

Harmful Addiction[†]

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Abstract

We construct an infinite horizon model of harmful addiction. Consumption is *compulsive* if it differs from what the individual would have chosen had commitment been available. A good is *addictive* if its consumption leads to more compulsive consumption of the same good. We analyze the welfare implications of drug policies and find that taxes on drugs decrease welfare while prohibitive policies may increase welfare. We also analyze the agent's demand for voluntary commitment ("rehab"). For appropriate parameters the model predicts a *cycle of addiction* where the agent periodically checks into rehab. Between these visits his drug consumption increases each period.

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1. Introduction

Substantial resources are spent to reduce the availability of and the demand for drugs. These efforts are justified by the belief that drug addiction is a serious health and social problem. What is special about drugs that could justify restricting its supply and demand?

Healthcare professionals often view addiction as a disease that impedes the agent's decision-making ability.¹ It is believed that after being struck by the disease, a person can no longer make the right decision.² The role of intervention is to “cure” (i.e., induce abstinence) or at least “control” (i.e., reduce consumption) the disease.

In this paper, we provide a model of addiction that captures some aspects of the healthcare professionals' view of addiction. We build on previous work (Gul and Pe-sendorfer (2001)) and allow utility to depend both on what the agent chooses and on the set of options from which the choice is made. This set may contain tempting alternatives that reduce the agent's welfare either by distorting his choice or by necessitating costly self-control or both. The central idea of this paper is to model drug consumption as a tempting choice that – in the case of an addictive drug – erodes self-control in future periods. Hence, the agent is more likely to succumb to an addictive drug if he has consumed the drug in the past.

To define harmful addiction, we first introduce the notion of *compulsive consumption*. An individual is *compulsive* if his choice differs from what he would have chosen had commitment been possible. An agent is more compulsive after consumption history A than after consumption history B if for every decision problem in which the agent is compulsive after B he is also compulsive after A. The drug is *addictive* if an increase in drug consumption makes the agent more compulsive. Hence, a harmful addiction is defined as a widening of the gap between the individual's choice and what he would have chosen before experiencing temptation. Healthcare professionals define addiction through the underlying physiological processes or by comparing the individuals consumption choices with external social standards of acceptability. In contrast, our notion of harmful addiction

¹ “Is alcoholism a disease? Yes, alcoholism is a disease. The craving that an alcoholic feels for alcohol can be as strong as the need for food or water. An alcoholic will continue to drink despite serious family, health, or legal problems” (Cited from the website of the National Institute on Alcohol Abuse and Alcoholism <http://www.niaaa.nih.gov/faq/q-a.htm#question2>.)

² There are numerous criticisms of the disease model of drug addiction (see for example, Davies (1992)).

relies only on revealed choice and compares the individual's behavior to his own behavior under different circumstances.

To see how our model works, consider an agent who must choose his level of drug consumption (d) and non-drug consumption (c), given his period t budget set B_t . For simplicity, we assume that there is no saving. The dynamic program below characterizes the agent's utility as a function of last period's drug consumption. Let $W(d_{t-1})$ denote the utility (value) function in period t , then

$$W(d_{t-1}) = \max_{(c,d) \in B_t} \{u(c,d) + \sigma(d_{t-1})v(d) + \delta W(d)\} - \max_{(\hat{c},\hat{d}) \in B_t} \sigma(d_{t-1})v(\hat{d})$$

We interpret $\sigma(d_{t-1})v$ as the temptation utility and call $u + \delta W$ the commitment utility. To understand this terminology, note that if all options were equally tempting – that is, resulted in the same v – then the v -terms in the above equation would drop-out. Therefore, such consumption problems would be evaluated according to $u + \delta W$. For example, when B_t consists of a single option, the overall utility of the current decision problem is the commitment utility $u + \delta W$ of that option. Commitment utility $u + \delta W$ is independent of past drug consumption, while temptation utility $\sigma(d_{t-1})v$ depends on last period's drug consumption. Addiction exerts its influence through this dependence.

The individual's choice (c, d) maximizes $u + \sigma(d_{t-1})v + \delta W$. This choice reflects the compromise between commitment utility and temptation. An individual is *compulsive* if the choice does not maximize commitment utility $u + \delta W$. A drug is *addictive* if an increase in drug consumption leads to more compulsive drug consumption.

In Proposition 1 we show that at higher values of σ the agent is more compulsive. Hence, if the drug is addictive the function σ is increasing. An increase in σ implies that the agent's utility function places a relatively smaller weight on commitment utility, and hence the gap between optimal commitment choices and actual choices widens. Proposition 2 shows that consumption of an addictive drug is reinforcing, that is, higher drug consumption in the current period leads to higher drug consumption in future periods.

The economics literature on addiction has focused on the analysis of drug demand. The key comparative static is that drug demand decreases as the future drug price increases. Becker, Grossman and Murphy (1994) found that sales of cigarettes in the current

period decrease if future prices increase. Becker, Grossman and Murphy conclude there are complementarities between current and future drug consumption. Gruber and Koszegi (2001) confirm the Becker, Grossman and Murphy finding after controlling for the difference between sales and consumption. The latter research finds evidence of greater sales and lower consumption in response to an anticipated price increase, capturing the consumers' desire to stockpile and to avoid the increased cost of addiction. As we show in section 3, our model is consistent with the empirical findings of Becker, Grossman and Murphy (1994) and Gruber and Koszegi (2001). Proposition 3 shows that if the drug is addictive, an increase in its future price decreases its current consumption.

Addicts frequently seek treatment for their addiction by enrolling in voluntary rehabilitation programs. To analyze the demand for drug treatment, we consider a decision problem in which the agent has the option of checking into a rehabilitation center. Rehabilitation centers provide temporary and costly commitment to zero drug consumption.

Drug programs offer many treatments that go beyond simple commitment. However, making drugs difficult to acquire seems to be a common feature of most treatment programs. These programs remove patients from their familiar surroundings, closely monitor their activities, and ensure that drugs are not available on the premises. In this way, they offer temporary commitment. Some rehabilitation programs use medications such as naltrexone to provide temporary commitment. Naltrexone blocks the opioid receptors in the brain and hence eliminates the euphoric effects of these drugs for up to 3 days after the last dose. Clearly, voluntary treatment programs cannot offer permanent commitment since agents can quit the program at any moment. For our purposes, the key feature is that *immediate* drug consumption is not possible.

In our analysis of the demand for drug treatment, the agent faces a fixed budget in every period and can choose to enter a rehabilitation center. The decision to enter rehab results in a commitment to zero drug consumption for the subsequent period but is costly in terms of the agent's non-drug consumption. Our model predicts a cycle of treatment and relapse that matches casual empiricism. Agents seek treatment when their drug consumption has reached its peak. After rehab, agents can be expected to follow a pattern of increasing drug consumption followed by another visit to the rehabilitation

center. We also demonstrate that the expectation of entering drug rehab in the next period increases current drug demand. Hence, agents will “go on a binge” just before seeking treatment.

Governments devote large resources to restrict the supply of addictive drugs. To analyze the welfare effects of such policies, we consider a simple decision problem in which the agent faces a fixed budget set in every period. We assume that the government can affect this budget set by changing the price of the drug (price policy) or by reducing the maximal feasible drug consumption (prohibitive policy). Many actual policies will change the price of the drug and the maximal feasible drug consumption simultaneously. We separate these two effects in order to identify the sources of welfare effects. An example of a policy with mostly prohibitive effects is a ban on drug consumption. A tax on the drug will have mostly price effects if it does not affect the maximal feasible drug consumption in a given period – that is, if drug consumption is a relatively small part of an agent’s budget. A *pure price policy* refers to a policy that has only price effects, whereas a *purely prohibitive policy* refers to a policy with only prohibitive effects.

We show that a pure price policy always reduces the agent’s welfare. A pure price policy makes it more costly to consume the drug but does not change the most tempting alternative. In response to a pure price policy, the agent will consume less of the drug and exercise more costly self-control. By a revealed preference argument, the increased cost of self-control is always larger than the possible utility gain from reduced drug consumption. The key feature of a pure price policy is that it distorts the agent’s consumption choice without removing temptations from his choice set. Such a policy will reduce drug consumption but also decrease welfare.

To examine the effect of a prohibitive policy, we focus on the special case where optimal drug consumption is zero whenever commitment is possible. We show that if the drug is addictive, a prohibitive policy increases welfare. The agent benefits from a prohibitive policy because it removes temptations from his choice set. A purely prohibitive policy does this without distorting his choice among the remaining alternatives.

We also examine how drug demand changes when a prohibitive policy is introduced. If the prohibitive policy is not binding then a reduction in the maximal allowed drug

consumption will increase drug demand. Current drug consumption makes self-control more costly in future periods. If the maximal feasible drug consumption is reduced then an increase in current drug consumption has a smaller effect on future self-control costs. Therefore, current drug consumption is more attractive.

Together, our welfare results show that welfare improvement stems from the commitment effect of a policy, while the price effect reduces welfare. Moreover, a reduced drug demand cannot be taken as an indication that the policy causing the reduction “works”. This distinction between drug consumption and welfare holds even though the drug is unambiguously “bad”, that is, the optimal drug consumption under commitment is zero.

The final section provides a foundation for the preferences analyzed in this paper. We provide axioms that imply the representation used in the text. The key difference between the current model and the representation in our earlier work is that in the current setting, preferences may depend on past (drug) consumption.

1.1 Related Literature

The standard economic model identifies addiction with intertemporal complementarities. Becker and Murphy (1986) view the consumption of an addictive good much like an investment that affects the utility of future consumption. For Becker and Murphy, addictive consumption is beneficial if, compared to alternative consumption choices, it entails a decrease in current utility in exchange for an increase in future utility. Conversely, addictive consumption is harmful if it entails an increase in current utility in exchange for a decrease in future utility. Hence, indulging in the consumption of a beneficial addictive good is exactly like investing; the agent forgoes current reward in exchange for higher future payoffs. Conversely, consumption of a good that is defined as a harmful addiction is like disinvesting.

Regardless of whether the addiction is harmful or beneficial, the availability of drugs is never bad in the Becker-Murphy model. Be it harmful or beneficial, their agents engage in the consumption of an addictive good if and only if the perceived trade-off between current and future utility warrants the consumption. Policy interventions cannot improve such agents’ welfare. This is analogous to the observation that restricting or forcing investment cannot improve the payoff of a profit maximizing firm. It is in this sense that the Becker

and Murphy preferences are standard; their analysis of addiction boils down to evaluating intertemporal cross-elasticities of drug demand.

There are at least two criticisms that can be levied at the Becker-Murphy model of addiction. First, in order to use their key distinction between harmful and beneficial addictions, we need to observe the timing of *utility* flows and not just of consumption. Since optimal choices rely only on the discounted present values of these flows and not on their timing, harmful and beneficial addictions cannot be distinguished through observed behavior.

Second, the Becker-Murphy formulation entails an a priori rejection of “the problem” of addiction. Their reliance on standard dynamic preferences ensures that regardless of the details of the subsequent analysis of demand, there will be no room for welfare enhancing drug policies. A harmful addiction is harmful in the same way that disinvestment is harmful; it increases current payoff at the expense of lower future payoffs. The model offers no argument for why individuals might be more likely to struggle with harmful addictions than with any other consumption decision.

The inability of standard economic models to identify addiction as a problem has led researchers to seek an alternative model of the decision-maker. O’Donoghue and Rabin (1999) and Gruber and Koszegi (2001) introduce models based on Strotz’s (1955) analysis of changing preferences and on the subsequent work by Phelps and Pollak (1968) and Laibson (1997). In this literature, the decision-maker is viewed as a sequence of distinct agents – called the (multi)selves. The utility function of each self exhibits a “presence bias” (Rabin (1998)) which implies that the period t choice is typically not optimal for other selves. Therefore, agents may have a desire for commitment and policies that restrict current consumption can be rationalized by appealing to the need for protecting the interests of past or future selves.

Bernheim and Rangel (2005) offer a different type of multi-selves model. In their model, there are two selves identified with different states of the brain.³ In the cold state the agent makes rational, long-run optimizing choices, anticipating the possibility that he may lose control to the hot state. The brain switches back and forth between these states

³ See also Laibson (2001), and Loewenstein (1996) for related work.

according to a stochastic process. For certain parameters, the Bernheim and Rangel model also generates addiction cycles, although their mechanism is different. In our model, the demand for temporary commitment generates cycles; Bernheim and Rangel's cycles are similar to cycles that may be obtained in a standard model. Low drug consumption (i.e., checking into rehab) in period t increases the marginal utility of drug consumption in period $t + 1$ so much that the agent prefers, even in his cold state, consuming on alternate days to consuming everyday.

