

# **Getting Started in Linear Regression using R**

## **(with some examples in Stata)**

(v. 1.0)

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**R****Stata**

## Using dataset “Prestige”\*

Used in the regression models in the following pages

```
# Dataset is in the following library  
  
library(car)  
  
# If not installed type  
  
install.packages("car")  
  
# Type help(Prestige) to access the codebook  
  
✓ education. Average education of occupational  
  incumbents, years, in 1971.  
✓ income. Average income of incumbents, dollars, in  
  1971.  
✓ women. Percentage of incumbents who are women.  
✓ prestige. Pineo-Porter prestige score for occupation,  
  from a social survey conducted in the mid-1960s.  
✓ census .Canadian Census occupational code.  
✓ type. Type of occupation. A factor with levels (note:  
  out of order): bc, Blue Collar; prof, Professional,  
  Managerial, and Technical; wc, White Collar.
```

```
/* Stata version here */  
  
use http://www.ats.ucla.edu/stat/stata/examples/ara/Prestige, clear  
  
/* Renaming/recoding variables to match the  
dataset's R version*/  
  
rename educat education  
rename percwomn women  
rename occ_code census  
recode occ_type (2=1 "bc") (4=2 "wc") (3=3  
"prof") (else=.), gen(type) label(type)  
label variable type "Type of occupation"  
drop occ_type  
replace type=3 if occtitle=="PILOTS"  
gen log2income=log10(income)/log10(2)
```

\*Fox, J. and Weisberg, S. (2011) *An R Companion to Applied Regression*, Second Edition, Sage.

NOTE: The R content presented in this document is mostly based on an early version of Fox, J. and Weisberg, S. (2011) *An R Companion to Applied Regression*, Second Edition, Sage; and from class notes from the ICPSR’s workshop *Introduction to the R Statistical Computing Environment* taught by John Fox during the summer of 2010.

## Linear regression

```
# R automatically process the log base 2 of income
in the equation

reg1 <- lm(prestige ~ education + log2(income) +
            women, data=Prestige)

summary(reg1)

(See output next page)
```

```
/* You need to create the log base 2 of income
first, type: */

gen log2income=log10(income)/log10(2)

/* Then run the regression */

regress prestige education log2income women
```

## Linear regression (heteroskedasticity-robust standard errors)

```
library(lmtest)
library(sandwich)
reg1$robse <- vcovHC(reg1, type="HC1")
coeftest(reg1, reg1$robse)
```

For cluster standard errors see the slide towards the end of this document.

```
regress prestige education log2income women,
           robust
```

## Predicted values/Residuals

```
# After running the regression

prestige_hat <- fitted(reg1) # predicted values
as.data.frame(prestige_hat)

Prestige_resid <- residuals(reg1) # residuals
as.data.frame(Prestige_resid)
```

```
/* After running the regression */

predict prestige_hat    /* Predicted values */

predict prestige_resid /* Residuals */
```

NOTE: For output interpretation (linear regression) please see <https://www.princeton.edu/~otorres/Regression101.pdf>

## Linear regression (output)

```
> reg1 <- lm(prestige ~ education + log2(income) + women, data=Prestige)
> summary(reg1)
```

Call:

```
lm(formula = prestige ~ education + log2(income) + women, data = Prestige)
```

Residuals:

Min	1Q	Median	3Q	Max
-17.3639	-4.4293	-0.1010	4.3160	19.1793

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-110.9658	14.8429	-7.476	3.27e-11 ***
education	3.7305	0.3544	10.527	< 2e-16 ***
log2(income)	9.3147	1.3265	7.022	2.90e-10 ***
women	0.0469	0.0299	1.568	0.12

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 7.093 on 98 degrees of freedom

Multiple R-squared: 0.8351, Adjusted R-squared: 0.83

F-statistic: 165.4 on 3 and 98 DF, p-value: < 2.2e-16

. regress prestige education log2income women

Source	SS	df	MS
Model	24965.5409	3	8321.84695
Residual	4929.88524	98	50.3049514
Total	29895.4261	101	295.994318

Number of obs	=	102
F( 3, 98)	=	165.43
Prob > F	=	0.0000
R-squared	=	0.8351
Adj R-squared	=	0.8300
Root MSE	=	7.0926

prestige	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
education	3.730508	.354383	10.53	0.000	3.027246 4.433769
log2income	9.314667	1.326515	7.02	0.000	6.682241 11.94709
women	.0468951	.0298989	1.57	0.120	-.0124382 .1062285
_cons	-110.9658	14.84293	-7.48	0.000	-140.4211 -81.51052

**R****Stata****Dummy regression with no interactions (analysis of covariance, fixed effects)**

```

reg2 <- lm(prestige ~ education + log2(income) +
            type, data = Prestige)

summary(reg2)

(See output next page)

# Reordering factor variables

Prestige$type <- with(Prestige, factor(type,
  levels=c("bc", "wc", "prof")))

```

Stata 11.x\*

```
regress prestige education log2income i.type
```

Stata 10.x

```
xi: regress prestige education log2income i.type
```

\*See <http://www.stata.com/help.cgi?whatsnew10to11>

**Dummy regression with no interactions (interpretation, see output next page)**

	bc	wc	prof
Intercept	-81.2	-81.2 - 1.44 = -82.64	-81.2 + 6.75 = -74.45
log2(income)	7.27	7.27	7.27
education	3.28	3.28	3.28

