Inequality and Unemployment in a Global Economy^{*}

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Abstract

This paper develops a new framework for examining the distributional consequences of trade liberalization that is consistent with increasing inequality in every country, growth in residual wage inequality, rising unemployment, and reallocation within and between industries. While the opening of trade yields welfare gains, unemployment and inequality within sectors are higher in the trade equilibrium than in the closed economy. In the open economy changes in trade openness have nonmonotonic effects on unemployment and inequality within sectors. As aggregate unemployment and inequality have within- and between-sector components, changes in sector composition following the opening of trade complicate its impact on aggregate unemployment and inequality. However, when countries are nearly symmetric, the sectoral composition effects reinforce the within-sector effects, and both aggregate inequality and aggregate unemployment rise with trade liberalization.

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1 Introduction

The existence of welfare gains from trade is one of the most central propositions of neoclassical economics. Equally central is the idea that while countries as a whole can gain from trade, particular individuals and groups within those countries can lose. Traditionally the key intellectual framework for examining the distributional consequences of trade has been the Stolper-Samuelson Theorem of the Heckscher-Ohlin model, according to which the interaction of country factor abundance and industry factor intensity determines the impact of trade on the distribution of income. Several limitations have, however, recently emerged concerning the use of this model as a framework for understanding the distributional consequences of trade liberalization.¹

While the Stolper-Samuelson Theorem predicts that trade liberalization leads to a rise in income inequality in skill-abundant developed countries and a decline in income inequality in laborabundant developing countries, recent empirical evidence suggests rising income inequality in both sets of countries following trade liberalization.² Additionally, whereas neoclassical trade theory emphasizes the return to skills as the prime driver of income inequality, a substantial component of the recent rise in income inequality is accounted for by residual wage inequality that is unexplained by observed personal characteristics.³ Similarly, although unemployment is in practice an important channel through which individuals can experience income loss from trade liberalization, the frictionless factor markets in neoclassical trade theory rule out equilibrium unemployment by assumption. Finally, while the mechanism through which trade affects income inequality in the Heckscher-Ohlin model is a reallocation of resources across industries that changes relative factor prices, recent empirical evidence from trade liberalization episodes suggests that much of the observed reallocation instead occurs across firms within industries.⁴

In this paper we develop an alternative intellectual framework for examining the distributional consequences of trade liberalization. Motivated by empirical evidence from micro datasets on firms and workers, our model incorporates three key features of product and labor markets that together enable us to make progress in addressing each of the limitations discussed above. First, heterogeneity in productivity across firms generates differences in firm revenue and profits, and as a result trade liberalization induces reallocations of resources across firms within industries as well as across industries. Second, heterogeneity in unobserved ability across workers, imperfect screening of worker ability by firms, and wage bargaining give rise to rent sharing within firms and wage variation across firms. Third, as a result of search and matching frictions in the labor market, equi-

¹The main competing framework within neoclassical trade theory is the specific factors model. Several of the limitations discussed below also apply to that model, including in particular the absence of unemployment and the emphasis on across-sector reallocation of resources.

²See for example the survey by Goldberg and Pavcnik (2007). For a neoclassical model in which trade can increase the return to skills in both developed and developing countries, see Feenstra and Hanson (1996) and Zhu and Trefler (2005).

³ For developed-country evidence, see Autor et al. (2008), Juhn et al. (1993), and Lemieux (2006). For evidence of a rise in residual inequality following trade liberalization, see Attanasio et al. (2004) and Menezes-Filho et al. (2008).

⁴See for example Attanasio et al. (2004) and Levinsohn (1999).

librium unemployment occurs and workers with the same observed characteristics receive different wages. The combination of these three features therefore generates residual wage inequality and unemployment that can increase in all countries following trade liberalization. Moreover, each of the features interact with one another to shape equilibrium labor and product market outcomes.

Our model builds upon the closed-economy framework of Helpman et al. (2008). We consider a world of two countries which can be asymmetric along a number of dimensions, although we emphasize asymmetry in labor market frictions. Labor is the sole factor of production and there are two sectors: a homogeneous-good sector that is chosen for the numeraire and a differentiated-good sector consisting of many horizontally-differentiated varieties. The homogeneous good is produced with a unit labor requirement under conditions of perfect competition, and therefore workers in this sector receive for certain a wage of one. In contrast, varieties of the differentiated product are produced under conditions of monopolistic competition and in the presence of labor market frictions, which give rise to equilibrium unemployment. As labor is mobile across sectors, the expected return to entering the differentiated sector, which equals the average wage times the probability of employment, has to equal the certain wage of one in the homogeneous-good sector if both goods are produced.

Within the differentiated sector, firms are heterogeneous in terms of their productivity and workers are heterogeneous in terms of an unobserved ability. Worker ability can be either match-specific or worker-specific, and in either case it is drawn from a known distribution but not costlessly observed when a worker is matched with a firm. Each firm in a differentiated sector incurs a search cost to match with workers and a screening cost to obtain information about their ability. By incurring the screening cost, the firm can determine those workers who have an ability below an endogenously-chosen threshold. Firm output depends on firm productivity, the measure of workers hired, and the average ability of the workers hired. Each firm therefore chooses a screening ability threshold trading off the increase in output from raising average worker ability against the reduction in output from hiring fewer workers and the costs incurred by screening. In equilibrium, more productive firms have larger revenues, sample more workers, screen to a higher ability threshold and employ workers with a higher average ability. The firm and its workers engage in strategic bargaining over the division of the surplus from production. As more productive firms employ workforces with higher average ability, which are more costly to replace, they pay higher equilibrium wages.

We consider a trade equilibrium with fixed and variable costs of trade in which only some firms export. As in Melitz (2003), the least productive firms cannot cover the fixed cost of production and exit, more productive firms serve the domestic market only, and the most productive firms pay the fixed exporting cost and serve both the domestic and foreign markets. The combination of a fixed cost of exporting and rent-sharing within firms results in a discrete jump in both firm revenue and wages at the productivity threshold for entry into the export market. The model therefore matches empirical findings that exporters pay higher wages than nonexporters within the same industry, even after controlling for firm size (see for example Bernard and Jensen 1995, 1997),

and empirical findings that a substantial part of these higher wages is explained by differences in workforce composition (see for example Kaplan and Verhoogen 2006, Schank, Schnabel and Wagner 2007, and Munch and Skaksen 2008).

One of our central results is that while the opening of trade is welfare improving, the distribution of wages in the differentiated sector is more unequal in the trade equilibrium than in autarky. This result holds for both countries. The intuition is as follows. As a high-productivity firm enters the export market, the resulting increase in its revenues leads the firm to screen workers more, so as to increase the average quality of its workforce. As a result, it becomes more costly for the firm to replace the workers, and wage bargaining leads the firm to share the larger revenue with its workers in the form of higher wages. Additionally, the positive probability of drawing a productivity high enough to export increases the expected value of entry, which leads more firms to enter the differentiated sector. This increased entry enhances product market competition and reduces revenue and wages at low productivity firms that serve only the domestic market. The increased wages of exporters and reduced wages of non-exporters following the opening of trade raise wage inequality within the differentiated sector relative to the autarky equilibrium. Once an economy is open to international trade, however, the relationship between sectoral wage inequality and trade openness is nonmonotonic. On the one hand, when nearly all firms serve only the domestic market, an increase in trade openness enhances wage inequality by expanding the small number of exporting firms that pay high wages. On the other hand, when nearly all firms export, an increase in trade openness depresses wage inequality by further reducing the small number of firms that serve only the domestic market and pay low wages. Therefore, once the economy is open to international trade, a given change in trade openness can have quite different effects on sectoral wage inequality depending on the initial fraction of firms that export.

Another key result is that unemployment in the differentiated sector is higher in the trade equilibrium than in autarky. Unemployment arises in this sector as a result of the search and screening frictions. Workers can be unemployed either because they are not matched with firms, or because once matched they are not hired as a result of their ability falling below the firm's screening ability cutoff. The opening of trade leads to change in industry composition, as low-productivity firms that serve only the domestic market exit and contract, while high-productivity firms that export expand. Since more productive firms screen to a higher ability cutoff, they hire a smaller fraction of sampled workers than less productive firms. Therefore, this change in composition towards more productive firms within the differentiated sector increases unemployment. Once the economy is open to international trade, however, trade openness can have a nonmonotonic relationship with unemployment as with wage inequality. The reason is that as the fraction of exporting firms increases, new entrants to the export market become less and less productive relative to existing exporters, until eventually all firms export. As less productive firms have less selective recruitment policies, this change in composition towards less productive firms within the group of exporters can potentially reduce unemployment.

Income inequality in our model depends on both wage inequality and unemployment. We use

the Theil index as our preferred measure of inequality, because it permits an exact decomposition of overall inequality into the contributions of within- and between-group inequality measures. We are therefore able to undertake this decomposition for the differentiated sector using the two groups of employed and unemployed workers. As the opening of the closed economy to trade raises both wage inequality and unemployment in the differentiated sector, it also increases income inequality in the differentiated sector. Similarly, once the economy is open to trade, the fact that changes in trade openness have ambiguous effects on wage inequality and unemployment implies that they also have ambiguous effects on income inequality. Therefore, once the economy is open to trade, a given change in trade openness can either increase or decrease income inequality within the differentiated sector.

Having examined how trade openness affects inequality and unemployment in the differentiated sector, we next turn to the impact of labor market frictions. Despite the model's richness, we show that its comparative statics can be characterized in the neighborhood of an equilibrium with small asymmetries between the two countries in which only a small fraction of firms export. In the environs of such an equilibrium, an increase in a country's labor market frictions leads to a contraction in the differentiated sector at home relative to the foreign country, which reduces the degree of product market competition in the home market relative to the foreign market. This change in relative product market competition makes serving the foreign market less attractive relative to serving the home market and reduces the fraction of home firms that export. In turn, the reduction in export participation of domestic firms decreases wage inequality within the differentiated sector at home. In contrast, if there are large asymmetries in labor market frictions between the two countries and a nonnegligible fraction of firms exports, there can be a nonmonotonic relationship between labor market frictions and sectoral wage inequality, which reflects the nonmonotonic relationship between trade openness and sectoral wage inequality discussed above.

Our general equilibrium focus also enables us to highlight the distinction between sectoral unemployment and inequality, as discussed above, and the aggregate values of these variables for the economy as a whole. The key difference between these two levels of analysis is that changes in sectoral composition need to be taken into account at the aggregate level.⁵ We again use the Theil index to decompose aggregate inequality into its within- and between-group components, where the groups are now the homogeneous and differentiated sectors. When countries are symmetric, the increase in average productivity in the differentiated sector, induced by the opening of trade between formerly closed economies, expands the share of the labor force employed in this sector in both countries. Therefore aggregate unemployment and income inequality rise in both countries, because of greater unemployment and income inequality in the differentiated sector, and also because of a larger share of the labor force employed in this sector, which has higher unemployment and income inequality than the homogeneous sector. When countries are asymmetric, the country with a comparative advantage in the differentiated sector experiences an increase in the share of its labor

⁵Recent empirical evidence that such compositional effects are important includes Lemieux (2006) for residual inequality and Blum (2008) for overall inequality.

force employed in this sector following the opening of trade, and hence exhibits a rise in aggregate unemployment and income inequality. In contrast, the country with a comparative disadvantage in the differentiated sector experiences a decrease in the share of its labor force employed in this sector following the opening of trade, yielding an ambiguous impact on aggregate unemployment and income inequality.

Our paper is related to recent research on firm heterogeneity in international trade, which builds on the influential framework developed by Melitz (2003), such as Antràs and Helpman (2004), Bernard et al. (2007), and Helpman et al. (2004).⁶ In this literature, the modelling of the labor market has traditionally been highly stylized; workers are identical and reallocation across firms is costless. As a result, these authors predict that firms pay workers with the same observed characteristics the same wage, irrespective of the firm's productivity, which sits awkwardly with a large empirical literature that finds a positive employer–size wage premium and rent–sharing within firms.⁷ In contrast, in our framework rent sharing leads to differences in wages across firms for workers with the same observed characteristics, which is consistent with the observed employer-size wage premium. Moreover, consistent with recent evidence from matched employee-employer data sets, the employer–size wage premium is driven by the endogenous sorting of workers across firms according to unobserved worker characteristics.⁸

Our research is also related to the literature on international trade and labor market frictions. One strand of this literature considers the implications for trade of theories of efficiency or fair wages, including Amiti and Davis (2008), Davis and Harrigan (2007), Egger and Kreickemeier (2007, 2008) and Grossman and Helpman (2008). Another strand of research, more closely related to our work, examines the consequences for trade of Diamond-Mortensen-Pissarides search and matching frictions, including Davidson et al. (1988, 1999), Felbermayr et al. (2008) and Helpman and Itskhoki (2008). Our main point of departure from most existing research on international trade and labor market frictions is the introduction of worker heterogeneity and imperfect screening of workers by firms, which generates residual wage inequality that is influenced by both trade liberalization and labor market frictions.

Our paper is also related to the large labor and macroeconomics literature concerned with search frictions in the labor market.⁹ A number of approaches have been taken in the search literature to explaining wage differences across workers. One influential line of research has followed Burdett and Mortensen (1998) and Mortensen (2003) in analyzing wage dispersion in models of wage posting and random search. Another important line of research has examined wage dispersion when both firms and workers are heterogeneous, including models of pure random search such as Shimer and

⁶For an alternative approach to modelling firm heterogeneity and trade, see Bernard et al. (2003).

⁷While wages vary across heterogeneous firms in Yeaple (2005), this variation arises because firms employ workers with heterogeneous observed characteristics.

⁸For example, using French matched employee-employer data, Abowd et al. (1999) find that around 90 percent of the employer–size wage premium is accounted for by the sorting of workers across firms according to unobserved worker characteristics. See the Abowd and Kramatz (1999) survey for a discussion of similar findings from other countries.

⁹See in particular Mortensen (1970), Pissarides (1974), Diamond (1982a,b), Mortensen and Pissarides (1994) and Pissarides (2000), as reviewed in Rogerson *et al.* (2005).

Smith (2000), Albrecht and Vroman (2002), Davidson et al. (2008), and models incorporating onthe-job-search such as Postel-Vinay and Robin (2002), Cahuc et al. (2006) and Lentz (2008). ¹⁰ In both lines of research, worker ability is assumed to be costlessly observable by firms when matching occurs. In contrast, our framework emphasizes the idea that a substantial component of worker ability cannot be directly observed, so that firms undertake costly investments in order to gain only imperfect information about worker ability. ¹¹ Given a common screening technology for all firms, more productive firms have an incentive to screen more intensively, because they have a greater return to hiring higher ability workers. In equilibrium, more productive firms have workforces of higher average ability, which increases the cost of replacing those workers in the bargaining game, and leads more productive firms to pay higher wages.

The remainder of the paper is structured as follows. Section 2 outlines the model and solves for general equilibrium. Section 3 presents our core results on the impact of international trade on welfare, wage inequality, unemployment, and income inequality. Section 4 examines the impact of trade impediments and labor market frictions on inequality and unemployment. Section 5 concludes, while the Appendix contains detailed derivations of the main results.

2 The Model

This section lays out the model and characterizes its equilibrium. We consider a world economy consisting of a home country and a foreign country. As our analysis focuses on asymmetries in labor market frictions, we assume that the home and foreign countries are identical in other respects. Nonetheless, our framework can also be used to consider other asymmetries, such as in the distributions of firm productivity and worker ability or in country size, as discussed further below.

Although we allow for firm and worker heterogeneity, search and matching, and wage bargaining, the general equilibrium of the two-country world remains tractable as a result of a number of simplifying assumptions that we make about preferences, production technology, and distribution functions. In particular, we adopt the specifications from the closed economy model of Helpman et al. (2008), where the reader can find a more detailed discussion of the properties of demand, the production technology, and labor market frictions.

Throughout the following we denote home variables without an asterisk and foreign variables with an asterisk. To simplify notation, we develop equilibrium relationships for the home country, with analogous expressions holding for foreign.

¹⁰A somewhat different line of research in Ohnsorge and Trefler (2007) has examined two-dimensional worker heterogeneity within the context of the Roy model.

¹¹See Jovanovic (1979), (1984) and Moscarini (2005) for models in which a worker's productivity in a job is revealed gradually over time with job tenure.

2.1 Preferences and Demand

Utility depends on consumption of a homogeneous and a differentiated product.¹² We assume that preferences between the two products are quasi-linear, while preferences across varieties of the differentiated product take a constant elasticity of substitution (CES) form. In particular, the utility function is

$$\mathbb{U} = q_0 + \frac{1}{\zeta} Q^{\zeta}, \qquad Q = \left[\int_{\omega \in \Omega} q(\omega)^{\beta} d\omega \right]^{\frac{1}{\beta}}, \qquad \zeta < \beta < 1, \tag{1}$$

where q_0 is consumption of the homogeneous good, Q denotes the real consumption index of varieties of the differentiated product, $q(\omega)$ represents consumption of variety ω , Ω denotes the set of varieties available for consumption, ζ controls the elasticity of substitution between the homogeneous and differentiated products, and β controls the elasticity of substitution between varieties of the differentiated product. The parameter restriction $\beta > \zeta$ ensures that brands of the differentiated product are better substitutes for each other than for the homogeneous good. We assume that the homogeneous good is costlessly traded between countries and choose it as our numeraire, so that $p_0 = p_0^* = 1$.

Given these preferences, the equilibrium revenue received by a firm in the differentiated sector can be written as follows:

$$r(\omega) = Q^{-(\beta - \zeta)} q(\omega)^{\beta}. \tag{2}$$

Tighter product market competition—reflected in a low sectoral price index P—leads to higher aggregate sectoral demand, Q, but to lower demand for each individual variety in the sector, and lower revenue, $r(\omega)$. These preferences also imply that the indirect utility function can be expressed in terms of aggregate income and consumer surplus from the differentiated good:

$$V = E + \frac{1-\zeta}{\zeta} Q^{\zeta} = E + \frac{1-\zeta}{\zeta} P^{-\frac{\zeta}{1-\zeta}}, \tag{3}$$

where E is income (expenditure), and consumer surplus from the differentiated product can be expressed either in terms of its real consumption index, Q, or its dual price index, P, as $(1 - \zeta) Q^{\zeta}/\zeta = (1 - \zeta) P^{-\frac{\zeta}{1-\zeta}}/\zeta$.¹⁴

While individual workers face idiosyncratic risk, we assume for analytical convenience that each country is populated by a continuum of identical families of measure one, each of which has the preferences of the representative consumer. As each family includes a measure of \bar{L} workers that maximize the family's utility, idiosyncratic risk is perfectly diversified within families.¹⁵

¹²While for clarity we focus on the case of a single differentiated sector, the introduction of multiple differentiated sectors is straightforward.

¹³See Helpman and Itskhoki (2008) for a discussion of trade costs in the homogeneous sector.

¹⁴The dual price index, P, is given by $P = \left[\int_{\omega \in \Omega} p(\omega)^{-\beta/(1-\beta)} d\omega \right]^{-(1-\beta)/\beta}$, where $p(\omega)$ is the price of variety ω .

¹⁵ Alternatively, if we assumed homothetic preferences over the consumption of the homogeneous and differentiated goods and constant relative risk aversion, the family interpretation would be useful, but not required. In this case the idiosyncratic risk across agents does not have to be fully diversified in equilibrium. See Helpman and Itskhoki

2.2 Technologies and Market Structure

The homogeneous good is produced using a constant returns to scale technology, with one unit of labor required for each unit of output, and there are no labor market frictions in this sector. We choose the factor endowments $\bar{L} = \bar{L}^*$ to be large enough so that in equilibrium both countries consume and produce the homogeneous good. Therefore, with the homogeneous good chosen as the numeraire, the wage in the homogeneous sector is equal to one in each country: $w_0 = w_0^* = 1$.

In the differentiated sector, both firm and worker productivity levels are heterogeneous. A worker's productivity in a differentiated-sector firm is assumed to depend on her ability a, which can be specific to the match between the firm and worker, or specific to the worker. In either case, this ability is unknown when the worker decides whether to seek employment in the homogeneous or differentiated sectors. Worker ability is assumed to be drawn from a Pareto distribution, with cumulative distribution function $G_a(a) = 1 - (a_{\min}/a)^k$ for $a \ge a_{\min} > 0$ and k > 2. This distribution is not only tractable, but together with our other assumptions yields a Pareto income distribution, which provides a reasonable approximation to observed income distributions (see for example Pen, 1971).

