Web Appendix to Firm Heterogeneity and Aggregate Welfare (Not for Publication)*

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1 Introduction

This web appendix contains the technical derivations of expressions for each section of the paper and additional supplementary material. In the interests of clarity, and to ensure that the web appendix is self-contained, we reproduce some material from the paper, but also include the intermediate steps for the derivation of expressions. The sections of the web appendix closely follow the sections of the paper with the same name.

2 Closed Economy

We compare the canonical heterogeneous and homogeneous firm models of Melitz (2003) and Krugman (1980). The homogeneous firm model is a special case of the heterogeneous firm model with a degenerate productivity distribution.

2.1 Heterogeneous Firm Model

The specification of preferences, production and entry is the same as Melitz (2003).[1] There is a continuum of firms that are heterogeneous in terms of their productivity $\varphi \in (0, \infty)$, which is drawn from a fixed distribution $g(\varphi)$ after incurring a sunk entry cost of f_e units of labor. Production involves a fixed production cost and a constant marginal cost that depends on firm productivity, so that $l(\varphi) = f_d + q(\varphi)/\varphi$ units of labor are required to supply $q(\varphi)$ units of output. Consumers have constant elasticity of substitution (CES) preferences defined over the differentiated varieties supplied by firms, so that the equilibrium revenue for a firm with productivity φ is:

$$r(\varphi) = RP^{\sigma-1}p(\varphi)^{1-\sigma}, \tag{1}$$

where R is aggregate revenue; P is the aggregate CES price index; and $p(\varphi)$ is the price chosen by a firm with productivity φ. Profit maximization implies that equilibrium prices are a constant mark-up over marginal cost:

$$p(\varphi) = \frac{\sigma}{\sigma-1}\frac{w}{\varphi}. \tag{2}$$

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[1] Following most of the subsequent international trade literature, including Arkolakis, Costinot and Rodriguez-Clare (2012), we consider a static version of Melitz (2003) in which there is zero probability of firm death.

Together the equilibrium revenue function (1) and pricing rule (2) imply that the relative revenues of firms depend only on their relative productivities:

$$\frac{r(\varphi_1)}{r(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}, \tag{3}$$

and equilibrium profits are a constant fraction of revenue minus the fixed production cost:

$$\pi(\varphi) = \frac{r(\varphi)}{\sigma} - wf_d.$$

Fixed production costs imply a productivity cutoff below which firms exit (φ_d^A) defined by the following zero-profit condition:

$$r_d(\varphi_d^A) = R\left(\frac{\sigma-1}{\sigma}P\varphi_d^A\right)^{\sigma-1} w^{1-\sigma} = \sigma w f_d, \tag{4}$$

where the superscript A denotes autarky.

The equilibrium value of this zero-profit productivity is uniquely determined by the free entry condition that requires that the probability of successful entry times average profits conditional on successful entry to equal the sunk entry cost:

$$\left[1 - G\left(\varphi_d^A\right)\right]\bar{\pi} = wf_e, \tag{5}$$

which using the relationship between the revenues of firms with different productivities (3) and the zero-profit condition (4) can be re-written as follows:

$$\begin{aligned}
\left[1 - G\left(\varphi_d^A\right)\right]\int_{\varphi_d^A}^{\infty}\left[\frac{r(\varphi)}{\sigma} - wf_d\right]\frac{g(\varphi)\,d\varphi}{1-G\left(\varphi_d^A\right)} &= wf_e, \\
\left[1 - G\left(\varphi_d^A\right)\right]\int_{\varphi_d^A}^{\infty}\left[\left(\frac{\varphi}{\varphi_d^A}\right)^{\sigma-1}\frac{r(\varphi_d)}{\sigma} - wf_d\right]\frac{g(\varphi)\,d\varphi}{1-G\left(\varphi_d^A\right)} &= wf_e, \\
\int_{\varphi_d^A}^{\infty}\left[\left(\frac{\varphi}{\varphi_d^A}\right)^{\sigma-1} - 1\right] f_d g(\varphi)\,d\varphi &= f_e,
\end{aligned}$$

which can be written more compactly as:

$$f_d J\left(\varphi_d^A\right) = f_e, \tag{6}$$

$$J\left(\varphi_d^A\right) = \int_{\varphi_d^A}^{\infty}\left[\left(\frac{\varphi}{\varphi_d^A}\right)^{\sigma-1} - 1\right] dG(\varphi) = \left[1 - G(\varphi_d^A)\right]\left[\left(\frac{\tilde{\varphi}_d^A}{\varphi_d^A}\right)^{\sigma-1} - 1\right], \tag{7}$$

where $\tilde{\varphi}_d^A$ is a weighted average of firm productivities:

$$\tilde{\varphi}_d^A = \left[\int_{\varphi_d^A}^{\infty}\varphi^{\sigma-1}\frac{g(\varphi)\,d\varphi}{1-G\left(\varphi_d^A\right)}\right]^{\frac{1}{\sigma-1}}. \tag{8}$$

Note that $\lim_{\varphi_d^A \to 0} J\left(\varphi_d^A\right) = \infty$, $\lim_{\varphi_d^A \to \infty} J\left(\varphi_d^A\right) = 0$, and $J\left(\varphi_d^A\right)$ is a monotonically decreasing function. It follows that the free entry condition (6) determines a unique equilibrium value of the autarkic zero-profit productivity φ_d^A independently of the other endogenous variables of the model.

The mass of entrants (M_e) equals the mass of producing firms (M) divided by the probability of successful entry $(1 - G(\varphi_d^A))$:

$$M_e = \frac{M}{1 - G(\varphi_d^A)} = \frac{R}{\bar{r}\left[1 - G(\varphi_d^A)\right]}. \tag{9}$$

Using the relationship between average firm revenue ($\bar{r}$) and average firm profits ($\bar{\pi}$):

$$\bar{r} = \sigma \left(\bar{\pi} + w f_d \right),$$

and the free entry condition (5) the mass of entrants can be re-expressed as:

$$M_e = \frac{R}{\sigma w \left[f_e + \left[1 - G \left(\varphi_d^A \right) \right] f_d \right]}. \tag{10}$$

We choose labor as the numeraire ($w = 1$). Using the relationship between the mass of entrants and mass of firms (9) in the free entry condition (5), we obtain:

$$M \bar{\pi} = M_e f_e = L_e,$$

which implies that total payments to labor used in entry equal total profits. Note that total payments to labor used in production equal total revenue minus total profits:

$$L_p = R - M \bar{\pi},$$

which together with labor market clearing implies that aggregate revenue equals total labor payments ($R = L$). Using this equality of aggregate revenue and total labor payments in the zero-profit condition (4), welfare can be written solely in terms of the zero-profit productivity (φ_d^A) and parameters:

$$\mathbb{W}_{\text{Het}}^A = \frac{w}{P} = \left(\frac{L}{\sigma f_d} \right)^{\frac{1}{\sigma - 1}} \frac{\sigma - 1}{\sigma} \varphi_d^A. \tag{11}$$

Therefore the zero-profit productivity is a sufficient statistic for welfare.

2.2 Homogeneous Firm Model

We construct a homogeneous firm model that replicates the same aggregate equilibrium as the heterogeneous firm equilibrium that we just described. Firms pay a sunk entry cost of f_e units of labor and draw a productivity of either zero or $\bar{\varphi}_d$ with exogenous probabilities $\bar{G}_d$ and $\left(1 - \bar{G}_d\right)$ respectively. Fixed production costs imply that only firms drawing a productivity of $\bar{\varphi}_d$ find it profitable to produce. Therefore producing firms are homogeneous and there is a degenerate productivity distribution conditional on production at $\bar{\varphi}_d$.

Note that the only difference between the homogeneous and heterogeneous firm models is the productivity distribution of entrants: the distribution $G(\cdot)$ is replaced by the degenerate distribution with parameters $\bar{G}_d$ and $\bar{\varphi}_d$. All other parameters are the same. This homogeneous firm model is isomorphic to Krugman (1980), in which the representative firm's productivity is set equal to $\bar{\varphi}_d$ and the fixed production cost is scaled to incorporate the expected value of entry costs ($F_d = f_d + f_e / \left[1 - \bar{G}_d \right]$). These values for the representative firm's productivity and the fixed production cost are exogenous and held constant in Krugman (1980). To simplify the exposition, we adopt this Krugman (1980) interpretation. The representative firm's production technology is:

$$l = \frac{q}{\bar{\varphi}_d} + F_d. \tag{12}$$

Consumers again have constant elasticity of substitution (CES) preferences defined over the differentiated varieties supplied by firms. Profit maximization implies that equilibrium prices are a constant markup over marginal cost:

$$p = \frac{\sigma}{\sigma - 1} \frac{w}{\bar{\varphi}_d}.$$

while profit maximization and free entry imply that equilibrium output and employment for the representative variety are proportional to the fixed production cost:

$$q = \bar{\varphi}_d F_d(\sigma - 1), \qquad l = \sigma F_d.$$

Using the common employment for each variety, the mass of firms can be determined from the labor market clearing condition:

$$M = \frac{L}{\sigma F_d}. \tag{13}$$

Using the equilibrium pricing rule and the mass of firms, the CES price index is:

$$P^{1-\sigma} = M\left(\frac{\sigma}{\sigma - 1}\frac{w}{\bar{\varphi}_d}\right)^{1-\sigma}, \tag{14}$$

where we again choose labor as the numeraire and hence $w = 1$.

Rearranging the price index (14), and using the mass of firms (13) and our choice of numeraire, welfare can be written in terms of productivity and other parameters:

$$\frac{w}{P} = \left(\frac{L}{\sigma F_d}\right)^{\frac{1}{\sigma-1}} \frac{\sigma - 1}{\sigma}\bar{\varphi}_d. \tag{15}$$

2.3 Aggregate Equilibrium Equivalence

We now pick the parameters $\bar{G}_d$ and $\bar{\varphi}_d$ of the degenerate productivity distribution with homogeneous firms such that the autarky equilibrium is isomorphic to the heterogeneous firm equilibrium, in the following sense:

Proposition 1 *Consider a homogeneous firm model that is a special case of the heterogeneous firm model with an exogenous probability of successful entry $\left[1 - \bar{G}_d\right] = \left[1 - G(\varphi_d^A)\right]$ and an exogenous degenerate distribution of productivity conditional on successful entry $\bar{\varphi}_d = \tilde{\varphi}_d^A$. Given the same value for all remaining parameters $\{f_d,\, f_e,\, L,\, \sigma\}$, all aggregate variables (welfare, wage, price index, mass of firms, and aggregate revenue) are the same in the closed economy equilibria of the two models.*

Proof. Combining $F_d = f_d + f_e/\left[1 - G(\varphi_d^A)\right]$ with the free entry condition (6) in the heterogeneous firm model, we obtain:

$$\frac{F_d}{f_d} = \left(\frac{\tilde{\varphi}_d^A}{\varphi_d^A}\right)^{\sigma-1}. \tag{16}$$

Substituting this result into closed economy welfare in the homogeneous firm model (15), we obtain the same welfare in the two models:

$$\mathbb{W}_{\text{Hom}}^A = \frac{w}{P} = \left(\frac{L}{\sigma f_d}\right)^{\frac{1}{\sigma-1}} \frac{\sigma - 1}{\sigma}\varphi_d^A = \mathbb{W}_{\text{Het}}^A.$$

Equal wages follow from our choice of numeraire ($w = 1$). Equal welfare and equal wages in turn imply equal price indices. Equal masses of firms follow from (13), (10) and (16). Equal aggregate revenue follows from $R = wL = L$ in both models. ■

3 Open Economy

We consider the canonical case of trade between two symmetric countries, as examined in Krugman (1980) and Melitz (2003). We assume that the heterogeneous and homogeneous firm models have the same trade costs, so that there is fixed exporting cost of f_x units of labor and an iceberg variable trade cost, where $\tau > 1$ units of a variety must be shipped from one country in order for one unit to arrive in the other country. We compare the effect of moving from the closed economy to the open economy on welfare in the two models, keeping all structural parameters other than the productivity distribution the same in the two models.

3.1 Heterogeneous Firm Model

Equilibrium firm revenues in the domestic and export markets are:

$$r_d(\varphi) = RP^{\sigma-1}p_d(\varphi)^{1-\sigma}, \qquad r_x(\varphi) = \tau^{1-\sigma}r_d(\varphi),$$

where the subscript d indicates the domestic market and the subscript x indicates the export market.