NOTE: "type" is a categorical or factor variable with three options: bc (blue collar), prof (professional, managerial, and technical) and wc (white collar). R automatically recognizes it as factor and treat it accordingly. In Stata you need to identify it with the "i." prefix (in Stata 10.x or older you need to add "xi:")

NOTE: For output interpretation (linear regression) please see <https://www.princeton.edu/~otorres/Regression101.pdf>

NOTE: For output interpretation (fixed effects) please see <https://www.princeton.edu/~otorres/Panel101.pdf>

## Dummy regression with interactions (output)

```
> reg2 <- lm(prestige ~ education + log2(income) + type, data = Prestige)
> summary(reg2)
```

Call:  
`lm(formula = prestige ~ education + log2(income) + type, data = Prestige)`

Residuals:

Min	1Q	Median	3Q	Max
-13.511	-3.746	1.011	4.356	18.438

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-81.2019	13.7431	-5.909	5.63e-08 ***
education	3.2845	0.6081	5.401	5.06e-07 ***
log2(income)	7.2694	1.1900	6.109	2.31e-08 ***
typewc	-1.4394	2.3780	-0.605	0.5465
typeprof	6.7509	3.6185	1.866	0.0652 .

Signif. codes: 0 '\*\*\*\*' 0.001 '\*\*\*' 0.01 '\*\*' 0.05 '\*' 0.1 '.' 1

Residual standard error: 6.637 on 93 degrees of freedom  
(4 observations deleted due to missingness)

Multiple R-squared: 0.8555, Adjusted R-squared: 0.8493  
F-statistic: 137.6 on 4 and 93 DF, p-value: < 2.2e-16

. regress prestige education log2income i.type

Source	SS	df	MS	Number of obs	=	98
Model	24250.5893	4	6062.64731	F( 4, 93)	=	137.64
Residual	4096.2858	93	44.0460839	Prob > F	=	0.0000
Total	28346.8751	97	292.235825	R-squared	=	0.8555
				Adj R-squared	=	0.8493
				Root MSE	=	6.6367

	prestige	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
education	3.284486	.608097	5.40	0.000	2.076926	4.492046
log2income	7.269361	1.189955	6.11	0.000	4.906346	9.632376
type						
2	-1.439403	2.377997	-0.61	0.546	-6.161635	3.282828
3	6.750887	3.618496	1.87	0.065	-.434729	13.9365
_cons	-81.20187	13.74306	-5.91	0.000	-108.4929	-53.91087

**R****Stata****Dummy regression with interactions**

```

reg3 <- lm(prestige ~ type*(education +
log2(income)), data = Prestige)

summary(reg3)

(See output next page)

# Other ways to run the same model

reg3a <- lm(prestige ~ education + log2(income) +
type + log2(income):type + education:type,
data = Prestige)

reg3b <- lm(prestige ~ education?type +
log2(income)*type, data = Prestige)

```

Stata 11.x\*

```

regress prestige i.type##c.education
i.type##c.log2income

```

Stata 10.x

```

xi: regress prestige i.type*education
i.type*log2income

```

\*See <http://www.stata.com/help.cgi?whatsnew10to11>

**Dummy regression with interactions (interpretation, see output next page)**

	bc	wc	prof
Intercept	-120.05	-120.05 +30.24 = -89.81	-120.05 + 85.16 = -34.89
log2(income)	11.08	11.08-5.653 = 5.425	11.08 - 6.536 = 4.542
education	2.34	2.34 + 3.64 = 5.98	2.34 + 0.697 = 3.037

NOTE: "type" is a categorical or factor variable with three options: bc (blue collar), prof (professional, managerial, and technical) and wc (white collar). R automatically recognizes it as factor and treat it accordingly. In Stata you need to identify it with the "i." prefix (in Stata 10.x or older you need to add "xi:")

NOTE: For output interpretation (linear regression) please see <https://www.princeton.edu/~otorres/Regression101.pdf>

NOTE: For output interpretation (fixed effects) please see <https://www.princeton.edu/~otorres/Regression101.pdf>

## Dummy regression with interactions (output)

```
> reg3 <- lm(prestige ~ type*(education + log2(income)), data = Prestige)
> summary(reg3)
```

Call:  
`lm(formula = prestige ~ type * (education + log2(income)), data = Prestige)`

Residuals:

Min	1Q	Median	3Q	Max
-13.970	-4.124	1.206	3.829	18.059

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-120.0459	20.1576	-5.955	5.07e-08 ***
typewc	30.2412	37.9788	0.796	0.42800
typeprof	85.1601	31.1810	2.731	0.00761 **
education	2.3357	0.9277	2.518	0.01360 *
log2(income)	11.0782	1.8063	6.133	2.32e-08 ***
typewc:education	3.6400	1.7589	2.069	0.04140 *
typeprof:education	0.6974	1.2895	0.541	0.58998
typewc:log2(income)	-5.6530	3.0519	-1.852	0.06730 .
typeprof:log2(income)	-6.5356	2.6167	-2.498	0.01434 *

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6.409 on 89 degrees of freedom

(4 observations deleted due to missingness)

