8764826
```

Another way to estimate the predicted probabilities is by setting initial conditions.

Getting predicted probabilities holding all predictors or independent variables to their means.

After estimating the logit model and creating the dataset with the mean values of the predictors, you can use the predict() function to estimate the predicted probabilities (for help/details type ?predict.glm), and add them to the allmean dataset.



When all predictor values are hold to their means, the probability of y = 1 is 83%.

Logit model: predicted probabilities with categorical variable

```
logit <- glm(y bin ~ x1+x2+x3+opinion, family=binomial(link="logit"), data=mydata)</pre>
```

To estimate the predicted probabilities, we need to set the initial conditions. Getting predicted probabilities holding all predictors or independent variables to their means for each category of categorical variable 'opinion':

```
Creating a new
allmean <- data.frame(x1=rep(mean(mydata$x1),4),</pre>
                                                                                                        dataset with the
                        x2=rep(mean(mydata$x2),4),
                                                                                                       mean values of the
                        x3=rep(mean(mydata$x3),4),
                                                                                                       predictors for each
                        opinion=as.factor(c("Str agree","Agree","Disag","Str disag")))
                                                                                                           category
allmean
 x1
                     xЗ
                          opinion
1 0.6480006 0.1338694 0.761851 Str agree
2 0.6480006 0.1338694 0.761851
                                     Agree
3 0.6480006 0.1338694 0.761851
                                     Disag
4 0.6480006 0.1338694 0.761851 Str disag
allmean <- cbind(allmean, predict(logit, newdata=allmean, type="response", se.fit=TRUE))
                                   The object with the
                                                                           Requesting predicted
                                                                                                  Standard error of
                                                       Dataset with the
                                    logit coefficients
                                                          conditions
                                                                              probabilities
                                                                                                   the prediction
                                   allmean
                                                                      opinion
                                                                                             se.fit residual.scale
                                            x1
                                                       x2
                                                                                     fit
                                   1 0.6480006 0.1338694 0.761851 Str agree 0.8764826 0.07394431
                                                                                                                  1
                                   2 0.6480006 0.1338694 0.761851
                                                                        Agree 0.5107928 0.15099064
                                                                                                                  1
                                   3 0.6480006 0.1338694 0.761851
                                                                        Disag 0.9077609 0.06734568
                                                                                                                  1
                                   4 0.6480006 0.1338694 0.761851 Str disag 0.9339310 0.06446677
                                                                                                                  1
(continue next page)
```

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Logit model: predicted probabilities with categorical variable

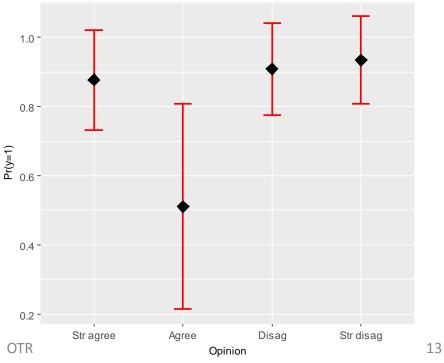
```
# Renaming "fit" and "se.fit" columns
names(allmean) [names(allmean) == "fit"] = "prob"
names(allmean) [names(allmean) == "se.fit"] = "se.prob"
# Estimating confidence intervals
allmean$11 = allmean$prob - 1.96*allmean$se.prob
allmean$ul = allmean$prob + 1.96*allmean$se.prob
allmean
                               opinion prob se.prob residual.scale
        x1 x2 x3
                                                                               11
                                                                                         ul
1 0.6480006 0.1338694 0.761851 Str agree 0.8764826 0.07394431
                                                                      1 0.7315518 1.0214134
2 0.6480006 0.1338694 0.761851 Agree 0.5107928 0.15099064
                                                                      1 0.2148511 0.8067344
3 0.6480006 0.1338694 0.761851 Disag 0.9077609 0.06734568
                                                                      1 0.7757634 1.0397585
4 0.6480006 0.1338694 0.761851 Str disag 0.9339310 0.06446677
                                                                       1 0.8075762 1.0602859
```

(continue next page)

Logit model: predicted probabilities with categorical variable

```
# Plotting predicted probabilities and confidence intervals using ggplot2
library(ggplot2)
ggplot(allmean, aes(x=opinion, y = prob)) +
  geom errorbar(aes(ymin = 11, ymax = u1), width = 0.2, lty=1, lwd=1, col="red") +
  geom point(shape=18, size=5, fill="black") +
  scale x discrete(limits = c("Str agree", "Agree", "Disag", "Str disag")) +
  labs(title= " Predicted probabilities", x="Opinion", y="Pr(y=1)", caption = "add footnote here") +
  theme(plot.title = element text(family = "sans", face="bold", size=13, hjust=0.5),
        axis.title = element text(family = "sans", size=9),
        plot.caption = element text(family = "sans", size=5))
```

Predicted probabilities



Logit model: marginal effects

Marginal effects show the change in probability when the predictor or independent variable increases by one unit. For continuous variables this represents the instantaneous change given that the 'unit' may be very small. For binary variables, the change is from 0 to 1, so one 'unit' as it is usually thought.

Ordinal logit model