Profit maximization implies that equilibrium prices are again a constant mark-up over marginal costs, with export prices a constant multiple of domestic prices because of the variable costs of trade:

$$p_d(\varphi) = \frac{\sigma}{\sigma-1}\frac{w}{\varphi}, \qquad p_x(\varphi) = \tau p_d(\varphi), \tag{17}$$

This equilibrium pricing rule implies that profits in each market are a constant proportion of revenues minus the fixed costs:

$$\pi_d(\varphi) = \frac{r_d(\varphi)}{\sigma} - wf_d, \qquad \pi_x(\varphi) = \frac{r_x(\varphi)}{\sigma} - wf_x,$$

where we assume that fixed exporting costs are incurred in the source country and we apportion the fixed production cost to the domestic market. The productivity cutoffs for serving the domestic market (φ_d^T) and export market (φ_x^T) are defined by the following zero-profit conditions:

$$r_d(\varphi_d^T) = R\left(\frac{\sigma-1}{\sigma}P\varphi_d^T\right)^{\sigma-1} w^{1-\sigma} = \sigma wf_d. \tag{18}$$

$$r_x(\varphi_x^T) = R\left(\frac{\sigma-1}{\sigma}P\varphi_x^T\right)^{\sigma-1} (\tau w)^{1-\sigma} = \sigma wf_x, \tag{19}$$

where the superscript T indicates the open economy equilibrium. Together these two zero-profit conditions imply that the export cutoff is a constant multiple of the domestic cutoff that depends on the fixed and variable costs of trade:

$$\varphi_x^T = \tau\left(\frac{f_x}{f_d}\right)^{\frac{1}{\sigma-1}}\varphi_d^T. \tag{20}$$

For sufficiently high fixed and variable trade costs ($\tau\left(f_x/f_d\right)^{\frac{1}{\sigma-1}} > 1$), only the most productive firms export, consistent with an extensive empirical literature (see for example the review in Bernard, Jensen, Redding and Schott 2007).

The free entry condition again equates the expected value of entry to the sunk entry cost:

$$\left[1 - G\left(\varphi_d^T\right)\right]\bar{\pi} = wf_e. \tag{21}$$

Noting that the relative revenues of firms within the same market depend solely on their relative productivities, and using the domestic cutoff condition (18) and the export cutoff condition (19), the free entry condition can be re-written as follows:

$$\left[1-G\left(\varphi_d^T\right)\right]\left[\begin{array}{c} \int_{\varphi_d^T}^{\infty}\left[\frac{r_d(\varphi)}{\sigma}-wf_d\right]\frac{g(\varphi)d\varphi}{1-G\left(\varphi_d^T\right)} \\ +\frac{1-G\left(\varphi_x^T\right)}{1-G\left(\varphi_d^T\right)}\int_{\varphi_x^T}^{\infty}\left[\frac{r_x(\varphi)}{\sigma}-wf_x\right]\frac{g(\varphi)d\varphi}{1-G(\varphi_x^T)} \end{array}\right] = wf_e,$$

$$\left[1-G\left(\varphi_d^T\right)\right]\left[\begin{array}{c} \int_{\varphi_d^T}^{\infty}\left[\left(\frac{\varphi}{\varphi_d^T}\right)^{\sigma-1}\frac{r_d(\varphi_d)}{\sigma}-wf_d\right]\frac{g(\varphi)d\varphi}{1-G\left(\varphi_d^T\right)} \\ +\frac{1-G\left(\varphi_x^T\right)}{1-G\left(\varphi_d^T\right)}\int_{\varphi_x^T}^{\infty}\left[\left(\frac{\varphi}{\varphi_x^T}\right)^{\sigma-1}\frac{r_x(\varphi_x)}{\sigma}-wf_x\right]\frac{g(\varphi)d\varphi}{1-G(\varphi_x^T)} \end{array}\right] = wf_e,$$

$$\left[\begin{array}{c} \int_{\varphi_d^T}^{\infty}\left[\left(\frac{\varphi}{\varphi_d^T}\right)^{\sigma-1}-1\right]f_d g(\varphi)\,d\varphi \\ +\int_{\varphi_x^T}^{\infty}\left[\left(\frac{\varphi}{\varphi_x^T}\right)^{\sigma-1}-1\right]f_x g(\varphi)\,d\varphi \end{array}\right] = f_e.$$

or equivalently:

$$f_d J\left(\varphi_d^T\right)+f_x J\left(\varphi_x^T\right)=f_e, \tag{22}$$

where $J(\cdot)$ is defined in (7) and weighted average productivity in the export market $(\tilde{\varphi}_x^T)$ is defined in an analogous way to weighted average productivity in the domestic market $(\tilde{\varphi}_d^T)$ in (8):

$$\tilde{\varphi}_x^T=\left[\int_{\varphi_x^T}^{\infty}\varphi^{\sigma-1}\frac{g(\varphi)\,d\varphi}{1-G\left(\varphi_x^T\right)}\right]^{\frac{1}{\sigma-1}}.$$

Using the relationship between the productivity cutoffs (20), the free entry condition can be written solely in terms of the domestic productivity cutoff:

$$\left[\begin{array}{c} \int_{\varphi_d^T}^{\infty}\left[\left(\frac{\varphi}{\varphi_d^T}\right)^{\sigma-1}-1\right]f_d g(\varphi)\,d\varphi \\ +\int_{\tau(f_x/f_d)^{1/(\sigma-1)}\varphi_d^T}^{\infty}\left[\left(\frac{\varphi}{\tau(f_x/f_d)^{1/(\sigma-1)}\varphi_d^T}\right)^{\sigma-1}-1\right]f_x g(\varphi)\,d\varphi \end{array}\right]=f_e. \tag{23}$$

Noting that the left-hand side converges towards infinity as φ_d^T converges towards zero; the left-hand side converges towards zero as φ_d^T converges towards infinity; and the left-hand side is monotonically decreasing in φ_d^T. It follows that the free-entry condition (23) determines a unique equilibrium value of φ_d^T independently of the other endogenous variables of the model. The unique equilibrium value of φ_x^T follows immediately from the relationship between the cutoffs (20). Since the left-hand sides of the closed and open economy free entry conditions ((6) and (23) respectively) are monotonically decreasing in φ_d, it also follows that the domestic cutoff in the open economy is greater than the domestic cutoff in the closed economy ($\varphi_d^T>\varphi_d^A$).

As in the closed economy, the mass of entrants (M_e) equals the mass of firms (M) divided by the probability of successful entry $(1-G\left(\varphi_d^T\right))$:

$$M_e=\frac{M}{1-G(\varphi_d^T)}=\frac{R}{\bar{r}\left[1-G(\varphi_d^T)\right]}. \tag{24}$$

Using the relationship between average firm revenue ($\bar{r}$) and average firm profits ($\bar{\pi}$):

$$\bar{r}=\sigma\left(\bar{\pi}+wf_d+\frac{1-G(\varphi_x^T)}{1-G(\varphi_d^T)}wf_x\right),$$

and the free entry condition (21) the mass of entrants can be re-expressed as:

$$M_e = \frac{R}{\sigma w \left[f_e + \left[1 - G\left(\varphi_d^T\right)\right] f_d + \left[1 - G\left(\varphi_x^T\right)\right] f_x\right]}.$$

and aggregate revenue equals total labor payments ($R = wL$).

Using the equilibrium pricing rule and the mass of firms, the CES price index in the open economy can be written as:

$$P^{1-\sigma} = M\left[\left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \chi\tau^{1-\sigma}\left(\tilde{\varphi}_x^T\right)^{\sigma-1}\right]\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} w^{1-\sigma}, \tag{25}$$

where $\chi = \left[1 - G\left(\varphi_x^T\right)\right] / \left[1 - G\left(\varphi_d^T\right)\right]$ is the proportion of exporting firms. We choose labor in one country as the numeraire, which with symmetric countries implies that the wage in each country is equal to one ($w = 1$).

Rearranging the price index (25), and using the mass of firms (24) and our choice of numeraire, welfare can be expressed in terms of productivity and parameters:

$$\mathbb{W}_{\text{Het}}^T = \frac{w}{P} = \left(\frac{L\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}}{\sigma\left[\frac{f_e}{1-G\left(\varphi_d^T\right)} + f_d + \chi f_x\right]}\right)^{\frac{1}{\sigma-1}} \left[\left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \chi\tau^{1-\sigma}\left(\tilde{\varphi}_x^T\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}. \tag{26}$$

In an open economy equilibrium with selection into export markets ($\varphi_x^T > \varphi_d^T$), the zero-profit condition for the domestic market (18) implies that open economy welfare can be written equivalently in terms of the domestic productivity cutoff and parameters:

$$\mathbb{W}_{\text{Het}}^T = \frac{w}{P} = \left(\frac{L}{\sigma f_d}\right)^{\frac{1}{\sigma-1}} \frac{\sigma-1}{\sigma}\varphi_d^T. \tag{27}$$

Comparing (11) and (27), and noting that the domestic cutoff is higher in the open economy than in the closed economy ($\varphi_d^T > \varphi_d^A$), there are necessarily welfare gains from trade.

In contrast, in an open economy equilibrium in which all firms export, the domestic and export productivity cutoffs are equal to one another ($\varphi_x^T = \varphi_d^T$), and are determined by the requirement that the sum of variable profits in the domestic and export markets is equal to the sum of fixed production and exporting costs. Using this zero-profit condition, open economy welfare again can be written equivalently in terms of the domestic productivity cutoff and parameters:

$$\mathbb{W}_{\text{Het}}^T = \frac{w}{P} = \left(\frac{\left(1+\tau^{1-\sigma}\right)L}{\sigma\left(f_d + f_x\right)}\right)^{\frac{1}{\sigma-1}} \frac{\sigma-1}{\sigma}\varphi_d^T. \tag{28}$$

Comparing (11) and (28), and noting that $f_x/f_d \leq \tau^{1-\sigma}$ in an open economy equilibrium in which all firms export, there are again necessarily welfare gains from trade.

3.2 Homogeneous Firm Model

In the homogeneous firm model, the probability of successful entry and productivity conditional on successful entry are exogenous and remain unchanged and equal to $\left[1 - \bar{G}_d\right]$ and $\bar{\varphi}_d$ respectively. For sufficiently high fixed and variable trade costs ($\tau^{\sigma-1} f_x/F_d > 1$), the representative firm does not find it profitable to export. In this case, welfare in the open economy equilibrium is necessarily higher in the heterogeneous firm model than in the homogeneous firm model, because the two models have the same closed economy welfare, there are welfare gains from trade, and trade only occurs in the heterogeneous firm model. In contrast, for sufficiently low fixed and variable trade costs ($\tau^{\sigma-1} f_x / F_d < 1$), the representative firm finds it profitable to export. In this case, there is positive trade in both models, and we now compare their relative welfare in such an open economy equilibrium.

Profit maximization again implies that equilibrium prices are a constant mark-up over marginal costs, with export prices a constant multiple of domestic prices because of the variable costs of trade:

$$p_d = \frac{\sigma}{\sigma - 1} \frac{w}{\bar{\varphi}_d}, \qquad p_x = \tau p_d. \tag{29}$$

Free entry implies that total revenue equals total costs. In an equilibrium in which the representative firm does not find it profitable to export, we have:

$$\frac{r_d(\bar{\varphi}_d)}{\sigma} = F_d. \tag{30}$$

In contrast, in an equilibrium in which the representative firm finds it profitable to export, we have:

$$\frac{\left(1 + \tau^{1-\sigma}\right) r_d(\bar{\varphi}_d)}{\sigma} = F_d + f_x. \tag{31}$$

Using these two free entry conditions (30) and (31), we can confirm that the representative firm exports if:

$$\tau^{\sigma-1} \frac{f_x}{F_d} < 1, \qquad F_d = f_d + \frac{f_e}{1 - \bar{G}_d}. \tag{32}$$

In an equilibrium in which the representative firm exports, profit maximization and free entry imply that equilibrium output and employment for the representative variety are proportional to fixed costs:

$$q = \bar{\varphi}_d (F_d + f_x)(\sigma - 1),$$

$$l = \sigma (F_d + f_x).$$

Therefore both output and employment rise for the representative firm following the opening of trade to cover the additional fixed costs of exporting.

From the labor market clearing condition, this rise in employment for the representative firm implies a fall in the mass of domestically-produced varieties:

$$M = \frac{L}{\sigma (F_d + f_x)}. \tag{33}$$

Using the equilibrium pricing rule and the mass of firms, the CES price index in the open economy is:

$$P^{1-\sigma} = \left[1 + \tau^{1-\sigma}\right] M \left(\frac{\sigma}{\sigma - 1} \frac{w}{\bar{\varphi}_d}\right)^{1-\sigma}, \tag{34}$$

where we again choose labor as the numeraire and hence $w = 1$.

Rearranging the price index (34), and using the mass of firms (33) and our choice of numeraire, welfare can be again expressed in terms of productivity and parameters:

$$\mathbb{W}^T_{\text{Hom}} = \frac{w}{P} = \left(\frac{\left[1 + \tau^{1-\sigma}\right] L}{\sigma (F_d + f_x)}\right)^{\frac{1}{\sigma-1}} \frac{\sigma - 1}{\sigma} \bar{\varphi}_d. \tag{35}$$

3.3 Relative Welfare in the Open Economy

In the homogeneous firm model, aggregate productivity is exogenous and hence constant by assumption. In contrast, in the heterogeneous firm model, aggregate productivity is the result of the endogenous entry and exit decisions of heterogeneous firms, which provides a new adjustment margin through which the economy can respond to the opening of trade. The presence of this new adjustment margin implies that the relative change in welfare following the opening of trade is strictly larger in the heterogeneous firm model than in the homogeneous firm model. Since the homogeneous firm model is a special case of the heterogeneous firm model, this comparison of the welfare gains from trade across the two models is equivalent to a comparative static within the heterogeneous firm model on the productivity distribution (from a non-degenerate to a degenerate distribution). This comparative static interpretation requires that we hold all other parameters equal when comparing the two models (same $f_d, f_e,\ f_x,\ \tau,\ L,\ \sigma$). We maintain this assumption throughout the paper whenever we compare the heterogeneous and homogeneous firm models. We pick the parameters of the degenerate productivity distribution under homogeneous firms ($\bar{G}_d$ and $\bar{\varphi}_d$) such that the autarky equilibrium is isomorphic to the heterogeneous firm one (as outlined in Proposition 1): $\bar{G}_d = G(\varphi_d^A)$ and $\bar{\varphi}_d = \tilde{\varphi}_d^A$.