Multiple R-squared: 0.871, Adjusted R-squared: 0.8595

F-statistic: 75.15 on 8 and 89 DF, p-value: < 2.2e-16

. regress prestige i.type##c.education i.type##c.log2income

Source	SS	df	MS	Number of obs
Model	24691.4782	8	3086.43477	98
Residual	3655.3969	89	41.0718753	F( 8, 89) = 75.15
Total	28346.8751	97	292.235825	Prob > F = 0.0000

prestige	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
type					
2	30.24117	37.97878	0.80	0.428	-45.22186 105.7042
3	85.16011	31.181	2.73	0.008	23.20414 147.1161
education	2.335673	.927729	2.52	0.014	.492295 4.179051
type# c.education					
2	3.640038	1.758948	2.07	0.041	.1450456 7.13503
3	.6973987	1.289508	0.54	0.590	-1.864827 3.259624
log2income	11.07821	1.806298	6.13	0.000	7.489136 14.66729
type# c.log2income					
2	-5.653036	3.051886	-1.85	0.067	-11.71707 .410996
3	-6.535558	2.616708	-2.50	0.014	-11.7349 -1.336215
_cons	-120.0459	20.1576	-5.96	0.000	-160.0986 -79.99318

# R

## Diagnostics for linear regression (residual plots, see next page for the graph)

```
library(car)

reg1 <- lm(prestige ~ education + income + type,
data = Prestige)

residualPlots(reg1)

      Test stat Pr(>|t|)
education   -0.684    0.496
income      -2.886    0.005
type          NA       NA
Tukey test   -2.610    0.009

# Using 'income' as is.
# Variable 'income' shows some patterns.

# Other options:

residualPlots(reg1, ~ 1, fitted=TRUE) #Residuals
                                         vs fitted only

residualPlots(reg1, ~ education, fitted=FALSE) #
                                         Residuals vs education only
```

```
# What to look for: No patterns, no problems.
# All p's should be non-significant.
# Model ok if residuals have mean=0 and variance=1 (Fox, 316)
# Tukey test null hypothesis: model is additive.
```

```
library(car)

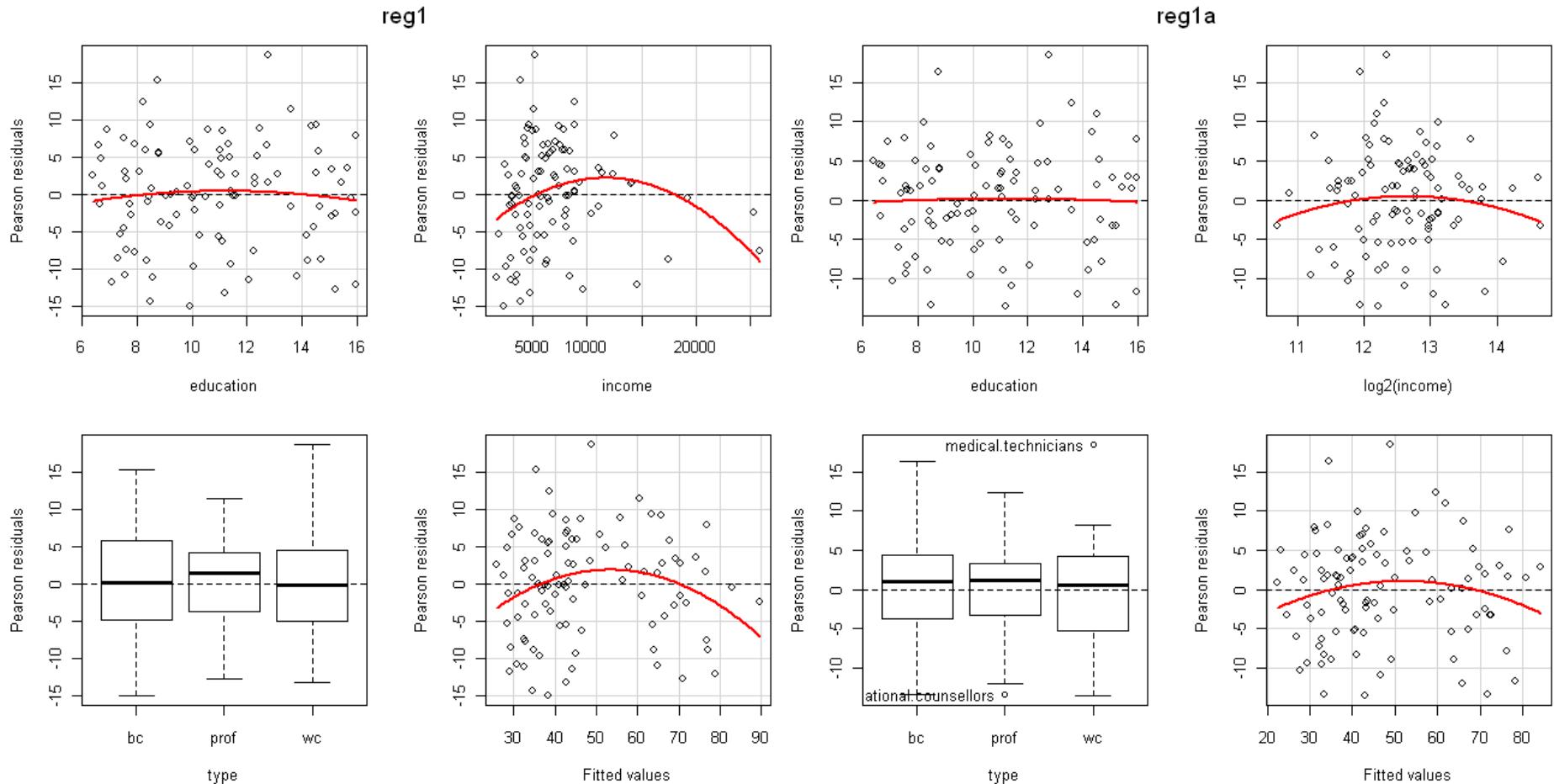
reg1a <- lm(prestige ~ education + log2(income) +
type, data = Prestige)

residualPlots(reg1a)

      Test stat Pr(>|t|)
education      -0.237    0.813
log2(income)   -1.044    0.299
type            NA       NA
Tukey test     -1.446    0.148

# Using 'log2(income)'.
# Model looks ok.
```