There is a competitive fringe of potential firms who can choose to enter the differentiated sector by paying an entry cost of f_e units of the homogeneous good. Once a firm incurs the sunk entry cost, it observes its productivity, θ , also drawn from a Pareto distribution, with the cumulative distribution function $G_{\theta}(\theta) = 1 - (\theta_{\min}/\theta)^z$ for $\theta \ge \theta_{\min} > 0$ and z > 2. Together with our other assumptions, this specification yields a Pareto firm-size distribution, which also provides a reasonable approximation to observed firm-size distributions (see for example Axtell, 2001). As in equilibrium all firms with the same productivity in the differentiated sector behave similarly, we index firms by θ from now onwards.

Firms in the differentiated sector produce horizontally differentiated varieties under conditions of monopolistic competition. Production of each variety involves a fixed production cost of f_d units of the homogeneous good. The amount of output of the variety produced, y, depends upon the productivity of the firm, θ , the average ability of its workers, \bar{a} , and the measure of workers hired, h. We assume that there are diminishing marginal returns to the measure of workers hired and that the production technology takes the following form:

$$y = \theta h^{\gamma} \bar{a}, \qquad 0 < \gamma < 1.$$

This production function can be interpreted as capturing either human capital externalities (e.g., production in teams in which the productivity of a worker depends on the average productivity of her team) or a managerial time constraint (e.g., a manager with a fixed amount of time who needs to allocate some time to each worker). Helpman et al. (2008) provide further discussion of these interpretations.

In addition to the fixed cost of production, a differentiated-sector firm incurs a fixed exporting (2008) for more details.

cost of f_x units of the homogeneous good in order to serve the foreign market. An exporting firm in the differentiated sector also incurs variable trade costs, which take the iceberg form, such that $\tau > 1$ units of a variety must be shipped in order for one unit to arrive in the other country.

Firms in the differentiated sector face labor market frictions. A firm that pays a search cost of bn units of the homogeneous good can randomly sample a measure of n workers, where the search cost b is endogenously determined by labor market frictions as discussed below. The firm can also screen the sampled workers and identify those with an ability below a_c (with $a_c \ge a_{\min}$) by paying a screening cost of ca_c^{δ}/δ units of the homogeneous good, where c > 0 and $\delta > 0$. Screening costs are increasing in the ability cutoff a_c chosen by the firm, because more complex and costlier tests are required for higher ability cutoffs. ¹⁷

As search is random, the ability distribution among workers sampled by a firm is described by the ex ante distribution function $G_a(a)$. With a Pareto distribution of worker ability, the measure of workers hired with abilities greater than a screening ability cutoff a_c is $h = n \left(a_{\min}/a_c \right)^k$, and the average ability of these workers is $\bar{a} = ka_c/(k-1)$. Therefore the production technology can be expressed as

$$y = \frac{k a_{\min}^{\gamma k}}{k - 1} \theta n^{\gamma} a_c^{1 - \gamma k}. \tag{4}$$

We focus on parameter values that satisfy $0 < \gamma < 1/k$, which implies that there are sufficiently strong diminishing returns to the measure of workers hired (low γ) relative to the dispersion of worker ability (high 1/k) such that firm output can be increased by not hiring the lowest-ability workers sampled by the firm. While hiring the lowest ability workers would increase firm output by raising employment, it would decrease firm output by reducing average worker ability. For $0 < \gamma < 1/k$, the second effect dominates, so that the marginal product of workers with an ability below $a_c(\theta)$ is negative. As a result some low ability workers are not hired, which is consistent with the view that firms screen job candidates in order to exclude those believed to be less able.

We assume that preferences, the production technology, and the distribution of productivity and worker ability are the same in both countries. However, the two countries can differ in the extent of labor market frictions: search cost, b, and screening cost, c.¹⁹

¹⁶In this formulation, there is a fixed cost of screening, even when the screening is not informative, i.e., when $a_c = a_{\min}$. We focus on interior equilibria in which firms of all productivities choose screening tests that are informative, $a_c > a_{\min}$, and so the fixed cost of screening is always incurred. As we show below, this is the case when the screening cost, c, is sufficiently small.

¹⁷There are therefore increasing returns to scale in screening. All results generalize immediately to the case where the screening costs are separable in a_c and n and linear in n.

¹⁸In contrast, when $\gamma > 1/k$, no firm screens and the model reduces to a model without screening, as studied in Helpman and Itskhoki (2008). For this reason, we do not discuss this case here.

¹⁹We also could allow for the size of the countries to be different, i.e., \bar{L} could differ from \bar{L}^* , but this difference is not important as long as we focus on equilibria in which each country consumes and produces the homogeneous and differentiated products, which are the equilibria we shall analyze.

2.3 Wages, Employment and Profits

A θ -firm allocates its output $y(\theta)$ between domestic and foreign sales, $y_d(\theta)$ and $y_x(\theta)$ respectively, to maximize its profits. With consumer love of variety and a fixed production cost, no firm will ever serve the export market without also serving the domestic market. Hence firms either serve only the domestic market or both markets. When both markets are served, profit maximization implies that the firm equates marginal revenues in the two markets, which from (2) implies $[y_d(\theta)/y_x(\theta)]^{\beta-1} = \tau^{-\beta} (Q/Q^*)^{-(\beta-\zeta)}$. Therefore a firm's total revenue can be expressed as follows:

$$r(\theta) \equiv r_d(\theta) + r_x(\theta) = \Upsilon(\theta)^{1-\beta} Q^{-(\beta-\zeta)} y(\theta)^{\beta}, \qquad (5)$$

where $r_d(\theta)$ is revenue from domestic sales, $r_x(\theta)$ is revenue from exporting, $y(\theta) = y_d(\theta) + y_x(\theta)$ is total firm output, and

$$\Upsilon(\theta) \equiv 1 + I_x(\theta) \tau^{-\frac{\beta}{1-\beta}} \left(\frac{Q^*}{Q}\right)^{-\frac{\beta-\zeta}{1-\beta}},\tag{6}$$

where $I_x(\theta)$ is an indicator variable that equals one if the firm exports and zero otherwise. We refer to $\Upsilon(\theta)$ as a "market access" variable, which depends on whether the firm chooses to serve both the domestic and foreign markets or only the domestic market.

The presence of labor market frictions in the differentiated sector implies that workers inside the firm are not interchangeable with workers outside the firm. As a result, hired workers have bargaining power. We assume that the firm and its hired h workers engage in strategic bargaining with equal weights in the manner proposed by Stole and Zwiebel (1996a,b). At the bargaining stage, the search and screening costs have been sunk by the firm, and the outside option of hired workers is unemployment, whose value we normalize to zero. Furthermore, the only information revealed by screening about worker ability is that each of the hired workers has an ability above the cutoff a_c , so that neither the firm nor workers know individual abilities.²¹ Therefore, the outcome of this bargaining game is that fraction $1/(1 + \beta \gamma)$ of the revenue (5) is retained by the firm while each worker gets fraction $\beta \gamma / (1 + \beta \gamma)$ of the average revenue per worker.²²

A firm chooses its total output to maximize its profits subject to the revenue function (5) and the production technology (4). This profit-maximization problem can be written as choosing the

²⁰Due to the iceberg trade cost τ , export revenue net of transport cost is given by $r_x(\theta) = (Q^*)^{-(\beta-\zeta)}(q_x(\theta)/\tau)^{\beta}$, where $q_x(\theta)$ is the 'free on board' quantity produced for the export market prior to transport cost being incurred.

²¹While we study a static model, in which workers do not know their ability before they decide which sector to enter and firms do not know the ability of individual workers, the same issues could also be examined in a dynamic specification in which workers and firms can update their priors on unobserved ability over time. As long as there remains imperfect information about unobserved worker ability, as for example in a setting with continuing birth and death of workers and firms, we expect the residual inequality emphasized by our model to also be a feature of the dynamic economy.

²²See the Appendix for further details.

measure of workers to sample, n, the screening ability cutoff, a_c , and the export status, I_x , to solve

$$\pi(\theta) \equiv \max_{\substack{n \geq 0, \\ a_c \geq a_{\min}, \\ I_x \in \{0,1\}}} \left\{ \frac{1}{1 + \beta \gamma} \left[1 + I_x \tau^{-\frac{\beta}{1-\beta}} \left(\frac{Q^*}{Q} \right)^{-\frac{\beta-\zeta}{1-\beta}} \right]^{1-\beta} Q^{-(\beta-\zeta)} \left(\frac{k a_{\min}^{\gamma k}}{k-1} \theta n^{\gamma} a_c^{1-\gamma k} \right)^{\beta} - \left(bn + \frac{c}{\delta} a_c^{\delta} + f_d + I_x f_x \right) \right\}.$$
 (7)

From the first-order conditions to this problem, the equilibrium measure of workers sampled and the screening ability cutoff are both increasing in total firm revenue:

$$\frac{\beta\gamma}{1+\beta\gamma}r(\theta) = bn(\theta),\tag{8}$$

$$\frac{\beta(1-\gamma k)}{1+\beta\gamma}r(\theta) = ca_c(\theta)^{\delta}.$$
 (9)

Therefore firms with larger revenue sample more workers and screen to a higher ability. The measure of workers hired, $h = n \left(a_{\min}/a_c \right)^k$, is increasing in the measure of workers sampled, n, but decreasing in the screening ability cutoff, a_c . Under the assumption $\delta > k$, firms with larger revenue also hire more workers, in line with empirical evidence. Finally, from the division of revenue in the bargaining game, the total wage bill is a constant share of revenue, which implies that wages are monotonically increasing in the screening ability cutoff:

$$w(\theta) = \frac{\beta \gamma}{1 + \beta \gamma} \frac{r(\theta)}{h(\theta)} = b \frac{n(\theta)}{h(\theta)} = b \left[\frac{a_c(\theta)}{a_{\min}} \right]^k.$$
 (10)

Intuitively, as a result of screening to a higher ability cutoff, a firm with larger revenue has a workforce of higher average ability. Since it is more costly to replace a workforce of higher average ability, this improves workers' bargaining position and leads firms with larger revenue to pay higher wages. From the discussion above, the assumption $\delta > k$ implies that firms with larger revenue hire more workers as well as pay higher wages, ensuring that the model is consistent with empirical findings of a positive employer-size wage effect (see for example the survey by Oi and Idson, 1999).

Using the first-order conditions (8) and (9), equilibrium total firm revenue can be written as the following increasing function of firm productivity:

$$r(\theta) \equiv r_d(\theta) + r_x(\theta) = \kappa_r \left[c^{-\frac{\beta(1-\gamma k)}{\delta}} b^{-\beta\gamma} \Upsilon(\theta)^{1-\beta} Q^{-(\beta-\zeta)} \theta^{\beta} \right]^{1/\Gamma}, \tag{11}$$

$$\Gamma \equiv 1 - \beta \gamma - \frac{\beta}{\delta} (1 - \gamma k) > 0, \tag{12}$$

where κ_r is a constant that depends on the parameters β, γ, δ, k and a_{\min} (see Appendix). The derived parameter Γ influences the equilibrium relationship between relative firm revenue and relative firm productivity, and depends upon the dispersion of worker ability (as captured by k),

the screening technology (through δ), the curvature of demand (as parameterized by β), and the extent of diminishing returns to the measure of workers hired (as captured by γ).

The first-order conditions (8) and (9) also imply that the measure of workers sampled, n, the screening ability cutoff, a_c , the measure of workers hired, h, total output, y, and wages, w, can all be written as increasing functions of firm revenue. Therefore, as firm revenue is increasing in productivity, more productive firms sample more workers, screen more intensively, hire more workers, produce more output and pay higher wages. Finally, total firm profits can be expressed in terms of total firm revenue and fixed production and exporting costs:

$$\pi\left(\theta\right) = \frac{\Gamma}{1 + \beta\gamma} r\left(\theta\right) - f_d - I_x\left(\theta\right) f_x. \tag{13}$$

As a firm either serves only the domestic market, or if it exports equates its marginal revenue between the domestic and export markets, revenue in each market can be expressed as the following fractions of total firm revenue (see Appendix):

$$r_d(\theta) = \frac{1}{\Upsilon(\theta)} r(\theta), \qquad r_x(\theta) = \frac{\Upsilon(\theta) - 1}{\Upsilon(\theta)} r(\theta).$$
 (14)

The presence of a fixed production cost implies that there is a zero-profit cutoff for productivity, θ_d , such that a firm drawing a productivity below θ_d exits without producing. Similarly, the presence of a fixed exporting cost implies that there is an exporting cutoff for productivity, θ_x , such that a firm drawing a productivity below θ_x does not find it profitable to serve the export market. As a large empirical literature finds evidence of selection into export markets, where only the most productive firms export, we focus on values of variable trade costs and the fixed production and fixed exporting costs for which $\theta_x > \theta_d > \theta_{\min}$, as discussed further below.²³ Therefore the least productive firms exit, more productive firms serve only the domestic market, and only the most productive firms export.

2.4 Open Economy Equilibrium

We reference the open economy equilibrium by six variables in each country: (i) the zero-profit productivity cutoff, below which firms exit, θ_d ; (ii) the exporting cutoff productivity, above which firms export, θ_x ; (iii) the domestic real consumption index for the differentiated product, Q; (iv) the measure of firms operating in the differentiated sector, M; (v) the measure of workers seeking employment in this sector, L; and (vi) the tightness of the labor market in this sector, $x \equiv N/L$, where N is the measure of workers sampled by differentiated-sector firms, and where the role of x is explained further below. All other endogenous variables, including employment in the homogeneous sector, $L_0 = \bar{L} - L$, and consumption of the homogeneous good, q_0 , can be determined as functions of these six variables. We characterize the equilibrium conditions that determine these variables for the home country. Analogous equilibrium conditions hold in the foreign country.

²³ For empirical evidence of selection into export markets, see for example Bernard and Jensen (1995) and Roberts and Tybout (1997).

2.4.1 Product Markets

In an equilibrium in which only some firms export, firms with a productivity just below the exporting cutoff supply the domestic market alone, with $\Upsilon(\theta_x^-) = 1$. In contrast, firms with a productivity just above the exporting cutoff supply both the domestic and export markets, with $\Upsilon(\theta_x^+) = \Upsilon_x$, where

$$\Upsilon_x = 1 + \tau^{-\frac{\beta}{1-\beta}} \left(\frac{Q^*}{Q}\right)^{-\frac{\beta-\zeta}{1-\beta}} > 1. \tag{15}$$

Therefore the revenue function, $r(\theta)$, is discontinuous at the export cutoff and jumps by the factor of proportionality $\Upsilon_x^{(1-\beta)/\Gamma} > 1$ as productivity rises from θ_x^- to θ_x^+ (see (11)).

The zero-profit cutoff productivity, θ_d , below which firms exit, is defined by the requirement that variable profits in the domestic market equal the fixed production cost; that is, $\pi_d(\theta_d) = 0$. Using (11) and (13), this condition can be expressed as

$$r(\theta_d) = \kappa_r \left[c^{-\frac{\beta(1-\gamma_k)}{\delta}} b^{-\beta\gamma} Q^{-(\beta-\zeta)} \theta_d^{\beta} \right]^{1/\Gamma} = f_d \frac{1+\beta\gamma}{\Gamma}, \tag{16}$$

where we substituted $\Upsilon(\theta_d) = \Upsilon_d = 1$ into the revenue function.

Similarly, the exporting cutoff productivity, θ_x , above which firms export, is determined by the requirement that at this productivity a firm is indifferent between serving only the domestic market and exporting. When a firm serves only the domestic market it makes revenue of $r\left(\theta_x^-\right)$ with $\Upsilon\left(\theta_x^-\right) = \Upsilon_d = 1$, while when a firm serves both the domestic and export markets it makes revenue of $r\left(\theta_x^+\right)$ with $\Upsilon\left(\theta_x^+\right) = \Upsilon_x > 1$. Therefore, using (11) and (13), this indifference condition implies

$$\frac{\Gamma}{1+\beta\gamma}\kappa_r \left[c^{-\frac{\beta(1-\gamma k)}{\delta}} b^{-\beta\gamma} Q^{-(\beta-\zeta)} \theta_x^{\beta} \right]^{1/\Gamma} \left[\Upsilon_x^{(1-\beta)/\Gamma} - 1 \right] = f_x. \tag{17}$$

This equation states that the incremental profit from serving the foreign market for a firm with productivity θ_x , given by the expression on the left-hand side, is equal to the fixed cost of entering the foreign market.

In equilibrium, we also require the free-entry condition to hold, which equates the expected value of entry to the sunk entry cost:

$$\int_{\theta_{d}}^{\theta_{x}} \pi_{d}(\theta) dG_{\theta}(\theta) + \int_{\theta_{x}}^{\infty} \pi_{d+x}(\theta) dG_{\theta}(\theta) = f_{e},$$

where the subscript d+x indicates the sum of profits from domestic sales and exports. Therefore the expected value of entry depends on profits from serving only the domestic market for $\theta \in [\theta_d, \theta_x)$ and profits from serving both markets for $\theta \in [\theta_x, \infty)$. From (11) the relative revenue for any two firms serving only the domestic market depends solely on their relative productivity, $r_d(\theta) = (\theta/\theta_d)^{\beta/\Gamma} r_d(\theta_d)$, while the zero-profit cutoff condition (16) implies $r_d(\theta_d) = f_d(1 + \beta \gamma) / \Gamma$. Similarly, from (11) the relative revenue of any two exporters depends solely on their relative

productivity, $r_{d+x}(\theta) = (\theta/\theta_x)^{\beta/\Gamma} r_{d+x}(\theta_x)$, while the exporting cutoff condition (17) implies $r_{d+x}(\theta_x) = r_d(\theta_x) + f_x(1+\beta\gamma)/\Gamma$. Using these relationships together with equilibrium profits (13), the free entry condition can be re-written as follows:

$$\int_{\theta_d}^{\infty} f_d \left[\left(\frac{\theta}{\theta_d} \right)^{\beta/\Gamma} - 1 \right] dG_{\theta}(\theta) + \int_{\theta_x}^{\infty} f_x \left[\left(\frac{\theta}{\theta_x} \right)^{\beta/\Gamma} - 1 \right] dG_{\theta}(\theta) = f_e.$$
 (18)

Now note that, given Q^* , and after substituting (15) into (17), conditions (16)-(18) characterize the equilibrium values of the home country's domestic and export cutoffs and real consumption of differentiated products, i.e., the vector (θ_d, θ_x, Q) . Similar equations characterize the equilibrium values of the foreign country's variables $(\theta_d^*, \theta_x^*, Q^*)$, given Q. Together, they allow us to solve for $(\theta_d, \theta_x, Q, \theta_d^*, \theta_x^*, Q^*)$. The equilibrium values of these six variables are independent of the other equilibrium conditions that we describe below. For this reason the model is bloc recursive; after solving these six variables one can proceed to solve the rest of the model.^{24,25}

Having solved this first bloc of equilibrium conditions, we can determine the mass of firms in the differentiated sector in each country, M and M^* , using the definition of the real consumption index in (1) and the requirement that consumption in each market equals output supplied to that market. In the home country this means that expenditure on differentiated products, $PQ = Q^{\zeta}$, has to equal the revenue of domestic and foreign firms that sell varieties of the differentiated product in the home market:

$$Q^{\zeta} = M \int_{\theta_d}^{\infty} r_d(\theta) \, dG_{\theta}(\theta) + M^* \int_{\theta_x^*}^{\infty} r_x^*(\theta) \, dG_{\theta}(\theta).$$
 (19)

Using the revenue functions in (14) and the cutoff conditions (16) and (17), we obtain an expression which, together with its counterpart for the foreign country, constitutes the second bloc of the equilibrium system that allows us to solve for the mass of firms in each country, (M, M^*) (see Appendix for more details).

2.4.2 Labor Markets

Following the standard Diamond-Mortensen-Pissarides model of search and unemployment, we assume that the search cost, b, depends on the tightness of the labor market, x:

$$b = \alpha_0 x^{\alpha_1}, \qquad \alpha_0 > 1, \qquad \alpha_1 > 0,$$

²⁴Note that this bloc of equilibrium conditions depends on the values of exogenous parameters, including screening costs (c and c^*) and variable and fixed trade costs (τ and f_x), as well as on the values of the endogenous search costs (b and b^*). We show below, however, that the value of the search cost is pinned down by exogenous labor market parameters and hence can also be taken as exogenous for this bloc of the model.