The key difference between our temptation model and the multiple selves models is in the role of commitment. In the temptation model, agents value commitment because it reduces temptation, while in the multiple-selves models, commitment is useful only if it alters behavior. The different role of commitment in the two models is related to their different predictions about addicts' attitudes toward drug taxes. For example, our model predicts that smokers would vote against a tax increase on cigarettes, while the Gruber and Koszegi (2001) model predicts the opposite. In their calibration, Gruber and Koszegi (2001) assess that smokers should vote for a tax of at least 1\$ per pack.⁴

Opinion polls provide some evidence regarding smokers' attitudes to cigarette taxes. A recent poll (February 15, 2002) asks Connecticut voters whether they support a 61 cent increase in cigarette taxes.⁵ After the proposed tax increase – which was subsequently passed – the Connecticut cigarette tax became 1.11\$ per pack. Among smokers, 66% opposed the tax increase, while 32% were in favor; among non-smokers, 19% opposed the tax increase, and 78% were in favor. It would be incorrect to conclude from this survey that 32% of smokers favor a cigarette tax increase because it reduces the cigarette consumption of their future selves. Presumably, most voters, including smokers, value a reduction in the budget deficit. Smokers may view paying more for their cigarettes a reasonable price for this reduction. However, the fact that a much smaller fraction of smokers favor the cigarette tax than non-smokers does suggest that many smokers do not consider the overall affect of the tax, including the reduced state budget deficit and reduced future cigarette consumption, to be welfare increasing.

⁴ This assumes that the agent votes in the current period on a tax change that will take effect in the next period.

⁵ See <http://www.quinnipiac.edu/x11362.xml?ReleaseID=411>. The Connecticut cigarette tax revenue was expected to be used primarily to reduce the budget deficit.

In a recent paper, Gruber and Mullainathan (2001) provide evidence that smokers are happier *after* cigarette taxes have been raised. However, increased ex post happiness does not mean that smokers would benefit from such a tax nor does it mean that they would vote for it. To see this, consider the following example. Suppose the government requires all individuals to undergo a painful surgical procedure that has a small benefit. Some time after the surgery surveys indicate an increase in happiness. Obviously this does not mean that the surgery was welfare improving since at the time of the survey the cost of surgery is in the past while the benefit is still being enjoyed. Ex post measures of happiness cannot shed light on whether the benefit of surgery outweighs the cost.

1.2 Welfare

Most economic analysis uses the individuals' choice behavior as a welfare criterion. Alternative x is deemed to be better for the agent than alternative y if and only if given the opportunity, the agent would choose x over y . In a standard setting, it is straightforward to implement this welfare criterion. A standard agent⁶ can be thought of as maximizing a utility function over consumption choices. Hence, maximizing welfare means maximizing this utility function.

We extend the idea of equating welfare with choice behavior to the setting analyzed in this paper. Note, however, that a standard utility function defined on consumption cannot adequately describe behavior in our model and therefore cannot capture welfare. The agent may value commitment and our welfare criterion must take that into account. For example, suppose x represents abstention from drug consumption in period 2, while y represents going on a drug binge in period 2. Suppose in period 1, the agent chooses to commit to x rather than delay the decision to period 2. We conclude that the agent is unambiguously better off when committed to x than when he has the choice between x and y in period 2. Suppose further that the agent chooses y if commitment is not possible and the choice between x and y must be made in period 2. We conclude that the agent is (unambiguously) better off choosing y in period 2 *if commitment is not possible*.

Doesn't period 2 behavior contradict our assertion that the agent is better off when committed to x ? The answer is no because in period 2, the agent can only choose between

⁶ By standard, we mean an agent described by a choice function satisfying Houthakker's axiom. See Kreps (1988).

x and y given that he is not committed. The agent prefers commitment to x but if such commitment is not possible, he finds it too costly to exercise self-control, and therefore chooses y . Hence, period 2 behavior tells us how to rank consumption given that there is no commitment, while period 1 behavior tells us how to rank options that involve commitment for period 2.⁷

Our notion of welfare refuses to second-guess the decisions of individuals. It assumes that agents make the best possible choices under the circumstances. Government policies can have a positive impact if they satisfy the consumers' demand for commitment but there is no room for paternalistic intervention.

The multiple-selves approach takes the view that each self should be treated like an autonomous individual. Hence, the period t self's choice of alternative x over y reflects only the fact that given the predicted behavior of the subsequent selves, x leads to a consumption stream that is better for the period t self than the one induced by y . Other selves may be and often are made worse-off by this choice. The role of government interventions is to protect the interests of past or future selves against actions of the current self.

In support of our single-self view, we note that the idea of a consistent preference corresponding to the agent's welfare seems to permeate our informal, everyday analysis of struggles with temptations. Consider the example of a smoker. Suppose, in period 0 he has decided to quit and thrown out his last pack of cigarettes. In period 1, he visits a friend who offers him a cigarette which he accepts. After the visit, his friend is reproached by the friend's spouse who asks: "Why did you do that? You knew he was trying to quit!" To this the friend responds: "It was his period 0 self that wanted to quit. Obviously, the period 1 self did not, since it accepted the cigarette that I offered. Why should I be concerned with the welfare of the period 0 self? After all, it was the period 1 self that was nice enough to pay us a visit." Should we consider this an adequate defence of the friend's actions? If we take the multi-selves view literally, we may have to. In contrast, our model takes the view that the agent is unambiguously harmed by his friend's decision

⁷ A referee makes the following objection to our welfare criterion: "After Odysseus tied himself to the mast ... he cried out furiously for his crew to untie him. That moment's Odysseus was presumably less happy than an untied Odysseus would have been." Our point is that there is little connection between welfare and happiness or comfort at a moment in time. Commitment may be painful just like surgery is painful. The fact that a patient is in pain sheds no light on whether the surgery improves his welfare.

to make cigarettes available. The agent's decision to smoke when cigarettes are available only indicates that exercising self-control is too costly. It does not invalidate his earlier desire for commitment.⁸

The idea that welfare is identified with choice behavior can be applied not just to the temptation model but to most other models of economic decision making, including the sophisticated $\beta - \delta$ model as analyzed by Laibson (1997).⁹ In Gul and Pesendorfer (2004) we provide a re-interpretation of the $\beta - \delta$ model that leads to a welfare criterion that equates welfare with choice behavior.

2. Model

We consider an environment with 2 goods, where $C = [0, 1]^2$ is the set of possible consumption vectors. A consumption bundle is a pair $(c, d) \in C$, where d is the consumption of the addictive good.

An agent is confronted with a dynamic decision problem. Every period $t = 1, 2, \dots$ he must take an action which results in a consumption for period t and constrains future actions. Dynamic decision problems can be described recursively as a set of alternatives where each alternative is a lottery over current consumption and continuation decision problems.¹⁰ Let Z denote the set of all decision problems and let x, y or z denote generic elements of Z . Generic choices (i.e., elements of a given z) are denoted μ, ν or η . A choice μ is a lottery over $C \times Z$, where $c \in C$ represents the realization of current consumption and $x \in Z$ represents the realized continuation decision problem. A deterministic choice yields a particular consumption (c, d) and a particular deterministic continuation problem z with certainty and is denoted (c, d, z) . Most of the analysis in this paper focuses on the set of *deterministic decision problems*, $\bar{Z} \subset Z$. Each $z \in \bar{Z}$ is a (compact) set of alternatives

⁸ As a referee points out, advocates of the multiple-selves view could argue that the friend's action to make cigarette's available is objectionable because it harms a future (e.g., period 2) self. In that case, making cigarettes available may be objectionable whether or not the agent threw away his last pack of cigarettes. In contrast, our view of welfare suggests that making cigarettes available is objectionable *because* the agent made an effort to commit in the previous period.

⁹ Equating welfare with choice behavior would not make sense in a model of naive time-inconsistent decision makers (see O'Donoghue and Rabin (1999)) who incorrectly predict their future behavior. That model explicitly assumes that agents make mistakes; for example, by rejecting commitment opportunities in the false belief that future behavior would maximize current utility.

¹⁰ See Gul and Pesendorfer (2004) for a detailed discussion of dynamic decision problems.

of the form (c, d, x) where (c, d) denotes current consumption and $x \in \bar{Z}$ denotes the deterministic continuation problem.

The set of decision problems Z serves as the domain of preferences for the agent. This formulation allows us to describe agents who struggle with temptation. For example, the agent may strictly prefer a decision problem in which some alternatives are unavailable because these alternatives present temptations that are hard to resist. Even when the agent makes the same ultimate choice from two distinct decision problems, he may have a strict preference for one decision problem because making the same choice from the other requires more self-control. Decision problems are the natural domain for identifying these phenomena. Below, we represent the individual's preferences over decision problems by a utility function. This utility function is analogous to the *indirect utility function* in standard consumer theory. The traditional indirect utility function is defined over decision problems that can be represented by a budget set. In contrast, our utility function is defined over a broader class of decision problems.

The preferences we analyze depend on the agent's past consumption. To capture this dependence, we index the individual's preferences by $s \in S$, the state in the initial period of the decision problem. The state s represents the relevant consumption history prior to the initial period of analysis. For simplicity, we assume that only drug consumption in the last period influences the agents preferences and set $S := [0, 1]$. We refer to the indexed family of preferences $\succeq := \{\succeq_s\}_{s \in S}$ simply as *the agent* or the preference \succeq . We say that the utility function $W : S \times Z \rightarrow \mathbb{R}$ represents the preference \succeq if, for all s , $x \succeq_s y$ iff $W(s, x) \geq W(s, y)$.

In section 6 (Theorem 2) we provide axioms for the utility function used in this paper. These axioms ensure that the decision-maker's preferences can be represented by a continuous function W of the following form:

$$W(s, z) = \max_{(c, d, x) \in z} [u(c, d) + \sigma(s)v(d) + \delta W(d, x)] - \max_{(\hat{c}, \hat{d}, \hat{x}) \in z} \sigma(s)v(\hat{d}) \quad (1)$$

where the function u is continuous and nonconstant, v is continuous and strictly increasing, σ is continuous and strictly positive, and $\delta \in (0, 1)$. Henceforth, these axioms are implicit

in any reference to a preference and it is understood that W, u, v, σ, δ refer to the functional form in equation (1).

Straightforward application of known dynamic programming results imply that for every (u, v, σ, δ) with u, v continuous, and $\delta \in (0, 1)$, there is a unique W that satisfies equation (1). We say that (u, v, σ, δ) represents the preference \succeq if the unique W that satisfies equation (1) represents \succeq .

Equation (1) implies that if the agent is committed to a single choice (i.e., $z = \{(c, d, x)\}$) then $W(z) = u(c, d) + \delta W(x)$. Therefore, we refer to $u + \delta W$ as the *commitment utility* of a particular choice. Note that commitment utility is independent of the state s .

Consider a deterministic decision problem that does not offer commitment and assume that (c, d, x) is the unique maximizer of the commitment utility $u + \delta W$ and (\hat{c}, \hat{d}, y) is the unique maximizer of v in z . In this case, it follows from equation (1) that removing (\hat{c}, \hat{d}, y) from the choice set would increase the agent's welfare. We refer to alternatives (\hat{c}, \hat{d}, y) that have this property as *temptations*. Temptations create a *preference for commitment*; that is, situations where the agent strictly prefers the decision problem x over z even though $x \subset z$.

The agent's choice from z in state s maximizes the function

$$u + \sigma(s)v + \delta W$$

If (c, d, x) is the choice from z and (\hat{c}, \hat{d}, y) maximizes v in z then the agent incurs a self-control cost of

$$-\sigma(s)[v(d) - v(\hat{d})]$$

This cost is zero if the choice maximizes v . Otherwise, it is positive. In our model, past consumption affects current behavior by changing the cost of self-control.

The set of optimal choices from z is $\mathcal{D}(s, z)$, while $\mathcal{C}(z)$ denotes the commitment utility maximizers. For any function $f : C \times Z \rightarrow \mathbb{R}$, let $E_\mu[f]$ be the expectation of f with respect to μ . Then,

$$\mathcal{D}(s, z) := \{\mu \in z | E_\mu[u + \sigma(s)v + \delta W] \geq E_\nu[u + \sigma(s)v + \delta W], \forall \nu \in z\}$$

$$\mathcal{C}(z) := \{\mu \in z | E_\mu[u + \delta W] \geq E_\nu[u + \delta W], \forall \nu \in z\}$$

When the agent chooses alternatives that do not maximize commitment utility it means that behavior is affected by temptations. We call such choices *compulsive*. This motivates the following definition.

Definition: *The preference \succeq_s is compulsive at z iff $\mathcal{D}(s, z) \setminus \mathcal{C}(z) \neq \emptyset$.*

The notion of compulsive consumption plays a central role in the clinical definition of addiction and in the definition we present below. What distinguishes addiction from other types of compulsive behavior is the fact that the compulsiveness associated with an addictive substance is “caused” (or made worse) by past consumption (or higher levels of past consumption) of the *same* substance. Below, we offer a criterion for ranking states with respect to compulsiveness. This criterion provides a formal, choice-theoretic definition of what it means for compulsiveness to get worse.

Definition: *A preference \succeq is more compulsive at \bar{s} than at s (denoted $\bar{s}\mathbf{C}s$) if \succeq_s is compulsive at z implies $\succeq_{\bar{s}}$ is compulsive at z .*

Note that our notion of more compulsive is analogous to the familiar notion of more risk averse: first we provide a criterion for a consumption choice to be compulsive. Then we define the agent to be more compulsive in situation \bar{s} than in s if the set of decision problems in which he makes a compulsive choice at s is contained in the set of decision problems in which he makes a compulsive choice at \bar{s} .