## Diagnostics for linear regression (residual plots graph)



# R

## Influential variables - Added-variable plots (see next page for the graph)

```
library(car)

reg1 <- lm(prestige ~ education + income + type, data = Prestige)

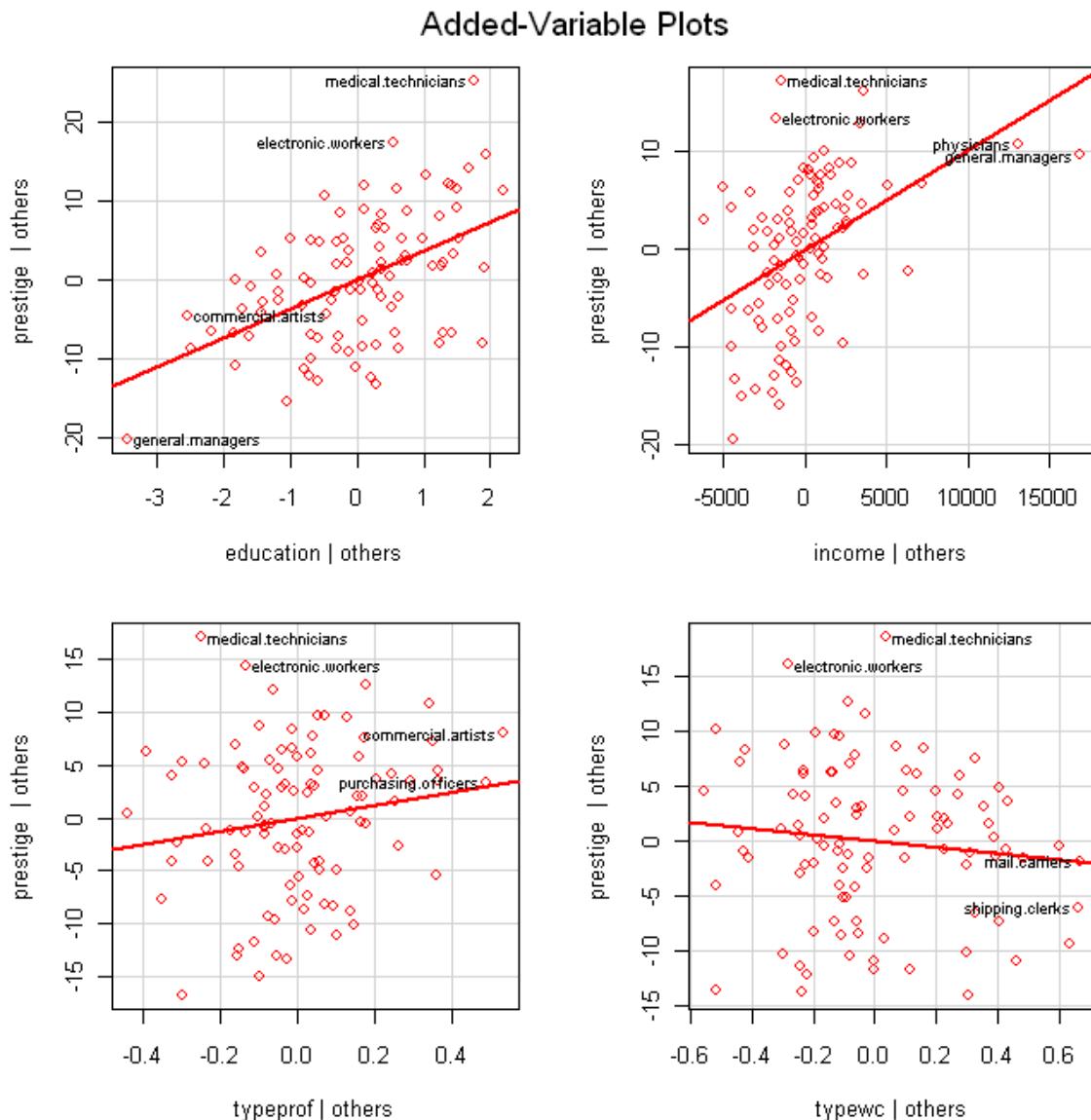
avPlots(reg1, id.n=2, id.cex=0.7)

# id.n - id most influential observation
# id.cex - font size for id.

# Graphs outcome vs predictor variables holding the rest constant (also called partial-regression
plots)
# Help identify the effect (or influence) of an observation on the regression coefficient of the
predictor variable
```

NOTE: For Stata version please see <https://www.princeton.edu/~otorres/Regression101.pdf>