Proposition 2 *The proportional welfare gains from trade are strictly larger in the heterogeneous firm model than in the homogeneous firm model ($\mathbb{W}_{Het}^T/\mathbb{W}_{Het}^A > \mathbb{W}_{Hom}^T/\mathbb{W}_{Hom}^A$), except in the special case with no fixed exporting cost. In this special case, the proportional welfare gains from trade are the same in the two models.*

Proof. We establish the proposition for the various possible types of open economy equilibria depending on parameter values. **(I)** First, we consider parameter values for which the representative firm does not find it profitable to export in the homogeneous firm model ($\tau\left(f_x/F_d\right)^{1/(\sigma-1)} > 1$). For these parameter values, the proposition follows immediately from the fact that the two models have the same closed economy welfare, there are welfare gains from trade, and trade only occurs in the heterogeneous firm model. **(II)** Second, we consider parameter values for which the representative firm exports in the homogeneous firm model and there is selection into export markets in the heterogeneous firm model ($0 < \tau\left(f_x/F_d\right)^{1/(\sigma-1)} < 1 < \tau\left(f_x/f_d\right)^{1/(\sigma-1)}$). From (26) and (35), open economy welfare is higher in the heterogeneous firm model than in the homogeneous firm model if the following inequality is satisfied:

$$\frac{\left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \chi\tau^{1-\sigma}\left(\tilde{\varphi}_x^T\right)^{\sigma-1}}{\frac{f_e}{1-G\left(\varphi_d^T\right)} + f_d + \chi f_x} > \frac{\left(1+\tau^{1-\sigma}\right)\left(\tilde{\varphi}_d^A\right)^{\sigma-1}}{F_d + f_x}. \tag{36}$$

To show that this inequality must be satisfied, we use the open economy free entry condition in the heterogeneous firm model, which implies:

$$f_d\int_{\varphi_d^T}^{\infty}\left[\left(\frac{\varphi}{\varphi_d^T}\right)^{\sigma-1} - 1\right]dG\left(\varphi\right) + f_x\int_{\varphi_x^T}^{\infty}\left[\left(\frac{\varphi}{\varphi_x^T}\right)^{\sigma-1} - 1\right]dG\left(\varphi\right) = f_e,$$

$$f_d\left[1-G\left(\varphi_d^T\right)\right]\left[\left(\frac{\tilde{\varphi}_d^T}{\varphi_d^T}\right)^{\sigma-1} - 1\right] + f_x\left[1-G\left(\varphi_x^T\right)\right]\left[\left(\frac{\tilde{\varphi}_x^T}{\varphi_x^T}\right)^{\sigma-1} - 1\right] = f_e,$$

$$f_d\left(\frac{\tilde{\varphi}_d^T}{\varphi_d^T}\right)^{\sigma-1} + f_x\frac{1-G\left(\varphi_x^T\right)}{1-G\left(\varphi_d^T\right)}\left(\frac{\tilde{\varphi}_x^T}{\varphi_x^T}\right)^{\sigma-1} = \frac{f_e}{1-G\left(\varphi_d^T\right)} + f_d + \chi f_x.$$

Using $\left(\varphi_x^T\right)^{\sigma-1} = \left(\varphi_d^T\right)^{\sigma-1}\tau^{\sigma-1}f_x/f_d$, we obtain:

$$\frac{f_d}{\left(\varphi_d^T\right)^{\sigma-1}}\left[\left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \chi\tau^{1-\sigma}\left(\tilde{\varphi}_x^T\right)^{\sigma-1}\right] = \frac{f_e}{1-G\left(\varphi_d^T\right)} + f_d + \chi f_x. \tag{37}$$

Note that the open economy free entry condition in the heterogeneous firm model also implies:

$$f_d \int_{\varphi_d^A}^{\infty} \left[\left(\frac{\varphi}{\varphi_d^T} \right)^{\sigma-1} - 1 \right] dG(\varphi) + f_x \int_{\varphi_d^A}^{\infty} \left[\left(\frac{\varphi}{\varphi_x^T} \right)^{\sigma-1} - 1 \right] dG(\varphi) < f_e, \tag{38}$$

since $\varphi_d^A < \varphi_d^T < \varphi_x^T$ and

$$\left[\left(\frac{\varphi}{\varphi_d^T} \right)^{\sigma-1} - 1 \right] < 0, \qquad \text{for} \qquad \varphi < \varphi_d^T,$$

$$\left[\left(\frac{\varphi}{\varphi_x^T} \right)^{\sigma-1} - 1 \right] < 0 \qquad \text{for} \qquad \varphi < \varphi_x^T.$$

Rewriting (38), we have:

$$f_d \left[1 - G\left(\varphi_d^A\right)\right] \left[\left(\frac{\tilde{\varphi}_d^A}{\varphi_d^T} \right)^{\sigma-1} - 1 \right] + f_x \left[1 - G\left(\varphi_d^A\right)\right] \left[\left(\frac{\tilde{\varphi}_d^A}{\varphi_x^T} \right)^{\sigma-1} - 1 \right] < f_e,$$

$$f_d \left(\frac{\tilde{\varphi}_d^A}{\varphi_d^T} \right)^{\sigma-1} + f_x \left(\frac{\tilde{\varphi}_d^A}{\varphi_x^T} \right)^{\sigma-1} < \frac{f_e}{1 - G\left(\varphi_d^A\right)} + f_d + f_x.$$

Using $\left(\varphi_x^T\right)^{\sigma-1} = \left(\varphi_d^T\right)^{\sigma-1} \tau^{\sigma-1} f_x / f_d$, we obtain:

$$\frac{f_d}{\left(\varphi_d^T\right)^{\sigma-1}} \left(1 + \tau^{1-\sigma}\right) \left(\tilde{\varphi}_d^A\right)^{\sigma-1} < \frac{f_e}{1 - G\left(\varphi_d^A\right)} + f_d + f_x. \tag{39}$$

From (37) and (39), we have:

$$\frac{\frac{f_d}{\left(\varphi_d^T\right)^{\sigma-1}} \left[\left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \chi \tau^{1-\sigma} \left(\tilde{\varphi}_x^T\right)^{\sigma-1} \right]}{\frac{f_e}{1 - G\left(\varphi_d^T\right)} + f_d + \chi f_x} = 1, \tag{40}$$

$$\frac{\frac{f_d}{\left(\varphi_d^T\right)^{\sigma-1}} \left(1 + \tau^{1-\sigma}\right) \left(\tilde{\varphi}_d^A\right)^{\sigma-1}}{\frac{f_e}{1 - G\left(\varphi_d^A\right)} + f_d + f_x} = \frac{\frac{f_d}{\left(\varphi_d^T\right)^{\sigma-1}} \left(1 + \tau^{1-\sigma}\right) \left(\tilde{\varphi}_d^A\right)^{\sigma-1}}{F_d + f_x} < 1,$$

which establishes that inequality (36) is satisfied. From (27) and (35), the condition for open economy welfare to be higher in the heterogeneous firm model than in the homogeneous firm model can be also written as:

$$\left(\frac{1}{f_d} \right)^{\frac{1}{\sigma-1}} \varphi_d^T > \left(\frac{1 + \tau^{1-\sigma}}{F_d + f_x} \right)^{\frac{1}{\sigma-1}} \tilde{\varphi}_d^A.$$

Using (36) and (40), this (equivalent) inequality is necessarily satisfied. Since closed economy welfare is the same in the two models, and open economy welfare is higher in the heterogeneous firm model than in the homogeneous firm model, it follows that the proportional welfare gains from trade are larger in the heterogeneous firm model ($\mathbb{W}_{\text{Het}}^T / \mathbb{W}_{\text{Het}}^A > \mathbb{W}_{\text{Hom}}^T / \mathbb{W}_{\text{Hom}}^A$). **(III)** Third, we consider parameter values for which the representative firm exports in the homogeneous firm model and all firms export in the heterogeneous firm model, but fixed exporting costs are still positive ($0 < \tau \left(f_x / F_d\right)^{1/(\sigma-1)} < \tau \left(f_x / f_d\right)^{1/(\sigma-1)} \leq 1$). This is simply a special case of **(II)** in which $\varphi_x^T = \varphi_d^T$, $\tilde{\varphi}_x^T = \tilde{\varphi}_d^T$ and $\frac{1-G\left(\varphi_x^T\right)}{1-G\left(\varphi_d^T\right)} = 1$. Therefore the same line of reasoning as in **(II)** can be used to show that the inequality (36) is satisfied and hence open economy welfare is higher in the heterogeneous firm model than in the homogeneous firm model. In this special case in which all firms export, the free entry condition in the open economy equilibrium of the heterogeneous firm model implies:

$$(f_d + f_x) \int_{\varphi_d^T}^{\infty} \left[\left(\frac{\varphi}{\varphi_d^T} \right)^{\sigma-1} - 1 \right] dG(\varphi) = f_e,$$

$$(f_d + f_x) \int_{\varphi_d^A}^{\infty} \left[\left(\frac{\varphi}{\varphi_d^T} \right)^{\sigma-1} - 1 \right] dG(\varphi) < f_e, \tag{41}$$

since $\varphi_d^A < \varphi_d^T$ and

$$\left[\left(\frac{\varphi}{\varphi_d^T} \right)^{\sigma-1} - 1 \right] < 0, \qquad \text{for} \qquad \varphi < \varphi_d^T.$$

Rewriting (41), we obtain:

$$(f_d + f_x) \left(\frac{\tilde{\varphi}_d^A}{\varphi_d^T} \right)^{\sigma-1} < \frac{f_e}{1 - G(\varphi_d^A)} + f_d + f_x = F_d + f_x. \tag{42}$$

From (28) and (35), the condition for open economy welfare to be higher in the heterogeneous firm model than in the homogeneous firm model can be also written as:

$$\left(\frac{1}{f_d + f_x} \right)^{\frac{1}{\sigma-1}} \varphi_d^T > \left(\frac{1}{F_d + f_x} \right)^{\frac{1}{\sigma-1}} \tilde{\varphi}_d^A.$$

From (42), this inequality is necessarily satisfied. Since closed economy welfare is the same in the two models, and open economy welfare is higher in the heterogeneous firm model than in the homogeneous firm model, it follows that the proportional welfare gains from trade are larger in the heterogeneous firm model ($\mathbb{W}_{\text{Het}}^T / \mathbb{W}_{\text{Het}}^A > \mathbb{W}_{\text{Hom}}^T / \mathbb{W}_{\text{Hom}}^A$). **(IV)** Finally, when fixed exporting costs are zero, we have $0 = \tau (f_x/F_d)^{1/(\sigma-1)} = \tau (f_x/f_d)^{1/(\sigma-1)}$. This is a special case of **(III)** in which $\varphi_x^T = \varphi_d^T = \varphi_d^A$, $\tilde{\varphi}_x^T = \tilde{\varphi}_d^T = \tilde{\varphi}_d^A$ and $\frac{1-G(\varphi_x^T)}{1-G(\varphi_d^T)} = 1$. In this special case of zero fixed exporting costs, the free entry condition in the open economy equilibrium of the heterogeneous firm model implies:

$$(f_d + f_x) \int_{\varphi_d^A}^{\infty} \left[\left(\frac{\varphi}{\varphi_d^T} \right)^{\sigma-1} - 1 \right] dG(\varphi) = f_e,$$

$$(f_d + f_x) \left(\frac{\tilde{\varphi}_d^A}{\varphi_d^T} \right)^{\sigma-1} = \frac{f_e}{1 - G(\varphi_d^A)} + f_d + f_x = F_d + f_x,$$

where we have used $\varphi_d^A = \varphi_d^T$. From (28) and (35), it follows immediately that open economy welfare is the same in the two models when fixed exporting costs are equal to zero. ■

In the special case with no fixed exporting cost, the domestic productivity cutoff does not respond to the opening of trade in the heterogeneous firm model. As a result, the additional adjustment margin provided by heterogeneous firms entry and exit decisions is inoperable, and the welfare gains from trade are the same in the two models. But this special case is uninteresting, because firm productivity dispersion plays no role in the heterogeneous firm model (the exit threshold and average productivity are the same in the closed and open economies). Furthermore, this special case stands at odds with an extensive body of empirical evidence that only some firms export, exporters are larger and more productive than non-exporters, and there are substantial fixed exporting costs.[2]

[2]For reviews of the extensive empirical literatures on firm export market participation, see Bernard, Jensen, Redding and Schott (2007) and Melitz and Redding (2012). For evidence of substantial fixed exporting costs, see Roberts and Tybout (1997) and Das, Roberts and Tybout (2007).

Since the proportional welfare gains from trade are strictly lower in the homogeneous firm model than in the heterogeneous firm model for positive fixed exporting costs, and since open economy welfare in the homogeneous firm model is monotonically decreasing in trade costs, we also obtain the following result.