Psychologists and health care professionals commonly refer to an individual as addicted if, after repeated self-administration of a drug, the individual develops a pattern of compulsive drug seeking and drug-taking behavior.¹¹ The clinical definition emphasizes a lack of control on the part of addicted subjects and suggests a conflict between what the addict *ought to consume* and what he *actually consumes*.

In our model, the agent is compulsive when the choice is different from the $u + \delta W$ optimal alternative. Thus, an agent is compulsive if behavior would change were commitment possible. In line with the clinical definition above, we say that the drug is addictive if higher current drug consumption makes the individual more compulsive; that

¹¹ See Robinson and Berridge (1993), pg. 248.

is, following an increase in drug consumption, there are more situations in which the agent makes a choice that does not maximize U . The definition below expresses this idea.

Definition: *The drug is addictive if $\bar{s}\mathbf{C}s$ for all $\bar{s} > s$ and $\succeq_1 \neq \succeq_0$.*

Proposition 1: (i) $\bar{s}\mathbf{C}s$ if and only if $\sigma(\bar{s}) > \sigma(s)$; (ii) the drug is addictive if and only if σ is non-decreasing with $\sigma(1) > \sigma(0)$.

Proposition 1 relates our definition of addiction to our representation of preferences. It shows that the function σ measures how compulsive the agent is and therefore the drug is addictive when σ is increasing.¹² The proof of Proposition 1 is in the appendix. Note that the “if” part of part (i) is straightforward since a higher σ implies a larger weight on temptation utility. For the “only if” part we need to show that when $\sigma(\bar{s}) > \sigma(s)$ there is a decision problem with the property that the agent is compulsive at \bar{s} but not at s . Such a decision problem can be constructed as long as $u + \delta W$ and $u + \sigma v + \delta W$ are not positive affine transformations of each other. Since u and v are not constant, $\delta > 0, \sigma > 0$ this holds in our case.

For $z \in \bar{Z}$, let $D(s, z)$ denote the individual’s current period drug demand in state s ; that is, $d \in D(s, z)$ if and only if there exists c, x such that $(c, d, x) \in \mathcal{D}(s, z)$. We write $D(\bar{s}, x) \geq D(s, y)$ if $\bar{d} \in D(\bar{s}, x), d \in D(s, y)$ implies $\bar{d} \geq d$. Proposition 2 shows that an increase in σ leads to higher drug demand in every decision problem.

Proposition 2: *If $\sigma(\bar{s}) \geq \sigma(s)$ then $D(\bar{s}, z) \geq D(s, z)$ for all $z \in \bar{Z}$.*

Proof: Let $(c, d, x) \in \mathcal{D}(s, z)$ and $(\bar{c}, \bar{d}, \bar{x}) \in \mathcal{D}(\bar{s}, z)$. Then,

$$\begin{aligned} u(c, d) + \sigma(s)v(d) + \delta W(d, x) &\geq u(\bar{c}, \bar{d}) + \sigma(s)v(\bar{d}) + \delta W(\bar{d}, \bar{x}) \\ u(\bar{c}, \bar{d}) + \sigma(\bar{s})v(\bar{d}) + \delta W(\bar{d}, \bar{x}) &\geq u(c, d) + \sigma(\bar{s})v(d) + \delta W(d, x) \end{aligned}$$

Hence,

$$(\sigma(\bar{s}) - \sigma(s))(v(\bar{d}) - v(d)) \geq 0$$

¹² Straightforward extensions of known uniqueness arguments (for example, the uniqueness result in Gul and Pesendorfer (2004)) ensure that if (u, v, σ, δ) represents some \succeq satisfying the conditions of Theorem 2 then $(\hat{u}, \hat{v}, \hat{\sigma}, \hat{\delta})$ represents the same \succeq if and only if there exist constants $a, b > 0, e$ and f such that $\hat{u} = au + e, \hat{v} = bv + f, \hat{\sigma} = \frac{a}{b}\sigma$, and $\hat{\delta} = \delta$. Hence, all of our assumptions (such as monotonicity, increasingness and differentiability etc.) and conclusions regarding (u, v, σ, δ) are properties of the underlying preference and not the particular representation.

and therefore $\sigma(\bar{s}) \geq \sigma(s)$ implies $D(\bar{s}, z) \geq D(s, z)$. \square

Psychologists use the term *reinforcement* to describe the fact that an increase in current drug consumption leads to an increase in future drug consumption. If $\epsilon > 0$ and $\sigma(d + \epsilon) > \sigma(d)$ then the ϵ increase is reinforcing. In particular, an additive increase in drug consumption is always reinforcing.

Our definition of addiction is appropriate for harmful addictions. A beneficial addiction can be modelled as the polar opposite of a harmful addiction. Consider an individual who tries to exercise regularly. If he could commit, he would choose to exercise a certain amount every period. If commitment is not possible, the agent is tempted to quit exercising. However, he is less tempted to quit if he has exercised in the previous period. Hence, past exercise *strengthens* the agent's self-control. To model this situation, let d represent the amount of exercise and let c represent consumption. The agent's utility function is as described in equation (1) with v *decreasing* in d and u increasing in both arguments. A beneficial addiction in this model corresponds to a σ that is decreasing in d . The remainder of the paper focuses on harmful addictions.

3. Addiction and Drug Demand

Our next objective is to analyze the implications of addiction on drug demand. In order to facilitate the comparative statics results in this and the subsequent sections, the following assumptions will sometimes be used. Assumption 1 requires u to not depend on drug consumption. It implies that the agent would commit to zero drug consumption if commitment were possible.

Assumption 1: $u(\cdot, d)$ is strictly increasing with $u(\cdot, d) = u(\cdot, \hat{d})$ for all d, \hat{d} .

When Assumption 1 is satisfied we write $u(c)$ instead of $u(c, d)$. Assumption 2 imposes curvature restrictions on u, v and σ . These restrictions are analogous to the standard curvature and differentiability assumptions in demand theory. The function σ plays a role similar to a cost function in a standard optimization problem. Therefore concavity of the objective function in the decision problems below is guaranteed when σ is convex.

Assumption 2: $u, v, -\sigma$ are twice differentiable and strictly concave.

We consider a simple stationary consumption problem. The individual cannot borrow or lend and can consume at most 1 unit of the drug in every period. Let $\mathbf{p} = p_1, \dots, p_t, \dots$ denote the sequence of prices. Given the price sequence \mathbf{p} , after date $\tau \geq 0$, the agent faces the price sequence $p_\tau, \dots, p_{\tau+k}, \dots$. We let \mathbf{p}_τ denote this sequence (i.e., $\mathbf{p}_1 = \mathbf{p}$). The individual is endowed with one unit of wealth and must choose consumption (c, d) from the budget set

$$B_t = \{(c, d) \in C \mid c + p_t d \leq 1\}$$

We assume that $p_t < 1$ for all t . Since $d \leq 1$, the maximal feasible drug consumption is 1 in every period independent of the price of the drug. Let $x(\mathbf{p})$ denote the dynamic decision problem confronting an agent who faces the price sequence \mathbf{p} . It is easy to verify that there is a unique optimal consumption plan for every $x(\mathbf{p})$. In particular, the current period drug demand $D(s, x(\mathbf{p}))$ is a singleton. We use $d(s, x(\mathbf{p}))$ to denote this demand. We define the period τ demand of the agent facing \mathbf{p} recursively, as follows:

$$\begin{aligned} d_1(s, \mathbf{p}) &= d(s, \mathbf{p}) \\ d_{\tau+1}(s, \mathbf{p}) &= d(d_\tau(s, \mathbf{p}), \mathbf{p}_{\tau+1}) \end{aligned}$$

To see how the two assumptions above facilitate comparative static analysis of addiction, recall that by Proposition 1, addictiveness implies that σ is non-decreasing. If, in addition, Assumption 2 holds then σ is a strictly increasing function. Assumption 1 ensures that u depends only on non-drug consumption. Hence, the objective function for the agent's choice from $x(\mathbf{p}_t)$ simplifies to

$$u(c_t) + \sigma(d_{t-1})v(d_t) + \delta W(d_t, x(\mathbf{p}_{t+1}))$$

Then, Assumption 2 enables us to identify the following first order condition for an interior solution for the optimal choice of d_t :

$$p_t u'(c_t) = \sigma(d_{t-1})v'(d_t) + \delta \sigma'(d_t)(v(d_{t+1}) - v(1))$$

To understand the above equation consider a marginal increase in drug consumption. This implies a reduction in current (non-drug) consumption and the left-hand side captures the

utility consequence of this reduction. The first term on the right hand side captures the current period utility change from the increase in drug consumption. Since v is increasing, this change is positive. The second term captures the effect of the increase in drug consumption on future utility. This effect works through a change in the cost of self-control in the next period. If σ is increasing (as in the case of an addictive drug) then the increase in the current drug consumption implies a higher self-control cost next period and the second term on the left hand side is negative. If σ is decreasing, then the increase in drug consumption implies a smaller self-control cost in the next period and the term is positive.

The next proposition analyzes the change in demand as a function of current and future prices.

Proposition 3: *Suppose that the drug is addictive and Assumptions 1 and 2 are satisfied. Then, $d(s, \mathbf{p})$ is nonincreasing in p_t for $t \geq 1$. If $0 < d_\tau(s, \mathbf{p}) < 1$ for $\tau \in \{1, \dots, t\}$ then $d(s, p_t, \dots, p_{t-1}, \cdot, p_{t+1}, \dots)$ is differentiable at p_t and $\partial d(s, \mathbf{p})/\partial p_t < 0$.*

Proof: First we prove the result for $t = 1$. Assume that $0 < d(s, \mathbf{p}) < 1$. The first order condition is

$$-p_1 u'(1 - p_1 d_1) + \sigma(s) v'(d_1) + \delta \sigma'(d_1)(v(d_2) - v(1)) = 0$$

Taking the total derivative we find

$$\partial d_1 / \partial p_1 = \frac{u'(c_1) - p_1 d_1 u''(c_1)}{p_1^2 u''(c_1) + \sigma(s) v''(d_1) + \delta \sigma''(d_1)(v(d_2) - v(1))} < 0$$

by Assumption 2. The weak monotonicity for boundary solutions is straightforward.

Next, assume that $d_\tau := d_\tau(s, \mathbf{p}), d_{\tau+1} = d_{\tau+1}(s, \mathbf{p})$ are interior for some τ such that $1 \leq \tau < t$. Note that

$$\frac{dd_\tau}{dd_{\tau+1}} = -\frac{\delta \sigma'(d_\tau) v'(d_{\tau+1})}{p_t^2 u''(c_\tau) + \sigma(d_{\tau-1}) v''(c_\tau) + \delta \sigma''(d_\tau)(v(d_{\tau+1}) - v(1))} > 0$$

by Assumption 2. Then, the fact that $\partial d(s, \mathbf{p})/\partial p_1 < 0$ and an inductive argument implies the result for the interior case. Weak monotonicity for the case of boundary solutions is again straightforward. \square

Proposition 3 applied to the case of $t = 1$ shows that the drug is a normal good under our assumptions. For $t > 1$, Proposition 3 shows that drug demand decreases if the future price of the drug increases.

To see why drug demand decreases in response to an increase in future drug prices, note that since the drug is a normal good, consumption in period t decreases as p_t increases. As a result, the period t cost of self-control, given by $\sigma(d_{t-1})(v(1) - v(d_t))$, increases. Since the drug is addictive, σ is increasing. Therefore, an increase in p_t renders period $t - 1$ drug consumption less attractive since a higher d_{t-1} means a higher marginal cost of self-control in period t . Hence, drug demand in period $t - 1$ decreases. Proceeding inductively, we conclude that drug demand in period 1 must also decrease.

The connection between current demand and future prices has been documented in the literature on drug demand. Hence, Proposition 3 shows that our model is consistent with this empirical finding. Both Becker, Grossman and Murphy (1994) and Gruber and Koszegi (2001) find support for the prediction that drug demand decreases as future prices increase.

Note that results analogous to Proposition 3 have been shown for other models of addiction. Becker, Grossman and Murphy (1994) show this result for quadratic utility for the Becker-Murphy model. Gruber and Koszegi (2001) analyze a decision problem very similar to the one analyzed in this section. They give conditions under which Proposition 3(ii) holds in a $\beta - \delta$ model of addiction with quadratic utility.

4. Drug Policy and Welfare

Drug policies affect consumers along two dimensions. On the one hand, they alter the availability of the drug, and hence the feasible level of drug consumption. On the other hand, they change the drug price, and hence the opportunity cost of drug consumption.

Consider, for example, a tax on cigarettes. A typical consumer can afford the maximal feasible cigarette consumption in a period before and after a moderate tax increase. Therefore, a moderate tax on cigarettes will affect the opportunity cost of drug consumption without changing the maximal feasible drug consumption in the current period. We call such a policy a price policy.