²⁵The Appendix provides closed-form expressions for these and other endogenous variables for the case of symmetric countries, and derives the parameter restrictions that ensure $\theta_x > \theta_d > \theta_{\min}$. Specifically, we show that $\theta_x > \theta_d$ requires that the fixed cost of exporting, f_x , is large relative to the fixed cost of production, f_d , while $\theta_d > \theta_{\min}$ requires that the fixed cost of production is large relative to the fixed cost of entry, f_e .

where labor market tightness depends on the measure of workers sampled by firms relative to the total measure of workers searching for employment in the differentiated sector, $x \equiv N/L$. As shown by Blanchard and Gali (2008), this relationship can be derived from a constant-returns-to-scale Cobb-Douglas matching function and a cost of posting vacancies.²⁶ The parameter α_0 is increasing in the cost of posting vacancies and decreasing in the productivity of the matching technology, while α_1 depends on the weight of vacancies in the Cobb-Douglas matching function.

In an incomplete specialization equilibrium, workers must be indifferent between searching for employment in the differentiated sector and receiving a certain wage of one in the homogeneous sector.²⁷ As the expected return to searching for employment in the differentiated sector equals the probability of being sampled times the expected wage conditional on being sampled, the requirement for workers to be indifferent can be written as:

$$xb=1$$
,

where we have used $h(\theta) = n(\theta) [a_{\min}/a_c(\theta)]^k$ and $w(\theta) = b [a_c(\theta)/a_{\min}]^k$. Thus the expected wage conditional on being sampled by a θ -firm, $w(\theta)h(\theta)/n(\theta) = b$, is constant across all firms, and workers have no incentive to direct their search towards certain types of firms.

Combining this indifference condition with the matching technology above, we can solve for the search cost, b, as a function of model parameters, and hence determine the equilibrium tightness of the labor market, x:

$$b = \alpha_0^{\frac{1}{1+\alpha_1}} > 1$$
 and $x = 1/b = \alpha_0^{-\frac{1}{1+\alpha_1}} < 1.$ (20)

Thus, as b depends solely on parameters of the model, we treat it in our discussion below as a derived parameter that summarizes the degree of search frictions in the differentiated sector. Recall that b is larger the less efficient the matching technology and the higher the cost of posting vacancies.

The mass of workers searching for employment in the differentiated sector, L, can be determined from the requirement that the sector's total wage bill equals L, which ensures that the ex ante expected wage for every worker searching for employment in the differentiated sector equals one, or

$$L = M \int_{\theta_d}^{\infty} w(\theta) h(\theta) dG_{\theta}(\theta) = M \frac{\beta \gamma}{1 + \beta \gamma} \int_{\theta_d}^{\infty} r(\theta) dG_{\theta}(\theta),$$
 (21)

where the second equality uses the fact that the wage bill is a constant share of revenue. As before, using the revenue functions in (14) and the cutoff conditions (16) and (17), we obtain an expression which, together with its counterpart for the foreign country, constitutes the third bloc of the equilibrium system that allows us to solve for the mass of workers in each country, (L, L^*) (see Appendix for more details).

This completes our description of the open economy equilibrium conditions; the solutions from

²⁶See also Helpman and Itskhoki (2008) for a derivation of labor costs when there are hiring costs, firing costs, and unemployment benefits.

²⁷This equilibrium condition is similar to Harris and Todaro (1970).

the previous two blocs of equations for $(\theta_d, \theta_x, Q, \theta_d^*, \theta_x^*, Q^*)$ and (M, M^*) together with (21) and a similar equation for the foreign country provide solutions for the measure of workers searching for jobs in the differentiated sector in each country, (L, L^*) .

2.4.3 Variation Across Firms

Given the equilibrium values for the domestic and exporting cutoff productivities, (θ_d, θ_x) , and consumption in the differentiated sector in the two countries, (Q, Q^*) , we can solve for all firm-specific variables for the home country. Specifically, they can be expressed as functions of the domestic and exporting cutoff productivities, θ_d and θ_x , and the market access variable, $\Upsilon(\theta)$, which depends on whether a firm chooses to serve the foreign market, as well as on the variable trade cost and the relative sizes of the two markets. In particular, we have (see the Appendix for details):

$$r(\theta) = \Upsilon(\theta)^{\frac{1-\beta}{\Gamma}} \cdot r_d \cdot \left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{\Gamma}}, \qquad r_d \equiv \frac{1+\beta\gamma}{\Gamma} f_d,$$

$$n(\theta) = \Upsilon(\theta)^{\frac{1-\beta}{\Gamma}} \cdot n_d \cdot \left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{\Gamma}}, \qquad n_d \equiv \frac{\beta\gamma}{\Gamma} \frac{f_d}{b},$$

$$a_c(\theta) = \Upsilon(\theta)^{\frac{1-\beta}{\Gamma\delta}} \cdot a_d \cdot \left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{\delta\Gamma}}, \qquad a_d \equiv \left[\frac{\beta(1-\gamma k)}{\Gamma} \frac{f_d}{c}\right]^{1/\delta},$$

$$h(\theta) = \Upsilon(\theta)^{\frac{1-\beta}{\Gamma}(1-k/\delta)} \cdot h_d \cdot \left(\frac{\theta}{\theta_d}\right)^{\frac{\beta(1-k/\delta)}{\Gamma}}, \qquad h_d \equiv \frac{\beta\gamma}{\Gamma} \frac{f_d}{b} \left[\frac{\beta(1-\gamma k)}{\Gamma} \frac{f_d}{ca_{\min}^{\delta}}\right]^{-k/\delta},$$

$$w(\theta) = \Upsilon(\theta)^{\frac{(1-\beta)k}{\Gamma\delta}} \cdot w_d \cdot \left(\frac{\theta}{\theta_d}\right)^{\frac{\beta k}{\delta\Gamma}}, \qquad w_d \equiv b \left[\frac{\beta(1-\gamma k)}{\Gamma} \frac{f_d}{ca_{\min}^{\delta}}\right]^{k/\delta},$$

where $\Upsilon(\theta)$ is given in (6).

Evidently, all these firm-specific variables are monotonically increasing in productivity and they experience a discrete upward jump at the exporting cutoff θ_x , at which point $\Upsilon(\theta)$ jumps from 1 to $\Upsilon_x > 1$. Therefore, as well as being more productive than nonexporters, exporters have larger revenues, sample more workers, screen to a higher ability cutoff, hire more workers, and pay higher wages. Also note that these variables depend on the foreign country only through $\Upsilon_x = 1 + \tau^{-\frac{\beta}{1-\beta}} \left(Q^*/Q\right)^{-\frac{\beta-\zeta}{1-\beta}}$, which depends on relative real consumption indices in the two countries, but does not directly depend on any other foreign variable. Similar firm-specific variables can be derived for the foreign country.²⁸

While there are differences in productivity and size between exporters and nonexporters in Melitz (2003) and Helpman and Itskhoki (2008), wages and workforce composition are the same across all firms in their models. In contrast, our framework not only generates differences in productivity and size between exporters and nonexporters, but it also provides a theoretical explanation

$$\beta(1-\gamma k)f_d > c\Gamma a_{\min}^{\delta}.$$

As $\gamma < 1/k$ was assumed above in order for any screening to occur, this condition is satisfied for a sufficiently small screening cost c.

²⁸ As we focus on an interior equilibrium in which all firms screen, we require $a_c(\theta_d) > a_{\min}$. From the solutions for firm-specific variables (22), this condition holds if and only if

for the empirical finding that exporters pay higher wages (e.g., Bernard and Jensen, 1995, 1997). Furthermore, the model is consistent with evidence from matched employee–employer datasets that a substantial part of the higher wages paid by exporters arises from differences in workforce composition (see for example Kaplan and Verhoogen 2006, Schank, Schnabel and Wagner 2007, and Munch and Skaksen 2008).²⁹ Finally, note that in our model revenue-based productivity, $r(\theta)/h(\theta)$, exhibits the same pattern as wages, i.e., it increases with the size of the firm and is higher for exporters than for nonexporters, consistent with empirical observations.

3 Trade Versus Autarky

In this section we derive the main results of the paper for the impact of trade on welfare, unemployment, and inequality. We show that every country gains from trade, yet trade leads to higher unemployment and more wage inequality in the differentiated sector than in autarky. The latter result also implies that aggregate unemployment and aggregate wage and income inequality are higher in the trade equilibrium than in autarky, as long as the two countries are sufficiently similar so that the compositional shift across sectors enhances the within-sectoral effect. In Section 4, we examine further how trade and labor market frictions influence unemployment and inequality in each country.

3.1 Gains from Trade

Our economies are distorted, because firms price above marginal cost in the differentiated sector, there are frictions in the labor market, and part of the labor force is unemployed, as discussed further below. Nevertheless, both countries gain from trade.

To show gains from trade, consider first the free-entry condition (18). This equation describes a downward-sloping relationship between the domestic and exporting productivity cutoffs for every distribution function $G_{\theta}(\theta)$. The economics of this relationship is as follows. Consider an increase in θ_x , which represents a reduction in export opportunities, in the sense that a higher productivity is now required to profitably export. As a result, expected profits from exporting for a new entrant are reduced. In order to induce firms to continue to enter, the reduction in expected profits from exporting has to be compensated for by an equal increase in the expected profits from serving the domestic market. This means that domestic sales need to be more profitable for lower productivity levels than they were before the decline in profits from exporting. In other words, θ_d has to decline.

For the Pareto distribution function $G_{\theta}(\theta) = 1 - (\theta_{\min}/\theta)^{z}$, $\theta \ge \theta_{\min}$ and z > 2, (18) becomes:

$$f_d \left(\frac{\theta_{\min}}{\theta_d}\right)^z + f_x \left(\frac{\theta_{\min}}{\theta_x}\right)^z = \frac{z\Gamma - \beta}{\beta} f_e.$$

²⁹On the one hand, more productive firms screen to a higher ability cutoff, and so exporters do not hire some low-ability workers who would be hired by a less productive firm serving only the domestic market (differences in workforce composition). On the other hand, exporters pay higher wages to those workers who would also be hired by a less productive firm serving only the domestic market (an exporter wage premium).

Evidently, the domestic cutoff is larger the smaller the export cutoff is. In autarky there are no exports and the domestic cutoff is obtained as the limit of $\theta_x \to \infty$. It therefore follows that the domestic cutoff is larger in a trade equilibrium than in autarky.

Next, examine condition (16) for the domestic cutoff; it holds in autarky and in every trade equilibrium. Along this condition, the real consumption Q and the domestic cutoff θ_d move together. The intuition is the following: higher Q corresponds to a more competitive market in which only more productive firms can make positive profits and survive. Therefore, we conclude that in equilibrium the real consumption index Q is higher the larger the domestic cutoff θ_d is. As a result, real consumption of differentiated products is higher in the trade equilibrium.

Finally, consider the indirect utility function (3). Family income E equals \bar{L} , because with incomplete specialization the expected wage of a family member who seeks a job in the differentiated sector is the same as the expected wage of a family member who seeks a job in the homogeneous sector, which is equal to one in both autarky and the trade equilibrium. Therefore the indirect utility of a family is

$$\mathbb{V} = \bar{L} + \frac{1 - \zeta}{\zeta} Q^{\zeta}$$

in autarky and in a trade equilibrium, except that real consumption of the differentiated product is higher in the trade equilibrium. It follows that welfare is higher in the trade equilibrium. A similar analysis applies to the foreign country. This establishes

Proposition 1 Every country gains from trade.

Proof. The proposition follows immediately from the arguments above.

While every country gains from trade, we show in the following subsections that the distribution of wages and the distribution of income (which accounts for both wage income and the zero income of the unemployed) are more equal in autarky than in the trade equilibrium, and moreover, unemployment is higher in the trade equilibrium. That is, while trade is beneficial in welfare terms, it negatively impacts wage and income inequality and unemployment, which are common indices of social disparity.

3.2 Wage Inequality in the Differentiated Sector

While all workers have the same ex ante expected income of one, the equilibrium of the model features ex post wage inequality across firms within sectors. Workers with the same observed characteristics receive different ex post wages depending on the employer with whom they are matched. In this section we characterize the distribution of wages within the differentiated sector, while in the following two sections we take account of unemployment and characterize the distribution of income among all individuals seeking employment in a sector.

The sectoral distribution of wages can be derived from the solutions for firm-specific variables in (22). Figure 1 displays the pattern of wages *across firms* for a particular set of parameter values showing that—while more productive firms pay higher wages in general—exporters pay especially

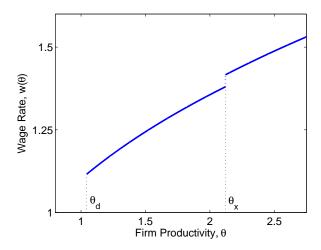


Figure 1: Wages as a function of firm productivity

high wages; the least productive exporter pays discretely higher wages than a nonexporter with slightly lower productivity.³⁰ The empirical implication is that exporters should pay higher wages than nonexporters within the same industry, even after controlling for firm characteristics such as productivity and size, which is a robust finding in the large empirical literature on firm export behavior following Bernard and Jensen (1995, 1997).

Combining the solution for firm-specific variables in (22) with the distribution of productivity across firms, $G_{\theta}(\theta) = 1 - (\theta_{\min}/\theta)^z$ for $\theta \geq \theta_{\min}$, we can compute the distribution of wages across workers, which depends on both the wages and employment of firms with different levels of productivity. To characterize the wage distribution, we use ρ to denote the ratio of the domestic to export cutoff productivities: $\rho \equiv \theta_d/\theta_x$. In this event the fraction of exporting firms equals ρ^z . This trade openness variable obtains values between zero and one: $\rho = 0$ when the export cutoff is infinite and no firm exports; $\rho = 1$ when the export cutoff converges on the domestic cutoff and all firms export; in between, $0 < \rho < 1$ and only a fraction of firms export.

Using this notation for relative productivity cutoffs, the distribution of wages across workers can be represented as a weighted average of the distribution of wages across domestic firms and exporters, with weights equal to the shares of employment in the two groups of firms:³¹

$$G_{w}(w) = \begin{cases} S_{h,d}G_{w,d}(w) & \text{for } w_{d} \leq w \leq w_{d}/\rho^{\frac{\beta k}{\delta \Gamma}}, \\ S_{h,d} & \text{for } w_{d}/\rho^{\frac{\beta k}{\delta \Gamma}} \leq w \leq w_{d}\Upsilon_{x}^{\frac{k(1-\beta)}{\delta \Gamma}}/\rho^{\frac{\beta k}{\delta \Gamma}}, \\ S_{h,d} + (1 - S_{h,d})G_{w,x}(w) & \text{for } w \geq w_{d}\Upsilon_{x}^{\frac{k(1-\beta)}{\delta \Gamma}}/\rho^{\frac{\beta k}{\delta \Gamma}}, \end{cases}$$
(23)

³⁰To derive closed form solutions for the model, we make a number of simplifying assumptions about functional form. Therefore this and subsequent figures are intended merely to illustrate the qualitative features of the model rather than its quantitative predictions. See the Appendix for a discussion of the parameter values used in all our figures.

³¹See the Appendix for a formal derivation of all the results of this section.

where $w_d = w(\theta_d)$ is the wage rate paid by the least productive firm, given in (22); $w_d/\rho^{\beta k/\delta\Gamma} = w(\theta_x^-)$ is the wage rate paid by the most productive firm that serves only the domestic market; $w_d \Upsilon_x^{k(1-\beta)/\delta\Gamma}/\rho^{\beta k/\delta\Gamma} = w(\theta_x^+)$ is the wage rate paid by the least productive exporting firm. The share of workers employed by firms that serve only the domestic market, $S_{h,d}$, can be evaluated using the Pareto productivity distribution and the solution for firm-specific variables (22) as

$$S_{h,d} = \frac{1 - \rho^{z - \frac{\beta(1 - k/\delta)}{\Gamma}}}{1 + \rho^{z - \frac{\beta(1 - k/\delta)}{\Gamma}} \left[\Upsilon_x^{\frac{(1 - \beta)(1 - k/\delta)}{\Gamma}} - 1 \right]}.$$

The distribution of wages across workers conditional on being employed by a domestic firm, $G_{w,d}(w)$, can be derived in the following way. As from (22) the relative wages paid by any two firms depend solely on their relative productivities, and productivity is Pareto distributed, $G_{w,d}(w)$ is a truncated Pareto distribution:

$$G_{w,d}(w) = \frac{1 - \left(\frac{w_d}{w}\right)^{1+1/\mu}}{1 - \rho^{z - \frac{\beta(1-k/\delta)}{\Gamma}}} \quad \text{for } w_d \le w \le w_d/\rho^{\frac{\beta k}{\delta \Gamma}}.$$
 (24)

Similarly, the distribution of wages across workers conditional on being employed by an exporter, $G_{w,x}(w)$, follows an untruncated Pareto distribution:

$$G_{w,x}(w) = 1 - \left[\frac{w_d}{w} \Upsilon_x^{\frac{k(1-\beta)}{\delta\Gamma}} \rho^{-\frac{\beta k}{\delta\Gamma}} \right]^{1+1/\mu} \qquad \text{for } w \ge w_d \Upsilon_x^{\frac{k(1-\beta)}{\delta\Gamma}} / \rho^{\frac{\beta k}{\delta\Gamma}}, \tag{25}$$

where the parameter μ is defined as

$$\mu = \frac{\beta k/\delta}{z\Gamma - \beta}.\tag{26}$$

The shape parameter of the wage distribution, μ , depends not only on the dispersion of worker ability k and the dispersion of firm productivity z, but also on the parameter δ of the screening cost, the demand parameter β , and the technology parameter γ , each of which enters Γ and influences the allocation of workers across firms. For the wage distribution to have a finite variance, we require $\mu < 1$, and we assume parameter values such that this inequality holds.³²

>From the expressions above we note that the sectoral wage distribution depends on the endogenous variables of the model only through our measure of openness to trade, $\rho \equiv \theta_d/\theta_x$, and the market access variable, $\Upsilon_x \equiv 1 + \tau^{-\beta/(1-\beta)} (Q^*/Q)^{-(\beta-\zeta)/(1-\beta)}$. While ρ determines the composition of firms in the sector between exporters and nonexporters, Υ_x determines the wage premium paid by exporters over nonexporters. These variables both depend on trade costs and labor market frictions in the two countries, as examined in Section 4. In addition to ρ and Υ_x , the sectoral wage distribution depends on exogenous parameters of the model and, in particular, on the derived

³²>From equation (26), $\mu < 1$ if and only if $\Gamma \equiv 1 - \beta \gamma - \frac{\beta}{\delta} (1 - \gamma k) > 2\beta/z$. Therefore we assume sufficiently large values of δ and sufficiently small values of β and γ for the inequality to hold, which implies that screening costs are sufficiently convex and revenue and output are sufficiently concave.

parameter μ . In this paper, we focus on how trade costs and labor market frictions affect unemployment and inequality through the endogenous variables ρ and Υ_x , keeping other parameters of the model fixed. In Helpman et al. (2008) we study the effects of the productivity and ability dispersion parameters (z and k) on unemployment and inequality in a closed economy.

As shown in Helpman et al. (2008), the shape parameter of a Pareto distribution uniquely determines the degree of inequality as measured by standard indexes of inequality, such as the coefficient of variation, the Gini coefficient, or the Theil index. We use the Theil index to measure inequality, because it permits an exact decomposition of aggregate inequality into within- and between-group components (see Bourguignon 1979). This type of decomposition is important, because there are several groups within each sector—the unemployed, workers employed by nonexporters, and workers employed by exporters—and aggregate inequality depends on the allocation of workers across sectors. The Theil index of an income distribution $G_{\varpi}(\varpi)$ is defined as

$$T = \int \frac{\overline{\omega}}{\overline{\omega}} \ln \left(\frac{\overline{\omega}}{\overline{\omega}} \right) dG_{\overline{\omega}} (\overline{\omega}), \qquad (27)$$

where ϖ is income, $\bar{\varpi}$ is mean income, $\varpi dG_{\varpi}(\varpi)/\bar{\varpi}$ is the income share of the ϖ -type individuals, while $\ln(\varpi/\bar{\varpi})$ is approximately equal to the proportional deviation of ϖ from mean income.