Proposition 3 *To achieve the same proportional welfare gains from trade requires strictly lower trade costs (either lower f_x and/or lower τ) in the homogeneous firm model than in the heterogeneous firm model, except in the special case of zero fixed exporting costs.*

Proof. The proposition follows immediately from $\mathbb{W}^T_{\text{Het}}/\mathbb{W}^A_{\text{Het}} > \mathbb{W}^T_{\text{Hom}}/\mathbb{W}^A_{\text{Hom}}$ in Proposition 2 and from $\frac{d\mathbb{W}^T_{\text{Hom}}}{df_x} < 0$ and $\frac{d\mathbb{W}^T_{\text{Hom}}}{d\tau} < 0$ in (35). ■

Although we chose the productivity of the representative firm ($\tilde{\varphi}^A_d$) to ensure the same closed economy welfare in both models, the ratio of open to closed economy welfare in the homogeneous firm model $\mathbb{W}^T_{\text{Hom}}/\mathbb{W}^A_{\text{Hom}}$ is independent of the representative firm's productivity (from (15) and (35)). It follows that both of the above propositions hold for any value of the representative firm's productivity. Both propositions also hold for general continuous productivity distributions.

4 Revealed Preference

The theoretical results throughout the paper are proved using the free entry condition in the market equilibrium of the heterogeneous firm model. But to provide further economic intuition for these results, we consider the problem of a social planner choosing the productivity cutoffs and the mass of entrants to maximize welfare in the heterogeneous firm model. We begin with the planner's problem in the closed economy, in which welfare in the homogeneous and heterogeneous firm models is the same. We next consider the planner's problem in the open economy. The planner is assumed to maximize world welfare, which with symmetric countries is equivalent to maximizing the welfare of the representative consumer in each country.[3] We show that the social planner's choices in the closed and open economies coincide with the market allocations, and hence the market allocations in the heterogeneous firm model are efficient.[4] We also show that the social planner in general chooses to adjust the productivity cutoffs following the opening of trade, even though it is feasible to leave them unchanged and replicate the open economy equilibrium of the homogeneous firm model. Therefore, by revealed preference, open economy welfare must be at least as high in the heterogeneous firm model as in the homogeneous firm model, and we show that it is in general higher.

4.1 Closed Economy

The real consumption index for the representative consumer is:

$$Q = \left[M_e \int_{\varphi^A_d}^{\infty} q(\varphi)^{(\sigma-1)/\sigma} dG(\varphi) \right]^{\sigma/(\sigma-1)}. \tag{43}$$

The social planner chooses the productivity cutoff φ^A_d, the output levels $q(\varphi)$ for all producing firms $\varphi \geq \varphi^A_d$, and the mass of entrants M_e to maximize Q subject to the aggregate labor constraint:

$$L = M_e \left\{ \int_{\varphi^A_d}^{\infty} \frac{q(\varphi)}{\varphi} dG(\varphi) + \left[1 - G(\varphi^A_d)\right] f_d + f_e \right\}, \tag{44}$$

[3] To highlight the efficiency properties of the market equilibrium, we assume a world planner, which abstracts from the incentives of national planners to manipulate the terms of trade between countries.

[4] For an analysis of how the efficiency of the monopolistically competitive equilibrium depends on the extent to which the elasticity of substitution between varieties is constant or variable, see Dixit and Stiglitz (1977) for homogeneous firm models and Dhingra and Morrow (2012) for heterogeneous firm models.

where the social planner faces the same productivity distribution $G(\varphi)$ and entry cost f_e per firm as in the market allocation.

The planner chooses the output levels $q(\varphi)$ to equate the marginal rates of transformations and marginal rates of substitution for firms with different productivities. The marginal rate of substitution between varieties for any two firms with productivities φ_1 and φ_2 is:

$$MRS = \left(\frac{q(\varphi_1)}{q(\varphi_2)}\right)^{1/\sigma}.$$

The marginal rate of transformation between varieties for any two firms with productivities φ_1 and φ_2 is:

$$MRT = \frac{\varphi_1}{\varphi_2}.$$

Efficiency requires:

$$MRS = MRT \iff \frac{q(\varphi_1)}{q(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma},$$

which yields the same relationship between relative quantities and relative productivities as in the market equilibrium. Using this relationship, we can rewrite the consumption index Q and the aggregate labor constraint as a function of the output level $\tilde{q}_d^A \equiv q(\tilde{\varphi}_d^A)$ of a firm with a weighted average productivity $\tilde{\varphi}_d^A$:

$$\begin{aligned} Q &= \left\{\left[1 - G(\varphi_d^A)\right] M_e\right\}^{\sigma/(\sigma-1)} \tilde{q}_d^A, \\ L &= \left[1 - G(\varphi_d^A)\right] M_e \left[\frac{\tilde{q}_d^A}{\tilde{\varphi}_d^A} + f_d + \frac{f_e}{1 - G(\varphi_d^A)}\right], \end{aligned}$$

where we have used $q(\varphi) = \left(\varphi/\tilde{\varphi}_d^A\right)^{\sigma} \tilde{q}_d^A$ and the definition of $\tilde{\varphi}_d^A$ in (8).

Using the aggregate labor constraint we can rewrite the real consumption index as:

$$Q = L^{\sigma/(\sigma-1)} \left[\frac{\tilde{q}_d^A}{\tilde{\varphi}_d^A} + f_d + \frac{f_e}{1 - G(\varphi_d^A)}\right]^{-\sigma/(\sigma-1)} \tilde{q}_d^A. \tag{45}$$

The social planner chooses the cutoff φ_d^A and quantity $\tilde{q}_d^A$ to maximize this consumption index. The trade-off faced by the social planner is that a lower productivity cutoff reduces expected entry costs conditional on successful entry, and thereby releases more labor for production. But this lower productivity cutoff involves firms of lower productivities producing, which reduces expected output conditional on successful entry. The first-order condition for $\tilde{q}_d^A$ yields:

$$\frac{\tilde{q}_d^A}{\tilde{\varphi}_d^A} = (\sigma - 1)\left[f_d + \frac{f_e}{1 - G(\varphi_d^A)}\right]. \tag{46}$$

The first-order condition for φ_d^A is:

$$\frac{\frac{d\tilde{\varphi}_d^A}{d\varphi_d^A}\tilde{q}_d^A}{\left(\tilde{\varphi}_d^A\right)^2} = \frac{g\left(\varphi_d^A\right) f_e}{\left[1 - G\left(\varphi_d^A\right)\right]^2}.$$

Noting that:

$$\frac{d\tilde{\varphi}_d^A}{d\varphi_d^A} = \frac{1}{\sigma - 1}\left[\left(\tilde{\varphi}_d^A\right)^{\sigma-1} - \left(\varphi_d^A\right)^{\sigma-1}\right] \frac{g\left(\varphi_d^A\right)}{1 - G\left(\varphi_d^A\right)} \left(\tilde{\varphi}_d^A\right)^{1-(\sigma-1)},$$

the first-order condition for φ_d^A can be re-written as:

$$\frac{\tilde{q}_d^A \left(\tilde{\varphi}_d^A\right)^{-\sigma}}{\sigma - 1}\left[\left(\tilde{\varphi}_d^A\right)^{\sigma-1} - \left(\varphi_d^A\right)^{\sigma-1}\right] = \frac{f_e}{1 - G\left(\varphi^A\right)}.$$

Substituting the optimal quantity $\tilde{q}_d^A$ from (46) delivers the planner's solution for the optimal cutoff φ_d^A:

$$\left[1-G(\varphi_d^A)\right]\left[\left(\frac{\tilde{\varphi}_d^A}{\varphi_d^A}\right)^{\sigma-1}-1\right]f_d=f_e, \tag{47}$$

which corresponds to the free entry condition in the market economy (6).

Therefore, the social planner chooses the same productivity cutoff φ_d^A (and the same values of all other endogenous variables) as in the market equilibrium of the heterogeneous firm model, which implies that the market equilibrium is efficient. In the heterogeneous firm model, changes in fixed costs and other parameters induce the social planner to change the productivity cutoff φ_d^A in addition to the output of a firm with weighted average productivity $\tilde{q}_d^A$ (and hence the mass of entrants M_e). In contrast, in the homogeneous firm model, productivity is constant by assumption, and hence changes in fixed costs and other parameters only induce changes in the mass of producing firms and output per firm.

4.2 Open Economy

The open economy real consumption index for the representative consumer is:

$$Q=\left[M_e\int_{\varphi_d^T}^{\infty}q_d\left(\varphi\right)^{(\sigma-1)/\sigma}dG\left(\varphi\right)+M_e\int_{\varphi_x^T}^{\infty}\left(\frac{q_x\left(\varphi\right)}{\tau}\right)^{(\sigma-1)/\sigma}dG\left(\varphi\right)\right]^{\sigma/(\sigma-1)}. \tag{48}$$

The social planner chooses the productivity cutoffs φ_d^T and φ_x^T, the output levels $q_d(\varphi)$ for the domestic market for all producing firms $\varphi\geq\varphi_d^T$, the output levels $q_x\left(\varphi\right)$ for the export market for all exporting firms $\varphi\geq\varphi_x^T$, and the mass of entrants M_e to maximize Q subject to the aggregate labor constraint:

$$L=M_e\left\{\begin{array}{c}\int_{\varphi_d^T}^{\infty}\frac{q_d(\varphi)}{\varphi}dG\left(\varphi\right)+\int_{\varphi_x^T}^{\infty}\frac{q_x(\varphi)}{\varphi}dG\left(\varphi\right)\\+\left[1-G(\varphi_d^T)\right]f_d+\left[1-G(\varphi_x^T)\right]f_x+f_e\end{array}\right\},$$

where the social planner faces the same productivity distribution $G(\varphi)$ and entry cost f_e per firm as in the market allocation.

The planner chooses the output levels $q_d(\varphi)$ and $q_x\left(\varphi\right)$ to equate the marginal rates of transformations and marginal rates of substitution for firms with different productivities:

$$\frac{q_d(\varphi_1)}{q_d(\varphi_2)}=\frac{q_x(\varphi_1)}{q_x(\varphi_2)}=\left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma},$$

$$\frac{q_x(\varphi_1)}{q_d(\varphi_2)}=\tau^{1-\sigma}\left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma},$$

which are the same relationships between relative quantities and relative productivities as in the market equilibrium.

Using these relationships, we can rewrite the consumption index Q and the aggregate labor constraint as a function of the mass of firms serving each market M_t and the output level $\tilde{q}_t^T\equiv q_d(\tilde{\varphi}_t^T)$ of a firm with a weighted average productivity $\tilde{\varphi}_t^T$:

$$Q=M_t^{\sigma/(\sigma-1)}\tilde{q}_t^T,$$

$$\begin{aligned}L&=M_t\left[\frac{\tilde{q}_t^T}{\tilde{\varphi}_t^T}+\frac{\left[1-G(\varphi_d^T)\right]f_d+\left[1-G(\varphi_x^T)\right]f_x+f_e}{\left[1-G(\varphi_d^T)\right]+\left[1-G(\varphi_x^T)\right]}\right],\\&=M_t\left[\frac{\tilde{q}_t^T}{\tilde{\varphi}_t^T}+\frac{1}{1+\chi}\left(f_d+\chi f_x+\frac{f_e}{1-G(\varphi_d^T)}\right)\right].\end{aligned}$$

The mass of firms serving each market (M_t) and weighted average productivity ($\tilde{\varphi}_t^T$) are defined as follows:

$$M_t = [1+\chi] M = [1+\chi] \left[1 - G\left(\varphi_d^T\right)\right] M_e, \tag{49}$$

$$\begin{aligned} \tilde{\varphi}_t^T &= \left\{ \frac{\left[1 - G\left(\varphi_d^T\right)\right] \left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \left[1 - G\left(\varphi_x^T\right)\right] \tau^{1-\sigma} \left(\tilde{\varphi}_x^T\right)^{\sigma-1}}{\left[1 - G\left(\varphi_d^T\right)\right] + \left[1 - G\left(\varphi_x^T\right)\right]} \right\}^{1/(\sigma-1)}, \\ &= \left\{ \frac{1}{1+\chi} \left[\left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \chi \tau^{1-\sigma} \left(\tilde{\varphi}_x^T\right)^{\sigma-1} \right] \right\}^{1/(\sigma-1)}. \end{aligned} \tag{50}$$

where χ is the proportion of exporting firms:

$$\chi = \frac{1 - G\left(\varphi_x^T\right)}{1 - G\left(\varphi_d^T\right)}.$$

Using the aggregate labor constraint, we can rewrite the real consumption index as:

$$Q = L^{\sigma/(\sigma-1)} \left[\frac{\tilde{q}_t^T}{\tilde{\varphi}_t^T} + \frac{\left[1 - G(\varphi_d^T)\right] f_d + \left[1 - G(\varphi_x^T)\right] f_x + f_e}{\left[1 - G(\varphi_d^T)\right] + \left[1 - G(\varphi_x^T)\right]} \right]^{-\sigma/(\sigma-1)} \tilde{q}_t^T$$

$$Q = L^{\sigma/(\sigma-1)} \left[\frac{\tilde{q}_t^T}{\tilde{\varphi}_t^T} + \frac{1}{1+\chi} \left(f_d + \chi f_x + \frac{f_e}{1 - G(\varphi_d^T)} \right) \right]^{-\sigma/(\sigma-1)} \tilde{q}_t^T. \tag{51}$$

The social planner chooses the productivity cutoffs φ_d^T and φ_x^T and quantity $\tilde{q}_t^T$ to maximize this consumption index. The trade-offs faced by the social planner are as follows. A lower domestic cutoff again reduces expected entry costs conditional on successful entry, and thereby releases more labor for production. But this lower domestic cutoff involves firms of lower productivities producing, which reduces expected output conditional on successful entry. A lower export cutoff for a given domestic cutoff increases the probability of exporting, which uses more labor in the fixed costs of exporting and reduces expected output conditional on exporting. But this lower export cutoff also increases the fraction of foreign varieties available to domestic consumers. The first-order condition for $\tilde{q}_t^T$ yields:

$$\begin{aligned} \frac{\tilde{q}_t^T}{\tilde{\varphi}_t^T} &= (\sigma-1) \left[\frac{\left[1 - G\left(\varphi_d^T\right)\right] f_d + \left[1 - G\left(\varphi_x^T\right)\right] f_x + f_e}{\left[1 - G\left(\varphi_d^T\right)\right] + \left[1 - G\left(\varphi_x^T\right)\right]} \right], \\ \frac{\tilde{q}_t^T}{\tilde{\varphi}_t^T} &= \frac{\sigma-1}{1+\chi} \left[f_d + \chi f_x + \frac{f_e}{\left[1 - G\left(\varphi_d^T\right)\right]} \right], \\ \left[1 - G(\varphi_d^T)\right] &\left[\frac{(1+\chi)\, \tilde{q}_t^T}{(\sigma-1)\, \tilde{\varphi}_t^T} - f_d - \chi f_x \right] = f_e, \end{aligned} \tag{52}$$

which equates the expected profits from entry to the sunk entry cost, because $\frac{(1+\chi)\tilde{q}_t^T}{(\sigma-1)\tilde{\varphi}_t^T} = \frac{(1+\chi)\tilde{r}_t}{\sigma}$ is expected variable profits conditional on successful entry.