The prohibition of a drug will affect the maximal feasible drug consumption for a typical consumer. We call a policy that reduces the maximal feasible drug consumption without changing the price of the drug (within the allowed range) a prohibitive policy. Often prohibitive policies are accompanied by a higher price of drugs. For analytical clarity we separate the prohibitive effects of a policy from the price effects in the analysis below.

A drug policy is a pair (τ, q) , where $\tau \geq 0$ is a per unit tax on the drug and $q \in [0, 1]$ is the maximal feasible drug consumption. We assume that tax revenues are distributed to the consumer in the form of a lump-sum transfer T . Let

$$B(\tau, q) = \{(c, d) \in [0, 1]^2 \mid c + (p + \tau)d \leq 1 + T, d \leq q\}$$

denote the individual's opportunity set under the policy (τ, q) . We assume that the agent faces a stationary decision problem in which he chooses some (c, d) from $B(\tau, q)$ in every period. To simplify the analysis, throughout this section, we assume that prices (and the parameter q) are constant across time. Let $y(\tau, q)$ denote the decision problem associated with the policy (τ, q) .

Any policy $(0, q)$ with $q < 1$ is a purely prohibitive policy since it reduces the maximal feasible drug consumption but does not affect the opportunity cost of drugs. A pure price policy is a policy $(\tau, 1)$ such that $p + \tau \leq 1 + T$. In this case, the maximal feasible drug consumption remains 1 in every period, but the opportunity cost of the drug is increased to $p + \tau$. If the tax is high enough, in particular, if $p + \tau > 1 + T$ then the policy $(\tau, 1)$ also has a prohibitive effect since it decreases the maximal drug consumption to $\frac{1+T}{p+\tau}$.

Propositions 4, 5 and 6 examine the welfare effects of prohibitive and price policies. Proposition 4 shows that a pure price policy can never increase welfare. This result is derived with a simple revealed preference argument and therefore does not require any additional assumptions.

Proposition 4: *If $p + \tau < 1 + T$ then $W(s, y(0, q)) \geq W(s, y(\tau, q))$ for all s .*

Proof: Let $s = d_0$ denote the initial state and $\{(c_t, d_t)_{t \geq 1}\}$ be the optimal consumption plan for the problem $y(\tau, 1)$. Note that $c_t + (p + \tau)d_t = 1 + T = 1 + \tau d_t$ and hence $c_t + pd_t = 1$. Therefore, $\{(c_t, d_t)_{t \geq 1}\}$ is a feasible choice from $y(0, 1)$. We conclude that

$$\begin{aligned} W(s, y(0, q)) &\geq \sum_{t=0}^{\infty} \delta^t (u(c_t, d_t) + \sigma(d_{t-1})v(d_t) - \sigma(d_{t-1})v(1)) \\ &= W(s, y(\tau, q)) \end{aligned}$$

□

A pure price policy does not affect the maximal feasible drug consumption, and therefore cannot eliminate temptations. It follows that such a policy cannot improve the agent's welfare. By distorting the optimal choice, a price policy reduces the agent's welfare. Proposition 4 stands in contrast to the findings of Gruber and Koszegi (2001). In their setting, a tax on drugs may increase welfare.

In our model, a policy can improve welfare *only if* it eliminates temptations. In section 6, Theorem 1, we provide a representation theorem that allows for a more general specification of temptation utility. In that model, temptation utility depends not only on current drug consumption, but also on non-drug consumption and the continuation problem. For that general model, it is possible to find specifications under which a price policy can increase welfare. We analyze the simpler model to capture the idea that the temptation associated with drugs is focused on current consumption of the drug.

Determining the best specification of temptation utility is an empirical issue. To distinguish various specifications for temptation utility one needs to examine the (policy) choices of consumers. Our specification would predict that addicts voluntarily seek commitment but vote against an increase in cigarette taxes. In contrast, a formulation for temptation utility that renders a price policy welfare improving leads to the prediction that addicts seek voluntary commitment but also vote for an increase in the tax on the drug.¹³

Recent opinion polls on cigarette taxes provide some evidence on this issue. A recent Gallup Poll¹⁴ finds that smokers overwhelmingly oppose raising taxes on cigarettes¹⁵ while

¹³ The model of Gruber and Koszegi (2001) also predicts that addicts seek voluntary commitment and vote for a tax on the drug.

¹⁴ Gallup News Service, August 15, 2002, www.gallup.com/poll/content/default.asp?ci=6610, subscription required.

¹⁵ Depending on the stated use of funds 70% or 80% of smokers are opposed to cigarette tax increases.

a slight majority of the overall population favors such a tax. In the introduction we cite a Quinnipiac University Poll with similar results.

Proposition 5 examines the welfare effect of a prohibitive policy. We consider the case where the drug is addictive and u does not depend on drug consumption (Assumption 1). Under those assumption, the agent is better off under the policy $(0, q)$ than under the policy (τ, \bar{q}) if $q < \bar{q}$. Hence, a more restrictive prohibitive policy together with a zero tax are always welfare improving.

Proposition 5: *Assume Assumption 1 is satisfied and the drug is addictive. If $\bar{q} > q$ and $p + \tau \leq 1 + T$ then $W(s, y(0, q)) > W(s, y(\tau, \bar{q}))$.*

Proof: Let $s = d_0 = \bar{d}_0$ be the initial state and let $\{(\bar{c}_t, \bar{d}_t)_{t \geq 1}\}$ denote the optimal consumption plan for the decision problem $y(\tau, \bar{q})$ at state s . Since Assumption 1 is satisfied we write $u(c)$ instead of $u(c, d)$. Define $\hat{d}_t = \min\{\bar{d}_t, q\}$ and set $\hat{c}_t = 1 - p\hat{d}_t$ for all $t \geq 1$. Clearly, $\hat{d}_t \leq \bar{d}_t$ for all $t \geq 1$ and therefore $\hat{c}_t \geq \bar{c}_t$. Hence, we have $u(\hat{c}_t) \geq u(\bar{c}_t)$, $\sigma(\hat{d}_t) \leq \sigma(\bar{d}_t)$. If $\bar{d}_t = \bar{q}_t$, then we have $\hat{d}_t = q$; if $\bar{d}_t < \bar{q}$ then $v(\hat{d}_t) - v(q) < v(\bar{d}_t) - v(\bar{q})$. In the former case, $u(\hat{c}_t) > u(\bar{c}_t)$ and $v(\hat{d}_t) - v(q) = 0$; in the latter case $\sigma(\hat{d}_{t-1})[v(\hat{d}_t) - v(q)] > \sigma(\bar{d}_{t-1})[v(\bar{d}_t) - v(\bar{q})]$. Hence,

$$\begin{aligned} W(s, y(0, q)) &\geq \sum_{t=0}^{\infty} \delta^t [u(\hat{c}_t, \hat{d}_t) + \sigma(\hat{d}_{t-1})v(\hat{d}_t) - \sigma(\hat{d}_{t-1})v_0(q)] \\ &> \sum_{t=0}^{\infty} \delta^t [(u(\bar{c}_t, \bar{d}_t) + \sigma(\bar{d}_{t-1})v(\bar{d}_t) - \sigma(\bar{d}_{t-1})v(\bar{q}))] \\ &= W(s, y(\tau, \bar{q})) \end{aligned}$$

□

In Proposition 5, we assume that Assumption 1 holds and therefore the level of drug consumption that maximizes commitment utility is zero. This ensures that a more restrictive prohibitive policy always increases utility in the current period. If the prohibitive policy does not bind, it reduces self-control costs. If prohibitive policy binds, it provides beneficial commitment to a lower drug consumption. This last conclusion may not be true if Assumption 1 is violated and u is increasing in drug consumption. In that case, a

prohibitive policy may harm the agent because the desirable level of drug consumption is infeasible.

In Proposition 5, we assume that the drug is addictive. This assumption ensures that a reduction in drug consumption lowers future self-control costs and therefore increases utility in future periods. To see why it is important for the drug to be addictive, consider an agent who is in state $s = .5$ in period 1. Suppose that abstaining ($d = 0$) or binging ($d = 1$) for one period will cause all temptation to go away in the next period but consuming intermediate levels will cause temptation to persist. Moreover, assume that the cost of self-control in the current state is very high. Then, it may be optimal for the agent to binge in the current period and abstain thereafter. In such a situation, a policy that reduces the maximal feasible level of drug consumption from 1 to $q = .5$ may reduce the agents welfare by forcing him to either incur the (reduced but still) high cost of self-control in the current period or remain addicted.

Next, we analyze the impact of prohibitive policies on the demand for drugs. Let $D(s, y(\tau, q))$ denote current period drug demand in state s given policy (τ, q) . Proposition 3 shows that drug demand is decreasing in the price of the drug if Assumptions 1 and 2 hold. Hence, a drug tax reduces drug demand. Below, we analyze the effect of prohibitive policies on drug demand.

Consider a purely prohibitive policy $(0, q)$. If the prohibitive policy is binding, that is, if $D(s, y(0, q)) = q$ then a reduction in the maximal allowed drug consumption q will obviously lead to a reduction in drug demand. Proposition 6 shows that if the policy is not binding then a reduction in q will lead to an *increase* in drug demand.

Proposition 6: *Suppose that the drug is addictive, Assumptions 1, 2 are satisfied, and $0 < d(s, y(0, q)) < q$ then $d(s, y(0, \cdot))$ is differentiable at q and $\partial d(s, y(0, q))/\partial q < 0$.*

Proof: Since the optimal consumption is interior, the first order necessary condition is

$$0 = -pu'(1 - pd_1) + \sigma(s)v'(d_1) + \sigma'(d_1)(v(d_2) - v(q)) \equiv A(d_1)$$

Taking the total derivative we get

$$dd_1 A'(d_1) - dq\sigma'(d_1)v'(q) = 0$$

Assumptions 1, 2 and addictiveness (see Proposition 1) implies that $A'(d_1) < 0$. Since $\sigma' > 0$ and $v' > 0$, the desired result follows. \square

A prohibitive policy reduces the utility cost of drug consumption by reducing future self-control costs. For this reason, drug demand increases as the prohibitive policy becomes more stringent.

Consider the following alternative interpretation of the model.¹⁶ Suppose that agents can consume a single indivisible unit of a variety of drugs, indexed by their severities $d \in [0, 1]$. Hence, harder drugs are more tempting and consumption of harder drugs leads to a bigger loss of self control. Assume also that harder drugs are more expensive. With this interpretation, Proposition 6 shows that banning the hardest drugs can induce agents to use harder drugs than they would otherwise use because the ban ensures that they cannot be tempted to use the hardest drugs in the future.

Assumption 1 implies that the agent (in period 0) would choose not to consume the drug in any period if perfect commitment were available. Hence, the fact that the drug is available is unambiguously “bad” for the consumer. Nevertheless, as our results show, policies that reduce drug consumption may reduce welfare, while policies that increases drug consumption may increase welfare.

5. Rehabilitation

In this section, we analyze a situation where the agent can choose to check into a rehabilitation center. In our interpretation, rehabilitation centers offer short term commitment to zero drug consumption.

As in the previous sections, we consider a simple decision problem that rules out intertemporal transfers of resources. The agent is either *in* or *out* of the rehabilitation center. If the agent is *out* he faces the budget set

$$B^o := \{(c, d) | c + pd = 1\}$$

¹⁶ We are grateful to a referee for providing this interpretation.

If the agent is *in* then he is committed to zero drug consumption and hence the choice set is

$$B^i(a) := \{(c, d) | c = 1 - a, d = 0\}$$

The parameter $a \in [0, 1]$ represents the cost of commitment (i.e., rehab).

The agent's decision problem is as follows. In each period $t \geq 1$, he finds himself either *in* and hence choosing from $B^i(a)$ or *out* and choosing from B^o . In addition, the agent must choose *in* or *out* for the next period. The decision problems $x^i(a), x^o(a)$ represent these two situations.

$$x^o(a) := \{(c, d, x) | (c, d) \in B^o, x \in \{x^o(a), x^i(a)\}\}$$

$$x^i(a) := \{(c, d, x) | (c, d) \in B^i(a), x \in \{x^o(a), x^i(a)\}\}$$

In period 1, $s = 0$ and the agent faces the decision problem $x^o(a)$.

The following propositions characterize optimal rehabilitation strategies for the addict. We write (c, d, j) with $j \in \{i, o\}$ to denote the choice from $x^k(a), k \in \{i, o\}$. An optimal policy for $x^k(a)$ is a sequence $(c_t, d_t, j_t), t = 1, 2, \dots$. We assume that the drug is addictive and that there is a unique optimal policy. Proposition 7 establishes that under these conditions, only three patterns of behavior can emerge. If the cost of rehab is too high, the addict never utilizes the program. If rehab is very inexpensive, the agent eventually enters rehab and once he is in he stays in. Between these two extremes, we observe a cycle of addiction and rehabilitation where the agent increases his drug consumption as long as he is not in rehab, then he enters rehab for one period, and afterwards restarts the cycle of increasing drug consumption.