One important property of the wage distributions (23)-(25) is that in the two limiting cases of $\rho = 0$ (no firm exports) and $\rho = 1$ (all firms export), the wage distribution is an untruncated Pareto with shape parameter $1+1/\mu$. For an untruncated Pareto distribution with shape parameter $1+1/\mu$, the Theil index is

$$T\left(1+\frac{1}{\mu}\right) = \mu - \ln\left(1+\mu\right). \tag{28}$$

It follows that in autarky the distribution of wages has the same degree of inequality as the distribution of wages in an open economy in which all firms export. Importantly, this result does not depend on how different the trading partners are in terms of labor market frictions, and the argument applies to the home and foreign country alike.³³ We have therefore shown

Lemma 1 In a trade equilibrium in which all firms export, wage inequality in the differentiated sector is the same as in autarky.

Proof. The lemma follows immediately from the fact that the Theil index for an untruncated Pareto distribution depends solely on the shape parameter of that distribution and is invariant to the lower limit of that distribution, as shown in the Appendix.

We next show that the Theil index of the open economy wage distribution (23) is larger than $\mu - \ln(1 + \mu)$ for $0 < \rho < 1$. This establishes that in a trade equilibrium in which some but not all firms export, there is more wage inequality in the differentiated sector than in autarky. To establish this result, consider Figure 2, which depicts the distribution function of wages $G_w(w)$, given in (23). This function equals zero for all wages lower than w_d , which is the lowest wage paid

³³It is easy to see that this result also holds in a world of many countries.

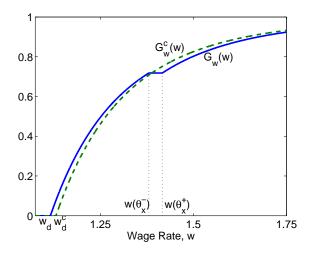


Figure 2: Cumulative distribution function of wages

in the industry; it rises for wages $w_d \leq w \leq w_d/\rho^{\beta k/\delta\Gamma}$ paid by nonexporters; it is flat for wages $w_d/\rho^{\beta k/\delta\Gamma} \leq w \leq w_d \Upsilon_x^{k(1-\beta)/\delta\Gamma}/\rho^{\beta k/\delta\Gamma}$; and it rises for wages $w \geq w_d \Upsilon_x^{k(1-\beta)/\delta\Gamma}/\rho^{\beta k/\delta\Gamma}$ paid by exporters.

Now construct a counterfactual untruncated Pareto wage distribution function, $G_w^c(w) = 1 - (w_d^c/w)^{1+1/\mu}$ for $w \ge w_d^c$, which has the same shape parameter as the conditional wage distributions for workers employed by nonexporters and exporters, and which has the same mean as the actual distribution of wages:

$$\int_{w_d}^{\infty} w dG_w(w) = \int_{w_d^c}^{\infty} w dG_w^c(w).$$

As the counterfactual wage distribution, $G_w^c(w)$, has the same shape parameter as $G_{w,d}(w)$ and $G_{w,x}(w)$, and has the same mean as the overall sectoral wage distribution, $G_w(w)$, it follows that:

$$w_d < w_d^c < w\left(\theta_x^+\right) = w_d \Upsilon_x^{\frac{k(1-\beta)}{\delta\Gamma}} \rho^{-\frac{\beta k}{\delta\Gamma}}.$$

That is, the lowest wage in the counterfactual distribution, w_d^c , lies strictly between the wage paid by the least productive firm in the industry, w_d , and the wage paid by the least productive exporter in the industry, $w\left(\theta_x^+\right)$. The intuition for these results is as follows. The lowest wage in the counterfactual distribution, w_d^c , cannot lie above $w\left(\theta_x^+\right)$, because with the same shape parameter as the wage distribution for workers employed by exporters, the entire counterfactual wage distribution would lie below the actual distribution of wages. This would imply that $G_w^c\left(w\right)$ would first-order stochastically dominate $G_w\left(w\right)$, contradicting the requirement that the two distributions have the same mean. Similarly, the lowest wage in the counterfactual distribution, w_d^c , cannot lie below w_d ,

because this would imply that the mean of $G_w^c(w)$ would be less than the mean of $G_w(w)$.³⁴

In addition, the slope of the counterfactual wage distribution is smaller than the slope of the actual wage distribution at $w(\theta_x^+)$, as shown in the Appendix. As a result, the relative location of the two distributions is as depicted in Figure 2: the actual and counterfactual distributions intersect only once and the actual distribution is above the counterfactual distribution for low wages and below it for high wages.³⁵ This last property is sufficient to establish the following result:

Lemma 2 Let $0 < \rho < 1$. Then the counterfactual wage distribution $G_w^c(w)$ strictly second-order stochastically dominates the actual wage distribution $G_w(w)$.

Proof. See the Appendix. ■

By construction, the actual and counterfactual wage distributions have the same mean \bar{w} , and therefore from the definition of the Theil index in (27) the difference in wage inequality between the actual and counterfactual wage distributions is

$$T_w - T_w^c = \frac{1}{\bar{w}} \int_{w_d}^{\infty} w \ln w \left[dG_w(w) - dG_w^c(w) \right].$$

Since the function $w \ln w$ is strictly convex and the counterfactual wage distribution, $G_w^c(w)$, strictly second-order stochastically dominates the actual wage distribution, $G_w(w)$, it follows that the Theil index of the actual wage distribution, T_w , is strictly greater than the Theil index of the counterfactual wage distribution is the same as the Theil index of the distribution of wages in a closed economy, since both are untruncated Pareto distributions with the same shape parameter, $1+1/\mu$, as noted above. Therefore wage inequality in the closed economy is strictly lower than in a trade equilibrium in which some but not all firms export $(0 < \rho < 1)$.³⁶ These results imply³⁷

$$w_d^c = \frac{1 + \left(\Upsilon_x^{\frac{1-\beta}{\Gamma}} - 1\right)\rho^{z-\frac{\beta}{\Gamma}}}{1 + \left[\Upsilon_x^{\frac{1-\beta}{\Gamma}\left(1-\frac{k}{\delta}\right)} - 1\right]\rho^{z-\frac{\beta}{\Gamma}\left(1-\frac{k}{\delta}\right)}} w_d.$$

³⁴See the Appendix for the formal derivation. The closed-form solution for w_d^c is

³⁵Note that the two distributions can intersect either above the wage rate at the most-productive nonexporting firm, $w(\theta_x^-)$, as shown in Figure 2, or below it. In both cases the actual and counterfactual distributions have the properties discussed in the text.

³⁶Note that this argument does not rely on the particular inequality index used. The degree of inequality in the counterfactual wage distribution is the same as in autarky since the shape parameter of the (untruncated) Pareto distribution is a sufficient statistic for inequality under all scale-independent measures of inequality. In addition, the counterfactual wage distribution second-order stochastically dominates the actual wage distribution in a trade equilibrium in which only some firms export, which is a general criterion of greater equality of outcomes. Putting these two arguments together implies that wage inequality in a trade equilibrium in which only some firms export is greater than in autarky for a general class of inequality measures.

³⁷In addition, the Appendix shows that wage inequality in the differentiated sector can be decomposed into within and between components for workers employed by exporters and nonexporters. While average inequality within groups decreases with trade, this effect is dominated by increasing inequality between the two groups of workers.

Proposition 2 Wage inequality is strictly greater in the trade equilibrium than in autarky when some but not all firms export, and the two distributions have the same degree of inequality when all firms export.

Proof. Follows from the above discussion.

This is a key proposition, which establishes that trade raises wage inequality in the differentiated sector. Moreover, in the limiting cases in which either no firm exports ($\rho = 0$) or all firms export ($\rho = 1$), wage inequality is the same, which implies that wage inequality is the same in autarky as in a trade equilibrium in which all firms export. As a result, a given change in fundamentals—such as trade costs or labor market frictions—that raises the fraction of exporting firms, raises wage inequality when a small fraction of firms export (low ρ) but reduces wage inequality when a large fraction of firms export (high ρ). In other words, the relationship between trade openness and wage inequality is nonmonotonic. Therefore, while wage inequality is higher in a trade equilibrium than in autarky, once the economy is open to trade a given change in trade openness can either raise or reduce wage inequality. We summarize these results as follows:

Corollary (to Proposition 2) An increase in the fraction of exporting firms raises wage inequality in the differentiated sector when the fraction of exporting firms is small, and reduces wage inequality in the differentiated sector when the fraction of exporting firms is large.

Proof. With a Pareto productivity distribution, the fraction of exporting firms is ρ^z . As $\rho \to 0$, wage inequality in the trade equilibrium converges to its autarky value, and from Proposition 2 a small increase in ρ raises wage inequality in the differentiated sector. As $\rho \to 1$, wage inequality in the trade equilibrium also converges to the same value as in autarky, and from Proposition 2 a small decrease in ρ raises wage inequality in the differentiated sector.

The nonmonotonic relationship between trade openness and wage inequality is illustrated in Figure 3.³⁸ The Theil index is the same for $\rho = 0$ and $\rho = 1$, and it is higher for values of ρ between these extremes. In the figure, the measure of inequality is single-peaked, so that inequality rises with ρ for all values of ρ below the value that maximizes inequality, and declines with ρ for all higher values.³⁹

3.3 Unemployment in the Differentiated Sector

The presence of search frictions in the differentiated sector gives rise to equilibrium unemployment. Workers can be unemployed either because they are not sampled by a firm, or because once sampled they are not hired as a result of their ability falling below the firm's ability cutoff. The rate of

³⁸ In this figure, we vary the fixed exporting cost f_x , holding constant the variable trade cost τ . With symmetric countries, this changes trade openness ρ , but leaves the market access variable Υ_x unchanged, because $Q = Q^*$ and hence $\Upsilon_x = 1 + \tau^{-\beta/(1-\beta)}$.

³⁹While T_w has a single peak in Figure 3, and this property has been found in all of our simulations, we have not been able to establish the existence of a single peak analytically.

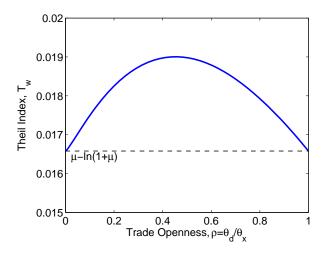


Figure 3: Theil index of sectoral wage inequality

unemployment in the differentiated sector, u, can therefore be expressed as one minus the product of the sectoral tightness of the labor market, $x \equiv N/L$, and the sectoral hiring rate, $\sigma \equiv H/N$, where H is the mass of employed workers, N is the mass of workers matched with firms before the screening stage, and L is the mass of workers searching for a job in the differentiated sector:

$$u = \frac{L - H}{L} = 1 - \frac{H}{N} \frac{N}{L} = 1 - \sigma x.$$
 (29)

In contrast, with no search frictions in the homogeneous sector, the rate of unemployment in that sector is equal to zero.⁴⁰

The sectoral tightness of the labor market, x = 1/b, was determined above (see (20)), while the sectoral hiring (or retention) rate, σ , can be expressed as

$$\sigma \equiv \frac{H}{N} = \frac{M \int_{\theta_d}^{\infty} h(\theta) dG_{\theta}(\theta)}{M \int_{\theta_d}^{\infty} n(\theta) dG_{\theta}(\theta)} = \frac{\int_{\theta_d}^{\infty} n(\theta) \left[a_{\min} / a_c(\theta) \right]^k dG_{\theta}(\theta)}{\int_{\theta_d}^{\infty} n(\theta) dG_{\theta}(\theta)}.$$

Using the solutions for firm-specific variables in equation (22), and evaluating the integrals in the expression for σ above using the Pareto productivity distribution, we can solve explicitly for the

⁴⁰The key simplifying feature introduced by the homogeneous sector is the determination of expected worker income and not the absence of unemployment in this sector. See Helpman and Itskhoki (2008) for an introduction of unemployment into the homogeneous sector. While sectoral unemployment in the model is defined in terms of workers who were unsuccessful in their search for employment in a sector, the empirical measures constructed by the Bureau of Labor Statistics (BLS) are defined in terms of workers who are currently unemployed and were previously employed in a sector. In a dynamic model with job destruction and a constant labor force in each sector, these measures would coincide. The BLS data reports significant variation of unemployment across sectors. For example, in 2007 Mining had an unemployment rate of 3.4%; Construction, 7.4%; and Manufacturing, 4.3% (see http://www.bls.gov/cps/cpsaat26.pdf, accessed on April 25, 2008).

sectoral hiring rate as a function of the relative productivity cutoffs ρ (see Appendix):

$$\sigma = \frac{1 + \left[\Upsilon_x^{\frac{(1-\beta)(1-k/\delta)}{\Gamma}} - 1\right] \rho^{z-\beta(1-k/\delta)/\Gamma}}{1 + \left[\Upsilon_x^{\frac{(1-\beta)}{\Gamma}} - 1\right] \rho^{z-\beta/\Gamma}} \sigma^A, \tag{30}$$

where

$$\sigma^{A} = \left[\frac{\Gamma}{\beta \left(1 - \gamma k \right)} \frac{c a_{\min}^{\delta}}{f_{d}} \right]^{k/\delta} \frac{1}{1 + \mu}$$

is the autarky hiring rate, obtained when $\rho \to 0$ (see also Helpman et al. 2008). Note that, as with sectoral wage inequality, the endogenous variables of the model affect sectoral unemployment only through the trade openness and market access variables, ρ and Υ_x . In a trade equilibrium in which some firms export $0 < \rho \le 1$, and the term in front of σ^A on the right-hand side of (30) is strictly less than one. Therefore $\sigma < \sigma^A$. As trade does not affect tightness in the labor market, x, but raises unemployment through a reduction of the hiring rate, σ , equations (29) and (30) imply

Proposition 3 In the differentiated sector, the hiring rate is strictly lower and the unemployment rate is strictly higher in a trade equilibrium than in autarky.

Proof. See the Appendix.

Therefore, moving from autarky to a trade equilibrium necessarily increases sectoral unemployment in the differentiated sector, and this holds for symmetric and asymmetric countries alike. The intuition for this result is as follows. Starting from autarky, the opening of trade increases the revenue and employment of high-productivity firms that enter the export market. As revenue and employment rise at high-productivity exporting firms, diminishing marginal returns to the number of employed workers lead these firms to become more selective in their recruitment policies, which increases equilibrium unemployment. Furthermore, the increase in revenue of high-productivity exporting firms leads to increased entry in the differentiated sector, which reduces the revenue and employment of low-productivity firms that serve only the domestic market. Although this decline in revenue and employment at low-productivity firms leads them to become less selective in their recruitment policies, there is a change in industry composition from low- to high-productivity firms. As more productive firms are more selective (screen to a higher ability cutoff), this change in industry composition raises sectoral unemployment. The net result is higher unemployment.

On the other hand, the relationship between trade openness and sectoral unemployment, like the relationship between trade openness and sectoral wage inequality, can be nonmonotonic. In other words, once the economy is open to trade, a given change in trade openness can either increase or decrease sectoral unemployment. The reason is as follows. Initially, only high-productivity firms find it profitable to export. A reduction in trade costs increases the revenue of these high-productivity exporters and reduces the revenue of low-productivity firms that continue to serve only the domestic market. Both of these effects raise sectoral unemployment, as explained above.

However, a reduction in trade costs also induces lower-productivity firms to enter the export market. As these new entrants to the export market are less productive than the incumbent exporters, they have less selective recruitment policies than the incumbents. Therefore there is a change in composition within the group of exporters from firms with more- to firms with less-selective recruitment policies, which reduces equilibrium unemployment. Depending on parameter values, this change in composition within the group of exporting firms can overwhelm the previously discussed effects. We have simulated examples in which the sectoral unemployment rate is monotonically increasing with the trade openness variable ρ , as well as examples in which this relationship has an inverted U-shape; unemployment increases initially with ρ and decreases after reaching a peak close to $\rho = 1.^{41}$

3.4 Income Inequality in the Differentiated Sector

The sectoral distribution of income depends not only on the distribution of wages across employed workers, but also on the probability of being unemployed. Recall that only a fraction $H/L = \sigma x$ of the workers seeking employment in the differentiated sector are hired, while the remaining fraction $1 - \sigma x$ become unemployed and receive zero income. To characterize sectoral income inequality, we use the property of the Theil index that it can be decomposed into within- and between-group components for the two groups of employed and unemployed workers. Using this property, the Theil index for income inequality in the differentiated sector, T_i , can be expressed solely in terms of the Theil index of sectoral wage inequality, T_w (derived above), and the unemployment rate, u (see Appendix):⁴²

$$T_{L} = T_{W} - \ln(1 - u). \tag{31}$$

That is, income inequality is increasing in inequality among employed wage-earners and in the unemployment rate. The first term on the right-hand side captures within-group inequality. As the unemployed all receive the same income of zero, they make no contribution to within-group inequality, which therefore equals the Theil index of wage inequality among the employed. The second term on the right-hand side represents between-group inequality, because the requirement that workers are indifferent across sectors implies that the average wage in the differentiated sector is inversely related to the unemployment rate.

As we have already established that both sectoral wage inequality and the sectoral unemployment rate are higher in a trade equilibrium than in autarky, it follows that the opening of trade also increases sectoral income inequality.

⁴¹See the Appendix for a discussion of the parameter values. As explained in footnote 35, this exercise corresponds to a movement in fixed exporting cost, f_x , when countries are symmetric. In response to a reduction in the variable trade cost, τ , there is an additional increase in the market access variable, Υ_x , on top of the increase in the openness variable, ρ . The Appendix proves that the sectoral unemployment rate increases monotonically in Υ_x . As a result, a reduction in τ is more likely to lead to an increase in sectoral unemployment than a reduction in f_x will. Moreover, in all our simulations, the sectoral unemployment rate was monotonically decreasing in τ , yet we have not been able to prove this result analytically. See Section 4 and the Appendix for more details.

⁴²A similar decomposition is available for the Gini coefficient (see Helpman et al. 2008).

Proposition 4 The distribution of income in the differentiated sector is more unequal in a trade equilibrium than in autarky.

Proof. The proposition follows immediately from Propositions 2 and 3 together with the expression for sectoral income inequality in (31), which is derived in the Appendix. ■

Now, once the economy is open to international trade, the fact that changes in trade openness have ambiguous effects on wage inequality and unemployment implies that they also have ambiguous effects on income inequality. Therefore, a change in trade openness can either increase or decrease income inequality within the differentiated sector.

Moving from autarky to the trade equilibrium raises income inequality in the differentiated sector through two channels. First, the partitioning of firms by productivity into nonexporters and exporters, and the discrete increase in wages paid by exporting firms raise sectoral wage inequality. Second, the change in firm composition towards more productive firms with more selective recruitment policies reduces the sectoral hiring rate and increases sectoral unemployment. Therefore, although the opening of trade leads to unambiguous welfare gains, there is an increase in social disparity. While some workers gain from the higher wages paid by exporting firms, other workers lose from the lower wages paid by firms serving only the domestic market and from the rise of unemployment.⁴³

3.5 Aggregate Unemployment and Inequality

Having characterized the relationship between trade, unemployment and inequality in the differentiated sector, we are now in a position to analyze the impact of trade on aggregate unemployment and inequality. The key difference between the analysis at the sectoral and aggregate levels is that, in the aggregate analysis, the impact of trade on sectoral composition, i.e., the allocation of workers across sectors, needs to be taken into account.

We begin by considering the aggregate unemployment rate, **u**, which can be expressed as a weighted average of the rates of unemployment in the homogeneous and differentiated sectors.⁴⁴ With no unemployment in the homogeneous sector, the aggregate rate of unemployment is therefore equal to the unemployment rate in the differentiated sector times the share of the labor force in this sector:

$$\mathbf{u} = \frac{L}{\bar{L}}u. \tag{32}$$

As we have already established that u is higher in the trade equilibrium than in autarky, a sufficient condition for the aggregate rate of unemployment to rise is for L to be higher in the trade equilibrium

⁴³Since the opening of trade leads to an increase in θ_d , firms that remain in business but cannot profitably export experience a reduction in revenues and hence in wages (from (22)). The workers employed by these firms can however still experience an increase in welfare, because the opening of trade reduces the price of the consumption bundle through its effect on the price index for the differentiated good.