The first-order condition for φ_d^T requires:

$$\begin{aligned} &-\frac{\tilde{q}_t^T \frac{d\tilde{\varphi}_t^T}{d\varphi_d^T}}{\left(\tilde{\varphi}_t^T\right)^2} - \frac{\left[1 - G\left(\varphi_x^T\right)\right] g\left(\varphi_d^T\right) f_d}{\left\{\left[1 - G\left(\varphi_d^T\right)\right] + \left[1 - G\left(\varphi_x^T\right)\right]\right\}^2} \\ &\qquad + \frac{\left[1 - G\left(\varphi_x^T\right)\right] g\left(\varphi_d^T\right) f_x}{\left\{\left[1 - G\left(\varphi_d^T\right)\right] + \left[1 - G\left(\varphi_x^T\right)\right]\right\}^2} + \frac{g\left(\varphi_d^T\right) f_e}{\left\{\left[1 - G\left(\varphi_d^T\right)\right] + \left[1 - G\left(\varphi_x^T\right)\right]\right\}^2} = 0. \end{aligned}$$

Using the first-order condition for $\tilde{q}_t^T$ and noting that:

$$\frac{d\tilde{\varphi}_t^T}{d\varphi_d^T} = \frac{1}{\sigma - 1} \left[\begin{array}{c} -\frac{[1-G(\varphi_x^T)]g(\varphi_d^T)(\tilde{\varphi}_d^T)^{\sigma-1}}{\{[1-G(\varphi_d^T)]+[1-G(\varphi_x^T)]\}^2} + \frac{\left[(\tilde{\varphi}_d^T)^{\sigma-1}-(\varphi_d^T)^{\sigma-1}\right]g(\varphi_d^T)}{[1-G(\varphi_d^T)]+[1-G(\varphi_x^T)]} \\ +\frac{[1-G(\varphi_x^T)]g(\varphi_d^T)\tau^{1-\sigma}(\tilde{\varphi}_x^T)^{\sigma-1}}{\{[1-G(\varphi_d^T)]+[1-G(\varphi_x^T)]\}^2} \end{array} \right] \left(\varphi_t^T\right)^{1-(\sigma-1)},$$

the first-order condition for φ_d^T can we written as follows:

$$\left(\frac{\varphi_d^T}{\tilde{\varphi}_t^T}\right)^{\sigma-1} \frac{1}{\sigma - 1} \frac{\tilde{q}_t^T}{\tilde{\varphi}_t^T} = \frac{r_d\left(\varphi_d^T\right)}{\sigma} = f_d, \tag{53}$$

which corresponds to the domestic cutoff condition for the open economy market equilibrium (18).

The first-order condition for φ_x^T requires:

$$-\frac{\tilde{q}_t^T \frac{d\tilde{\varphi}_t^T}{d\varphi_x^T}}{\left(\tilde{\varphi}_t^T\right)^2} + \frac{\left[1 - G\left(\varphi_d^T\right)\right] g\left(\varphi_x^T\right) f_d}{\left\{\left[1 - G\left(\varphi_d^T\right)\right] + \left[1 - G\left(\varphi_x^T\right)\right]\right\}^2} - \frac{\left[1 - G\left(\varphi_d^T\right)\right] g\left(\varphi_x^T\right) f_x}{\left\{\left[1 - G\left(\varphi_d^T\right)\right] + \left[1 - G\left(\varphi_x^T\right)\right]\right\}^2} + \frac{g\left(\varphi_x^T\right) f_e}{\left\{\left[1 - G\left(\varphi_d^T\right)\right] + \left[1 - G\left(\varphi_x^T\right)\right]\right\}^2} = 0.$$

Using the first-order condition for $\tilde{q}_t^T$ and noting that:

$$\frac{d\tilde{\varphi}_t^T}{d\varphi_x^T} = \frac{1}{\sigma - 1} \left[\begin{array}{c} \frac{[1-G(\varphi_d^T)]g(\varphi_x^T)(\tilde{\varphi}_d^T)^{\sigma-1}}{\{[1-G(\varphi_d^T)]+[1-G(\varphi_x^T)]\}^2} - \frac{[1-G(\varphi_d^T)]g(\varphi_x^T)\tau^{1-\sigma}(\tilde{\varphi}_x^T)^{\sigma-1}}{\{[1-G(\varphi_d^T)]+[1-G(\varphi_x^T)]\}^2} \\ +\frac{\tau^{1-\sigma}\left[(\tilde{\varphi}_x^T)^{\sigma-1}-(\varphi_x^T)^{\sigma-1}\right]g(\varphi_x^T)}{[1-G(\varphi_d^T)]+[1-G(\varphi_x^T)]} \end{array} \right] \left(\varphi_t^T\right)^{1-(\sigma-1)},$$

the first-order condition for φ_x^T can be expressed as:

$$\left(\frac{\varphi_x^T}{\tilde{\varphi}_t^T}\right)^{\sigma-1} \frac{\tau^{1-\sigma}}{\sigma - 1} \frac{\tilde{q}_t^T}{\tilde{\varphi}_t^T} = \frac{r_x\left(\varphi_x^T\right)}{\sigma} = f_x, \tag{54}$$

which corresponds to the export cutoff condition for the open economy market equilibrium (19).

From the two cutoff conditions (53) and (54), the relationship between the domestic and export cutoffs is the same as in the open economy market equilibrium:

$$\varphi_x^T = \tau \left(\frac{f_x}{f_d}\right)^{\frac{1}{\sigma-1}} \varphi_d^T,$$

and the free entry condition (52) can be written in the same form as in the open economy market equilibrium (22):

$$\left[1 - G(\varphi_d^T)\right] \left[\left(\frac{\tilde{\varphi}_d^T}{\varphi_d^T}\right)^{\sigma-1} - 1\right] f_d + \left[1 - G(\varphi_x^T)\right] \left[\left(\frac{\tilde{\varphi}_x^T}{\varphi_x^T}\right)^{\sigma-1} - 1\right] f_x = f_e. \tag{55}$$

Since the free entry, domestic cutoff and export cutoff conditions for the social planner are the same as those for the open economy market equilibrium, the social planner chooses the same productivity cutoffs φ_d^T and φ_x^T (and the same value of all other endogenous variables) as in the open economy market equilibrium of the heterogeneous firm model. Hence the open economy market equilibrium of the heterogeneous firm model is efficient.

4.3 Open Versus Closed Economy

Recall that the heterogeneous and homogeneous firm models are specified to yield the same values of all aggregate variables (including welfare) in the closed economy. Furthermore, it is technologically feasible for the social planner to choose the same domestic productivity cutoff in the open economy as in the closed economy ($\varphi_d^T = \varphi_d^A$) and to choose all firms to export ($\varphi_x^T = \varphi_d^T$). In this hypothetical allocation, the heterogeneous and homogeneous firm models would also generate the same values of all aggregate variables (including welfare) in the open economy.

However, comparing the closed and open economy free entry conditions (47) and (55), the social planner in general chooses a different domestic productivity cutoff in the open economy than in the closed economy ($\varphi_d^T \neq \varphi_d^A$) and in general restricts exporting to a proper subset of more productive firms ($\varphi_d^T < \varphi_x^T < \infty$). Since the social planner chooses different productivity cutoffs in the open economy when it is technological feasible to choose the same productivity cutoffs as in the closed economy, revealed preference implies that these different productivity cutoffs must yield at least as high level of welfare as the hypothetical allocation with the same productivity cutoffs. Furthermore, the social planner's objective (51) is globally concave in $\{\varphi_d^T, \varphi_x^T, \tilde{q}_t^T\}$, which implies that the different productivity cutoffs must yield strictly higher welfare than the hypothetical allocation with the same productivity cutoffs. Since the social planner's choice corresponds to the open economy equilibrium of the heterogeneous firm model, and the hypothetical allocation corresponds to the open economy equilibrium of the homogeneous firm model, it follows that open economy welfare is strictly higher in the heterogeneous firm model than in the homogeneous firm model whenever the productivity cutoffs differ in the open and closed economies.

This revealed preference argument implies larger welfare gains in the heterogeneous firm model both when there is selection into export markets ($\tau\left(f_x/f_d\right)^{1/(\sigma-1)} > 1$ and $\varphi_x^T > \varphi_d^T$) and when all firms export ($\tau\left(f_x/f_d\right)^{1/(\sigma-1)} \leq 1$ and $\varphi_x^T = \varphi_d^T$), as long as fixed exporting costs are positive ($f_x > 0$). Comparing the free entry conditions in the open and closed economies (47) and (55), the social planner chooses different domestic productivity cutoffs in the open and closed economies ($\varphi_d^T \neq \varphi_d^A$) for positive fixed exporting costs. The reason is that the social planner's objective in the open economy (51) features a different combination of fixed and variable costs from her objective in the closed economy (45). The social planner's optimal response to these different fixed and variable costs is to change the range of productivities for which firms serve each market as well as the mass of firms and average output per firm. Thus the greater welfare gains in the heterogeneous firm model reflect the presence of an additional adjustment margin (the firm productivity ranges) relative to the homogeneous firm model. In the heterogeneous firm model, the social planner can allocate low-productivity firms to serve only the domestic market and reallocate resources to higher-productivity exporting firms. Therefore the welfare that the social planner can achieve in a model with this additional adjustment margin must be at least as high (and in general higher) than in a model without it.

While we have focused on opening the closed economy to trade, an analogous analysis can be undertaken for the reverse comparative static of closing the open economy. Choosing the degenerate productivity distribution in the homogeneous firm model such that the two models to have the same welfare in an initial open economy equilibrium, the welfare costs of closing the open economy are smaller in the heterogeneous firm model than in the homogeneous firm. Again the welfare that the social planner can achieve in the heterogeneous firm model with its additional adjustment margin must be at least as high (and in general higher) than in the homogeneous model. As a result, whether we consider increases or reductions in trade costs, welfare in the two models is the same for the calibrated value of trade costs, but is strictly higher in the heterogeneous firm model than in the homogeneous firm model for all other values of trade costs.

5 Alternative Revealed Preference Derivation

In this section, we provide a complementary alternative derivation of the result that the market equilibrium of the heterogeneous firm model is efficient, which is used as part of our revealed preference argument above. This alternative derivation formulates the social planner's problem as a Lagrangian and uses the first-order conditions for this Lagrangian to show that the socially optimal allocation replicates the allocation in the market equilibrium.

5.1 Closed Economy

5.1.1 Social Planner's Problem

We first show that in the closed economy the social planner chooses the same values of $\{h(\varphi),\ \varphi_d^A,\ M_e,\ Q\}$ as in the market equilibrium, where $h(\varphi)$ denotes variable labor input. The social planner's problem is to maximize the welfare of the representative consumer subject to the constraints of the entry and production technologies. The welfare of the representative consumer is determined by the real consumption index:

$$Q = \left[M_e \int_{\varphi_d^A}^{\infty} (\varphi h(\varphi))^{\rho}\, dG(\varphi) \right]^{\frac{1}{\rho}}. \tag{56}$$

where $\sigma = \frac{1}{1-\rho}$. Therefore the social planner maximizes the following Lagrangian:

$$\begin{aligned}\max_{\{h(\cdot),\varphi_d^A,M_e\}} & \left[M_e \int_{\varphi_d^A}^{\infty} (\varphi h(\varphi))^{\rho}\, dG(\varphi) \right]^{\frac{1}{\rho}} \\ & - \lambda \left[M_e \int_{\varphi_d^A}^{\infty} h(\varphi)\, dG(\varphi) + M_e\left[1 - G(\varphi_d)\right] f_d + M_e f_e - L \right],\end{aligned}$$

where λ is the multiplier on the social planner's constraint.