Proposition 7: *Suppose the drug is addictive and that (c_t, d_t, j_t) is the unique optimal policy for the decision problem $x^o(a)$ in state $s = 0$. Then, (c_t, d_t, j_t) satisfies one of the following:*

- (i) $j_t = i$ for all t and $d_t = 0$ for all $t > 1$;
- (ii) $j_t = o$ for all t and $d_t \leq d_{t+1}$ for all t ;

(iii) there is $N \in \{2, 3, \dots\}$ and $(\hat{c}_n, \hat{d}_n, \hat{j}_n), n = 1, \dots, N$ such that for all $t = kN + n, k = 0, 1, \dots, (c_t, d_t, j_t) = (\hat{c}_n, \hat{d}_n, \hat{j}_n)$, where, $j_{N-1} = i, \hat{d}_N = 0$ and $0 < \hat{d}_1 < \dots < \hat{d}_{N-1}$.

Proof: Let (c_t, d_t, i_t) denote the unique optimal policy. Note that

$$W(s, x^i(a)) = W(0, x^i(a))$$

since the agent is committed to zero drug consumption in $x^i(a)$. Note also that $u(c, d) + \sigma(s)(v(d) - v(1)) + \delta \max[W(d, x^i(a)), W(d, x^o(a))]$ is non-increasing in s since σ is increasing and $v(d) \leq v(1)$ for all d . Therefore, $W(s, x^o(a))$ is non-increasing in s .

First, consider the case where $W(0, x^i(a)) > W(0, x^o(a))$. Hence, the agent prefers to be “in” when the state is 0. It follows from the above argument that $W(s, x^i(a)) > W(s, x^o(a))$ for all s . This implies that the agent chooses $j_t = i$ for all $t = 1, 2, \dots$. Hence, case (i) applies.

Next, consider the case where $W(d_t, x^i(a)) < W(d_t, x^o(a))$ for all d_t . In that case, the agent chooses $j_t = o$ for all $t = 1, 2, \dots$. Note that the agent’s consumption plan is optimal for the stationary decision problem in which he faces the budget set B^o in every period. Let $x = \{(c, d, x) | (c, d) \in B^o\}$ denote the corresponding decision problem. By Proposition 2, the drug demand from x is monotonically increasing in s . This in turn implies that d_t is non-decreasing and case (ii) applies.

Since we assumed a unique optimal solution, it remains to show that if $W(0, x^o(a)) > W(0, x^i(a))$ and $W(d_t, x^i(a)) > W(d_t, x^o(a))$ for some d_t , then case (iii) applies. It follows from $W(0, x^o(a)) > W(0, x^i(a))$ that $j_1 = o$. Let $N = t + 1$, where t is the smallest integer such that $W(d_t, x^i(a)) > W(d_t, x^o(a))$. Hence, $j_{N-1} = i$. Note that $W(0, x^o(a)) > W(0, x^i(a))$ implies $j_N = o$ and therefore, in period $N+1$, the state is 0 and the agent makes an optimal choice from the decision problem $x^o(a)$. This is the same state and decision problem that he faced in period 1. It follows from the uniqueness of the optimal policy that optimal choices in periods $N+1, \dots, 2N$ are identical to the choices in periods $1, \dots, N$ and that $j_n = o$ for $n < N-1$. It remains to show that $0 < d_1 < \dots < d_{N-1}$. Proposition 2 implies that the drug demand in $x^o(a)$ is non-decreasing in the state. Hence, it follows that $0 \leq d_1 \leq \dots \leq d_{N-1}$. Because the optimal policy is unique, drug demand must be strictly

increasing. To see this, first suppose $d_t = d_{t+1}$, $t < N - 1$. Then, (c_t, d_t, o) is an optimal choice from $x^o(a)$ at state d_t . But this contradicts the uniqueness of the optimal choice and the fact that $j_t = i$ for some t . If $d_1 = 0$ then it must be that $W(d, x^i(a)) \geq W(d, x^o(a))$ again contradiction. \square

In Proposition 7, we have assumed that the optimal solution is unique. Alternatively, we could have imposed Assumptions 1 and 2, which would imply that drug demand is strictly increasing in the state. Then, the optimal solution would be unique for generic values of a .

The following proposition demonstrates that cheaper rehabilitation centers may increase drug consumption in some periods. More precisely, suppose the initial cost of rehab is so high that in the current state it is not optimal to choose i . Now, assume that this cost is lowered so that i becomes the optimal choice. Then, drug consumption in the current period will increase. We say that $x^o(a)$ has interior optima for s if at state s , optimal drug consumption is strictly between zero and one whenever the agent is not in rehab.

Proposition 8: *Suppose the drug is addictive and Assumptions 1 and 2 are satisfied. If $\bar{a} > a$, (c, d, i) is an optimal choice from $x^o(a)$ in state s and (\bar{c}, \bar{d}, o) is an optimal choice from $x^o(\bar{a})$ in state s then $d \geq \bar{d}$. A strict inequality holds if $x^o(\bar{a})$ has interior optima for s .*

Proof: Since (c, d, i) is an optimal choice from $x^o(a)$ it follows that next period the agent is committed to a zero drug consumption. Hence, d solves

$$\max_{d' \in [0,1]} u(1 - pd') + \sigma(s)v(d')$$

Similarly, \bar{d} solves

$$\max_{d' \in [0,1]} u(1 - pd') + \sigma(s)v(d') + \delta\sigma(d')(v(\hat{d}) - v(1))$$

where \hat{d} is an optimal drug consumption in the next period given the choice problem $x^o(\bar{a})$. If $\hat{d} = 1$ then the two maximization problems are identical and hence $\bar{d} = d$. Note that $\sigma' > 0$ by Assumption 2 and the fact that the drug is addictive. Therefore, for $\hat{d} < 1$

$$-pu'(1 - pd') + \sigma(s)v'(d') + \delta\sigma'(d')(v(\hat{d}) - v(1)) < -pu'(1 - pd') + \sigma(s)v'(d')$$

for all $d' \in [0, 1]$. It follows that $d \geq \bar{d}$. If solutions to $x^o(\bar{a})$ are interior then $0 < \hat{d} < 1$ and $0 < \bar{d} < 1$. In that case it is easy to see that $\bar{d} > d$. \square

Although drug demand may increase as a result of less expensive rehab, the agent's welfare increases as rehab becomes cheaper.

Proposition 9: *If $u(\cdot, d)$ is nondecreasing and $\bar{a} > a$ then $W(s, x^o(a)) \geq W(s, x^o(\bar{a}))$.*

Proof: Let $(\bar{c}_t, \bar{d}_t, \bar{j}_t)$ denote an optimal policy for the decision problem $x^o(\bar{a})$. We have

$$W(s, x^o(\bar{a})) = \sum_{t=1}^{\infty} \delta^{t-1} (u(\bar{c}_1, \bar{d}_1) + v(\bar{d}_1) - \bar{v}_t^{\max})$$

where \bar{v}_t^{\max} denotes the maximal feasible drug consumption in period t . Note that $\bar{v}_t^{\max} = 1$ in period 1. In all other periods it is 1 if $\bar{j}_{t-1} = o$ and 0 if $\bar{j}_{t-1} = i$. Since $\bar{a} > a$ there is a feasible policy $(\hat{c}_t, \hat{d}_t, \hat{j}_t)$ for $x^o(a)$ with $\hat{d}_t = \bar{d}_t, \hat{j}_t = \bar{j}_t$ and $\hat{c}_t \geq \bar{c}_t$. The utility of this policy is

$$\sum_{t=1}^{\infty} \delta^{t-1} (u(\hat{c}_t, \hat{d}_t) + v(\hat{d}_t) - \bar{v}_t^{\max}) \geq W(s, x^o(a))$$

Hence $W(s, x^o(a)) \geq W(s, x^o(\bar{a}))$. \square

As in the previous section, we find that the success of policy or treatment options cannot be determined by examining their effect on drug consumption. Reducing the cost of rehab a makes the agent unambiguously better off even though it may increase drug consumption. However, the fact that welfare goes up as the cost of rehab decreases does not mean that governments should subsidize rehab. If the parameter a represents the actual resource cost of the rehab unit to society then a subsidy of the cost of rehab financed with lump sum taxation cannot improve the agent's welfare. The agent's demand for rehab at the resource cost is welfare maximizing.

6. Representation Theorems

In this section we provide two representation theorems. Theorem 1 axiomatizes a representation that is more general than the one used in the applications above. In particular, the representation allows for a more general state space and a more general specification of temptation utility. Theorem 2 provides additional axioms that yield the representation used throughout the previous sections.

The set of consumptions in each period is $C = [0, 1]^2$ and $b \in C$ denotes a generic consumption vector. For any subset X of a metric space, we let $\Delta(X)$ denote the set of all probability measures on the Borel σ -algebra of X and $\mathcal{K}(X)$ denote the set of all nonempty, compact subsets of X . An infinite horizon decision problem (denoted $z \in Z$) can be identified with an element in $\mathcal{K}(\Delta(C \times Z))$ and conversely each element in $\mathcal{K}(\Delta(C \times Z))$ identifies a decision problem $z \in Z$. For formal definitions of Z and the map that associates each element of Z with its equivalent recursive description as an element of $\mathcal{K}(\Delta(C \times Z))$, we refer the reader to Gul and Pesendorfer (2004). In what follows, only the recursive definition is used and hence without risk of confusion we identify the sets Z and $\mathcal{K}(\Delta(C \times Z))$. In Gul and Pesendorfer (2004), we note that since C is a compact metric space, $Z, \Delta(C \times Z)$ and $\mathcal{K}(\Delta(C \times Z))$ are compact metric spaces as well. We write Δ instead of $\Delta(C \times Z)$ when there is no risk of confusion.

The individual's preferences are defined on Z and are indexed by $s \in S$, the state in the initial period of the decision problem. The state s represents the relevant consumption history prior to the initial period. We assume that there is a finite number K such that consumption in only the last K periods influences the agents preferences. Therefore, without loss of generality we set $S := C^K$ where K is the minimal length of the individual's consumption history that allows us to describe \succeq .¹⁷ We refer to the indexed family of preferences $\succeq := \{\succeq_s\}_{s \in S}$ simply as *the agent* or the preference \succeq .

For any state $s = (b_1, \dots, b_K)$ let sb denote the state (b_2, \dots, b_K, b) . We impose the following axioms on \succeq_s for every $s \in S$.

Axiom 1: (Preference Relation) \succeq_s is a complete and transitive binary relation.

¹⁷ That is, there is a pair of states, $(s = (b_1, \dots, b_K), \hat{s} = (\hat{b}_1, \dots, \hat{b}_K))$ that differ only in their first component ($b_1 \neq \hat{b}_1, b_t = \hat{b}_t, t \geq 2$) and lead to different preferences ($\succeq_s \neq \succeq_{\hat{s}}$).

Axiom 2: (*Strong Continuity*) The sets $\{x \mid x \succeq_s z\}$ and $\{x \mid z \succeq_s x\}$ are closed in Z .

Axiom 3: (*Independence*) $\{\mu\} \succ_s \{\nu\}$ implies $\{\alpha\mu + (1-\alpha)\eta\} \succ_s \{\alpha\nu + (1-\alpha)\eta\} \forall \alpha \in (0, 1)$.

Axioms 1 – 3 are standard. In Axiom 4, we deviate from standard choice theory and allow for the possibility that adding options to a decision problem makes the consumer strictly worse-off. For a detailed discussion of Axiom 4, we refer the reader to our earlier paper (Gul and Pesendorfer 2001).

Axiom 4: (*Set Betweenness*) $x \succeq_s y$ implies $x \succeq_s x \cup y \succeq_s y$.

Next, we make a separability assumption. For $z \in Z$, let $bz \in Z$ denote the decision problem $\{(b, z)\}$, that is, the degenerate decision problem that yields c in the current period and the continuation problem z . Thus $b_1 b_2 \dots b_K z$ is a degenerate decision problem that yields the consumption (b_1, \dots, b_K) in the first K periods and the continuation problem z in period $K + 1$. For $s = (b_1, \dots, b_K)$, we write sz instead of $b_1 b_2 \dots b_K z$. Axiom 5 considers decision problems of the form $\{(b, sz)\}$ and requires that preferences are not affected by the correlation between current consumption c and the $K + 1$ period continuation problem z .

Axiom 5: (*Separability*) $\{\frac{1}{2}(b, sz) + \frac{1}{2}(\hat{c}, s\hat{z})\} \sim_s \{\frac{1}{2}(b, s\hat{z}) + \frac{1}{2}(\hat{c}, sz)\}$.

Axiom 6 requires preferences to be stationary. Consider the degenerate lotteries, (b, x) and (b, y) , each leading to the same period 1 consumption c . Stationarity requires that $\{(b, x)\}$ is preferred to $\{(b, y)\}$ in state s if and only if the continuation problem x is preferred to the continuation problem y in state sb .

Axiom 6: (*Stationarity*) $\{(b, x)\} \succeq_s \{(b, y)\}$ iff $x \succeq_{sb} y$.