```
# Getting sample data
library(foreign)
mydata <- read.dta("https://dss.princeton.edu/training/Panel101.dta")</pre>
# Loading library -MASS-
library (MASS)
# Running the ordered logit model
m1 <- polr(opinion ~ x1 + x2 + x3, data=mydata, Hess=TRUE)
Store results
              Outcome
                         Predictors
                                         Data source
                                                         Required for SE
summary (m1)
Call:
polr(formula = opinion \sim x1 + x2 + x3, data = mydata, Hess = TRUE)
Coefficients:
     Value Std. Error t value
x1 0.98140
               0.5641 1.7397
x2 0.24936
               0.2086 1.1954
x3 0.09089
                0.1549 0.5867
Intercepts:
                 Value
                         Std. Error t value
Str agree | Agree -0.2054 0.4682
                                     -0.4388
Agree | Disag
                  0.7370 0.4697
                                      1.5690
Disag|Str disag 1.9951 0.5204
                                      3.8335
Residual Deviance: 189.6382
AIC: 201.6382
```

Ordinal logit model: p-values

Getting coefficients and p-values

Ordered logit model

The stargazer() function from the package -stargazer allows a publication quality of the logit model.

The model will be saved in the working directory under the name `m1.htm' which you can open with Word or any other word processor.

```
library(stargazer)
stargazer(m1, type="html", out="m1.htm")
```

1	Dependent variable:
_	opinion
x1	0.981*
	(0.564)
x 2	0.249
	(0.209)
x3	0.091
	(0.155)
Observations	70
Note:	*p**p***p<0.01

NOTE: Use the option type = "text" if you want to see the results directly in the RStudio console.

Ordered logit model: odds ratios

Relative risk ratios allow an easier interpretation of the logit coefficients. They are the exponentiated value of the logit coefficients.

	ependent variable:
_	opinion
x1	2.668*
	(0.564)
x 2	1.283
	(0.209)
x3	1.095
	(0.155)
Observations	70
Note:	*p**p***p<0.01

Keeping all other variables constant, when x1 increases one unit, it is 2.668 times more likely to be in a higher category. In other words, the odds of moving to a higher category in the outcome variable is 166% when x1 move one unit (2.66-1). The coefficient is significant.

Ordinal logit model: predicted probabilities

Use "probs" for predicted probabilities

```
m1.pred <- predict(m1, type="probs")
summary(m1.pred)</pre>
```

Str ag	ree	Agre	е	Disa	3	Str dis	sag
Min.	:0.1040	Min.	:0.1255	Min.	:0.1458	Min.	:0.07418
1st Qu	.:0.2307	1st Qu	.:0.2038	1st Qu	.:0.2511	1st Qu	.:0.17350
Median	:0.2628	Median	:0.2144	Median	:0.2851	Median	:0.23705
Mean	:0.2869	Mean	:0.2124	Mean	:0.2715	Mean	:0.22923
3rd Qu	.:0.3458	3rd Qu	.:0.2271	3rd Qu	.:0.2949	3rd Qu	.:0.26968
Max.	:0.5802	Max.	:0.2313	Max.	:0.3045	Max.	:0.48832

The bold numbers are the predicted probabilities of each category when all predictors are at their mean value

Ordinal logit model: predicted probabilities

```
# At specific values, example x1 and x2 at their means, and x3 = 1 and x3 = 2.
# Use "probs" for predicted probabilities given specific predictors
setup1 <- data.frame(x1=rep(mean(mydata$x1),2),</pre>
                       x2=rep(mean(mydata$x2),2),
                       x3=c(1,2)
                                                              Setup for new predicted
setup1
                                                              probabilities
         x1
              x2 x3
1 0.6480006 0.1338694 1
2 0.6480006 0.1338694 2
setup1[, c("pred.prob")] <- predict(m1, newdata=setup1, type="probs")</pre>
setup1
       x1 x2 x3 pred.prob.Str agree pred.prob.Agree pred.prob.Disag pred.prob.Str disag
1 0.6480006 0.1338694 1
                              0.2757495
                                           0.2184382
                                                         0.2804806
                                                                           0.2253318
2 0.6480006 0.1338694 2
                              0.2579719
                                           0.2135235
                                                         0.2869123
                                                                          0.2415923
# Use "class" for the predicted category
setup1[, c("pred.prob")] <- predict(m1, newdata=setup1, type="class")</pre>
setup1
         x1 x2 x3 pred.prob
1 0.6480006 0.1338694 1
                               Disag
                                        These are the predicted categories given the new data
2 0.6480006 0.1338694 2
```

Ordinal logit model: marginal effects

```
# Load package "erer", use function ocMe() for marginal effects
library(erer)
x <- ocME(m1, x.mean=TRUE)</pre>
X
  effect.Str agree effect.Agree effect.Disag effect.Str disag
           -0.198
                  -0.047 0.076
                                                    0.169
x1
×2.
           -0.050
                      -0.012 0.019
                                                    0.043
           -0.018
                       -0.004 0.007
                                                    0.016
xЗ
# Type the following if you want t and p-values
```

x\$out

Multinomial logit model

Loading the required packages