5.1.2 First-order Conditions

The first-order condition for M_e is :

$$\frac{1}{\rho M_e} Q - \frac{\lambda}{M_e} \left[M_e \int_{\varphi_d^A}^{\infty} h(\varphi)\, dG(\varphi) + M_e \left[1 - G\left(\varphi_d^A\right)\right] f_d + M_e f_e \right] = 0. \tag{57}$$

The first-order condition for $h(\varphi)$ is:

$$M_e \varphi^{\rho} h(\varphi)^{\rho-1} g(\varphi)\, Q^{1-\rho} - \lambda M_e g(\varphi) = 0. \tag{58}$$

The first-order condition for φ_d^A is:

$$-\frac{M_e}{\rho} \left(\varphi_d^A h(\varphi_d)\right)^{\rho} g(\varphi_d)\, Q^{1-\rho} + \lambda M_e h\left(\varphi_d^A\right) g\left(\varphi_d^A\right) + \lambda g\left(\varphi_d^A\right) M_e f_d = 0. \tag{59}$$

The first-order condition for λ is:

$$\left[M_e \int_{\varphi_d^A}^{\infty} h(\varphi)\, dG(\varphi) + M_e \left[1 - G\left(\varphi_d^A\right)\right] f_d + M_e f_e - L \right] = 0. \tag{60}$$

5.1.3 Lagrange Multiplier

Using the first-order condition for λ (60), the first-order condition for M_e (57) becomes:

$$\frac{1}{\rho M_e}Q - \frac{\lambda}{M_e}L = 0,$$

$$\lambda = \frac{Q}{\rho L}. \tag{61}$$

5.1.4 Employment

Using the first-order condition for λ (60), the first-order condition for $h(\varphi)$ (58) can be written:

$$h(\varphi) = \varphi^{\frac{\rho}{1-\rho}} Q^{-\frac{\rho}{1-\rho}} (\rho L)^{\frac{1}{1-\rho}}. \tag{62}$$

or equivalently:

$$h(\varphi) = \varphi^{\sigma-1} Q^{-(\sigma-1)} \left(\frac{\sigma-1}{\sigma}L\right)^{\sigma}, \tag{63}$$

5.1.5 Labor Market Clearing

Using the solution for $h(\varphi)$ from (62) in the first-order condition for λ (60), the labor market clearing condition can be written as follows:

$$L = M_e \int_{\varphi_d^A}^{\infty} h(\varphi)\, dG(\varphi) + M_e \left[1 - G\left(\varphi_d^A\right)\right] f_d + M_e f_e.$$

$$L = M_e Q^{-(\sigma-1)} (\rho L)^{\sigma} \left[1 - G\left(\varphi_d^A\right)\right] \left(\tilde{\varphi}_d^A\right)^{\sigma-1} + M_e \left[1 - G\left(\varphi_d^A\right)\right] f_d + M_e f_e. \tag{64}$$

$$\left(\tilde{\varphi}_d^A\right)^{\sigma-1} = \int_{\varphi_d^A}^{\infty} \varphi^{\sigma-1} \frac{dG(\varphi)}{1 - G\left(\varphi_d^A\right)}.$$

5.1.6 Real Consumption Index and Mass of Firms

Using the solution for $h(\varphi)$ from (62) in the real consumption index (56) we get:

$$Q = M_e^{\frac{1}{\sigma-1}} \rho L \left[1 - G\left(\varphi_d^A\right)\right]^{\frac{1}{\sigma-1}} \tilde{\varphi}_d^A. \tag{65}$$

Substituting this expression for Q into the labor market clearing condition (64), we obtain the following expression for the mass of firms as a function of φ_d^A and parameters:

$$M_e = \frac{(1-\rho)L}{\left[1 - G\left(\varphi_d^A\right)\right] f_d + f_e}. \tag{66}$$

Using this result in (65), the real consumption index can also be written in terms of φ_d^A and parameters:

$$Q = \frac{\rho (1-\rho)^{\frac{1}{\sigma-1}} L^{\frac{\sigma}{\sigma-1}} \left[1 - G\left(\varphi_d^A\right)\right]^{\frac{1}{\sigma-1}} \tilde{\varphi}_d^A}{\left[\left[1 - G\left(\varphi_d^A\right)\right] f_d + f_e\right]^{\frac{1}{\sigma-1}}}. \tag{67}$$

5.1.7 Productivity Cutoff Condition

Using the first-order condition for λ (60) and the solution for $h(\varphi)$ in (62), the first-order condition for φ_d^A can be written as:

$$\frac{1}{\sigma-1} \left(\varphi_d^A\right)^{\sigma-1} Q^{-(\sigma-1)} (\rho L)^{\sigma} = f_d. \tag{68}$$

5.1.8 Free Entry

To derive the analogue of the free entry condition for the social planner, note that the labor market clearing condition (64) gives us one expression for L/M_e:

$$\frac{L}{M_e} = Q^{-(\sigma-1)} (\rho L)^{\sigma} \left[1 - G\left(\varphi_d^A\right)\right] \left(\tilde{\varphi}_d^A\right)^{\sigma-1} + \left[1 - G\left(\varphi_d^A\right)\right] f_d + f_e,$$

while the mass of firms (66) gives us another expression for L/M_e:

$$\frac{L}{M_e} = \frac{\left[1 - G\left(\varphi_d^A\right)\right] f_d + f_e}{1-\rho} = \sigma \left[1 - G\left(\varphi_d^A\right)\right] f_d + \sigma f_e.$$

Equating these two expressions, we obtain the analogue of the free entry condition for the social planner:

$$\frac{1}{\sigma-1} \left[1 - G\left(\varphi_d^A\right)\right] \left(\tilde{\varphi}_d^A\right)^{\sigma-1} Q^{-(\sigma-1)} (\rho L)^{\sigma} - \left[1 - G\left(\varphi_d^A\right)\right] f_d = f_e. \tag{69}$$

Using the productivity cutoff condition (68) to substitute for $\frac{1}{\sigma-1} Q^{-(\sigma-1)} (\rho L)^{\sigma}$, the analogue of the free entry condition becomes:

$$f_d \left[1 - G\left(\varphi_d^A\right)\right] \left[\left(\frac{\tilde{\varphi}_d^A}{\varphi_d^A}\right)^{\sigma-1} - 1\right] = f_e. \tag{70}$$

The social planner's choices of $\{h(\varphi),\ \varphi_d^A,\ M_e,\ Q\}$ are determined by (63), (70), (66) and (67). This is the same system of equations that characterizes the open economy market equilibrium. It follows that the welfare-maximizing social planner chooses the same allocation $\{h(\varphi),\ \varphi_d^A,\ M_e,\ Q\}$ as the market equilibrium and that the market equilibrium is efficient.

5.2 Open Economy

5.2.1 Social Planner's Problem

We now show that in the open economy the social planner chooses the same values of $\{h_d(\varphi),\ h_x(\varphi),\ \varphi_d^T,\ \varphi_x^T,\ M_e,\ Q\}$ as in the market equilibrium, where $h_d(\varphi)$ denotes variable labor input for the domestic market and $h_x(\varphi)$ denotes variable labor input for the export market. The social planner's problem is to maximize the joint welfare of the two countries subject to the constraints of the entry and production technologies. Since the two countries are symmetric, the social planner maximizes the welfare of the representative consumer in each country, which is determined by the real consumption index:

$$Q = \left[M_e \int_{\varphi_d^T}^{\infty} (\varphi h_d(\varphi))^{\rho} \, dG(\varphi) + M_e \tau^{-\rho} \int_{\varphi_x^T}^{\infty} (\varphi h_x(\varphi))^{\rho} \, dG(\varphi)\right]^{\frac{1}{\rho}}. \tag{71}$$

Therefore the social planner maximizes the following Lagrangian:

$$\max_{\left\{h_d(\cdot), h_x(\cdot), \varphi_d^T, \varphi_x^T, M_e\right\}} \left[M_e \int_{\varphi_d^T}^{\infty} (\varphi h_d(\varphi))^{\rho} \, dG(\varphi) + M_e \tau^{-\rho} \int_{\varphi_x^T}^{\infty} (\varphi h_x(\varphi))^{\rho} \, dG(\varphi)\right]^{\frac{1}{\rho}}$$
$$- \lambda \left[\begin{array}{c} M_e \int_{\varphi_d^T}^{\infty} h_d(\varphi) \, dG(\varphi) + M_e \int_{\varphi_x^T}^{\infty} h_x(\varphi) \, dG(\varphi) + M_e \left[1 - G\left(\varphi_d^T\right)\right] f_d \\ + M_e \left[1 - G\left(\varphi_x^T\right)\right] f_x + M_e f_e - L \end{array}\right],$$

where λ is the multiplier on the constraint.

5.2.2 First-order Conditions

The first-order condition for M_e is :

$$\frac{1}{\rho M_e}Q-\frac{\lambda}{M_e}\left[\begin{array}{c} M_e\int_{\varphi_d^T}^{\infty}h_d\left(\varphi\right)dG\left(\varphi\right)+M_e\int_{\varphi_x^T}^{\infty}h_x\left(\varphi\right)dG\left(\varphi\right)+M_e\left[1-G\left(\varphi_d^T\right)\right]f_d \\ +M_e\left[1-G\left(\varphi_x^T\right)\right]f_x+M_ef_e \end{array}\right]=0. \tag{72}$$

The first-order condition for $h_d\left(\varphi\right)$ is:

$$M_e\varphi^{\rho}h_d\left(\varphi\right)^{\rho-1}g\left(\varphi\right)Q^{1-\rho}-\lambda M_eg\left(\varphi\right)=0. \tag{73}$$

The first-order condition for $h_x\left(\varphi\right)$ is:

$$\tau^{-\rho}M_e\varphi^{\rho}h_x\left(\varphi\right)^{\rho-1}g\left(\varphi\right)Q^{1-\rho}-\lambda M_eg\left(\varphi\right)=0. \tag{74}$$

The first-order condition for φ_d^T is:

$$-\frac{M_e}{\rho}\left(\varphi_d^Th_d\left(\varphi_d^T\right)\right)^{\rho}g\left(\varphi_d^T\right)Q^{1-\rho}+\lambda M_eh_d\left(\varphi_d^T\right)g\left(\varphi_d^T\right)+\lambda g\left(\varphi_d^T\right)M_ef_d=0. \tag{75}$$

The first-order condition for φ_x^T is:

$$-\frac{M_e}{\rho}\tau^{-\rho}\left(\varphi_x^Th_d\left(\varphi_x^T\right)\right)^{\rho}g\left(\varphi_x^T\right)Q^{1-\rho}+\lambda M_eh_x\left(\varphi_x^T\right)g\left(\varphi_x^T\right)+\lambda g\left(\varphi_x^T\right)M_ef_x=0. \tag{76}$$

The first-order condition for λ is:

$$\left[\begin{array}{c} M_e\int_{\varphi_d^T}^{\infty}h_d\left(\varphi\right)dG\left(\varphi\right)+M_e\int_{\varphi_x^T}^{\infty}h_x\left(\varphi\right)dG\left(\varphi\right)+M_e\left[1-G\left(\varphi_d^T\right)\right]f_d \\ +M_e\left[1-G\left(\varphi_x^T\right)\right]f_x+M_ef_e-L \end{array}\right]=0. \tag{77}$$

5.2.3 Lagrange Multiplier

Using the first-order condition for λ (77), the first-order condition for M_E (72) becomes:

$$\frac{1}{\rho M_e}Q-\frac{\lambda}{M_e}L=0,$$

$$\lambda=\frac{Q}{\rho L}. \tag{78}$$

5.2.4 Employment

Using the first-order condition for λ (77), the first-order condition for $h_d\left(\varphi\right)$ (73) can be written:

$$h_d\left(\varphi\right)=\varphi^{\frac{\rho}{1-\rho}}Q^{-\frac{\rho}{1-\rho}}\left(\rho L\right)^{\frac{1}{1-\rho}}. \tag{79}$$

or equivalently:

$$h_d\left(\varphi\right)=\varphi^{\sigma-1}Q^{-(\sigma-1)}\left(\frac{\sigma-1}{\sigma}L\right)^{\sigma}, \tag{80}$$

Using the first-order condition for λ (77), the first-order condition for $h_x\left(\varphi\right)$ (74) can be written:

$$h_x\left(\varphi\right)=\tau^{-\frac{\rho}{1-\rho}}\varphi^{\frac{\rho}{1-\rho}}Q^{-\frac{\rho}{1-\rho}}\left(\rho L\right)^{\frac{1}{1-\rho}}. \tag{81}$$

or equivalently:

$$h_x\left(\varphi\right)=\tau^{-(\sigma-1)}\varphi^{\sigma-1}Q^{-(\sigma-1)}\left(\frac{\sigma-1}{\sigma}L\right)^{\sigma}, \tag{82}$$