Note that Axiom 6 implies that the conditional preferences at time $K + 1$, after consuming s in the first K periods, is the same as the initial preference \succeq_s . Together, Axioms 5 and 6 restrict the manner in which past consumption influences future preferences. Axiom 5 ensures that correlation between consumption prior to period $t - K$ and the decision problem in period t does not affect preferences, whereas Axiom 6 ensures that the realization of consumption prior to period $t - K$ does not affect the individuals ranking of consumption flows after period t .

Axiom 7 requires individuals to be indifferent as to the timing of resolution of uncertainty. In a standard, expected utility environment, this indifference is implicit in the assumption that the domain of preference is the set of lotteries over consumption paths. Our domain of preferences is the set of all decision problems, and in this richer structure a separate assumption is required to rule out agents that are not indifferent to the timing of resolution of uncertainty.¹⁸

Consider the lotteries $\mu = \alpha(b, x) + (1 - \alpha)(b, y)$ and $\nu = (b, \alpha x + (1 - \alpha)y)$. The lottery μ returns the consumption c together with the continuation problem x with probability α and the consumption c with the continuation problem y with probability $1 - \alpha$. In contrast, ν returns c together with the continuation problem $\alpha x + (1 - \alpha)y$ with probability 1. Hence, μ resolves the uncertainty about x and y in the current period, whereas ν resolves this uncertainty in the future. If $\{\mu\} \sim_s \{\nu\}$ then the agent is indifferent as to the timing of the resolution of uncertainty.

Axiom 7: (Indifference to Timing) $\{\alpha(b, x) + (1 - \alpha)(b, y)\} \sim_s \{(b, \alpha x + (1 - \alpha)y)\}$.

Definition: The preference \succeq_s is regular if there exists x, \hat{x}, y , and $\hat{y} \in Z$ such that $\hat{x} \subset x, \hat{y} \subset y, x \succ_s \hat{x}$ and $\hat{y} \succ_s y$. The preference \succeq is regular if each \succeq_s is regular.

Hence \succeq_s is not regular if it either displays no preference for commitment (i.e., is standard) or if it always prefers fewer options. Theorem 1 below establishes that all regular preferences that satisfy Axioms 1 – 7 can be represented as a discounted sum of state-dependent utilities minus state-dependent self-control costs. We say that the function $W : S \times Z \rightarrow \mathbb{R}$ represents \succeq when $x \succeq_s y$ iff $W(s, x) \geq W(s, y)$ for all s .

Theorem 1: If \succeq is regular and satisfies Axioms 1 – 7, then there exists $\delta \in (0, 1)$, continuous functions $u : S \times C \rightarrow \mathbb{R}$, $V : S \times C \times Z \rightarrow \mathbb{R}$, $W : S \times Z \rightarrow \mathbb{R}$ such that

$$W(s, z) = \max_{\mu \in \mathcal{Z}} \int [u(s, b) + \delta W(sb, z) + V(s, b, z)] d\mu(b, z) - \max_{\nu \in \mathcal{Z}} \int V(s, b, z) d\nu(b, z)$$

for all $s \in S, \nu \in \Delta$ and W represents \succeq . For any $\delta \in (0, 1)$, continuous u, V there exists a unique function W that satisfies the equation above and the \succeq represented by this W satisfies Axioms 1 – 7.

¹⁸ Such agents are model in Kreps and Porteus (1978).

The two main steps of the proof of Theorem 1 entail showing that a preference relation (over decision problems) that satisfies continuity, independence, set betweenness, stationarity and indifference to timing of resolution of uncertainty has a representation of the form

$$W(s, z) = \max_{\mu \in z} \{U(s, \mu) + V(s, \mu)\} - \max_{\nu} V(s, \nu) \quad (2)$$

and then using stationarity and separability to show that U is of the form $U = u + \delta W$. In Gul and Pesendorfer (2004), we offer a related proof under stronger stationarity and separability axioms, yielding a representation of state-independent preferences.

Next, we provide additional assumptions that are needed to characterize the preferences used in our analysis of addiction; that is, those represented by a utility function W satisfying equation (1).

Assumption I below is taken from Gul and Pesendorfer (2004). It requires that two alternatives, ν, η , offer the same temptation if they have the same marginal distribution over current consumption. For any $\mu \in \Delta(C \times Z)$, μ^1 denotes the marginal on the first coordinate (current consumption) and μ^2 denotes the marginal on the second coordinate (the continuation problem).

Assumption I: (*Temptation by Immediate Consumption*) For $\mu, \nu \in \Delta$, suppose $\nu^1 = \eta^1$. If $\{\mu\} \succ_s \{\mu, \nu\} \succ_s \{\nu\}$ and $\{\mu\} \succ_s \{\mu, \eta\} \succ_s \{\eta\}$ then $\{\mu, \nu\} \sim_s \{\mu, \eta\}$.

To understand Assumption I, note that $\{\mu\} \succ_s \{\mu, \nu\} \succ_s \{\nu\}$ represents a situation where the agent is tempted by ν but chooses μ from $\{\mu, \nu\}$.¹⁹ Similarly, $\{\mu\} \succ_s \{\mu, \eta\} \succ_s \{\eta\}$ means that the agent is tempted by η but chooses μ . Hence, the agent makes the same choice in both situations. If $\nu^1 = \eta^1$ then immediate temptation means that the agent experiences the same temptation in the two situations, and therefore is indifferent between them; $\{\mu, \nu\} \sim_s \{\mu, \eta\}$.

Assumption N below ensures that the good c is *neutral*, i.e., causes no temptation and has no dynamic effects. That is, only good d is tempting and only past consumption of d affects future rankings of decision problems.

¹⁹ This follows from a straightforward application of the representation in (2).

Assumption N: Let $b = (c, d)$ and $\hat{b} = (\hat{c}, \hat{d})$. If $d = \hat{d}$ then $\{(b, z), (\hat{b}, \hat{z})\} \succeq_s \{(b, z)\}$ and $\succeq_{sb} = \succeq_{s\hat{b}}$. If $\hat{d} > d$ and $\{(b, z)\} \succ_s \{(\hat{b}, \hat{z})\}$ then $\{(b, z)\} \succ_s \{(b, z), (\hat{b}, \hat{z})\}$.

The first statement in Assumption N ensures that there is no temptation so long as the options differ only with respect to current consumption of non-drugs. The second statement means that future preferences are the same so long as the current state and current consumption of drugs are the same. Finally, the third statement implies that higher current drug consumption is always tempting.

To state the final assumption, we first define what it means for the agent to have the same preference for commitment at two states.

Definition: The preference \succeq_s has a preference for commitment at z , if there is $x \subset z$ such that $x \succ_s z$; \succeq has the same preference for commitment at \hat{s} and at s if \succeq_s has a preference for commitment at z iff $\succeq_{\hat{s}}$ has a preference for commitment at z .

Assumption P says that the agent's preference for commitment does not change as the state changes. In other words, whether or not an alternative constitutes a temptation is independent of the state.

Assumption P: The agent has the same preference for commitment at all s .

Theorem 2: The preference \succeq is regular and satisfies Axioms 1 – 7, I, N and P if and only if (i) $S = [0, 1]$, and (ii) there are continuous functions $v, \sigma : [0, 1] \rightarrow \mathbb{R}$, $u : C \rightarrow \mathbb{R}$, and $\delta \in (0, 1)$, such that for $z \in \bar{Z}$

$$W(s, z) = \max_{(c, d, x) \in z} \{u(c, d) + \sigma(s)v(d) + \delta W(c, d, x)\} - \max_{(\hat{c}, \hat{d}, y) \in z} \sigma(s)v(\hat{d})$$

and W represents \succeq . (iii) u is nonconstant, v is strictly increasing, $\sigma > 0$, and s is the previous period's drug consumption.

Proof: See Appendix.

To see the role of the assumptions in Theorem 2, consider the representation provided in Theorem 1. Adding Assumption I ensures that $V(s, \cdot)$ depends only on current consumption. Then, Assumption N guarantees that $V(s, \cdot)$ depends only on current drug consumption and is strictly increasing in d . Finally, Assumption P implies that $U = u + \delta W$ is independent of the state and that the state is equal to last period's drug consumption.

7. Conclusion

Most studies on drug abuse assert that addiction should be considered a disease.²⁰ In our approach, drug abuse is identified with the discrepancy between what the agent would want to commit to, as reflected by maximizing commitment utility, and what he ends-up consuming by maximizing the sum of commitment utility and temptation utility. We provide straightforward choice experiments for measuring this discrepancy. Our approach is silent on the question of whether addiction is a disease or a part of the normal variation of preferences across individuals.

While our approach is compatible with the disease conception of addiction, there are important differences between our formulation and the typical disease model. Consider the following example: the opiate antagonist naltrexone blocks the opioid receptors in the brain and hence the euphoric effects of these drugs for up to 3 days after the last dose. Naltrexone is used in the treatment of heroin and morphine. However, with the exception of highly motivated addicts such as parolees, probationers and healthcare professionals, most addicts receiving naltrexone tend to stop taking their medicine and relapse. Addicts often report that they stop taking naltrexone because it prevents “getting high”. Doctors call this a “compliance problem” with naltrexone. For them, this is simply a limitation on the usefulness naltrexone, the same way that toxicity might be a limitation on the usefulness of some other medication.

In our model, there can be two reasons for an addict to discontinue naltrexone and resume heroin consumption: either 3 days is not the right time horizon for commitment or the addict does not wish to commit. The former would suggest a need for longer acting versions of Naltrexone, while the latter would mean that there is neither a need nor any room for treatment of this addict. In fact, by our definition, an individual who is unwilling to commit to reducing his drug consumption, for any length of time, at any future date is not an addict. Hence, where the disease model of addiction finds a compliance problem our model suggests that there may be no problem at all.

Our focus was on psychoactive drugs but the model presented in this paper can also be applied to other types of compulsive behavior such as over-eating and other forms of

²⁰ To emphasize the organic basis of the condition the term “disease of the brain” is often used.

dependency. Moreover, as we illustrate at the end of section 2, the model can easily be modified to describe beneficial addictions, such as physical exercise or practicing a musical instrument. In that case, the agent would like to commit to some activity every period. If the agent is not committed, he is tempted not to do it. Doing the activity now strengthens the agent's self-control and makes it more likely that he will do it in the future. Like harmful addictions, beneficial addictions generate a demand for commitment. However, in this case, agents seek commitment *before* they are addicted.²¹ Once the beneficial addiction is established, the agent can rely on self-control to continue exercising. In contrast, in the case of a harmful addiction, individuals seek commitment when they are most addicted.

²¹ Individuals who start exercising often hire personal trainers or commit to a year long health club subscription. The desire to commit may partially explain such behavior.

8. Appendix

8.1 Proof of Proposition 1:

To prove the “if” part, let $\sigma(\bar{s}) \geq \sigma(s)$ and let $\mu \in \mathcal{D}(\bar{s}, z) \cap \mathcal{C}(z)$. Then

$$\int (u(\hat{c}, \hat{d}) + \sigma(\bar{s})v(\hat{d}) + \delta W(\hat{d}, \hat{z}))d\mu(\hat{c}, \hat{d}, \hat{z}) \geq \int (u(\hat{c}, \hat{d}) + \sigma(\bar{s})v(\hat{d}) + \delta W(\hat{d}, \hat{z}))d\nu(\hat{c}, \hat{d}, \hat{z})$$

and

$$\int (u(\hat{c}, \hat{d}) + \delta W(\hat{d}, \hat{z}))d\mu(\hat{c}, \hat{d}, \hat{z}) \geq \int (u(\hat{c}, \hat{d}) + \delta W(\hat{d}, \hat{z}))d\nu(\hat{c}, \hat{d}, \hat{z})$$

for all $\nu \in z$. Since $\sigma > 0$ there is $\alpha \in (0, 1]$ such that $\sigma(s) = \alpha\sigma(\bar{s})$. Taking a convex combination of the above two inequalities we conclude that $\mu \in \mathcal{D}(s, z) \cap \mathcal{C}(z)$. Hence, if $\succeq_{\bar{s}}$ is not compulsive then \succeq_s is not compulsive. Obviously $\succeq_s \neq \succeq_{\bar{s}}$ if $\sigma(\bar{s}) > \sigma(s)$.

To prove the “only if” part we can repeat the argument of Lemma 12 from Gul and Pesendorfer (2001) in the current setting to obtain the following fact.

Fact: \succeq is more compulsive at \bar{s} than at s only if $U + \sigma(s)v = \beta_1 U + \beta_2(U + \sigma(\bar{s})v) + \beta_3$ for some $\beta_1, \beta_2 \in \mathbb{R}_+$, $\beta_3 \in \mathbb{R}$ and for all μ .

Note that

$$U + \sigma(s)v = U + \frac{\sigma(s)}{\sigma(\bar{s})}\sigma(\bar{s})v$$

Hence, $\beta_1 = 1$ and $\beta_2 = \frac{\sigma(s)}{\sigma(\bar{s})}$. Since, $\beta_1, \beta_2 > 0$, we conclude $\sigma(\bar{s}) > \sigma(s)$. □

8.2 Proof of Theorem 1

It is easy to show that if \succeq satisfies Axioms 3, 6 and 7 then it also satisfies the following stronger version of the independence axiom:

Axiom 3*: $x \succ_s y$, $\alpha \in (0, 1)$ implies $\alpha x + (1 - \alpha)z \succ_s \alpha y + (1 - \alpha)z$.