⁴⁴We use bold symbols to denote aggregate variables; thus u is the rate of unemployment in the differentiated sector, while \mathbf{u} is the aggregate rate of unemployment.

than in autarky. In general, the opening of trade can either raise or reduce the labor force in the differentiated sector. Furthermore, the reduction in L can outweigh the increase in u so as to reduce aggregate unemployment. Therefore, aggregate unemployment in the trade equilibrium can be either higher or lower than in autarky. However, if the two countries are sufficiently similar, the increase in average productivity in the differentiated sector caused by the opening of trade expands the labor force in this sector in both countries, and hence raises aggregate unemployment in both countries. We note that this condition is sufficient but not necessary, as aggregate unemployment can rise following the opening of trade even if the labor force in the differentiated sector contracts.⁴⁵

To characterize aggregate income inequality, we again exploit the property of the Theil index that it can be decomposed into within and between-group components, where the groups are now the labor forces in the homogeneous and differentiated sectors. As average (or expected) income in both sectors is equal to one, between-group income inequality is equal to zero. Hence aggregate income inequality depends solely on within-group inequality, which is equal to the weighted average of the Theil indices of the two sectors, using income shares (which equal labor shares) as weights. Additionally, since all workers in the homogeneous sector receive the same income of one, the Theil index of income inequality in the homogeneous sector equals zero. Therefore, the Theil index of aggregate income inequality, \mathbf{T}_{ι} , is simply equal to the Theil index of income inequality in the differentiated sector, T_{ι} , times the share of the labor force in this sector (see Appendix):

$$\mathbf{T}_{\iota} = \frac{L}{\bar{L}} T_{\iota}. \tag{33}$$

This expression has a similar form to the expression for aggregate unemployment in (32), and can therefore be analyzed in the same way. In general, aggregate income inequality can be either higher or lower in the trade equilibrium than in autarky. However, a sufficient condition for aggregate income inequality to rise as a result of opening to foreign trade is for L to rise, because income inequality in the differentiated sector is higher in the trade equilibrium than in autarky (Proposition 4). As long as the two countries are sufficiently similar, the opening of trade expands the labor force in the differentiated sector in both countries, and thereby raises aggregate inequality in both countries. We summarize these results in

Proposition 5 As long as countries are sufficiently similar, aggregate unemployment and aggregate income inequality are higher in both countries in the trade equilibrium than in autarky.

Proof. See the Appendix.

To characterize aggregate wage inequality, we use the result in (31) that links income inequality

 $^{^{45}}$ Empirically, relocations within sectors appear to be larger than relocations across sectors. Under these circumstances compositional effects on unemployment and inequality should be limited, and unemployment and inequality should be shaped by within-sectoral effects. Also note that the impact of L on aggregate unemployment depends on which sector has the higher sectoral rate of unemployment. While we have assumed for simplicity that the homogeneous sector has has zero unemployment, if it instead had a positive rate of unemployment, an increase in L would raise the aggregate rate of unemployment if and only if the unemployment rate in the homogeneous sector were lower than in the differentiated sector. See Helpman and Itskhoki (2008) for an analysis of sectoral compositional effects.

to wage inequality and the unemployment rate. This result holds at the aggregate and sectoral levels alike, and therefore $\mathbf{T}_{\iota} = \mathbf{T}_{w} - \ln(1-\mathbf{u})$ and $T_{\iota} = T_{w} - \ln(1-u)$. Combining these expressions with aggregate income inequality (33) and aggregate unemployment (32), we obtain:

$$\mathbf{T}_w = \mathbf{T}_\iota + \ln(1 - \mathbf{u}) = \frac{L}{\overline{L}} T_w + \ln\left(1 - \frac{L}{\overline{L}}u\right) - \frac{L}{\overline{L}} \ln(1 - u). \tag{34}$$

Comparing (33) with (34) one can see that the analysis of aggregate wage inequality involves additional considerations to the analysis of aggregate income inequality. In particular, although there is zero between-group income inequality for workers in the homogeneous and differentiated sectors, because average income is the same across sectors, there is positive between-group wage inequality because of the higher average wage in the differentiated sector. As a result, the opening of trade affects aggregate wage inequality in (34) through several channels. First, from Proposition 2, the opening of trade raises wage inequality in the differentiated sector, T_w , which increases aggregate wage inequality. Second, from Proposition 3, the opening of trade raises unemployment in the differentiated sector, u, which also increases aggregate wage inequality. The intuition for the second result is that in order for workers to remain indifferent between sectors, the higher unemployment rate in the differentiated sector must be compensated for by a higher average wage, which increases the wage gap between the homogeneous- and differentiated-good sectors. Third, the opening of trade affects aggregate wage inequality through the share of the labor force in the differentiated sector, L/\bar{L} . While in general L/\bar{L} can either rise or fall following the opening of trade, we know that it rises in both countries when they are sufficiently similar. However, from (34), such an expansion in the labor force in the differentiated sector can itself either increase or diminish aggregate wage inequality, depending on the initial size of the differentiated sector. When the differentiated sector is small, i.e., L/\bar{L} is small, an increase in L raises aggregate wage inequality. Intuitively, the increase in L shifts workers towards the high average wage sector, which raises aggregate wage inequality when this sector accounts for a small share of the labor force. In contrast, when the differentiated sector is large, i.e., L/\bar{L} is large, an increase in L raises aggregate wage inequality for low rates of unemployment (low average wages) in the differentiated sector, but reduces aggregate wage inequality for high rates of unemployment (relatively high average wages) in the differentiated sector. 46 Therefore, even in the case of symmetric countries, the model predicts a nuanced relationship between the opening of trade and aggregate wage inequality. On the other hand, when sectoral composition effects are small, aggregate wage inequality is higher in a trade equilibrium than in autarky.

$$rac{\partial \mathbf{T}_w}{\partial \left(L/\bar{L}
ight)} = T_w - \ln\left(1-u
ight) - rac{u}{1-\left(L/\bar{L}
ight)u}.$$

For L=0 the right-hand side of this equation is positive, because $-\ln(1-u)-u>0$ for all 0< u<1. At the other extreme, when $L=\bar{L}$, the right-hand side of this equation is positive for low values of u and negative for values of u close to one, because $-\ln(1-u)-u/(1-u)$ is negative for all 0< u<1, but it is close to zero for small values of u and it approaches minus infinity when u approaches 1.

4 Determinants of Unemployment and Inequality

While previous sections examined how opening up to foreign trade influences a country's unemployment and inequality, in this section we examine how, in a trade equilibrium, unemployment and inequality are influenced by exogenous parameters. Despite the model's richness, its comparative statics can be characterized in closed-form for small asymmetries in the neighborhood of a symmetric equilibrium, as shown formally in the Appendix. While we concentrate our discussion on trade costs and labor market frictions, the comparative statics analysis undertaken in the Appendix can also be used to consider the impact of other parameters.

We have already established that sectoral unemployment and inequality have a nonmonotonic relationship with openness to trade. Additionally, we have shown that aggregate unemployment and inequality can move in quite different ways from their sectoral counterparts, depending on changes in sectoral composition. Both of these results suggest that the impact of a change in model parameters on unemployment and inequality is likely to be ambiguous at both the sectoral and aggregate levels, as will indeed prove to be the case. Nonetheless, in the neighborhood of an equilibrium in which only a small fraction of firms export ($\rho \approx 0$) and asymmetries between the two countries are small ($b \approx b^*$ and $c \approx c^*$), unambiguous predictions for the effects of the variable trade cost and labor market frictions on unemployment and inequality can be derived. Although $\rho \approx 0$ is a special case, it is an interesting special case because the evidence shows that in most sectors only a small fraction of firms export. In the remainder of this section we derive comparative statics results in the neighborhood of such an equilibrium, which also illuminates forces at work in the model. Having completed this characterization, we return to discuss the more general relationships between unemployment, inequality and model parameters.

As a first step, recall from Section 3 that sectoral wage inequality, T_w , and unemployment, u, only depend on endogenous variables through trade openness, ρ , and market access, Υ_x . These latter two variables are linked through the equilibrium conditions of the model. Specifically, dividing the exporting cutoff condition (16) by the zero-profit cutoff condition (17), we obtain

$$\Upsilon_x^{\frac{1-\beta}{\Gamma}} = 1 + \frac{f_x}{f_d} \rho^{\frac{\beta}{\Gamma}}.$$
 (35)

This expression implies that if f_x is held constant, ρ and Υ_x move in the same direction in response to changes in the variable trade cost and both countries' labor market frictions. Therefore, holding the fixed cost of exporting constant, we can characterize sectoral wage inequality and unemployment in terms of trade openness, ρ , which is monotonically related to the fraction of firms that export, ρ^z .⁴⁷

We use this result to examine the comparative statics of changes in the variable trade cost and labor market frictions. Our analysis proceeds in two stages. First, we use (35) to obtain an expression for sectoral wage inequality and unemployment in terms of trade openness, ρ , for an equilibrium in which only a small fraction of firms export ($\rho \approx 0$). Second, we use the model's

⁴⁷ As is evident from (35), changes in the fixed cost of trade affect the relationship between ρ and Υ_x .

comparative statics for small asymmetries between the two countries ($b \approx b^*$ and $c \approx c^*$) to determine the impact of changes in the variable trade cost and labor market frictions on trade openness, ρ . Combining these two stages, we can relate sectoral wage inequality and unemployment to the exogenous parameters of the model.

For the first stage, we take a Taylor series approximation of sectoral wage inequality, T_w , and the sectoral hiring rate, σ , which implies

Lemma 3
$$T_w \sim \mu - \ln(1 + \mu) - \frac{\beta k f_x}{\delta \gamma f_d} \rho^z \ln \rho \text{ and } \sigma \sim \sigma^A \left(1 - \frac{f_x}{f_d} \rho^z\right) \text{ when } \rho \approx 0.$$

Proof. See the Appendix. ■

In an equilibrium in which a small fraction of firms export, Lemma 3 implies that sectoral wage inequality is increasing in the fraction of firms that export. Since z > 2, it follows that sectoral wage inequality is monotonically increasing in trade openness in the neighborhood of such an equilibrium. In an equilibrium of this type, the sectoral hiring rate, σ , is increasing in the autarkic hiring rate, σ^A , and decreasing in the fraction of firms that export, ρ^z . As sectoral unemployment, $u = 1 - \sigma x$, is decreasing in the sectoral hiring rate, it follows that sectoral unemployment is monotonically increasing in trade openness in the neighborhood of such an equilibrium.

Having established that sectoral wage inequality and sectoral unemployment are both increasing in trade openness when a small fraction of firms export, we now turn to examining the relationship between trade openness and the exogenous parameters of the model. We show

Lemma 4 When the two countries are nearly symmetric ($b \approx b^*$ and $c \approx c^*$), ρ decreases in τ , b/b^* and c/c^* .

Proof. See the Appendix.

>From Lemma 4, a lower variable cost of trade, lower home labor market frictions, and higher foreign labor market frictions increase home's trade openness and its fraction of firms that export. The intuition for these results is as follows. First, a lower variable trade cost, τ , raises export market revenue relative to domestic market revenue, which increases the fraction of home firms that export (a rise in ρ). Second, lower home labor market frictions increase real consumption in the differentiated sector in home relative to that sector in foreign. This in turn intensifies product market competition in the home market relative to that in the foreign market, which makes exporting more attractive and increases the fraction of home firms that export (a rise in ρ). Third, lower labor market frictions in the foreign country have precisely the opposite effect (a fall in ρ). Indeed, Lemma 4 implies that when the two countries are nearly symmetric, trade openness depends on relative labor market frictions in the two countries.⁴⁸ Finally, the two dimensions of labor market friction, search cost, b, and screening cost, c, have similar effects on trade openness

⁴⁸Note that a proportional change in labor market frictions in two symmetric countries, which keeps $b = b^*$ and $c = c^*$, has no effect on trade openness, because in this case $\Upsilon_x = 1 + \tau^{-\beta/(1-\beta)}$ is independent of labor market frictions, and therefore from (35), ρ is also independent of labor market frictions.

in this case, because an increase in either dimension of labor market friction leads to a reduction in the size of the differentiated sector.

Now that we have examined how sectoral wage inequality and unemployment are related to trade openness, and have also examined how trade openness is related to the exogenous parameters of the model, we are in a position to state the following:

Proposition 6 Consider an equilibrium in which a small fraction of firms export in the differentiated sector $(\rho \approx 0)$ and the two countries face similar levels of labor market frictions $(b \approx b^*)$ and $c \approx c^*$. Then: (i) a reduction in the variable trade cost increases wage inequality and unemployment in the differentiated sector of every country; (ii) a rise in the foreign country's labor market frictions or a reduction in the home country's labor market frictions increase wage inequality in the home country's differentiated sector and reduce wage inequality in the foreign country's differentiated sector; (iii) a rise in the foreign country's labor market frictions raises unemployment in the home country, while a rise in the home country's labor market frictions raises unemployment in the foreign country; and (iv) a rise in a country's labor market frictions can raise or reduce its own rate of unemployment.

Proof. The impacts of changes in the variable trade cost and labor market frictions on sectoral wage inequality and unemployment in the differentiated sector follow from Lemmas 3 and 4 together with the determinants of labor market tightness in (20) and unemployment in (30). ■

The comparative statics for sectoral wage inequality in Proposition 6 are intuitive. Lower variable trade costs, lower home labor market frictions, or higher foreign labor market frictions raise the fraction of home firms that export. Starting from an equilibrium in which a small fraction of firms export, this increase in the fraction of exporters raises home sectoral wage inequality, because exporters pay higher wages than nonexporters.

The comparative statics of sectoral unemployment in Proposition 6 are more subtle. The variable trade cost and foreign labor market frictions only affect sectoral unemployment in home through trade openness and the fraction of firms that export. Therefore a lower variable trade cost and higher foreign labor market frictions increase sectoral unemployment in home by raising the fraction of firms that export. In contrast, home's labor market frictions affect its sectoral unemployment rate through its autarkic sectoral hiring rate, σ^A , the tightness of its labor market, x, and its fraction of firms that export, ρ^z . As a result, the impact of a reduction in home's labor market friction depends on whether this reduction arises from a lower screening cost, c, or a lower search cost, b. On the one hand, a lower home screening cost reduces the autarkic sectoral hiring rate and increases the fraction of firms that export, both of which raise sectoral unemployment in the home country. On the other hand, a lower home search cost increases the tightness of the labor market and the fraction of firms that export. While the increase in tightness reduces sectoral unemployment, the increase in the fraction of firms that export raises sectoral unemployment. The net effect of a lower home search cost on sectoral unemployment is therefore ambiguous.

Having established how changes in the exogenous parameters affect sectoral wage inequality and unemployment, it is straightforward to derive their effect on sectoral income inequality (31). Lower variable trade costs, higher foreign labor market frictions and lower home screening frictions raise home sectoral income inequality, because they increase both sectoral wage inequality and unemployment. In contrast, lower home search frictions have an ambiguous effect on home sectoral income inequality, because they raise wage inequality but have an ambiguous effect on sectoral unemployment.

To examine how the exogenous parameters of the model influence aggregate unemployment and aggregate income inequality, we need to take into account changes in sectoral composition. In the Appendix, we derive comparative statics for the size of the differentiated sector in the neighborhood of an equilibrium with only small asymmetries between the two countries ($b \approx b^*$ and $c \approx c^*$). From this analysis, lower variable trade costs increase aggregate unemployment and income inequality in both countries. The reason is that they not only increase sectoral unemployment and income inequality, as discussed above, but also increase the share of the labor force employed in the differentiated sector, which has higher unemployment and income inequality than the homogeneous sector.

In contrast, relative labor market frictions in the two countries affect sectoral composition through comparative advantage. A reduction in labor market frictions in home relative to those in foreign causes the share of the labor force employed in the differentiated sector to expand in home and contract in foreign. Therefore higher foreign labor market frictions and lower home screening frictions increase home aggregate unemployment and income inequality. These parameter changes not only raise home sectoral unemployment and income inequality, as discussed above, but also raise the share of the home country's labor force employed in the high unemployment and high income inequality sector. In contrast, lower home search frictions have an ambiguous effect on aggregate unemployment and income inequality. Although they increase the share of the labor force employed in the high unemployment and high income inequality sector, they have an ambiguous effect on unemployment and income inequality within this sector, as discussed above.

Finally, comparative statics can also be derived for aggregate wage inequality, but are somewhat more nuanced. As discussed in Section 3.5, a change in the share of the labor force employed in the differentiated sector has an ambiguous effect on aggregate wage inequality, depending on the initial share of this sector in the labor force. Therefore the impact of changes in parameters on aggregate wage inequality is in general ambiguous. This completes our characterization of the model's comparative statics for equilibria in which only a small fraction of firms export ($\rho \approx 0$) and asymmetries between the two countries are small ($b \approx b^*$ and $c \approx c^*$).

We now turn to consider the case in which an arbitrary fraction of firms export and the two countries have arbitrary levels of labor market frictions. In this case there is a nonmonotonic relationship between sectoral unemployment and inequality and the exogenous parameters of the

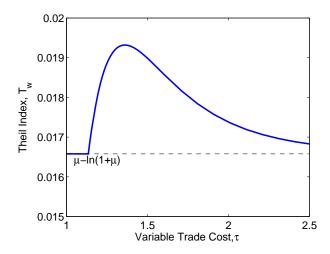


Figure 4: Wage inequality as a function of the variable trade cost

model.⁴⁹ This nonmonotonicity arises in both stages of our analysis: the impact of trade openness on sectoral wage inequality and unemployment, and the impact of exogenous parameters on trade openness. For the first stage of the analysis, the Corollary to Proposition 2 has already established that a rise in trade openness can increase sectoral wage inequality when the fraction of exporting firms is small and can decrease sectoral wage inequality when the fraction of exporting firms is large. For the second stage of the analysis, the nonmonotonic impact of exogenous parameters on trade openness can be seen from the relationship in (35) between ρ and Υ_x . As market access depends on relative real consumption of the differentiated product in the two countries (see (15)), it depends on the full general equilibrium of the model. Therefore the relationship between trade openness and any one parameter depends, through the full general equilibrium of the model, on the values of all the other parameters. In Figures 4 and 5 we provide examples for particular parameter values in which sectoral wage inequality has a nonmonotonic relationship with the variable trade cost, τ , and the home country's search cost, b, respectively.⁵⁰

These results have several implications for empirical work on the relationship between openness to trade, unemployment, and inequality. The robust predictions of the model are the propositions concerning the opening of a closed economy to international trade in Section 3, which were derived without making assumptions about the fraction of exporting firms or the level of labor market frictions in the two countries. Once an economy is open to international trade, however, the relationships between unemployment, inequality, and trade openness become subtle and depend on the initial equilibrium. Helpman and Itskhoki (2008) pointed out already that cross-country differences in rates of unemployment need not be positively correlated with cross-country differences in labor market frictions, and this warning also applies in our more general model. Moreover, because we

⁴⁹Nevertheless, it is possible to derive some general relationships between unemployment and the model's parameters, as discussed in the Appendix.

⁵⁰See the Appendix for further details.

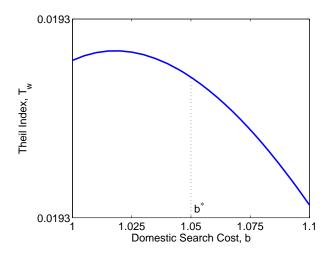


Figure 5: Wage inequality as a function of the home country's search cost

allow firms to screen workers, differences in screening costs across countries have an independent effect on unemployment, which can be in the opposite direction to search costs. As for inequality, which has not been studied by Helpman and Itskhoki (2008), our theoretical findings imply that estimates of the relationship between the degree of openness and inequality need, at a minimum, to allow for nonlinearities with positive and negative effects. In other words, the marginal impact of trade openness on inequality should be allowed to vary across countries conditional on their exposure to foreign trade. Formulating estimates that do not allow for this type of heterogeneity is likely to lead to misleading results.

5 Conclusion

The relationship between globalization and inequality is one of the most contested topics in economics. Traditionally, research has approached this issue from the perspective of neoclassical trade theory with its emphasis on specialization across industries and changes in relative factor rewards. In this paper we propose an alternative framework that explicitly recognizes heterogeneity across firms and workers within industries as well as labor market frictions. Both features are realistic aspects of economies and their inclusion yields interesting predictions for the effects of globalization on inequality. In contrast to traditional trade theory, our framework predicts that trade liberalization can enhance income inequality in both developed and developing countries; these changes are driven by residual inequality that is unexplained by observed characteristics; income inequality is influenced by wage inequality and unemployment; and both dimensions of income inequality depend on reallocations of workers across firms within industries as well as reallocations of workers across industries.