5.2.5 Labor Market Clearing

Using the solutions for $h_d(\varphi)$ and $h_x(\varphi)$ from (79) and (81) in the first-order condition for λ (77), the labor market clearing condition can be written as follows:

$$\begin{aligned} L = {} & M_e \int_{\varphi_d^T}^{\infty} h_d(\varphi)\, dG(\varphi) + M_e \int_{\varphi_x^T}^{\infty} h_x(\varphi)\, dG(\varphi) + M_e \left[1 - G\left(\varphi_d^T\right)\right] f_d \\ & + M_e \left[1 - G\left(\varphi_x^T\right)\right] f_x + M_e f_e. \end{aligned}$$

$$\begin{aligned} L = {} & M_e Q^{-(\sigma-1)} (\rho L)^{\sigma} \left[\left[1 - G\left(\varphi_d^T\right)\right] \left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \tau^{-(\sigma-1)} \left[1 - G\left(\varphi_x^T\right)\right] \left(\tilde{\varphi}_x^T\right)^{\sigma-1}\right] \\ & + M_e \left[1 - G\left(\varphi_d^T\right)\right] f_d + M_e \left[1 - G\left(\varphi_x^T\right)\right] f_x + M_e f_e, \end{aligned} \tag{83}$$

where

$$\left(\tilde{\varphi}_d^T\right)^{\sigma-1} = \int_{\varphi_d^T}^{\infty} \varphi^{\sigma-1} \frac{dG(\varphi)}{1 - G\left(\varphi_d^T\right)}, \qquad \left(\tilde{\varphi}_x^T\right)^{\sigma-1} = \int_{\varphi_x^T}^{\infty} \varphi^{\sigma-1} \frac{dG(\varphi)}{1 - G\left(\varphi_x^T\right)}.$$

5.2.6 Real Consumption Index and Mass of Firms

Using the solutions for $h_d(\varphi)$ and $h_x(\varphi)$ from (79) and (81) in the real consumption index (71) we get:

$$Q = M_e^{\frac{1}{\sigma-1}} \rho L \left[\left[1 - G\left(\varphi_d^T\right)\right] \left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \tau^{-(\sigma-1)} \left[1 - G\left(\varphi_x^T\right)\right] \left(\tilde{\varphi}_x^T\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}. \tag{84}$$

Substituting this expression for Q into the labor market clearing condition (83), we obtain the following expression for the mass of firms as a function of φ_d^T, φ_x^T and parameters:

$$M_e = \frac{(1-\rho) L}{\left[1 - G\left(\varphi_d^T\right)\right] f_d + \left[1 - G\left(\varphi_x^T\right)\right] f_x + f_e}. \tag{85}$$

Using this result in (84), the real consumption index can also be written in terms of φ_d^T and parameters:

$$Q = \frac{\rho (1-\rho)^{\frac{1}{\sigma-1}} L^{\frac{\sigma}{\sigma-1}} \left[\left[1 - G\left(\varphi_d^T\right)\right] \left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \tau^{-(\sigma-1)} \left[1 - G\left(\varphi_x^T\right)\right] \left(\tilde{\varphi}_x^T\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}}{\left[\left[1 - G\left(\varphi_d^T\right)\right] f_d + \left[1 - G\left(\varphi_x^T\right)\right] f_x + f_e\right]^{\frac{1}{\sigma-1}}}. \tag{86}$$

5.2.7 Productivity Cutoff Conditions

Using the first-order condition for λ (77) and the solution for $h_d(\varphi)$ in (79), the first-order condition for φ_d^T can be written as:

$$\frac{1}{\sigma-1} \left(\varphi_d^T\right)^{\sigma-1} Q^{-(\sigma-1)} (\rho L)^{\sigma} = f_d. \tag{87}$$

Using the first-order condition for λ (77) and the solution for $h_x(\varphi)$ in (81), the first-order condition for φ_x^* can be written as:

$$\frac{1}{\sigma-1} \tau^{-(\sigma-1)} \left(\varphi_x^T\right)^{\sigma-1} Q^{-(\sigma-1)} (\rho L)^{\sigma} = f_x. \tag{88}$$

Together these cutoff conditions imply that the relative productivity cutoffs satisfy:

$$\left(\frac{\varphi_x^T}{\varphi_d^T}\right)^{\sigma-1} = \tau^{\sigma-1} \frac{f_x}{f_d}. \tag{89}$$

5.2.8 Free Entry

To derive the analogue of the free entry condition for the social planner, note that the labor market clearing condition (83) gives us one expression for L/M_e:

$$\frac{L}{M_e} = Q^{-(\sigma-1)} (\rho L)^{\sigma} \left[\left[1 - G\left(\varphi_d^T\right)\right] \left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \tau^{-(\sigma-1)} \left[1 - G\left(\varphi_x^T\right)\right] \left(\tilde{\varphi}_x^T\right)^{\sigma-1} \right] \\ + \left[1 - G\left(\varphi_d^T\right)\right] f_d + \left[1 - G\left(\varphi_x^T\right)\right] f_x + f_e,$$

while the mass of firms (85) gives us another expression for L/M_e:

$$\frac{L}{M_e} = \frac{\left[1 - G\left(\varphi_d^T\right)\right] f_d + \left[1 - G\left(\varphi_x^T\right)\right] f_x + f_e}{1-\rho} \\ = \sigma \left[1 - G\left(\varphi_d^T\right)\right] f_d + \sigma \left[1 - G\left(\varphi_x^T\right)\right] f_x + \sigma f_e.$$

Equating these two expressions, we obtain the analogue of the free entry condition for the social planner:

$$\frac{1}{\sigma-1} Q^{-(\sigma-1)} (\rho L)^{\sigma} \left[\left[1 - G\left(\varphi_d^T\right)\right] \left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \tau^{-(\sigma-1)} \left[1 - G\left(\varphi_x^T\right)\right] \left(\tilde{\varphi}_x^T\right)^{\sigma-1} \right] \\ - \left[1 - G\left(\varphi_d^T\right)\right] f_d - \left[1 - G\left(\varphi_x^T\right)\right] f_x = f_e.$$

Using the productivity cutoff conditions (87) and (88) to substitute for $\frac{1}{\sigma-1} Q^{-(\sigma-1)} (\rho L)^{\sigma}$, the analogue of the free entry condition can be written as:

$$f_d \left[1 - G\left(\varphi_d^T\right)\right] \left[\left(\frac{\tilde{\varphi}_d^T}{\varphi_d^T}\right)^{\sigma-1} - 1 \right] + f_x \left[1 - G\left(\varphi_x^T\right)\right] \left[\left(\frac{\tilde{\varphi}_x^T}{\varphi_x^T}\right)^{\sigma-1} - 1 \right] = f_e. \tag{90}$$

The social planner's choices of $\{h_d(\varphi),\ h_x(\varphi),\ \varphi_d^T,\ \varphi_x^T,\ M_e,\ Q\}$ are determined by (80), (82), (90), (89), (85) and (86). This is the same system of equations that characterizes the open economy market equilibrium. It follows that the welfare-maximizing social planner chooses the same allocation $\{h_d(\varphi)$, $h_x(\varphi)$, φ_d^T, φ_x^T, M_e, $Q\}$ as the market equilibrium and that the market equilibrium is efficient.

6 Trade Shares and Welfare

No additional derivations required.

7 Trade Liberalization in the Open Economy

No additional derivations required.

8 Pareto Productivity Distribution

In this section, we examine the implications of assuming a Pareto productivity distribution. We first examine the determinants of the domestic trade share and the mass of entrants with a general continuous productivity distribution. We next examine these relationships under the assumption of a Pareto productivity distribution.

8.1 General Productivity Distribution

8.1.1 Domestic Trade Share

Each country's trade share with itself in the heterogeneous firm model is:

$$\begin{aligned}\lambda = \frac{X_d}{X} &= \frac{M_e}{L}\int_{\varphi_d^T}^{\infty} r_d(\varphi) g(\varphi)\, d\varphi, \\ &= \frac{M_e}{L}\int_{\varphi_d^T}^{\infty} \left(\frac{\varphi}{\varphi_d^T}\right)^{\sigma-1} \sigma f_d g(\varphi)\, d\varphi, \\ &= \frac{M_e}{L}\left[1 - G(\varphi_d^T)\right]\left(\frac{\tilde{\varphi}_d^T}{\varphi_d^T}\right)^{\sigma-1}\sigma f_d.\end{aligned} \tag{91}$$

8.1.2 Mass Entrants

Free entry implies a relationship between average revenue, the productivity cutoffs and fixed costs in the heterogeneous firm model. We derive this relationship using the free entry condition:

$$\bar{\pi} = \frac{f_e}{1 - G\left(\varphi_d^T\right)}, \tag{92}$$

and the relationship between average profits and average revenue conditional upon entry:

$$\bar{r} = \frac{\sigma}{1 - G\left(\varphi_d^T\right)}\left[f_e + \left[1 - G\left(\varphi_d^T\right)\right] f_d + \left[1 - G\left(\varphi_x^T\right)\right] f_x\right], \tag{93}$$

which together imply:

$$\left[1 - G\left(\varphi_d^T\right)\right]\left[\left(\frac{\tilde{\varphi}_d^T}{\varphi_d^T}\right)^{\sigma-1} - 1\right] f_d + \left[1 - G\left(\varphi_x^T\right)\right]\left[\left(\frac{\tilde{\varphi}_x^T}{\varphi_x^T}\right)^{\sigma-1} - 1\right] f_x = f_e. \tag{94}$$

Additionally, the mass of entrants is related to the domestic productivity cutoff and average revenue as follows:

$$\left[1 - G\left(\varphi_d^T\right)\right] M_e = \frac{R}{\bar{r}} = \frac{L}{\bar{r}}. \tag{95}$$

8.2 Pareto Productivity Distribution

Assume that productivity in the heterogeneous firm model is drawn from a Pareto distribution:

$$g(\varphi) = k\left(\varphi_{\min}\right)^k \varphi^{-(k+1)}, \qquad k > \sigma - 1, \qquad \varphi_{\min} > 0. \tag{96}$$

8.2.1 Productivity Cutoffs

Under this distributional assumption, we derive closed form solutions for the productivity cutoffs and weighted average productivity in the closed and open economies of the heterogeneous firm model. The closed economy domestic productivity cutoff is:

$$\left(\varphi_d^A\right)^k = \frac{\sigma - 1}{k - (\sigma - 1)}\frac{f_d}{f_e}\varphi_{\min}^k, \tag{97}$$

while the open economy domestic productivity cutoff is:

$$\left(\varphi_d^T\right)^k = \frac{\sigma - 1}{k - (\sigma - 1)}\left[\frac{f_d + \tau^{-k}\left(\frac{f_x}{f_d}\right)^{\frac{-k}{\sigma-1}} f_x}{f_e}\right]\varphi_{\min}^k. \tag{98}$$

Weighted average productivity in the domestic market is related to the domestic productivity cutoff as follows:

$$\left(\frac{\tilde{\varphi}_d^A}{\varphi_d^A}\right)^{\sigma-1} = \left(\frac{\tilde{\varphi}_d^T}{\varphi_d^T}\right)^{\sigma-1} = \frac{k}{k-(\sigma-1)}. \tag{99}$$

The exporting productivity cutoff in the open economy equilibrium is related to the domestic productivity cutoff through (20): $\left(\varphi_x^T\right)^{\sigma-1} = \tau^{\sigma-1}\frac{f_x}{f_d}\left(\varphi_d^T\right)^{\sigma-1}$. Weighted average productivity in the export market is related to the exporting productivity cutoff as follows:

$$\left(\frac{\tilde{\varphi}_x^T}{\varphi_x^T}\right)^{\sigma-1} = \frac{k}{k-(\sigma-1)}. \tag{100}$$

8.2.2 Domestic Trade Share

Using these closed form solutions for the Pareto productivity distribution, each country's trade share with itself (91) in the heterogeneous firm model becomes:

$$\lambda = \frac{X_d}{X} = \frac{M_e}{L}\sigma f_d \left(\frac{\varphi_{\min}}{\varphi_d^T}\right)^k \frac{k}{k-(\sigma-1)}, \tag{101}$$

8.2.3 Mass of Entrants

With a Pareto productivity distribution the relationship between average revenue, the productivity cutoffs and fixed costs (94) in the heterogeneous firm model simplifies to:

$$\left[1 - G\left(\varphi_d^T\right)\right] f_d + \left[1 - G\left(\varphi_x^T\right)\right] f_x = \frac{k-(\sigma-1)}{\sigma-1} f_e. \tag{102}$$

Using this result in average revenue (93), we obtain:

$$\bar{r} = \frac{f_e}{1 - G\left(\varphi_d^T\right)} \frac{\sigma k}{\sigma - 1}, \tag{103}$$

which implies that the mass of entrants (95) is given by:

$$M_e = \frac{L}{f_e}\frac{\sigma-1}{\sigma k}, \tag{104}$$

which depends solely on labor supply and other parameters.