## Added-variable plots – Influential variables (graph)



# R

## Outliers – QQ-Plots (see next page for the graph)

```
library(car)

reg1 <- lm(prestige ~ education + income + type, data = Prestige)

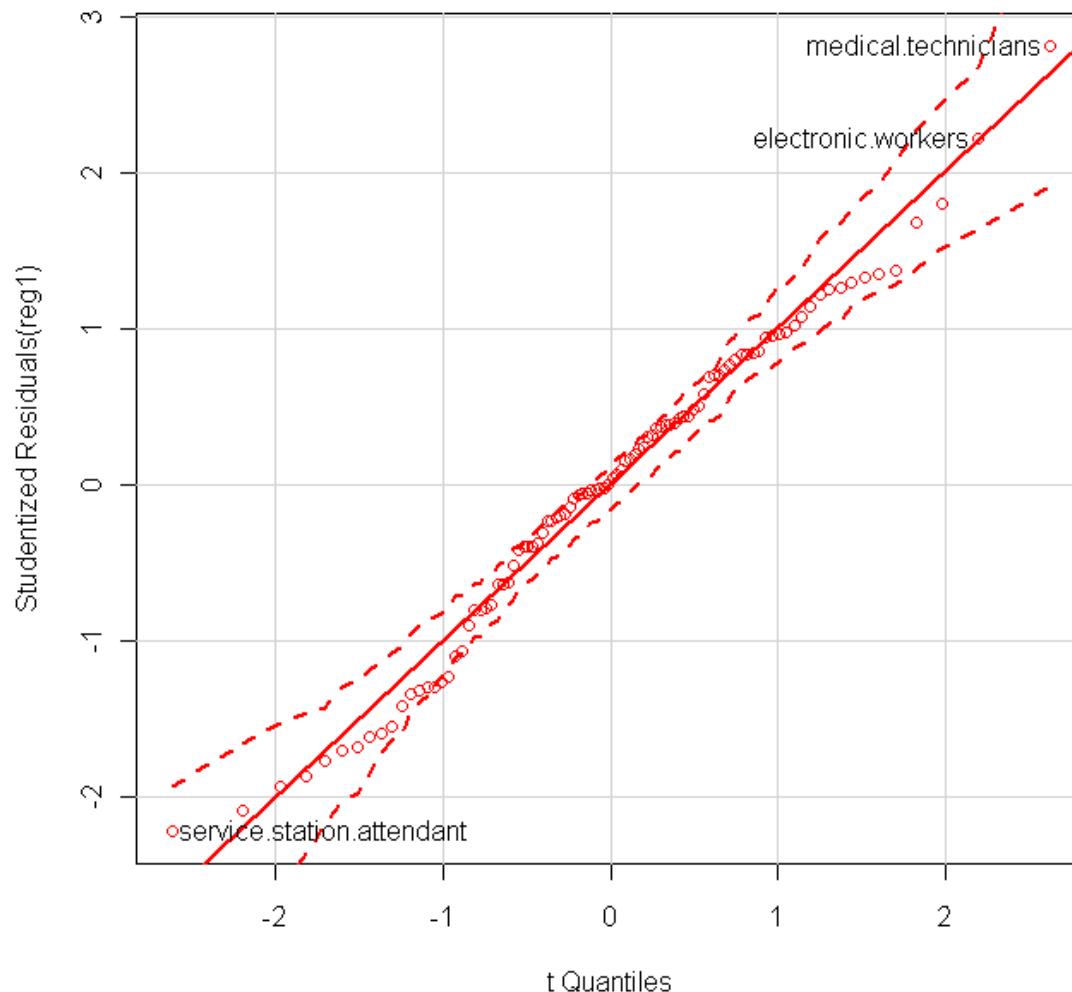
qqPlot(reg1, id.n=3)

[1] "medical.technicians"      "electronic.workers"
[3] "service.station.attendant"

# id.n - id observations with high residuals
```

NOTE: For Stata version please see <https://www.princeton.edu/~otorres/Regression101.pdf>

## Added-variable plots – Influential variables (graph)



## Outliers – Bonferonni test

```
library(car)

reg1 <- lm(prestige ~ education + income + type, data = Prestige)

outlierTest(reg1)

No Studentized residuals with Bonferonni p < 0.05
Largest |rstudent|:
            rstudent unadjusted p-value Bonferonni p
medical.technicians 2.821091      0.0058632      0.57459

# Null for the Bonferonni adjusted outlier test is the observation is an outlier. Here observation
# related to 'medical.technicians' is an outlier.
```