```
library(foreign)
library(nnet)
library(stargazer)
# Getting the sample data from UCLA
mydata = read.dta("http://www.ats.ucla.edu/stat/data/hsb2.dta")
# Checking the output (dependent) variable
table(mydata$ses)
   low middle
                high
           95
                  58
    47
# By default the first category is the reference.
# To change it so 'middle' is the reference type
mydata$ses2 = relevel(mydata$ses, ref = "middle")
```

NOTE: This section is based on the UCLA website http://www.ats.ucla.edu/stat/r/dae/mlogit.htm, applied to data from the page http://www.ats.ucla.edu/stat/stata/output/stata mlogit output.htm. Results here reproduce the output in the latter to compare, and to provide an additional source to interpret outcomes.

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Multinomial logit model

Running the multinomial logit model using the multinom() function

multi1 = multinom(ses2 ~ science + socst + female, data=mydata)

Store results

Outcome

Predictors

Data source

summary(multi1)

Call:

multinom(formula = ses2 ~ science + socst + female, data = mydata)

Coefficients:

(Intercept) science socst femalefemale low 1.912288 -0.02356494 -0.03892428 0.81659717 high -4.057284 0.02292179 0.04300323 -0.03287211

Std. Errors:

(Intercept) science socst femalefemale low 1.127255 0.02097468 0.01951649 0.3909804 high 1.222937 0.02087182 0.01988933 0.3500151

Residual Deviance: 388.0697

AIC: 404.0697

These are the logit coefficients relative to the reference category. For example, under 'science', the -0.02 suggests that for one unit increase in 'science' score, the logit coefficient for 'low' relative to 'middle' will go down by that amount, -0.02.

In other words, if your science score increases one unit, your chances of staying in the middle ses category are higher compared to staying in low ses.

Multinomial logit model

The multinom() function does not provide p-values, you can get significance of the coefficients using the stargazer() function from the package -stargazer.

The model will be saved in the working directory under the name 'multi1.htm' which you can open with Word or any other word processor.

library(stargazer)
stargazer(multi1, type="html", out="multi1.htm")

	Dependen	nt variable:
	low	high
	(1)	(2)
science	-0.024	0.023
	(0.021)	(0.021)
socst	-0.039**	0.043**
	(0.020)	(0.020)
femalefemale	0.817**	-0.033
	(0.391)	(0.350)
Constant	1.912*	-4.057***
	(1.127)	(1.223)
Akaike Inf. Crit	. 404.070	404.070
Note:	*p**	p***p<0.01

NOTE: Use the option type = "text" if you want to see the results directly in the RStudio console.

Multinomial logit model: relative risk ratios

Relative risk ratios allow an easier interpretation of the logit coefficients. They are the exponentiated value of the logit coefficients.

	Dependent variable:	
	low	high
	(1)	(2)
science	0.977	1.023
'	(0.021)	(0.021)
socst	0.962**	1.044**
	(0.020)	(0.020)
femalefemale	2.263**	0.968
	(0.391)	(0.350)
Constant	6.769*	0.017***
	(1.127)	(1.223)
Akaike Inf. Crit.	404.070	404.070
Note:	*p**	p***p<0.01

Keeping all other variables constant, if your science score increases one unit, you are 0.97 times more likely to stay in the low ses category as compared to the middle ses category (the risk or odds is 3% lower). The coefficient, however, is not significant.

Keeping all other variables constant, if your science score increases one unit, you are 1.02 times more likely to stay in the high ses category as compared to the middle ses category (the risk or odds is 2% higher). The coefficient, however, is not significant.

NOTE: Use the option type = "text" if you want to see the results directly in the RStudio console.

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Ordinal logit model: predicted probabilities

At specific values, example science and socst at their means for males and females. # Use "probs" for predicted probabilities given specific predictors allmean <- data.frame(science=rep(mean(mydata\$science),2),</pre> socst=rep (mean (mydata\$socst), 2), female = c("male", "female")) Setup for new predicted allmean probabilities science socst female 51.85 52.405 male 51.85 52.405 female 2 allmean[, c("pred.prob")] <- predict(multi1, newdata=allmean, type="probs")</pre> allmean science socst female pred.prob.middle pred.prob.low pred.prob.high 51.85 52.405 male 0.5555769 0.1441171 0.3003061 51.85 52.405 female 0.2478890 0.4739293 0.2781816 # Use "class" for the predicted category allmean[, c("pred.prob")] <- predict(multi1, newdata=allmean, type="class")</pre> allmean science socst female pred.prob 1 51.85 52.405 male middle These are the predicted categories given the new data

51.85 52.405 female

middle

Sources

Greene, *Econometric Analysis*, 7th. ed.

Gelman and Hill, Data Analysis Using Regression and Multilevel/Hierarchical Models, 2007

UCLA, http://www.ats.ucla.edu/stat/r/dae/

StatsExchange, http://stats.stackexchange.com/

R packages:

-mfx- http://cran.r-project.org/web/packages/mfx/mfx.pdf

-erer- http://cran.r-project.org/web/packages/erer/erer.pdf