Our central theoretical results relate to the opening of a closed economy to international trade. While both countries experience welfare gains from trade, unemployment and inequality within the differentiated sector are higher in a trade equilibrium in which only some firms export than in autarky. The intuition is that trade changes industry composition by reallocating resources from low- to high-productivity firms, which pay discretely higher wages and have more selective recruitment policies. As a result, both wage inequality and unemployment within the differentiated sector rise. Once an economy is open to international trade, however, the relationship between wage inequality and trade openness is nonmonotonic. On the one hand, when nearly all firms serve solely the domestic market, an increase in trade openness raises wage inequality by expanding the small number of exporting firms that pay a wage premium. On the other hand, when nearly all firms export, an increase in trade openness reduces wage inequality by further reducing the small number of firms that serve only the domestic market and pay low wages. In the trade equilibrium, the fraction of exporting firms and the wage premium paid by exporters depend on labor market frictions in the two countries. Labor market frictions therefore affect unemployment and inequality within the differentiated sector as well as the allocation of resources across sectors. Aggregate unemployment and inequality depend on the allocation of resources across sectors as well as on unemployment and inequality within sectors, complicating the empirical relationship across countries between unemployment, inequality, and trade openness.

While our model enables us to explore how trade liberalization affects unemployment and inequality in general equilibrium, it is necessarily an abstraction, and there remain a number of areas for further research. In our model, the effect of trade liberalization on unemployment and inequality varies with a worker's unobserved ability. The reason is that more productive high-wage firms also have more selective recruitment policies, and therefore do not employ some low-ability workers who are employed by less productive low-wage firms. As a result, high-ability workers face different wage and employment distributions than low-ability workers, and therefore are differentially affected by trade liberalization. This relationship between worker ability and the effects of trade liberalization on unemployment and inequality is itself worthy of further inquiry. Additionally, while we focus on changes in residual inequality, because this has been shown to be empirically important and has received little attention in existing work in international trade, it would also be interesting to consider multiple factors of production that differ in observed characteristics. Finally, the model could be extended to consider other dimensions of international integration, such as foreign direct investment. The tractability of our framework lends itself to these and other extensions.

Appendix

A Complete Closed-Form Solution

A.1 Division of Revenue in the Bargaining Game

Let $w(\theta, h)$ be the equilibrium wage that a θ -firm has to pay as a function of the measure of workers hired h. Then, following Stole and Zwiebel (1996a,b), this function satisfies the differential equation:

$$\frac{\partial}{\partial h} \left[r(\theta, h) - w(\theta, h) h \right] = w(\theta, h)$$

when the workers' outside option is zero, where $r(\theta, h)$ is the revenue from sales of the firm's variety when it hires h workers. Using the functional forms in the text this differential equation yields the solution

$$w(\theta, h) = \frac{\beta \gamma}{1 + \beta \gamma} \frac{r(\theta, h)}{h}.$$

The worker's share of surplus is increasing in $\beta\gamma$, that is decreasing in the concavity of the revenue function in h, where β comes from concavity of demand and γ comes from concavity of the production technology. A more concave revenue function implies a smaller effect of the departure of any given worker on firm revenue. See Helpman and Itskhoki (2008) for a derivation of equilibrium wages in a model with firing costs, unemployment benefits and unequal bargaining weights, and Blanchard and Giavazzi (2003) for a similar result in a different framework.

A.2 Problem of the Firm

Combining the two first-order conditions (8) and (9) we obtain a relationship between $n(\theta)$ and $a_c(\theta)$:

$$(1 - \gamma k)bn(\theta) = \gamma ca_c(\theta)^{\delta}.$$

Using the definition of $r(\theta)$, we can solve explicitly for

$$\begin{split} n(\theta) &= \phi_1 \phi_2^{\beta(1-\gamma k)} c^{-\frac{\beta(1-\gamma k)}{\delta \Gamma}} b^{-\frac{\beta\gamma+\Gamma}{\Gamma}} \Upsilon\left(\theta\right)^{\frac{1-\beta}{\Gamma}} Q^{-\frac{\beta-\zeta}{\Gamma}} \theta^{\frac{\beta}{\Gamma}}, \\ a_c(\theta) &= \phi_1^{1/\delta} \phi_2^{1-\beta\gamma} c^{-\frac{1-\beta\gamma}{\delta \Gamma}} b^{-\frac{\beta\gamma}{\delta \Gamma}} \Upsilon\left(\theta\right)^{\frac{1-\beta}{\delta \Gamma}} Q^{-\frac{\beta-\zeta}{\delta \Gamma}} \theta^{\frac{\beta}{\delta \Gamma}}, \end{split}$$

where we have introduced two constants:

$$\phi_1 \equiv \left[\frac{\beta \gamma}{1 + \beta \gamma} \left(\frac{k a_{\min}^{\gamma k}}{k - 1} \right)^{\beta} \right]^{\frac{1}{\Gamma}}$$
 and $\phi_2 \equiv \left(\frac{1 - \gamma k}{\gamma} \right)^{\frac{1}{\delta \Gamma}}$

and, as in the text, $\Upsilon(\theta) = \Upsilon_d = 1$ for $\theta \in [\theta_d, \theta_x)$ and $\Upsilon(\theta) = \Upsilon_x = 1 + \tau^{-\beta/(1-\beta)} (Q^*/Q)^{-(\beta-\zeta)/(1-\beta)}$ for

 $\theta \geq \theta_x$. Also we solve for

$$\begin{split} \frac{\beta\gamma}{1+\beta\gamma}r(\theta) &= bn(\theta) = \phi_1\phi_2^{\beta(1-\gamma k)}c^{-\frac{\beta(1-\gamma k)}{\delta\Gamma}}b^{-\frac{\beta\gamma}{\Gamma}}\Upsilon\left(\theta\right)^{\frac{1-\beta}{\Gamma}}Q^{-\frac{\beta-\zeta}{\Gamma}}\theta^{\frac{\beta}{\Gamma}},\\ \pi(\theta) + f_d + I_x(\theta)f_x &= \frac{\Gamma}{1+\beta\gamma}r(\theta) = \frac{\Gamma}{\beta\gamma}\phi_1\phi_2^{\beta(1-\gamma k)}c^{-\frac{\beta(1-\gamma k)}{\delta\Gamma}}b^{-\frac{\beta\gamma}{\Gamma}}\Upsilon\left(\theta\right)^{\frac{1-\beta}{\Gamma}}Q^{-\frac{\beta-\zeta}{\Gamma}}\theta^{\frac{\beta}{\Gamma}},\\ h(\theta) &= n(\theta)\left(\frac{a_{\min}}{a_c(\theta)}\right)^k = a_{\min}^k\phi_1^{(1-k/\delta)}\phi_2^{-(k-\beta)}c^{\frac{k-\beta}{\delta\Gamma}}b^{-\frac{1-\beta/\delta}{\Gamma}}\Upsilon\left(\theta\right)^{\frac{(1-\beta)(1-k/\delta)}{\Gamma}}Q^{-\frac{(\beta-\zeta)(1-k/\delta)}{\Gamma}}\theta^{\frac{\beta(1-k/\delta)}{\Gamma}}, \end{split}$$

so that $\kappa_r \equiv \phi_1 \phi_2^{\beta(1-\gamma k)}$. Finally, we solve for the wage rate:

$$w(\theta) = \frac{\beta \gamma}{1 + \beta \gamma} \frac{r(\theta)}{h(\theta)} = b \frac{n(\theta)}{h(\theta)} = b \left(\frac{a_c(\theta)}{a_{\min}}\right)^k$$
$$= a_{\min}^{-k} \phi_1^{k/\delta} \phi_2^{(1-\beta\gamma)k} c^{-\frac{(1-\beta\gamma)k}{\delta\Gamma}} b^{\frac{1-\beta\gamma-\beta/\delta}{\Gamma}} \Upsilon(\theta)^{\frac{(1-\beta)k}{\delta\Gamma}} Q^{-\frac{(\beta-\zeta)k}{\delta\Gamma}} \theta^{\frac{\beta k}{\delta\Gamma}}.$$

Note that we have the following relationship, which proves useful in further derivations:

$$w(\theta)h(\theta) = bn(\theta) = \frac{\beta\gamma}{1+\beta\gamma}r(\theta).$$

Now, using the zero-profit cutoff condition,

$$\pi(\theta_d) = \frac{\Gamma}{1 + \beta \gamma} r(\theta) = \frac{\Gamma}{\beta \gamma} \phi_1 \phi_2^{\beta(1 - \gamma k)} c^{-\frac{\beta(1 - \gamma k)}{\delta \Gamma}} b^{-\frac{\beta \gamma}{\Gamma}} Q^{-\frac{\beta - \zeta}{\Gamma}} \theta_d^{\frac{\beta}{\Gamma}} - f_d = 0,$$

we can express all firm-level variables solely as functions of θ/θ_d , $\Upsilon(\theta)$ and exogenous parameters of the model. Doing so results in expression (22) in the text. Further, taking the ratio of the two conditions for domestic and exporting cutoffs (16) and (17), we obtain a relationship between θ_d , θ_x and Υ_x :

$$\left(\frac{\theta_x}{\theta_d}\right)^{\beta/\Gamma} \left[\Upsilon_x^{(1-\beta)/\Gamma} - 1\right] = \frac{f_x}{f_d}.$$
(36)

Finally, we derive the split of the revenue between domestic sales and exporting given in (14). The total revenue of the firm is the sum of revenues in the two markets provided that the firm decides to export:

$$r(\theta) = Q^{-(\beta-\zeta)} q_d(\theta)^{\beta} + \tau^{-\beta} Q^{*-(\beta-\zeta)} q_x(\theta)^{\beta},$$

where $q(\theta) = q_d(\theta) + q_x(\theta)$ is the total output of the firm which it splits between the two markets. Maximization of revenue $r(\theta)$ with respect to $q_d(\theta)$ and $q_x(\theta)$ given the output level $q(\theta)$ results in the following output division:

$$\frac{q_x(\theta)}{q_d(\theta)} = \tau^{-\frac{\beta}{1-\beta}} \left(\frac{Q^*}{Q}\right)^{-\frac{\beta-\zeta}{1-\beta}} = \Upsilon_x - 1,$$

which equalizes marginal revenues in the two markets. Therefore, the ratio of exporting to domestic revenues is:

$$\frac{r_x(\theta)}{r_d(\theta)} = \tau^{-\beta} \left(\frac{Q^*}{Q}\right)^{-(\beta-\zeta)} \frac{q_x(\theta)^{\beta}}{q_d(\theta)^{\beta}} = \frac{q_x(\theta)}{q_d(\theta)} = \Upsilon_x - 1.$$

As a result, $r_d(\theta) = r(\theta)/\Upsilon_x$ and $r_x(\theta) = (\Upsilon_x - 1)r(\theta)/\Upsilon_x$ for exporting firms, i.e. for $\theta \ge \theta_x$. Since for nonexporting firms, $r_d(\theta) = r(\theta)$ and $\Upsilon(\theta) = 1$, the general optimal division rule in (14) follows.

A.3 General Equilibrium Conditions

As discussed in the main text, the first block of equilibrium conditions consists of two cutoff conditions—for domestic production and for exporting—and the free entry condition in each country; they are given by (16), (17) and (18) respectively. We provide here more details about the second block of the equilibrium system which solves for the number of firm-entrants M and the measure of workers searching for a job in the differentiated sector L. Using the equilibrium expressions for revenue from domestic sales and exports derived above, we can rewrite (19) as

$$Q^{\zeta} = \frac{1 + \beta \gamma}{\Gamma} \left[M f_d \int_{\theta_d}^{\infty} \Upsilon(\theta)^{\frac{1 - \beta}{\Gamma} - 1} \left(\frac{\theta}{\theta_d} \right)^{\frac{\beta}{\Gamma}} dG_{\theta}(\theta) + M^* f_x \frac{\Upsilon_x^{* \frac{1 - \beta}{\Gamma}}}{\Upsilon_x^{* \frac{1 - \beta}{\Gamma}} - 1} \frac{\Upsilon_x^* - 1}{\Upsilon_x^*} \int_{\theta_x^*}^{\infty} \left(\frac{\theta}{\theta_x^*} \right)^{\frac{\beta}{\Gamma}} dG_{\theta}(\theta) \right].$$

Similarly, we can rewrite (21) as

$$L = \frac{\beta \gamma}{\Gamma} M \left[f_d \int_{\theta_d}^{\infty} \left(\frac{\theta}{\theta_d} \right)^{\frac{\beta}{\Gamma}} dG_{\theta}(\theta) + f_x \int_{\theta_x}^{\infty} \left(\frac{\theta}{\theta_x} \right)^{\frac{\beta}{\Gamma}} dG_{\theta}(\theta) \right] = z \gamma f_e M,$$

where we have evaluated the integrals in the square brackets using the Pareto distribution and applied the free entry condition (18). This condition implies that L/M is constant in any equilibrium and L and M are equivalent measures of the size of the differentiated sector. Finally, observe that in a symmetric case the expression for Q^{ζ} can be considerably simplified and the two expressions above become identical up to a factor of $\beta\gamma/(1+\beta\gamma)$, as we discuss below.

A.4 Symmetric Countries Closed Form Solutions

Evaluating the integrals in the free entry condition (18) using a Pareto productivity distribution, we obtain:

$$\left(\frac{\beta}{z\Gamma - \beta}\right) f_d \left(\frac{\theta_{\min}}{\theta_d}\right)^z \left[1 + \frac{f_x}{f_d} \left(\frac{\theta_d}{\theta_x}\right)^z\right] = f_e.$$

Using (36), we can rewrite this as

$$\left(\frac{\beta}{z\Gamma - \beta}\right) f_d \left(\frac{\theta_{\min}}{\theta_d}\right)^z \left[1 + \left(\frac{f_x}{f_d}\right)^{\frac{z\Gamma - \beta}{\beta}} \left[\Upsilon_x^{\frac{1 - \beta}{\Gamma}} - 1\right]^{z\Gamma/\beta}\right] = f_e$$
(37)

Since in a symmetric equilibrium $Q = Q^*$, we have $\Upsilon_x = 1 + \tau^{-\beta/(1-\beta)}$. Therefore, (37) defines θ_d in a symmetric equilibrium. After solving for θ_d , θ_x can be obtained from (36).

Using (36) and (37), we can now derive the conditions on the parameters that ensure $\theta_x > \theta_d > \theta_{\min}$ in a symmetric equilibrium. Note that the square bracket in (37) is always greater than 1. Therefore, it is enough to require that

$$f_d > f_e \frac{z\Gamma - \beta}{\beta}$$

to ensure that $\theta_d > \theta_{\min}$ in any symmetric equilibrium. As stated in the text, high enough f_d always ensures

it. Next note from (36) that since $\Upsilon_x^{(1-\beta)/\Gamma} < 1$, it is enough to require that $f_x \geq f_d$ to ensure $\theta_x > \theta_d$ in any symmetric equilibrium, i.e., choose f_x high enough. Note that the same condition applies in Melitz (2003). Numerical simulations suggest that a much weaker condition is generally sufficient in this model.

Once we have established the equilibrium value of θ_d , we can solve for the real consumption index, Q, from the domestic productivity cutoff condition (16):

$$Q^{\beta-\zeta} = \left(\frac{\kappa_r \Gamma/f_d}{1+\beta\gamma}\right)^{\Gamma} c^{-\beta(1-\gamma k)/\delta} b^{-\beta\gamma} \theta_d^{\beta}. \tag{38}$$

Note that b and c do not affect θ_d in the symmetric equilibrium, however, they impact real consumption Q proportionally in both countries. Trade costs, on opposite, do not alter the relationship between Q and θ_d in (38), however, they reduce both θ_d and Q via free entry condition (37).

Finally, knowing Q, we can solve for the mass of firm entrants M and the measure of workers searching for a job in the differentiated sector L. Under symmetric countries the following two equalities hold:

$$L = \frac{\beta \gamma}{1 + \beta \gamma} Q^{\zeta} = z \gamma f_e M.$$

This completes the solution in the symmetric countries case.

A.5 Comparative Statics for Nearly Symmetric Countries

To obtain comparative statics, we take the first block of the equilibrium system (16)-(18) together with the definition of Υ_x in (15) and log-differentiate it around a symmetric equilibrium characterized above. We obtain the following log-differentiated system:

$$\begin{split} \hat{\theta}_{d} &= \frac{\beta - \zeta}{\beta} \hat{Q} + (1 - \gamma k) / \delta \hat{c} + \gamma \hat{b}, \\ \hat{\theta}_{d} &= \hat{\theta}_{x} - \frac{\Upsilon_{x} - 1}{\Upsilon_{x}} \frac{\Upsilon_{x}^{\frac{1 - \beta}{\Gamma}} - 1}{\Upsilon_{x}^{\frac{1 - \beta}{\Gamma}}} \left[\hat{\tau} + \frac{\beta - \zeta}{\beta} (\hat{Q}^{*} - \hat{Q}) \right], \\ \delta_{d} \hat{\theta}_{d} + \delta_{x} \hat{\theta}_{x} &= 0, \end{split}$$

where $\delta_s \equiv f_s \int_{\theta_s}^{\infty} (\theta/\theta_s)^{\beta/\Gamma} dG_{\theta}(\theta)$ for s = d, x. The first of these equations comes from the domestic productivity cutoff condition (16); the second equation comes from the exporting productivity cutoff condition (17), which also takes into account (16) and the definition of Υ_x in (15); finally, the third condition is the log-linearized version of the free entry condition (18).

To solve the system, we can eliminate $\hat{\theta}_d$ and $\hat{\theta}_x$ and rearrange to obtain:

$$(1 - \Phi) \hat{Q} + \Phi \hat{Q}^* = -\frac{\beta}{\beta - \zeta} \left[(1 - \gamma k) / \delta \hat{c} + \gamma \hat{b} + \Phi \hat{\tau} \right],$$

where

$$\Phi \equiv \frac{\delta_x}{\delta_d + \delta_x} \cdot \frac{\Upsilon_x^{\frac{1-\beta}{\Gamma}}}{\Upsilon_x^{\frac{1-\beta}{\Gamma}} - 1} \frac{\Upsilon_x - 1}{\Upsilon_x}.$$

A symmetric condition holds for the foreign country. Using them together we have a system of two equations

in two unknowns, (\hat{Q}, \hat{Q}^*) , which yields:

$$\hat{Q} = -\frac{\beta/(\beta-\zeta)}{1-\Phi-\Phi^*} \left[(1-\Phi^*) \left((1-\gamma k)/\delta \hat{c} + \gamma \hat{b} \right) - \Phi \left((1-\gamma k)/\delta \hat{c}^* + \gamma \hat{b}^* \right) + \Phi (1-2\Phi^*) \hat{\tau} \right].$$

Plugging this back into the equilibrium system, we obtain the comparative statics for production and exporting cutoffs:

$$\hat{\theta}_d = -\frac{\beta}{\beta - \zeta} \frac{\Phi}{1 - \Phi - \Phi^*} \left[(1 - \gamma k) / \delta(\hat{c} - \hat{c}^*) + \gamma(\hat{b} - \hat{b}^*) + (1 - 2\Phi^*) \hat{\tau} \right], \qquad \hat{\theta}_x = -\frac{\delta_d}{\delta_x} \hat{\theta}_d.$$

Stability of the equilibrium system requires $\Phi + \Phi^* < 1$ and for unambiguous comparative statics we need $\Phi < 1/2$ in both countries. We show below that this is indeed the case around the symmetric equilibrium, and now discuss the implications of these comparative statics assuming that these conditions hold.

First, note that the two cutoffs move in opposite directions which is the immediate implication of the free entry condition (18). Next observe that the cutoffs do not respond to proportional changes in the labor market frictions which hold c/c^* and b/b^* constant. Reduction in trade impediments increases real consumption Q and the domestic productivity cutoff θ_d , while it lowers the export productivity cutoff θ_x . This implies that both countries gain from continuous reductions in trade barriers independently of labor market frictions. Finally, improvements in labor market frictions enhance welfare at home (Q), but lower it abroad (Q^*) , while a proportional reduction in labor market frictions increases welfare in both countries. This is a generalization of the results in Helpman and Itskhoki (2007).