## High leverage (*hat*) points (graph next page)

```
library(car)

reg1 <- lm(prestige ~ education + income + type, data = Prestige)

influenceIndexPlot(reg1, id.n=3)

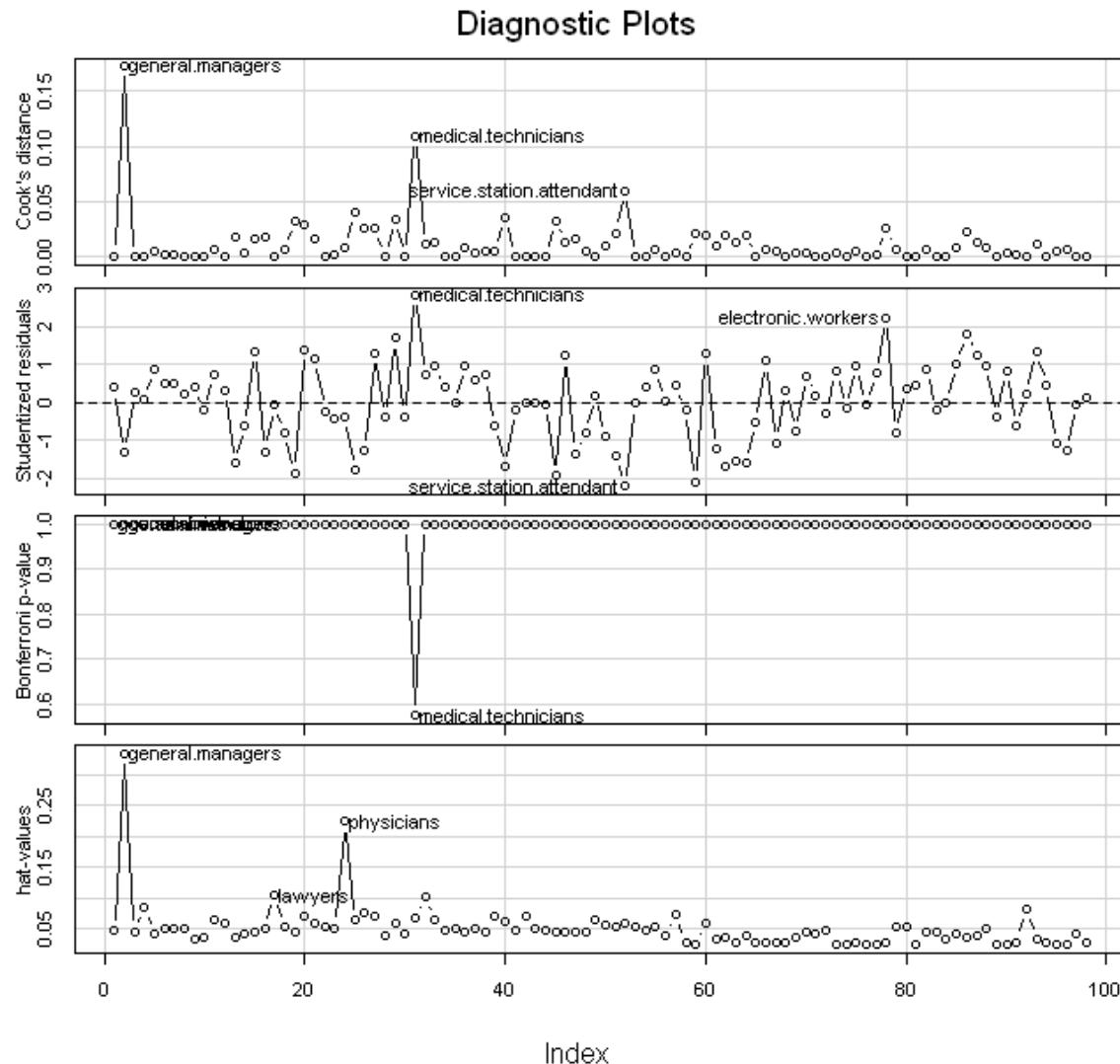
# Cook's distance measures how much an observation influences the overall model or predicted values
# Studentized residuals are the residuals divided by their estimated standard deviation as a way to
# standardize
# Bonferroni test to identify outliers
# Hat-points identify influential observations (have a high impact on the predictor variables)
```

NOTE: If an observation is an outlier and influential (high leverage) then that observation can change the fit of the linear model, it is advisable to remove it. To remove a case(s) type

```
reg1a <- update(prestige.reg4, subset=rownames(Prestige) != "general.managers")
reg1b <- update(prestige.reg4, subset= !(rownames(Prestige) %in% c("general.managers", "medical.technicians")))
```

NOTE: For Stata version please see <https://www.princeton.edu/~otorres/Regression101.pdf>