Theorem 1 of Gul and Pesendorfer (2001) establishes that \succeq_s satisfies Axioms 1, 2, 4 and 3* if and only if there exist $W(s, \cdot), U(s, \cdot), V(s, \cdot)$ such that

$$W(s, z) := \max_{\mu \in z} \{U(s, \mu) + V(s, \mu)\} - \max_{\nu \in z} V(s, \nu) \quad (*)$$

for all $z \in Z$ and \hat{W} represents \succeq . Moreover, the functions $W(s, \cdot), U(s, \cdot), V(s, \cdot)$ are continuous and linear in their second arguments. We refer to the triple $(U(s, \cdot), V(s, \cdot), W(s, \cdot))$ as a representation of \succeq_s . The additional content of Theorem 1 is that we may choose functions (U, V, W) that are continuous in s such that $(U(s, \cdot), V(s, \cdot), W(s, \cdot))$ is a representation of \succeq_s for each s and $U(s, \cdot)$ satisfies

$$U(s, \mu) = \int [u(s, b) + \delta W(sb, z)] d\mu(c, z)$$

for some continuous function u and $\delta \in (0, 1)$.

Fix \bar{s} and let $(\hat{W}(\bar{s}, \cdot), \hat{U}(\bar{s}, \cdot), \hat{V}(\bar{s}, \cdot))$ be a representation of $\succeq_{\bar{s}}$. Define W to be the following function:

$$W(s, y) := \hat{W}(\bar{s}, sy) \tag{**}$$

Observe that W is well defined and continuous in both arguments since \hat{W} is continuous in its second argument. In the following Lemmas, the function W is the function defined in (**).

Lemma 1: *W represents \succeq . Moreover, there exist continuous functions U, V such that*

$$W(s, z) := \max_{\mu \in z} \{U(s, \mu) + V(s, \mu)\} - \max_{\nu \in z} V(s, \nu)$$

and W, U, V are linear in their second arguments.

Proof: Axiom 6 implies $W(s, x) \geq W(s, y)$ iff $\hat{W}(s, x) \geq \hat{W}(s, y)$. Therefore, W represents \succeq . Note that \hat{W} is linear in its second argument. Let $z = \alpha x + (1 - \alpha)y$. Axiom 7 and linearity of \hat{W} in its second argument imply that

$$\begin{aligned} W(s, z) &= \hat{W}(\bar{s}, sz) \\ &= \hat{W}(\bar{s}, \{\alpha sx + (1 - \alpha)sy\}) \\ &= \alpha \hat{W}(\bar{s}, sx) + (1 - \alpha) \hat{W}(\bar{s}, sy) \\ &= \alpha W(s, x) + (1 - \alpha) W(s, y) \end{aligned}$$

Thus, W is linear in its second argument. It follows that $W(s, z) = \alpha(s) \hat{W}(s, z) + \beta(s)$ for some $\alpha, \beta : S \rightarrow \mathbb{R}$ such that $\alpha(s) \geq 0$. Since \succeq is regular, $\alpha(s) > 0$ for all s . Hence, $U = \alpha \hat{U} + \beta, V = \alpha \hat{V}$ and the W have the desired properties. \square

Lemma 2: Let $\hat{W}(s, \cdot)$ represent \succeq_s . Then,

$$\hat{W}(s, b_1 \dots b_l \hat{s} \bar{z}) - \hat{W}(s, b_1 \dots b_l \hat{s} z) = \hat{W}(s, \bar{b}_1 \dots \bar{b}_l \hat{s} \bar{z}) - \hat{W}(s, \bar{b}_1 \dots \bar{b}_l \hat{s} z)$$

for all l , $(\bar{b}_1, \dots, \bar{b}_l), (b_1, \dots, b_l) \in C^{l+1}$, $\hat{s} \in C^K$, $z, \bar{z} \in Z$.

Proof: Note that by Axiom 5,

$$\frac{1}{2}(\bar{b}_1, b_2 \dots b_l \hat{s} \bar{z}) + \frac{1}{2}(b_1, b_2 \dots b_l \hat{s} z) \sim_s \frac{1}{2}(b_1, b_2 \dots b_l \hat{s} \bar{z}) + \frac{1}{2}(\bar{b}_1, b_2 \dots b_l \hat{s} z)$$

Assume that the assertion holds for $l' \leq l - 1$. Then, Axiom 6 implies that

$$\frac{1}{2}(\bar{b}_1, \bar{b}_2 \dots \bar{b}_l \hat{s} \bar{z}) + \frac{1}{2}(\bar{b}_1, b_2 \dots b_l \hat{s} z) \sim_s \frac{1}{2}(\bar{b}_1, b_2 \dots b_l \hat{s} \bar{z}) + \frac{1}{2}(\bar{b}_1, \bar{b}_2 \dots \bar{b}_l \hat{s} z)$$

Since \hat{W} represents \succeq_s , we conclude

$$\begin{aligned} \hat{W}(s, b_1 \dots b_l \hat{s} \bar{z}) - \hat{W}(s, b_1 \dots b_l \hat{s} z) &= \hat{W}(s, \bar{b}_1 b_2 \dots b_l \hat{s} \bar{z}) - \hat{W}(s, \bar{b}_1 b_2 \dots b_l \hat{s} z) \\ &= \hat{W}(s, \bar{b}_1 \dots \bar{b}_l \hat{s} \bar{z}) - \hat{W}(s, \bar{b}_1 \dots \bar{b}_l \hat{s} z) \end{aligned}$$

and hence the assertion holds for $l' \leq \bar{l}$. Observe that Axiom 5 implies that the Lemma holds for $l = 1$. \square

Lemma 3: $W(s', sx) - W(s', sy) = W(s'', sx) - W(s'', sy)$ for all s', s'', x, y .

Proof: Recall that

$$W(s', sx) = \hat{W}(\bar{s}, s' sx)$$

for some \hat{W} such that $\hat{W}(\bar{s}, \cdot)$ represents $\succeq_{\bar{s}}$. Lemma 2 implies that

$$\hat{W}(\bar{s}, s' sx) - \hat{W}(\bar{s}, s' sy) = \hat{W}(\bar{s}, s'' sx) - \hat{W}(\bar{s}, s'' sy) \quad (\ddagger)$$

Substituting W for \hat{W} in equation (\ddagger) then proves the Lemma. \square

Lemma 3: There exist $\delta : S \times C \rightarrow (0, \infty)$ and $u : S \times C \rightarrow \mathbb{R}$ such that $U(s, \nu) = \int [u(s, b) + \delta(s, b)W(sb, z)]d\nu(b, z)$ for all $s \in S, \nu \in \Delta$.

Proof: Since $U(s, \cdot)$ is linear and continuous, it has an integral representation. That is;

$$U(s, \nu) = \int U(s, b, z) d\nu(b, z)$$

By Axiom 6, $U(s, b, \cdot)$ and $W(sb, \cdot)$ yield the same linear preferences over Z . By regularity, neither function is constant. It follows that $U(s, b, \cdot)$ is a strictly positive affine transformation of $W(sb, \cdot)$. Hence, for some u, δ ,

$$U(s, b, \cdot) = u(s, b) + \delta(s, b)W(sb, y)$$

where $\delta(s, b) > 0$ for all $s \in S, b \in C$. Therefore,

$$U(s, \nu) = \int [u(s, b) + \delta(s, b)W(sb, y)] d\nu(b, z)$$

as desired □

Lemma 4: *The function $\delta(\cdot)$ in Lemma 3 is constant.*

Proof: Suppose δ is not constant. Let $k \in 1, \dots, K+1$ denote the smallest integer such that $\delta(b_1, \dots, b_{K+1}) = \delta(\tilde{b}_1, \dots, \tilde{b}_{K+1})$ for all $(b_1, \dots, b_{K+1}), (\tilde{b}_1, \dots, \tilde{b}_{K+1})$ with $b_n = \tilde{b}_n$ for $n \leq k$. Then, it is straightforward to show that there exist $(s, b_{K+1}) = (b_1, \dots, b_{K+1})$ and $(s^*, b_{K+1}^*) = (b_1^*, \dots, b_{K+1}^*)$ such that $b_n = b_n^*, n \neq k$ and $\delta(b_1, \dots, b_{K+1}) > \delta(b_1^*, \dots, b_{K+1}^*)$.

Pick any $b \in C$. Let $s' = (b, \dots, b, b_1, b_2, \dots, b_{k-1})$. Fix any \hat{s} . By regularity there are $y_h, y_l \in Z$ such that $W(\hat{s}, y_h) > W(\hat{s}, y_l)$. Let $y_{hh} = b_k \dots b_{K+1} \hat{s} y_h, y_{hl} = b_k \dots b_{K+1} \hat{s} y_l$ and $y_{lh} = b_k^* \dots b_{K+1}^* \hat{s} y_h, y_{ll} = b_k^* \dots b_{K+1}^* \hat{s} y_l$. Let $x = .5y_{hh} + .5y_{ll}$ and $z = .5y_{hl} + .5y_{lh}$. By Lemma 2, $W(s', x) = W(s', z)$.

Applying Lemma 3 repeatedly and using the fact that $\delta(s, b) = \delta(\tilde{s}, \tilde{b})$ for $(s, b), (\tilde{s}, \tilde{b})$ with $b_n = \tilde{b}_n, n \leq k$ establishes $W(s', x) - W(s', z) = 0$ iff

$$\begin{aligned} & \delta(s, b_{K+1})W(sb_{K+1}, \hat{s}y_h) + \delta(s^*, b_{K+1}^*)W(s^*b_{K+1}^*, \hat{s}y_l) = \\ & \delta(s, b_{K+1})W(sb_{K+1}, \hat{s}y_l) + \delta(s^*, b_{K+1}^*)W(s^*b_{K+1}^*, \hat{s}y_h) \end{aligned}$$

Rearranging, this implies

$$\begin{aligned} & \delta(s, b_{K+1})(W(sb_{K+1}, \hat{s}y_h) - W(sb_{K+1}, \hat{s}y_l)) = \\ & \delta(s^*, b_{K+1}^*)(W(s^*b_{K+1}^*, \hat{s}y_h) - W(s^*b_{K+1}^*, \hat{s}y_l)) \end{aligned}$$

Observe that $W(s, \hat{s}y_h) - W(s, \hat{s}y_l) > 0$ by construction and hence Lemma 3 implies the desired contradiction. \square

Lemma 5: *Let $\delta \in \mathbb{R}$ denote the constant function in Lemma 4. Then, $0 < \delta < 1$.*

Proof: That $\delta > 0$ has already been established. Pick any $b \in C$ and let $s = (b, b, \dots, b)$. Let z_b denote the unique $z \in Z$ such that $z = \{b, z\}$. Pick $y_1 \in Z$ such that $W(s, y_1) \neq W(s, z)$. By regularity, such a y_1 exists. Define $y_n \in Z$ inductively as $y_n = \{(b, y_{n-1})\}$ and note that y_n converges to z . Hence, by continuity, $W(s, z) - W(s, y_n)$ must converge to 0. But, by Lemma 3, $W(s, z) - W(s, y_n) = \delta^{n-1}(W(s, y_1) - W(s, z)) \neq 0$. Hence, $\delta < 1$. \square

Lemmas 1–5 establish that there is a continuous representation (U, V, W) that satisfies $U(s, \mu) = u(\mu^1) + \int \delta W(sb, z) d\mu(b, z)$.

To conclude the proof, let $\delta \in (0, 1)$ and $u : S \times C \rightarrow \mathbb{R}$ and $V : S \times C \times Z \rightarrow \mathbb{R}$ be continuous functions.

Lemma 6 (A Fixed-Point Theorem): *If B is a closed subset of a Banach space with norm $\|\cdot\|$ and $T : B \rightarrow B$ is a contraction mapping (i.e., for some integer m and scalar $\alpha \in (0, 1)$, $\|T^m(W) - T^m(W')\| \leq \alpha \|W - W'\|$ for all $W, W' \in B$), then there is a unique $W^* \in B$ such that $T(W^*) = W^*$.*

Proof: See [Bertsekas and Shreve (1978), p. 55] who note that the theorem in Ortega and Rheinbold (1970) can be generalized to Banach spaces. \square

Let \mathcal{W} be the Banach space of all continuous, real-valued functions on $S \times Z$ (endowed with the sup norm). The operator $T : \mathcal{W} \rightarrow \mathcal{W}$, where

$$TW(s, z) = \max_{\mu \in \mathcal{Z}} \int [u(s, b) + V(s, b, z) + \delta W(sb, x)] d\mu(b, x) - \max_{\nu \in \mathcal{Z}} \int V(s, b, z) d\nu(b, z)$$

is well-defined and is a contraction mapping. Hence, by Lemma 6, there exists a unique $W \in \mathcal{W}$ such that $T(W) = W$.