We now discuss the implications of these comparative statics for the size of the labor force that seeks a job in the differentiated sector, L. This comparative statics is needed to evaluate the compositional effects on unemployment and inequality analyzed in Section 3.5. First of all, by adding (19) and (21) across the countries, we obtain a relationship between the worldwide expenditure on the differentiated good and the worldwide measure of workers attaching themselves to the differentiated sector:

$$L + L^* = \frac{\beta \gamma}{1 + \beta \gamma} \left(Q^{\zeta} + Q^{*\zeta} \right).$$

When countries are symmetric, this condition holds for each country separately, but when they are asymmetric and there is net trade in differentiated goods this conditions holds only for the world economy. Since reduction in trade barriers raises Q in both countries, it has to raise the worldwide size of the labor force in the differentiated sector, $L + L^*$. When countries are nearly symmetric, both L and L^* increase in response to a fall in τ . When the asymmetries between countries are large, one can show that L increases in the country with more flexible labor market institutions which has comparative advantage in the differentiated sector. In addition, it is possible to show that under certain conditions a reduction in b and c or an increase in b^* and c^* increase L and reduce L^* . We omit this tedious proof for brevity.

To complete the analysis, we show that $\Phi < 1/2$ in the environ of a symmetric equilibrium. Using the definition of δ_s , we have:

$$\frac{\delta_x}{\delta_d + \delta_x} = \frac{f_x \theta_x^{-z}}{f_d \theta_d^{-z} + f_x \theta_x^{-z}} = \frac{1}{1 + \left(\frac{f_x}{f_d}\right)^{\frac{z\Gamma - \beta}{\beta}} \left(\Upsilon_x^{(1-\beta)/\Gamma} - 1\right)^{-z\Gamma/\beta}} < \frac{1}{2},$$

where the second equality comes from (36) and the inequality holds due to $\Upsilon_x^{(1-\beta)/\Gamma} - 1 < 1$ and our assumption that $f_x \geq f_d$. This however is not enough to guarantee $\Phi < 1/2$ since the remaining term is greater than 1. Altogether we have:

$$\Phi = \frac{1}{1 + \left(\frac{f_x}{f_d}\right)^{\frac{z\Gamma - \beta}{\beta}} \left(\Upsilon_x^{(1-\beta)/\Gamma} - 1\right)^{-z\Gamma/\beta} \frac{\Upsilon_x^{\frac{1-\beta}{\Gamma}}}{\Upsilon_x^{\frac{1-\beta}{\Gamma}}} - 1} \frac{\Upsilon_x - 1}{\Upsilon_x}.$$

Note that when countries are symmetric, $\Upsilon_x = 1 + \tau^{-\beta/(1-\beta)}$ and, therefore, $\Phi < 1/2$ that constitutes a valid restriction on the exogenous parameters of the model. Moreover, as trade costs τ or f_x increase, Φ decreases towards 0. As a result, there always exists a lower bound on f_x and τ such that for any higher trade costs $\Phi < 1/2$ is satisfied. To see how restrictive this requirement is, let us fix $f_x = f_d$ (recall that this is the lowest level of f_x in the Melitz (2003) model which ensures $\theta_x > \theta_d$ when τ is low). We also set the variable trade cost, τ , to its lowest value of 1 and verify numerically that $\Phi < 1/2$ holds for all admissible values of z, β and Γ .⁵¹ Thus, we conclude that the condition $\Phi < 1/2$ is not restrictive and, in particular, is implied by the empirically motivated assumption that $\theta_x > \theta_d$. By continuity, the same arguments apply when the asymmetries between countries are small and numerical simulations suggest that generally $\Phi < 1/2$ in both countries even when asymmetries are large.

B Derivation of Results in Section 3

B.1 Derivation of Results in Section 3.2

B.1.1 Wage Distribution among Workers in Exporting and Nonexporting Firms

The share of workers employed by firms that serve only the domestic market is from (22) and the Pareto productivity distribution:

$$S_{h,d} = 1 - \frac{\int_{\theta_x}^{\infty} h\left(\theta\right) dG_{\theta}\left(\theta\right)}{\int_{\theta_d}^{\infty} h\left(\theta\right) dG_{\theta}\left(\theta\right)} = \frac{1 - \rho^{z - \frac{\beta(1 - k/\delta)}{\Gamma}}}{1 + \rho^{z - \frac{\beta(1 - k/\delta)}{\Gamma}} \left[\Upsilon_x^{\frac{(1 - \beta)(1 - k/\delta)}{\Gamma}} - 1\right]},$$

where $\rho \equiv \theta_d/\theta_x$. To compute the distribution of wages across workers employed by non-exporting firms, note that the fraction of workers receiving a particular wage $w(\theta) \in [w_d, w_d/\rho^{\beta k/\delta\Gamma})$ is proportional to $h(\theta)dG_{\theta}(\theta)$. In other words, we have:

$$G_{w,d}(w) = \frac{M \int_{\theta_d}^{\theta_{w,d}(w)} h(\theta) dG_{\theta}(\theta)}{M \int_{\theta_d}^{\theta_x} h(\theta) dG_{\theta}(\theta)} = 1 - \frac{\int_{\theta_{w,d}(w)}^{\theta_x} h(\theta) dG_{\theta}(\theta)}{\int_{\theta_d}^{\theta_x} h(\theta) dG_{\theta}(\theta)} \quad \text{for } w \in \left[w_d, w_d / \rho^{\beta k / \delta \Gamma} \right],$$

where $\theta_{w,d}(\cdot)$ is the inverse of $w(\cdot)$ and equal to $\theta_{w,d}(w) = \theta_d(w/w_d)^{\delta\Gamma/(\beta k)}$. Finally, for $w < w_d$, $G_{w,d}(w) = 0$, and for $w > w_d/\rho^{\beta k/\delta\Gamma}$, $G_{w,d}(w) = 1$. Using the Pareto productivity distribution, the distribution of

⁵¹Note that Φ monotonically decreases in z. Therefore, we set it to its lowest value of z=2. The restriction on the parameters of the model imply $\Gamma > 1 - \beta/2$. We evaluate Φ for all $\beta \in (0,1)$ and $\Gamma \in (1 - \beta/2,1)$ on a very detailed grid. $\Phi \to 1/2$ when $\beta \to 0$, but otherwise is separated from 1/2 and for the most part takes values around 0.2.

wages across workers employed by domestic firms is the following truncated Pareto distribution:

$$G_{w,d}(w) = \frac{1 - \left(\frac{\theta_d}{\theta_{w,d}(w)}\right)^{z - \frac{\beta(1-k/\delta)}{\Gamma}}}{1 - \left(\frac{\theta_d}{\theta_x}\right)^{z - \frac{\beta(1-k/\delta)}{\Gamma}}} = \frac{1 - \left(\frac{w_d}{w}\right)^{1+1/\mu}}{1 - \rho^{z - \frac{\beta(1-k/\delta)}{\Gamma}}}, \quad \text{for } w \in \left[w_d, w_d/\rho^{\beta k/\delta\Gamma}\right],$$

where $\mu \equiv \beta k / [\delta(z\Gamma - \beta)]$.

The distribution of wages across workers employed by exporters can be computed in the same way:

$$G_{w,x}(w) = \frac{M \int_{\theta_x}^{\theta_{w,x}(w)} h(\theta) dG_{\theta}(\theta)}{M \int_{\theta_x}^{\infty} h(\theta) dG_{\theta}(\theta)} = 1 - \frac{\int_{\theta_{w,x}(w)}^{\infty} h(\theta) dG_{\theta}(\theta)}{\int_{\theta_x}^{\infty} h(\theta) dG_{\theta}(\theta)} \quad \text{for} \quad w \in \left[w_d \Upsilon_x^{\frac{k(1-\beta)}{\delta \Gamma}} / \rho^{\beta k/\delta \Gamma}, \infty \right),$$

where $\theta_{w,x}(\cdot)$ is the inverse of $w(\cdot)$ and equal to $\theta_{w,x}(w) = \theta_d \left(w/w_d \right)^{\delta\Gamma/(\beta k)} \Upsilon_x^{\frac{k(1-\beta)}{\delta\Gamma}}/\rho^{\beta k/\delta\Gamma}$. Finally, for $w < w_d \Upsilon_x^{\frac{k(1-\beta)}{\delta\Gamma}}/\rho^{\beta k/\delta\Gamma}$, $G_{w,x}(w) = 0$. Using the Pareto productivity distribution, the distribution of wages across workers employed by exporters is the following untruncated Pareto distribution:

$$G_{w,x}(w) = 1 - \left(\frac{\theta_x}{\theta_w(w)}\right)^{z - \frac{\beta(1 - k/\delta)}{\Gamma}} = 1 - \left(\frac{w_d}{w} \Upsilon_x^{\frac{k(1 - \beta)}{\delta \Gamma}} \rho^{-\frac{\beta k}{\delta \Gamma}}\right)^{1 + 1/\mu}, \quad \text{for} \quad w \in \left[w_d \Upsilon_x^{\frac{k(1 - \beta)}{\delta \Gamma}} / \rho^{\beta k/\delta \Gamma}, \infty\right).$$

Combining $S_{h,d}$, $G_{w,d}(\cdot)$ and $G_{w,x}(\cdot)$ together we obtain the unconditional wage distribution among workers employed in the differentiated sector, $G_w(w)$, as defined in (23).

B.1.2 Theil Index of Sectoral Wage Inequality among Workers in Exporting and Nonexporting Firms

Using definition (27), the Theil index of sectoral wage inequality across workers employed by exporters is:

$$T_{w,x} = \int_{w_x}^{\infty} \frac{w}{\bar{w}_x} \ln\left(\frac{w}{\bar{w}_x}\right) dG_{w,x}(w) = \int_{w_x}^{\infty} \frac{w}{\bar{w}_x} \ln w dG_{w,x}(w) - \ln \bar{w}_x,$$

where $\bar{w}_x = (1+\mu)w_x$ is the mean of the wage distribution $G_{w,x}(\cdot)$ and $w_x = w_d \Upsilon_x^{\frac{k(1-\beta)}{\delta\Gamma}}/\rho^{\beta k/\delta\Gamma}$. Computing the integral in the formula above we obtain:⁵²

$$T_{w,x} = \frac{1}{\mu} w_x^{1/\mu} \int_{w_x}^{\infty} w^{-(1+1/\mu)} \ln w dw - \ln w_x - \ln(1+\mu) = \mu - \ln(1+\mu).$$

Note that $T_{w,x}$ is monotonically increasing in μ and $T_{w,x} = 0$ when $\mu = 0$. Importantly, the Theil Index for the untruncated Pareto distribution $G_{w,x}(w)$ does not depend on the lower limit of the wage distribution w_x and depends only on the shape parameter, μ . As the distribution of wages under autarky is also an untruncated Pareto with a lower limit of w_d rather than w_x , it follows that the Theil Index for wage inequality under autarky takes the same value as the Theil index for wage inequality in a trade equilibrium in which all firms export. This establishes the result in (28) and proves Lemma 1.

⁵² We use the facts that $G'_{w,x}(w) = (1+1/\mu)w_x^{1+1/\mu}w^{-2-1/\mu}$ and $\int x^{-\alpha} \ln x dx = \frac{x^{1-\alpha}}{1-\alpha} \left(\ln x - \frac{1}{1-\alpha}\right)$ up to a constant (where $\alpha > 1$).

Next we compute the Theil index of wage inequality for workers employed in non-exporting firms:

$$T_{w,d} = \int_{w_d}^{w_x} \frac{w}{\bar{w}_d} \ln w dG_{w,d}(w) - \ln \bar{w}_d$$
$$\mu - \ln(1+\mu) + \frac{\beta k}{\delta \Gamma} \frac{\rho^{z-\beta/\Gamma} \ln \rho}{1 - \rho^{z-\beta/\Gamma}} + \ln \left(\frac{1 - \rho^{z-\beta(1-k/\delta)/\Gamma}}{1 - \rho^{z-\beta/\Gamma}} \right),$$

where

$$\bar{w}_d = \frac{1 - \rho^{z - \beta/\Gamma}}{1 - \rho^{z - \beta(1 - k/\delta)/\Gamma}} (1 + \mu) w_d$$

is the mean of $G_{w,d}(\cdot)$. As $\rho \to 1$ (i.e., there are no nonexporting firms), $T_{w,d} \to 0$, and as $\rho \to 0$ (i.e., all firms are nonexporting), $T_{w,d} \to \mu - \ln(1+\mu)$ as the conditional distribution of wages converges to the autarkic distribution. For intermediate values of $\rho \in (0,1)$, $0 < T_{w,d} < \mu - \ln(1+\mu)$. Therefore, there is less inequality among workers employed in non-exporting firms than among workers employed in exporting firms.

B.1.3 Within and Between Inequality

We can decompose the wage inequality index of all employed in the differentiated sector into the within and between components for the two groups of workers—employed in exporting and non-exporting firms. The Theil index allows decomposing overall inequality into within- and between-group components in the following way:

$$T_{\varpi} = T_{\varpi,W} + T_{\varpi,B} = \sum_{j} \left(\frac{\phi_{j} \bar{\varpi}_{j}}{\bar{\varpi}} \right) T_{j} + \sum_{j} \phi_{j} \left(\frac{\bar{\varpi}_{j}}{\bar{\varpi}} \right) \ln \left(\frac{\bar{\varpi}_{j}}{\bar{\varpi}} \right), \tag{39}$$

where ϖ is an income measure, j indexes the groups, ϕ_j is the population weight of group j, $\bar{\varpi}_j$ is the average income in group j, $\bar{\varpi}$ is the group-wide average income, and T_j is the Theil index for group j computed according to (27).⁵³ We can now easily compute the within component of inequality as the income-weighted average inequality within the two groups:

$$T_{w,W} = S_{w,d}T_{w,d} + (1 - S_{w,d})T_{w,x},$$

where

$$S_{w,d} = \frac{\bar{w}_d S_{h,d}}{\bar{w}_d S_{h,d} + \bar{w}_x S_{h,x}} = \frac{1 - \rho^{z - \beta/\Gamma}}{1 + \rho^{z - \beta/\Gamma} \left[\Upsilon_x^{(1-\beta)/\Gamma} - 1 \right]}$$

is the income share of workers employed in the nonexporting firms. Note that in autarky $S_{w,d} = 1$ and $T_{w,d} = \mu - \ln(1 + \mu)$, while when all firms export $S_{w,d} = 0$. The inequality among workers in the exporting firms is always $T_{w,x} = \mu - \ln(1 + \mu)$ and in the nonexporting firms $T_{w,d} < \mu - \ln(1 + \mu)$ when $0 < \rho < 1$. The immediate implication is

Proposition 7 The within component of sectoral wage inequality is lower in any open economy equilibrium than in autarky and is the same as in autarky when all firms export. By consequence, the inequality component between workers employed in exporting and non-exporting firms is higher in any trade equilibrium and the movements in the between component dominate the movements in the within component.

⁵³A direct calculation confirms that the two alternative definitions of the aggregate Theil index, (27) and (39), are equivalent and consistent with each other (see also Bourguignon, 1979).

Proof: Since $S_{w,d} \in [0,1]$ and $T_{w,d} \leq \mu - \ln(1+\mu)$ and $T_{w,x} = \mu - \ln(1+\mu)$, it follows that $T_{w,W} \leq \mu - \ln(1+\mu)$ with equality holding for $\rho = 0$ and $\rho = 1$ and strict inequality otherwise. From Proposition 2 we know that sectoral wage inequality, $T_w = T_{w,W} + T_{w,B}$, is higher in any trade equilibrium than in autarky. This necessarily implies that the between component of inequality has to be larger in any trade equilibrium than in autarky; moreover, movements in the between component dominate those in the within component.

This proposition has an interesting implication that the source behind the inequality increase in open economy relative to autarky is the growth in inequality between workers employed in the exporting firms which pay high wages and nonexporting firms which pay low wages.

B.1.4 Actual and Counterfactual Wage Distributions

Define the following notation:

$$\begin{split} &\eta_1 \equiv \Upsilon_x^{\frac{(1-\beta)(1-k/\delta)}{\Gamma}} - 1, \qquad \eta_2 \equiv \Upsilon_x^{\frac{1-\beta}{\Gamma}} - 1, \\ &\vartheta_1 \equiv z - \frac{\beta(1-k/\delta)}{\Gamma}, \qquad \qquad \vartheta_2 \equiv z - \frac{\beta}{\Gamma}. \end{split}$$

Using this notation, the lowest wage paid by exporters and the highest wage paid by domestic firms can be written as:

$$w(\theta_x^+) = w(\theta_x^-) \frac{1+\eta_2}{1+\eta_1}$$
 and $w(\theta_x^-) = w_d \rho^{\theta_2-\theta_1}$.

Similarly, using this notation, the actual wage distribution (23) can be written as:

$$G_{w}(w) = \begin{cases} \frac{1}{1+\eta_{1}\rho^{\vartheta_{1}}} \left[1 - (w_{d}/w)^{1+1/\mu} \right], & w_{d} \leq w \leq w(\theta_{x}^{-}), \\ (1-\rho^{\vartheta_{1}})/(1+\eta_{1}\rho^{\vartheta_{1}}), & w(\theta_{x}^{-}) \leq w \leq w(\theta_{x}^{+}), \\ \frac{1-\rho^{\vartheta_{1}}}{1+\eta_{1}\rho^{\vartheta_{1}}} + \frac{(1+\eta_{1})\rho^{\vartheta_{1}}}{1+\eta_{1}\rho^{\vartheta_{1}}} \left[1 - \left(w(\theta_{x}^{+})/w \right)^{1+1/\mu} \right], & w \geq w(\theta_{x}^{+}) \end{cases}$$

$$(40)$$

and the mean of this distribution can be written as

$$\bar{w} = (1+\mu)w_d \frac{1+\eta_2 \rho^{\vartheta_2}}{1+\eta_1 \rho^{\vartheta_1}}.$$

The counterfactual wage distribution is defined as:

$$G_w^c(w) = 1 - (w_d^c/w)^{1+1/\mu}, \qquad w \ge w_d^c,$$
 (41)

where in order for the mean of the counterfactual distribution to equal \bar{w} , its lower limit must satisfy:

$$w_d^c = \frac{1 + \eta_2 \rho^{\vartheta_2}}{1 + \eta_1 \rho^{\vartheta_1}} w_d.$$

Therefore we can establish the following result:

$$w_d^c > w_d$$
 since $\frac{1 + \eta_2 \rho^{\vartheta_2}}{1 + \eta_1 \rho^{\vartheta_1}} > 1$ for $0 < \rho < 1$,

as $0 < \eta_1 < \eta_2 < 1$ and $1 < z/2 < \vartheta_2 < \vartheta_1 < z$. Similarly, we can establish:

$$w_d^c < w(\theta_x^+)$$
 since $\frac{1 + \eta_2 \rho^{\vartheta_2}}{1 + \eta_1 \rho^{\vartheta_1}} < \frac{1 + \eta_2}{1 + \eta_1} \rho^{\vartheta_2 - \vartheta_1} = \frac{(1 + \eta_2) \rho^{\vartheta_2}}{(1 + \eta_1) \rho^{\vartheta_1}}$

with the inequality being satisfied since:

$$\frac{1+\eta_2\rho^{\vartheta_2}}{1+\eta_1\rho^{\vartheta_1}} = \frac{(1+\eta_2)\rho^{\vartheta_2} + (1-\rho^{\vartheta_2})}{(1+\eta_1)\rho^{\vartheta_1} + (1-\rho^{\vartheta_1})} = \frac{(1+\eta_2)\rho^{\vartheta_2}}{(1+\eta_1)\rho^{\vartheta_1}} \cdot \frac{1+\frac{1-\rho^{\vartheta_2}}{(1+\eta_2)\rho^{\vartheta_2}}}{1+\frac{1-\rho^{\vartheta_1}}{(1+\eta_1)\rho^{\vartheta_1}}} < \frac{(1+\eta_2)\rho^{\vartheta_2}}{(1+\eta_1)\rho^{\vartheta_1}},$$

as $\rho^{\vartheta_1} < \rho^{\vartheta_2} < 1$ and $(1 + \eta_2) > (1 + \eta_1)$. Note that in general we can have either $w_d^c > w(\theta_x^-)$ or $w_d^c < w(\theta_x^-)$, but the same arguments apply in both cases.