## High leverage (*hat*) points (graph)



# R

## Influence Plots (see next page for a graph)

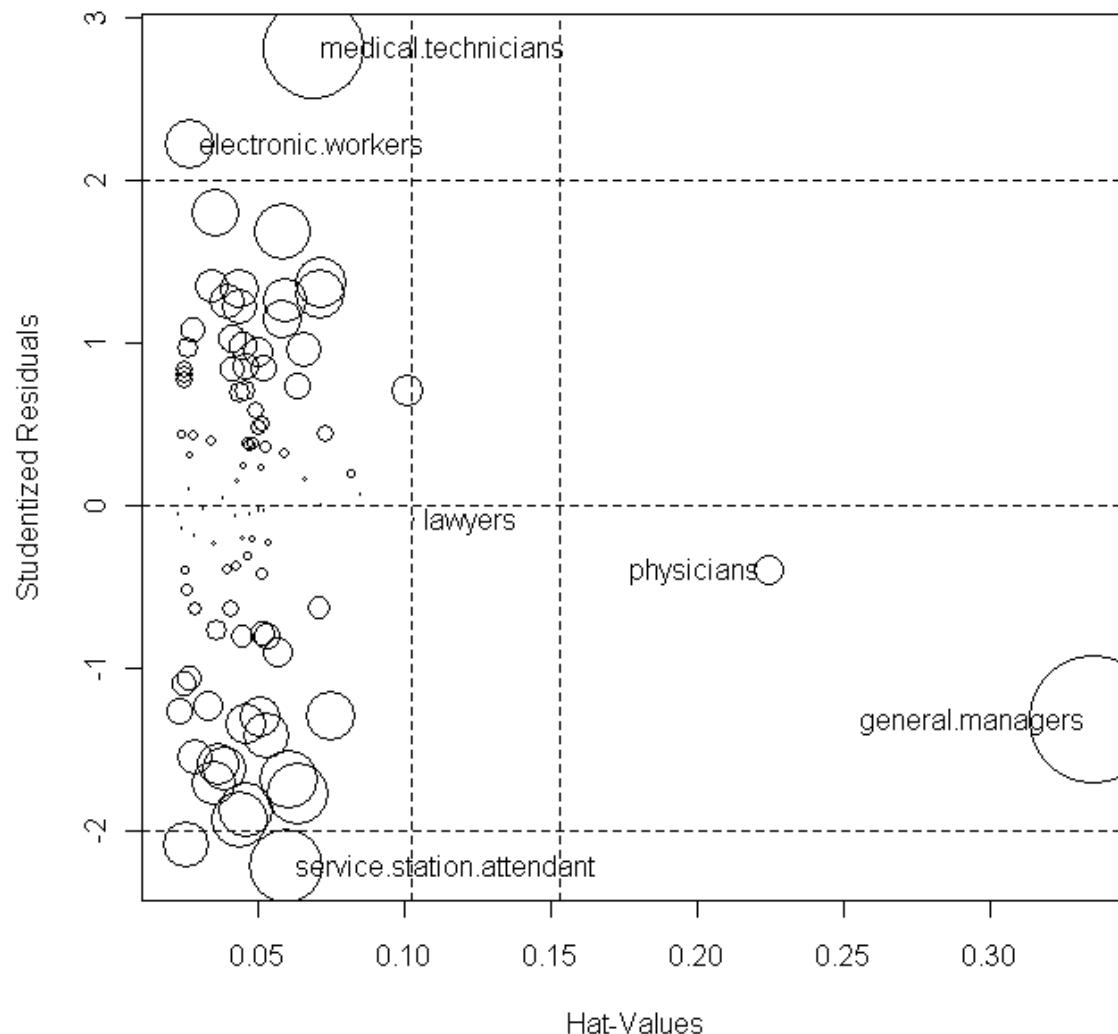
```
library(car)

reg1 <- lm(prestige ~ education + income + type, data = Prestige)

influencePlot(reg1, id.n=3)

# Creates a bubble-plot combining the display of Studentized residuals, hat-values, and Cook's
# distance (represented in the circles).
```

## Influence plot



# R

## Testing for normality (see graph next page)

```
library(car)

reg1 <- lm(prestige ~ education + income + type, data = Prestige)

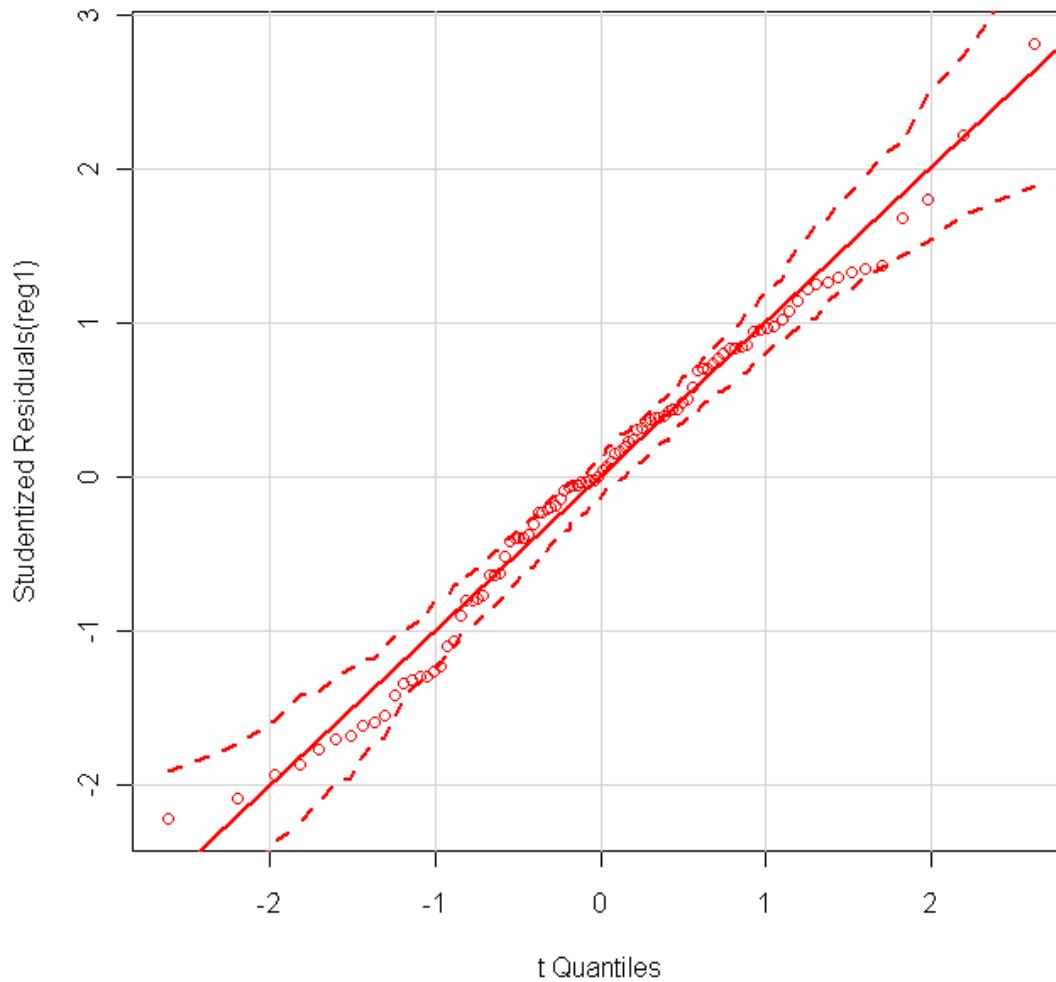
qqPlot(reg1)

# Look for the tails, points should be close to the line or within the confidence intervals.
# Quantile plots compare the Studentized residuals vs a t-distribution
# Other tests: shapiro.test(), mshapiro.test() in library(mvnormtest)-library(ts)
```