8.2.4 Welfare

Rearranging the domestic trade share under a Pareto productivity distribution (101), we obtain the following relationship between the domestic productivity cutoff and each country's trade share with itself in the heterogeneous firm model:

$$\varphi_d^T = \left[\frac{M_e}{\lambda L}\right]^{\frac{1}{k}} (\sigma f_d)^{\frac{1}{k}} \left[\frac{k}{k-(\sigma-1)}\right]^{\frac{1}{k}} \varphi_{\min}. \tag{105}$$

Using this result in welfare in the open economy equilibrium of the heterogeneous firm model (27), we can relate welfare to each country's trade share with itself:

$$\mathbb{W}_{\text{Het}}^T = \frac{w}{P} = \left(\frac{L}{\sigma f_d}\right)^{\frac{k-(\sigma-1)}{k(\sigma-1)}} \frac{\sigma-1}{\sigma} \left[\frac{k}{k-(\sigma-1)}\right]^{\frac{1}{k}} \varphi_{\min} M_e^{1/k} \lambda^{-1/k}. \tag{106}$$

Noting that under a Pareto productivity distribution the mass of entrants (104) is proportional to labor supply, we obtain:

$$\mathbb{W}^{T}_{\text{Het}} = \frac{w}{P} = \lambda^{-1/k} L^{\frac{1}{\sigma-1}} \left[\frac{\varphi^{k}_{\min} f_d^{1-\frac{k}{\sigma-1}}}{f_e \left(\frac{\sigma}{\sigma-1}\right)^{k} \sigma^{\frac{k}{\sigma-1}}} \frac{\sigma-1}{k-(\sigma-1)} \right]^{\frac{1}{k}}. \tag{107}$$

Therefore, under this assumption of a Pareto productivity distribution, knowing a country's domestic trade share (λ) and the shape parameter of the productivity distribution (k) is sufficient to determine the welfare gains from trade in the heterogeneous firm model:

$$\frac{\mathbb{W}^{T}_{\text{Het}}}{\mathbb{W}^{A}_{\text{Het}}} = \left[\frac{1}{\lambda^{T}_{\text{Het}}} \right]^{\frac{1}{k}}. \tag{108}$$

With a Pareto productivity distribution, the elasticity of the domestic trade share with respect to variable trade costs is:

$$\frac{d\lambda_{\text{Het}}}{d\tau} \frac{\tau}{\lambda_{\text{Het}}} = \begin{cases} k\left(1-\lambda_{\text{Het}}\right) & \tau\left(f_x/f_d\right)^{1/(\sigma-1)} \geq 1 \\ (\sigma-1)\left(1-\lambda_{\text{Het}}\right) & \tau\left(f_x/f_d\right)^{1/(\sigma-1)} < 1 \end{cases}, \tag{109}$$

In contrast, in the homogeneous firm model, the domestic trade share and welfare in the closed and open economies ((15) and (35) respectively) imply that the welfare gains from trade can be expressed as:

$$\frac{\mathbb{W}^{T}_{\text{Hom}}}{\mathbb{W}^{A}_{\text{Hom}}} = \left[\frac{F_d}{\lambda^{T}_{\text{Hom}}\left(F_d + f_x\right)} \right]^{\frac{1}{\sigma-1}}. \tag{110}$$

In this homogeneous firm model, knowing a country's domestic trade share (λ) and the elasticity of substitution (σ) is sufficient to determine the welfare gains from trade only in the absence of fixed export costs. Otherwise, the ratio of those fixed export costs to the remaining fixed costs is also needed to compute those welfare gains. In the homogeneous firm model, the elasticity of the domestic trade share with respect to variable trade costs remains as specified in Section 5 of the paper.

Proposition 6 *Assuming a Pareto productivity distribution with shape parameter $k > \sigma - 1$ and positive fixed exporting costs, the greater the dispersion of firm productivity (smaller k),* ***(a)*** *the larger the proportional welfare gains from opening the closed economy to trade in the heterogeneous firm model (larger $\mathbb{W}^{T}_{Het}/\mathbb{W}^{A}_{Het}$),* ***(b)*** *the larger (smaller) the proportional welfare gains (costs) from a reduction (increase) in variable trade costs.*

Proof. **(a)** First, consider parameter values for which the representative firm exports in the homogeneous firm model and there is selection into export markets in the heterogeneous firm model ($0 < \tau\left(f_x/F_d\right)^{1/(\sigma-1)} < 1 < \tau\left(f_x/f_d\right)^{1/(\sigma-1)}$). From (11) and (27), we have:

$$\frac{\mathbb{W}^{T}_{\text{Het}}}{\mathbb{W}^{A}_{\text{Het}}} = \frac{\varphi^{T}_{d}}{\varphi^{A}_{d}}. \tag{111}$$

In the special case of a Pareto productivity distribution and for these parameter values for which there is selection into export markets in the heterogeneous firm model, we have:

$$\frac{\varphi^{T}_{d}}{\varphi^{A}_{d}} = \left[1 + \left(\frac{1}{\tau\left(f_x/f_d\right)^{1/(\sigma-1)}} \right)^{k} \frac{f_x}{f_d} \right]^{1/k},$$

which can be written as:

$$\ln\left(\frac{\varphi_d^T}{\varphi_d^A}\right) = k^{-1}\ln\left[1+\left(\frac{1}{\tau\left(f_x/f_d\right)^{1/(\sigma-1)}}\right)^k\frac{f_x}{f_d}\right].$$

Note that

$$\frac{d\ln\left(\varphi_d^T/\varphi_d^A\right)}{dk} = -k^{-2}\ln\left[1+\left(\frac{1}{\tau(f_x/f_d)^{1/(\sigma-1)}}\right)^k\frac{f_x}{f_d}\right]-\frac{k^{-1}\ln\left(\tau(f_x/f_d)^{1/(\sigma-1)}\right)\left(\frac{1}{\tau(f_x/f_d)^{1/(\sigma-1)}}\right)^k\frac{f_x}{f_d}}{\left[1+\left(\frac{1}{\tau(f_x/f_d)^{1/(\sigma-1)}}\right)^k\frac{f_x}{f_d}\right]}<0, \tag{112}$$

where we have used $d\left(a^x\right)/dx = (\ln a)\, a^x$. Since a smaller value of k corresponds to greater productivity dispersion, it follows that greater productivity dispersion implies larger φ_d^T/φ_d^A. Second, consider parameter values for which the representative firm exports in the homogeneous firm model and all firms export in the heterogeneous firm model, but fixed exporting costs are still positive $(0<\tau\left(f_x/F_d\right)^{1/(\sigma-1)}<\tau\left(f_x/f_d\right)^{1/(\sigma-1)}\leq 1)$. From (11) and (28), we have:

$$\frac{\mathbb{W}_{\text{Het}}^T}{\mathbb{W}_{\text{Het}}^A} = \left(\frac{\left(1+\tau^{1-\sigma}\right)f_d}{f_d+f_x}\right)^{\frac{1}{\sigma-1}}\frac{\varphi_d^T}{\varphi_d^A}.$$

In the special case of a Pareto productivity distribution and for these parameter values for which all firms export in the heterogeneous firm model, we have:

$$\frac{\varphi_d^T}{\varphi_d^A} = \left[1+\frac{f_x}{f_d}\right]^{1/k},$$

which can be written as:

$$\ln\left(\frac{\varphi_d^T}{\varphi_d^A}\right) = k^{-1}\ln\left[1+\frac{f_x}{f_d}\right].$$

Note that

$$\frac{d\ln\left(\varphi_d^T/\varphi_d^A\right)}{dk} = -k^{-2}\ln\left[1+\frac{f_x}{f_d}\right]<0. \tag{113}$$

Since a smaller value of k corresponds to greater productivity dispersion, it follows that greater productivity dispersion again implies larger φ_d^T/φ_d^A. Taking (112) and (113) together and using (111), it follows that greater dispersion of firm productivity (smaller k) implies larger proportional welfare gains from opening the closed economy to trade. **(b)** Consider parameter values for which there is selection into export markets in the open economy equilibrium of the heterogeneous firm model $(\tau\left(f_x/f_d\right)^{1/(\sigma-1)}>1)$. In the special case of a Pareto productivity distribution, we have:

$$\varphi_d^T = \left(\frac{\sigma-1}{k-(\sigma-1)}\right)^{1/k}\left[\frac{f_d+\left(\frac{1}{\tau(f_x/f_d)^{1/(\sigma-1)}}\right)^k f_x}{f_e}\right]^{1/k}\varphi_{\min}.$$

We have:

$$\begin{aligned}\frac{d\varphi_d^T}{d\tau}\frac{\tau}{\varphi_d^T}d\tau &= -\frac{\left(\frac{1}{\tau(f_x/f_d)^{1/(\sigma-1)}}\right)^k f_x}{f_d+\left(\frac{1}{\tau(f_x/f_d)^{1/(\sigma-1)}}\right)^k f_x}d\tau \\ &= -\xi d\tau.\end{aligned}$$

Hence:

$$\frac{d\left(\frac{d\varphi_d^T}{d\tau}\frac{\tau}{\varphi_d^T}d\tau\right)}{dk}=\frac{\ln\left(\tau\left(f_x/f_d\right)^{1/(\sigma-1)}\right)\left(\frac{1}{\tau(f_x/f_d)^{1/(\sigma-1)}}\right)^k f_x}{f_d+\left(\frac{1}{\tau(f_x/f_d)^{1/(\sigma-1)}}\right)^k f_x}\left(1-\xi\right)d\tau,$$

which implies:

$$\frac{d\left(\frac{d\varphi_d^T}{d\tau}\frac{\tau}{\varphi_d^T}d\tau\right)}{dk}<0 \qquad \text{for} \qquad d\tau<0,$$
$$\frac{d\left(\frac{d\varphi_d^T}{d\tau}\frac{\tau}{\varphi_d^T}d\tau\right)}{dk}>0 \qquad \text{for} \qquad d\tau>0.$$

Therefore greater dispersion of firm productivity (smaller k) implies a larger elasticity of the domestic productivity cutoff with respect to reductions in variable trade costs, which from (27) implies greater proportional welfare gains from reductions in variable trade costs. By the same reasoning, greater dispersion of firm productivity (smaller k) implies a smaller elasticity of the domestic productivity cutoff with respect to increases in variable trade costs, which from (27) implies smaller proportional welfare costs from increases in variable trade costs. ■

9 Quantitative Relevance

In this section, we use the assumption of a Pareto productivity distribution to show that the differences in the aggregate implications of heterogeneous and homogeneous firms are of quantitative relevance. We choose standard values for the model's parameters based on central estimates from the existing empirical literature and moments of the U.S. data.

We set the elasticity of substitution between varieties $\sigma=4$, which is consistent with the estimates using plant-level U.S. manufacturing data in Bernard, Eaton, Jensen and Kortum (2003). Since productivity is Pareto distributed and firm revenue is a power function of firm productivity, firm revenue also has a Pareto distribution: $G\left(r\right)=1-\left(\frac{r_d}{r}\right)^{\frac{k}{\sigma-1}}$, where r_d is the revenue of the least productive firm. Existing empirical estimates suggest that the firm size distribution is well approximated by a Pareto distribution with a shape parameter close to one (see for example Axtell 2001). Therefore, we set the Pareto shape parameter for firm productivity $k=4.25$, which ensures a Pareto shape parameter for firm revenue close to one and that log firm revenue has a finite mean ($\frac{k}{\sigma-1}=1.42>1$). A choice for the Pareto scale parameter is equivalent to a choice of units in which to measure productivity, and hence we normalize $\varphi_{\min}=1$.

The general equilibrium of the model under the assumption of a Pareto distribution can be summarized by the following system of equations:

$$\varphi_d^T=\left[\frac{\sigma-1}{k-(\sigma-1)}\left(\frac{f_d+\tau^{-k}\left(\frac{f_x}{f_d}\right)^{\frac{-k}{\sigma-1}}f_x}{f_e}\right)\varphi_{\min}^k\right]^{\frac{1}{k}}.$$

$$\varphi_x^T=\tau\left(\frac{f_x}{f_d}\right)^{\frac{1}{\sigma-1}}\varphi_d^T,$$

$$M_e=\frac{L}{f_e}\frac{\sigma-1}{\sigma k},$$

$$M=\left(\frac{\varphi_{\min}}{\varphi_d^T}\right)^k M_e,$$

$$R = L,$$

$$\bar{r} = \frac{f_e}{1 - G\left(\varphi_d^T\right)} \frac{\sigma k}{\sigma - 1}.$$

Inspecting this system of equations, it is clear that scaling L and $\{f_e,\, f_d,\, f_x\}$ up or down by the same proportion leaves the productivity cutoffs $\{\varphi_d,\, \varphi_x\}$ and the mass of entrants unchanged (M_e), and merely scales average firm size ($\bar{r}$) up or down by the same proportion. Therefore we set L equal to the U.S. labor force and normalize f_d to one. We choose values for $\{\tau,\, f_e,\, f_x\}$ to match moments of the U.S. data. We calibrate τ to match the average fraction of exports in firm sales in U.S. manufacturing ($\frac{\tau^{1-\sigma}}{1+\tau^{1-\sigma}} = 0.14$ as reported in Bernard, Jensen, Redding and Schott 2007), which implies $\tau = 1.83$ (which is in line with the estimate of 1.7 in Anderson and van Wincoop 2004). Given our choice for the parameters $\{\sigma,\, k,\, \varphi_{\min},\, f_d,\, \tau\}$, we choose $\{f_e,\, f_x\}$ to ensure that the model is consistent with the annual exit rate for U.S. firms with more than 500 employees (0.0055 as reported in Atkeson and Burstein 2010) and the average fraction of U.S. manufacturing firms that export (0.18 as reported in Bernard, Jensen, Redding and Schott 2007). We match both these moments with $f_e = 0.0145$ and $f_x = 0.545$.

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