We can also show that the slope of the counterfactual wage distribution is smaller than the slope of the actual wage distribution at $w\left(\theta_{x}^{+}\right)^{+}$: $g_{w}\left(w(\theta_{x}^{+})\right) > g_{w}^{c}\left(w(\theta_{x}^{+})\right)$. Since the truncations of $G_{w}(w)$ and $G_{w}^{c}(w)$ at $w(\theta_{x}^{+})$ are both Pareto with shape parameter $(1+1/\mu)$, we can show that $g_{w}\left(w(\theta_{x}^{+})\right) > g_{w}^{c}\left(w(\theta_{x}^{+})\right)$ by establishing that $1 - G_{w}\left(w(\theta_{x}^{+})\right) > 1 - G_{w}^{c}\left(w(\theta_{x}^{+})\right)$. From (40) and (41), this implies:

$$1 - \frac{1 - \rho^{\vartheta_1}}{1 + \eta_1 \rho^{\vartheta_1}} > \left(\frac{w_d^c}{w(\theta_x^+)}\right)^{1 + 1/\mu} \qquad \Leftrightarrow \qquad \frac{(1 + \eta_1)\rho^{\vartheta_1}}{1 + \eta_1 \rho^{\vartheta_1}} > \left(\frac{1 + \eta_2 \rho^{\vartheta_2}}{1 + \eta_1 \rho^{\vartheta_1}} \frac{1 + \eta_1}{1 + \eta_2}\right)^{\frac{\vartheta_1}{\vartheta_1 - \vartheta_2}} \rho^{\vartheta_1}$$

$$\Leftrightarrow \qquad \phi(\rho) \equiv \left(\frac{1 + \eta_1 \rho^{\vartheta_1}}{1 + \eta_1}\right)^{\vartheta_2} - \left(\frac{1 + \eta_2 \rho^{\vartheta_2}}{1 + \eta_2}\right)^{\vartheta_1} > 0.$$

To show that $\phi(\rho) > 0$ for all $\rho \in [0,1)$, note that:

$$\phi\left(0\right) \equiv \left(\frac{1}{1+\eta_{1}}\right)^{\vartheta_{2}} - \left(\frac{1}{1+\eta_{2}}\right)^{\vartheta_{1}} > 0,$$

as $\eta_1 < \eta_2$ and $\vartheta_1 > \vartheta_2$. Note also that $\phi(1) = 1 - 1 = 0$. Consider now the derivative of $\phi(\rho)$ for $\rho \in (0, 1]$:

$$\phi'(\rho) = \frac{\eta_1\eta_2}{\rho} \left[\left(\frac{1+\eta_1\rho^{\vartheta_1}}{1+\eta_1} \right)^{\vartheta_2} \frac{\eta_1\rho^{\vartheta_1}}{1+\eta_1\rho^{\vartheta_1}} - \left(\frac{1+\eta_2\rho^{\vartheta_2}}{1+\eta_2} \right)^{\vartheta_1} \frac{\eta_2\rho^{\vartheta_2}}{1+\eta_2\rho^{\vartheta_2}} \right].$$

Note that

$$\frac{\eta_2 \rho^{\vartheta_2}}{1 + \eta_2 \rho^{\vartheta_2}} > \frac{\eta_1 \rho^{\vartheta_1}}{1 + \eta_1 \rho^{\vartheta_1}},$$

since $\eta_1 < \eta_2$ and $\rho^{\vartheta_1} < \rho^{\vartheta_2}$. As a result, whenever $\phi(\rho) \le 0$, we also necessarily have $\phi'(\rho) < 0$. Therefore, if there exists ρ' such that $\phi(\rho') = 0$, then $\phi(\rho) < 0$ for all $\rho > \rho'$. But since $\phi(1) = 0$, this implies that $\phi(\rho) > 0$ for all $\rho \in (0,1)$.

We now establish that $G_w^c(w)$ second-order stochastically dominates $G_w(w)$ for $\rho \in (0,1)$. Since the two distributions have the same mean, we have from the definition of the Theil index (27):

$$T_{G_w(\cdot)} - T_{G_w^c(\cdot)} = \frac{1}{\bar{w}} \int_{w_{-}}^{\infty} w \ln w \left[dG_w(w) - dG_w^c(w) \right],$$

where $dG_w^c(w) = 0$ for $w \in [w_d, w_d^c)$. We introduce the following notation:

$$\Delta = \bar{w} \cdot \left(T_{G_w(\cdot)} - T_{G_w^c(\cdot)} \right),\,$$

and it remains to show that $\Delta > 0$. Note that $w \ln w$ is a convex function. Therefore, it remains to show that $G_w(w)$ is second-order stochastically dominated by $G_w^c(w)$. Using the fact that the truncations of $G_w(w)$ and $G_w^c(w)$ at $w(\theta_x^+)$ are both Pareto with shape parameter $(1 + 1/\mu)$, and the result above that $g_w\left(w(\theta_x^+)\right) > g_w^c\left(w(\theta_x^+)\right)$, we know that this inequality holds for all $w > w(\theta_x^+)$. We have two cases:

1. $w(\theta_r^-) \le w_d^c < w(\theta_r^+)$:

$$g_w(w) - g_w^c(w) = \begin{cases} > 0, & w_d \le w < w(\theta_x^-), \\ = 0, & w(\theta_x^-) \le w < w_d^c, \\ < 0, & w_d^c \le w < w(\theta_x^+), \\ > 0, & w \ge w(\theta_x^+). \end{cases}$$

2. $w_d^c < w(\theta_r^-)$:

$$g_w(w) - g_w^c(w) = \begin{cases} > 0, & w_d \le w < w_d^c, \\ \le 0, & w_d^c \le w < w(\theta_x^-), \\ < 0, & w(\theta_x^-) \le w < w(\theta_x^+), \\ > 0, & w \ge w(\theta_x^+). \end{cases}$$

Importantly, $g_w(w) - g_w^c(w)$ takes either only positive or only negative values in the range $[w_d^c, w(\theta_x^-))$, since for this range

$$g_w(w) - g_w^c(w) = (C - C^c) w^{-(2+1/\mu)}$$

where C and C^c are positive constants.

Note that in both cases the above characterization of $g_w(w) - g_w^c(w)$ implies that this difference of density functions is positive for low values of w, negative for intermediate values of w, and again positive for larger values of w. This immediately implies that the cumulative distribution functions intersect only once in the range where the difference of density functions is negative (see Figure 2 in the text), which is a sufficient condition to establish that indeed $G_w^c(w)$ second-order stochastically dominates $G_w(w)$ (see, for example, Mas-Colell, Whinston and Green 1995, p.195).

B.2 Derivation of Results in Section 3.3

Using the solution for firm-specific variables (22), the expression for the sectoral hiring rate σ in Section 3.3 can be rewritten as:

$$\sigma = \frac{h_d \int_{\theta_d}^{\theta_x} \left(\frac{\theta}{\theta_d}\right)^{\frac{\beta(1-k/\delta)}{\Gamma}} dG_{\theta}\left(\theta\right) + h_x \int_{\theta_x}^{\infty} \left(\frac{\theta}{\theta_x}\right)^{\frac{\beta(1-k/\delta)}{\Gamma}} dG_{\theta}\left(\theta\right)}{n_d \int_{\theta_d}^{\theta_x} \left(\frac{\theta}{\theta_d}\right)^{\beta/\Gamma} dG_{\theta}\left(\theta\right) + n_x \int_{\theta_x}^{\infty} \left(\frac{\theta}{\theta_x}\right)^{\beta/\Gamma} dG_{\theta}\left(\theta\right)},$$

where $h_x = \Upsilon_x^{\frac{(1-\beta)(1-k/\delta)}{\Gamma}} (\theta_x/\theta_d)^{\frac{\beta(1-k/\delta)}{\Gamma}} h_d$ and $n_x = \Upsilon_x^{\frac{1-\beta}{\Gamma}} (\theta_x/\theta_d)^{\frac{\beta}{\Gamma}} n_d$. Evaluating the integrals using the Pareto productivity distribution yields (30) in the main text. Using the notation introduced in Appendix Section B.1, (30) can be re-written as:

$$\sigma = \frac{1 + \eta_1 \rho^{\vartheta_1}}{1 + \eta_2 \rho^{\vartheta_2}} \sigma_A < \sigma_A,$$

since $0 < \rho \le 1$, $\eta_1 < \eta_2$ and $\vartheta_1 > \vartheta_2$. Therefore, the hiring rate is lower in any open economy equilibrium than in autarky. Since the sectoral unemployment rate is equal to $u = 1 - \sigma x$, where x = 1/b does not depend on the degree of openness, the sectoral unemployment rate is higher in any open economy equilibrium than in autarky.

B.3 Derivation of Results in Section 3.4

Using the decomposition of the Theil index introduced in (39), we can write $T_{\iota} = T_{\iota W} + T_{\iota B}$, where we split the workers attached to the differentiated sector into employed and unemployed. Consider first the within-group component, $T_{\iota W}$. All unemployed receive the same income of zero so that the Theil index for them is $T_u = 0$. Additionally, the share of unemployed in income is zero and the share of employed is 1. Therefore, the within-group component of income inequality is:

$$T_{\iota W} = 0 \cdot T_{\iota \iota} + 1 \cdot T_{\iota \iota} = T_{\iota \iota}.$$

Next consider the between-group component, $T_{\iota B}$. We have:

$$T_{\iota B} = 0 \cdot \ln 0 + 1 \cdot \ln[1/(1-u)] = -\ln(1-u),$$

where $1/(1-u) = \bar{w}/\bar{\iota}$ is the ratio of the average income of the employed to the average income in the population, since $\bar{\iota} = u \cdot 0 + (1-u) \cdot \bar{w}$. Combining these expressions for the within- and between-group components yields the expression for sectoral income inequality in (31) in the main text. In fact, this constitutes a proof of a more general result:

Lemma 5 Let u be the share of unemployed in the population with no income and T_w be the Theil index of wage inequality among the employed with wages constituting the only source of income. Then the Theil index of income inequality in the population is given by $T_t = T_w - \ln(1-u)$.

B.4 Derivation of Results in Section 3.5

We showed in Section A.5 that L increases in both countries as variable trade costs, τ , fall if countries are nearly symmetric in their labor market frictions. This implies that trade shifts resources towards the differentiated sector which constitutes the compositional effect discussed in the text.

Next we compute the aggregate Theil index of income inequality using the decomposition provided in (39). Consider the between component first. Since average income in both sectors is the same and equal to 1 (due to the workers' indifference condition between the sectors), we have $\mathbf{T}_{\iota B} = 0$. Further, L/\bar{L} constitutes the income share of the workers attached to the differentiated sector and T_{ι} , defined in (31), is the Theil index of income inequality in the differentiated sector. In addition, all workers in the homogeneous-good sector receive a constant wage of one; therefore, the Theil index of income inequality in the homogeneous sector is zero. Combining these arguments together, we have the within component: $\mathbf{T}_{\iota W} = L/\bar{L} \cdot T_{\iota}$. As a result, the aggregate index of income inequality is

$$\mathbf{T}_{\iota} = \mathbf{T}_{\iota W} + \mathbf{T}_{\iota B} = rac{L}{\overline{L}} T_{\iota},$$

as stated in (33).

As explained in the text, the aggregate index of wage inequality can be derived from the aggregate index of income inequality by twice applying Lemma 5. The same result obtains if one uses decomposition formula (39) and partitions the employed workers by sector. For more on this derivation and for the discussion of nonmonotonicity of the compositional effect see Helpman, Itskhoki and Redding (2008).

C Derivation of Results in Section 4

We now derive the Taylor approximations to the sectoral hiring rate and Theil index of wage inequality around $\rho = 0$. Consider first the hiring rate defined in (30). Note that the autarkic hiring rate, σ^A , does not depend on any parameters characterizing trade openness of the sector, including ρ and Υ_x . Therefore, we need to consider only the behavior of the hiring rate in the open economy relative to that in autarky:

$$\varphi \equiv \sigma/\sigma^A = \frac{1 + \left[\Upsilon_x^{\frac{(1-\beta)(1-k/\delta)}{\Gamma}} - 1\right] \rho^{z-\beta(1-k/\delta)/\Gamma}}{1 + \left[\Upsilon_x^{\frac{(1-\beta)}{\Gamma}} - 1\right] \rho^{z-\beta/\Gamma}}.$$

One can show that φ decreases monotonically in Υ_x and decreases in ρ when ρ is low, but may be decreasing or increasing in ρ when ρ is high (see Figure 4 for both examples). Movements in τ , however, affect both ρ and Υ_x at the same time.⁵⁴ To make further progress, we use the relationship between Υ_x and ρ provided in (35) to substitute for Υ_x :

$$\varphi = \frac{1 + \left[\left(1 + \frac{f_x}{f_d} \rho^{\beta/\Gamma} \right)^{1 - k/\delta} - 1 \right] \rho^{z - \beta(1 - k/\delta)/\Gamma}}{1 + \frac{f_x}{f_d} \rho^z}.$$

Using this representation, the relative hiring rate is a function of ρ (as long as f_x is constant, so that ρ changes in response to the variable trade cost or the labor market frictions). Around $\rho = 0$, we have the following two results:

$$\left(1 + \frac{f_x}{f_d}\rho^{\beta/\Gamma}\right)^{1-k/\delta} = 1 + (1 - k/\delta)\frac{f_x}{f_d}\rho^{\beta/\Gamma} + O(\rho^{2\beta/\Gamma}),$$

$$\frac{1}{1 + \frac{f_x}{f_d}\rho^z} = 1 - \frac{f_x}{f_d}\rho^z + O(\rho^{2z}),$$

where $O(\cdot)$ denotes "the same order of magnitudes as", i.e., $\lim_{\lambda \to 0} |O(\lambda)/\lambda| < \infty$. Using these results, we have:

$$\varphi = 1 - \frac{f_x}{f_d} \rho^z + (1 - k/\delta) \frac{f_x}{f_d} \rho^{z + \frac{\beta k}{\delta \Gamma}} + O\left(\rho^{z + \frac{\beta}{\Gamma}(1 + k/\delta)}\right) = 1 - \frac{f_x}{f_d} \rho^z + O\left(\rho^{z + \frac{\beta k}{\delta \Gamma}}\right),$$

which establishes the claim in Lemma 3.

Next we look at the Theil index of wage inequality. To compute a closed form of this index, we partition the population of workers in the differentiated sector into those employed by exporting firms and those employed by nonexporting firms, and then apply the decomposition formula (39). Using the result in Section B.1 of the Appendix, we can write the between- and within-components of sectoral wage inequality

⁵⁴We were unable to prove that the hiring rate is monotonically decreasing as τ falls. However, we showed that it is necessarily the case both when $\rho \approx 0$ and $\rho \approx 1$. Our conjecture is that the effect is monotonic for all parameter values.

as:

$$T_{wW} = S_{w,d} T_{w,d} + (1 - S_{w,d}) T_{w,x},$$

$$T_{wB} = S_{w,d} \ln \frac{\bar{w}_d}{\bar{w}} + (1 - S_{w,d}) \ln \frac{\bar{w}_x}{\bar{w}},$$

with the overall wage inequality in the sector given by $T_w = T_{wW} + T_{wB}$. Note that all the ingredients in these formulas were already defined and provided explicitly in Section B.1. Putting everything together and substituting out Υ_x using (35), we have the following closed form expression for the Theil index:

$$T_{w} = \mu - \ln(1+\mu) - \frac{\beta k}{\delta \Gamma} \frac{\frac{f_{x}}{f_{d}} \rho^{z} \ln \rho}{1 + \frac{f_{x}}{f_{d}} \rho^{z}} + \frac{\frac{k}{\delta} \rho^{z - \frac{\beta}{\Gamma}} \left(1 + \frac{f_{x}}{f_{d}} \rho^{\frac{\beta}{\Gamma}}\right) \ln \left(1 + \frac{f_{x}}{f_{d}} \rho^{\frac{\beta}{\Gamma}}\right)}{1 + \frac{f_{x}}{f_{d}} \rho^{z}} - \ln \left(1 + \frac{f_{x}}{f_{d}} \rho^{z}\right) + \ln \left\{1 + \rho^{z - \frac{\beta(1-k/\delta)}{\Gamma}} \left[\left(1 + \frac{f_{x}}{f_{d}} \rho^{\frac{\beta}{\Gamma}}\right)^{1-k/\delta} - 1\right]\right\}.$$

As with the hiring rate, we take the Taylor approximation to this expression term-by-term around $\rho = 0$ to obtain:

$$T_{w} = \mu - \ln(1+\mu) \underbrace{-\frac{\beta k}{\delta \Gamma} \frac{f_{x}}{f_{d}} \rho^{z} \ln \rho + O\left(\rho^{2z} \ln \rho\right)}_{-\frac{f_{x}}{f_{d}} \rho^{z} + O\left(\rho^{2z}\right) + \left(1 - k/\delta\right) \frac{f_{x}}{f_{d}} \rho^{z + \frac{\beta k}{\delta \Gamma}} + O\left(\rho^{z + \frac{\beta}{\Gamma}(1+k/\delta)}\right)$$
$$= \mu - \ln(1+\mu) - \frac{\beta k}{\delta \Gamma} \frac{f_{x}}{f_{d}} \rho^{z} \ln \rho + O\left(\rho^{z}\right),$$

which establishes the second claim in Lemma 3. Note that the derivative of $-\rho^z \ln \rho$ is $-\rho^{z-1}(\ln \rho + 1) > 0$ for $\rho \approx 0$.

Next consider the effects of τ , b/b^* and c/c^* on ρ , as described in Lemma 4. Section A.5 of the Appendix derives the comparative statics for θ_d and θ_x with respect to these parameters (the response of L to these parameters is also discussed there). From that section we know that θ_d increases and θ_x decreases as τ , b/b^* and c/c^* fall. This immediately implies that $\rho = \theta_d/\theta_x$ moves in the same direction with θ_d , as stated in Lemma 4.

D Simulation Parameters

In Figures 1-5, we illustrate the qualitative features of the model by displaying solutions for particular parameter values. In this appendix, we discuss the choice of parameter values used in the figures.

We set $\beta = 0.75$ and $\zeta = 0.5$. This corresponds to an elasticity of substitution of 4 between varieties within the differentiated sector and an elasticity of substitution of 2 across sectors. These numbers are broadly consistent with the estimates in Bernard, Eaton, Jensen and Kortum (2003) and Broda and Weinstein (2006).

Next, we set $\tau = 1.5$, which implies a variable trade cost of 50% in line with the estimates in Anderson and van Wincoop (2004). This also implies an exporter wage premium of 4.5% after controlling for size difference, consistent with the findings in Bernard, Jensen, Redding and Schott (2007). The fixed costs are

set such that $f_x/f_d = 0.2$ and $f_d/f_e = 1.6$, which results in 10% of firms exiting and 18% of firms exporting, also consistent with the evidence.

We set the shape parameter of the ability distribution to k=2, following the calibration in Saez (2001). We set $\delta/k=3.5$, which results in an unconditional wage-size premium of 36%, consistent with the evidence in Oi and Idson (1999). Further, we set z=2.6 which implies a coefficient of variation of firm productivity of 0.80. Finally, we set $\gamma=1/3$, which results in a coefficient of variation of revenue per worker of 0.25. Both these numbers are broadly consistent with the findings in Hsieh and Klenow (2008) for the U.S. economy. These parameters imply $\mu=0.19$ and an economy-wide Gini coefficient of income distribution of about 0.30 (on par with the Gini coefficients in Western Europe, but lower than in the U.S.).

We set $a_{\min} = \theta_{\min} = 1$ which are mere normalizations. We consider a symmetric equilibrium with $c = c^* = 0.28$, which ensures $a_d \gtrsim a_{\min}$, i.e., that even the least productive firm screens, but it is almost indifferent between screening and not screening. This results in $\sigma^A = 0.85$ and $\sigma = 0.82$. As a result, in autarky 15% of the sampled workers are not hired due to screening, while in the trade equilibrium 18% of the sampled workers are not hired due to screening. In addition, we set $b = b^* = 1.05$, so that 5% of the workers searching for jobs in the differentiated sector are not matched with any firm. This results in a 19% sectoral unemployment rate in autarky and a 22% sectoral unemployment rate in the trade equilibrium. Finally, we set $\bar{L} = 1$, so that in equilibrium close to a third of labor income is derived from the differentiated sector and the aggregate rate of unemployment is 6.7%.

For Figures 1-2 we use the baseline parameters. In Figures 3-5, respectively, we vary the fixed exporting cost, f_x , the variable trade cost, τ , and the domestic search cost, b, holding constant all other parameters.

Although we have chosen parameters consistent with some features of the data, our simulations are designed to illustrate the theoretical results; they do not represent a calibration of the model. A proper calibration requires more flexible functional forms or a richer model.

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