NOTE: For Stata version please see <https://www.princeton.edu/~otorres/Regression101.pdf>

R

## Influence plot



# R

## Testing for heteroskedasticity

```
library(car)

reg1 <- lm(prestige ~ education + income + type, data = Prestige)

ncvTest(reg1)

Non-constant Variance Score Test
Variance formula: ~ fitted.values
Chisquare = 0.09830307    Df = 1    p = 0.7538756

# Breush/Pagan and Cook/Weisberg score test for non-constant error variance. Null is constant variance
# See also residualPlots(reg1).
```

NOTE: For Stata version please see <https://www.princeton.edu/~otorres/Regression101.pdf>

# R

## Testing for multicollinearity

```
library(car)

reg1 <- lm(prestige ~ education + income + type, data = Prestige)

vif(reg1)

      GVIF     Df    GVIF^(1/(2*Df))
education 5.973932  1      2.444163
income    1.681325  1      1.296659
type      6.102131  2      1.571703

# A gvif> 4 suggests collinearity.

# "When there are strong linear relationships among the predictors in a regression analysis, the precision of the estimated regression coefficients in linear models declines compared to what it would have been were the predictors uncorrelated with each other" (Fox:359)
```

NOTE: For Stata version please see <https://www.princeton.edu/~otorres/Regression101.pdf>

## Linear regression (cluster-robust standard errors)

**R**

```
library(car)
library(lmtest)
library(multiwayvcov)

# Need to remove missing before clustering

p = na.omit(Prestige)

# Regular regression using lm()

reg1 = lm(prestige ~ education + log2(income)
          + women, data = p)

# Cluster standard errors by 'type'

reg1$cse <- cluster.vcov(reg1, p$type)

coeftest(reg1, reg1$cse)
```

NOTE: See output next page

**Stata**

```
reg prestige education log2income ///
women, vce(cluster type)
```

NOTE: See output next page

# Linear regression (cluster-robust standard errors)

R

```
summary(reg1) # Without cluster SE
```

Call:

```
lm(formula = prestige ~ education + log2(income) + women, data = p)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-16.8202	-4.7019	0.0696	4.2245	17.6833

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-129.16790	18.95716	-6.814	8.97e-10 ***
education	3.59404	0.38431	9.352	4.39e-15 ***
log2(income)	10.81688	1.68605	6.416	5.62e-09 ***
women	0.06481	0.03270	1.982	0.0504 .
---				

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6.828 on 94 degrees of freedom

Multiple R-squared: 0.8454, Adjusted R-squared:

0.8405

F-statistic: 171.4 on 3 and 94 DF, p-value: < 2.2e-16

```
coeftest(reg1, reg1$cse) # Cluster Standard errors
```

t test of coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-129.167902	47.025065	-2.7468	0.0072132 **
education	3.594044	1.003023	3.5832	0.0005401 ***
log2(income)	10.816884	4.406736	2.4546	0.0159431 *
women	0.064813	0.067722	0.9571	0.3409945
---				

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

\* Without cluster SE

```
. reg prestige education log2income women
```

Source	SS	df	MS	Number of obs	=	102
Model	24965.5409	3	8321.84695	F( 3, 98 )	=	165.43
Residual	4929.88524	98	50.3049514	Prob > F	=	0.0000
Total	29895.4261	101	295.994318	R-squared	=	0.8351

	prestige	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
education	3.730508	.354383	10.53	0.000	3.027246	4.433769
log2income	9.314667	1.326515	7.02	0.000	6.682241	11.94709
women	.0468951	.0298989	1.57	0.120	-.0124382	.1062285
_cons	-110.9658	14.84293	-7.48	0.000	-140.4211	-81.51052

\* Cluster standard errors

```
. reg prestige education log2income women, vce(cluster type)
```

	Linear regression					Number of obs	=	98
F( 1, 2 )	=	.						
Prob > F	=	.						
R-squared	=	0.8454						
Root MSE	=	6.8278						

	(Std. Err. adjusted for 3 clusters in type)					
	Robust					
	prestige	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
education	3.594044	1.003023	3.58	0.070	-.7216167	7.909704
log2income	10.81688	4.406738	2.45	0.134	-.8.143777	29.77755
women	.0648133	.0677216	0.96	0.440	-.2265692	.3561957
_cons	-129.1679	47.02508	-2.75	0.111	-.331.5005	73.16469

## References/Useful links

- ESS <https://economics.princeton.edu/undergraduate-program/ess/#>
- John Fox's site <http://socserv.mcmaster.ca/jfox/>
- Quick-R <http://www.statmethods.net/>
- UCLA Resources to learn and use R <http://www.ats.ucla.edu/stat/R/>
- UCLA Resources to learn and use Stata <http://www.ats.ucla.edu/stat/stata/>
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## References/Recommended books

- *An R Companion to Applied Regression*, Second Edition / John Fox , Sanford Weisberg, Sage Publications, 2011
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