For any W, u, v, δ such that

$$W(s, z) = \max_{\mu \in \mathcal{Z}} \left\{ \int [u(s, b) + V(s, b, z) + \delta W(sb, x)] d\mu(b, x) \right\} - \max_{\nu \in \mathcal{Z}} \int V(s, b, z) d\nu(b, z)$$

define \succeq_s by $x \succeq_s y$ iff $W(s, x) \geq W(s, y)$. Verifying that \succeq_s satisfies Axioms 1 – 7 is straightforward. \square

8.3 Proof of Theorem 2

By Theorem 1, \succeq can be represented by a continuous \hat{W} where

$$\hat{W}(s, z) = \max_{\mu \in z} \left\{ \int [\hat{u}(s, \mu^1) + \delta \hat{W}(sb, x)] d\mu(b, x) + \hat{V}(s, \mu) \right\} - \max_{\nu \in z} \hat{V}(s, \nu) \quad (3)$$

for some continuous u, v and $\delta \in (0, 1)$. Moreover, $\hat{W}, \hat{u}, \hat{V}$ are linear in their second argument. Let $\hat{U}(s, \mu) = \int (\hat{u}(s, b) + \delta \hat{W}(sb, x)) d\mu(b, x)$.

Lemma 6: $\hat{V}(s, \mu) = \hat{V}(s, \nu)$ if $\mu^1 = \nu^1$.

Proof: If $\hat{V}(s, \cdot) = \alpha \hat{U}(s, \cdot) + \beta$ for some $\alpha \leq -1$, then $x \succeq_s y$ for all $x \subset y$ contradicting regularity. If $\hat{V}(s, \cdot) = \alpha \hat{U}(s, \cdot) + \beta$ for some $\alpha \geq 0$ then $x \succeq_s y$ for all $y \subset x \in Z$ and \succeq is not regular. Hence, for each $s \in S$ there are two possibilities: either $\hat{V}(s, \cdot)$ is not an affine transformation of $\hat{U}(s, \cdot)$ or there exists $\alpha \in (-1, 0)$ such that $\hat{V}(s, \cdot) = \alpha \hat{U}(s, \cdot) + \beta$. In either case, regularity implies that there exist $\mu^s, \nu^s \in \Delta$ such that $\hat{U}(s, \mu^s) + \hat{V}(s, \mu^s) > \hat{U}(s, \nu^s) + \hat{V}(s, \mu^s)$ and $\hat{V}(s, \mu^s) < \hat{V}(s, \nu^s)$.

Take any $\nu, \hat{\nu} \in \Delta$ such that $\nu^1 = \hat{\nu}^1$. By continuity, there exists $\alpha > 0$ small enough so that

$$\hat{U}(s, \mu^s) + \hat{V}(s, \mu^s) > \hat{U}(s, \alpha\nu + (1 - \alpha)\nu^s) + \hat{V}(s, \alpha\nu + (1 - \alpha)\nu^s)$$

$$\hat{U}(s, \mu^s) + \hat{V}(s, \mu^s) > \hat{U}(s, \alpha\hat{\nu} + (1 - \alpha)\nu^s) + \hat{V}(s, \alpha\hat{\nu} + (1 - \alpha)\nu^s)$$

$$\hat{V}(s, \mu^s) < \hat{V}(s, \alpha\nu + (1 - \alpha)\nu^s)$$

$$\hat{V}(s, \mu^s) < \hat{V}(s, \alpha\hat{\nu} + (1 - \alpha)\nu^s)$$

Then, Assumption I implies $\{\alpha\nu + (1 - \alpha)\nu^s, \mu^s\} \sim_s \{\alpha\hat{\nu} + (1 - \alpha)\nu^s, \mu^s\}$. Since \hat{W} represents \succeq we have $\hat{V}(s, \alpha\nu + (1 - \alpha)\nu^s) = \hat{V}(s, \alpha\hat{\nu} + (1 - \alpha)\nu^s)$ and since \hat{V} is linear, we conclude $\hat{V}(s, \nu) = \hat{V}(s, \hat{\nu})$ as desired. \square

By Lemma 6, there is a function $\hat{v} : S \times \Delta(C) \rightarrow \mathbb{R}$ such that $\hat{V}(s, \mu) = \hat{v}(s, \mu^1)$. Regularity implies that neither $\hat{U}(s, \cdot)$ nor $\hat{v}(s, \cdot)$ is constant. Moreover, (since $\delta > 0$) this

implies that $\hat{v}(s, \cdot)$ is not an affine transformation of $\hat{U}(s, \cdot)$. Hence, we may apply Theorem 7 of Gul and Pesendorfer (2001) to yield the following implications:

Fact: (Theorem 7 (Gul and Pesendorfer (2001))) $\hat{s}Ps$ iff for some $\alpha_u, \alpha_v \in [0, 1], \gamma > 0, \gamma_u, \gamma_v \in \mathbb{R}$

$$\begin{aligned}\gamma \hat{U}(s, \mu) &= \alpha_u \hat{U}(\hat{s}, \mu) + (1 - \alpha_u) \hat{v}(\hat{s}, \mu^1) + \gamma_u \\ \gamma \hat{v}(s, \mu^1) &= \alpha_v \hat{U}(\hat{s}, \mu) + (1 - \alpha_v) \hat{v}(\hat{s}, \mu^1) + \gamma_v\end{aligned}$$

for all μ .

Pick any $s_0 \in S$. It follows from Assumption P and the Fact above that for all $s \in S$

$$\begin{aligned}\hat{U}(s, \mu) &= \alpha(s) \hat{U}(s_0, \mu) + \hat{\gamma}_u(s) \\ \hat{v}(s, \mu^1) &= \beta(s) \hat{v}(s_0, \mu^1) + \hat{\gamma}_v(s)\end{aligned}\tag{4}$$

for some functions $\alpha, \beta, \hat{\gamma}_u, \hat{\gamma}_v$ such that $\alpha(s) > 0, \beta(s) > 0$ for all s . Note that \hat{U} and \hat{v} are continuous and hence $\alpha, \beta, \gamma_u, \gamma_v$ are continuous.

Combining (3) and (4) yields,

$$\begin{aligned}\int [\hat{u}(s, b) + \delta \hat{W}(sb, z)] d\nu(b, z) &= \\ \int [\alpha(s) \hat{u}(s_0, b) + \gamma_u(s) + \alpha(s) \delta \hat{W}(s_0 b, z)] d\nu(b, z)\end{aligned}\tag{5}$$

The only terms on either side of (5) that depend on ν^2 are $\delta \hat{W}(sb, z)$ and $\alpha(s) \delta \hat{W}(s_0 b, z)$. Since regularity implies that neither of these terms is constant it follows that

$$\hat{W}(sb, \cdot) = \alpha(s) \hat{W}(s_0 b, \cdot) + A(s, b)$$

Lemma 2 (in the proof of Theorem 1) then implies that $\alpha(s) = 1$ for all s . It follows that $\hat{W}(s_0 b, \cdot)$ represents \succeq_{sb} . Hence, $K = 1$. That is, $sb = b$ for all s, b . Henceforth, we write b instead of sb .

Let $W(b, z) = \hat{W}(b, z) - \gamma_u(b)$, $u(b) = \hat{u}(s_0, b) + \delta \gamma_u(b)$ for all b . Let $v(\cdot) = \hat{v}(s_0, \cdot)$.

Then,

$$\begin{aligned}
W(b, z) &= \hat{W}(b, z) - \gamma_u(b) = \max_{\mu \in z} \{\hat{U}(b, \mu) + \hat{v}(b, \mu)\} - \max_{\nu \in z} \hat{v}(b, \nu) - \gamma_u(b) \\
&= \max_{\mu \in z} \{\hat{U}(s_0, \mu) + \beta_v(b) \hat{v}(s_0, \mu)\} - \max_{\nu \in z} \beta_v(b) \hat{v}(s_0, \nu) \\
&= \max_{\mu \in z} \int [\hat{u}(s_0, b') + \beta_v(b) \hat{v}(s_0, b') + \delta \hat{W}(b', x)] d\mu(b', x) \\
&\quad - \max_{\nu \in z} \beta_v(b) \hat{v}(s_0, \nu) \\
&= \max_{\mu \in z} \int [u(b') + \beta_v(b) v(b') + \delta W(b', x)] d\mu(b', x) \\
&\quad - \max_{\nu \in z} \beta_v(b) \hat{v}(\nu^1)
\end{aligned}$$

Define $\sigma(b) := \beta_v(b)$. By Assumption N, $v(c, d) = v(\hat{c}, \hat{d})$ if $d = \hat{d}$. Assumption N also implies that $v(c, d)$ is strictly increasing in d . Finally, Assumption N implies that $\sigma(c, d) = \sigma(\hat{c}, \hat{d})$ if $d = \hat{d}$. Hence, u, v, σ, δ satisfy all the desired properties.

Establishing that the preference represented by W with (u, v, σ, δ) satisfying the conditions (i – iii) of the theorem satisfies Axioms 1 – 7, I, N and P is straightforward. Hence, to conclude the proof of the converse, we need to show only that the \succeq represented is regular. Since u is nonconstant, there exists (\bar{c}, \bar{d}) and (c, d) such that $u(c, d) > u(\bar{c}, \bar{d})$. Pick any $x \in Z$ and let $\bar{z} = \{(\bar{c}, \bar{d}, x)\}$ and $z = \{(c, d, x)\}$. Then, it follows from the representation of Theorem 2 that $W(s, z \cup \bar{z}) = W(s, \bar{z}) > W(s, z)$. Next, let $\bar{y} = (\hat{c}, \hat{d}, \bar{z})$ and $y = (\hat{c}, \hat{d} + \epsilon, z)$ for \bar{z}, z as defined above and some \hat{c}, \hat{d} and some $\epsilon > 0$. It follows from the continuity and increasingness of v that for ϵ sufficiently small, $W(s, \bar{y}) > W(s, y \cup \bar{y})$ proving that \succeq is regular. \square

References

1. Bertsekas D. P. and S. E. Shreve, *Stochastic Optimal Control: The Discrete Time Case*, 1978, Academic Press, New York.
2. Becker, G. S., and K. M. Murphy, “A Theory of Rational Addiction,” *Journal of Political Economy*, 1988, 675–700.
3. Becker, G. S., Grossman M. and K. M. Murphy, “An Empirical Analysis of Cigarette Addiction,” *American Economic Review*, 1994, 84, 396–418.
4. Bernheim, B. D., and A. Rangel, “Addiction and Cue-Conditioned Cognitive Processes,” forthcoming *American Economic Review*, 2005.
5. Davies, J. B. *The Myth of Addiction; the Application of the Psychological Theory of Attribution to Illicit Drug Use*, 1992, Harwood Academic Publishers, Chur.
6. Gruber, J. and B. Koszegi, “Is Addiction Rational? Theory and Evidence,” *Quarterly Journal of Economics*, 2001, 116, 1261–1303.
7. Gruber, J. and S. Mullainathan, “Do Cigarette Taxes Make Smokers Happier”, mimeo March 2001.
8. Gul, F. and W. Pesendorfer, “Temptation and Self-Control,” *Econometrica*, 2001, 69, 1403–1436..
9. Gul, F. and W. Pesendorfer, “Self-Control and the Theory of Consumption,” *Econometrica*, 2004, 72, 119–158.
10. Gul, F. and W. Pesendorfer, “The Revealed Preference Theory of Changing Tastes,” forthcoming *Review of Economic Studies*, 2004.
11. Kreps, D., *Notes on the Theory of Choice*. Westview Press, Boulder and London 1988.
12. Kreps, D. and E. Porteus (1978): “Temporal Resolution of Uncertainty and Dynamic Choice Theory,” *Econometrica*, 46, 185–200.
13. Laibson, D., “Golden Eggs and Hyperbolic Discounting,” *Quarterly Journal of Economics*, 1997, 112, 443–477.
14. Laibson, D., “A Cue Theory of Consumption,” *Quarterly Journal of Economics*, 2001, 66, 81–120.
15. Loewenstein G., “Out of Control: Visceral Influences on Behavior,” *Organizational Behavior and Human Decision Processes*, 1996, 65, 272–292.
16. O’Donoghue, T. and M. Rabin, “Addiction and Self-Control,” in *Addiction: Entries and Exits*, 1999, J. Elster, ed., Russel Sage Foundation.

17. Ortega J.M., and W.L. Rheinbold, *Iterative Solutions of Nonlinear Equations in Several Variables*, 1970, Academic Press, New York.
18. Phelps, E. S. and R. A. Pollak “On Second-Best National Saving and Game Equilibrium Growth,” *Review of Economic Studies*, 1968, 35, 185–199.
19. Rabin M., “Psychology and Economics,” *Journal of Economic Literature* 36, 1998, 11–46.
20. Robinson T. E. and K. C. Berridge, “The Neural Basis of Drug Craving: an Incentive-Sensitization Theory of Addiction”, *Brain Research Reviews*, 1993, 18, 247–291.
21. Strotz, R. H., “Myopia and Inconsistency in Dynamic Utility Maximization,” *Review of Economic Studies*, 1955, 23, 165–180.