Web Appendix for Task Specialization in U.S. Cities from 1880-2000: Not for Publication

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1 Introduction

This appendix contains the technical derivations of expressions and additional supplementary material for the main paper.

2 Theoretical Model

In this section, we outline a theoretical model that we use to rationalize the empirical results reported below. First, the model explains a secular reallocation of employment over time towards more interactive occupations in terms of differences in productivity growth across occupations and inelastic demand. Second, the model predicts that this reallocation is stronger in metro areas than in non-metro areas because of falling trade costs and specialization according to comparative advantage. To the extent that densely-populated locations are relatively more productive in interactive tasks (because agglomeration forces are stronger for interactive tasks), reductions in task trade costs induce densely-populated locations to specialize in more-interactive occupations, while more sparsely-populated locations specialize in less-interactive occupations. Third, and for the same reason, the model predicts that the reallocation towards more interactive occupations is stronger in denser metro areas. Fourth, the model explains how this allocation of tasks across areas that differ in their population density could have taken place both between sectors and within sectors. In this respect our model is more general than existing models that focus on differential changes across sectors (e.g. manufacturing vs. services) or within sectors (e.g. relocation of headquarters). Fifth, the model accounts for our finding that in 1880, when trade in tasks was still costly, metro areas were not more interactive than non-metro areas. Finally, the model

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explains faster population growth over our sample period in metro than in non-metro areas in terms of faster productivity growth in metro areas.

While some existing models may be consistent with some of these stylized facts, we are unaware of any model that can account for all of them. One of the challenges in developing such a model is retaining tractability despite allowing for rich patterns of specialization across occupations, sectors and locations. To ensure that the model remains tractable, we use a stochastic formulation of productivity differences across occupations, sectors and locations. The distribution of population across locations is determined as the outcome of a tension between congestion forces (an inelastic supply of land) and agglomeration forces (productivity differences that depend on production externalities). Variation in the magnitude of these productivity differences across occupations and sectors gives rise to specialization according to Ricardian comparative advantage within and across sectors.

2.1 Preferences and Endowments

The economy consists of many locations indexed by $n \in N$. Each location n is endowed with an exogenous supply of land $\bar{H}_n$. The economy as a whole is endowed with a measure of workers $\bar{L}$, who are perfectly mobile across locations.

Workers' preferences are defined over a goods consumption index (C_n) and residential land use (H_n) and are assumed to take the Cobb-Douglas form:[1]

$$U_n = C_n^{\alpha} H_n^{1-\alpha}, \qquad 0 < \alpha < 1. \tag{1}$$

The goods consumption index (C_n) is assumed to be a constant elasticity of substitution (CES) function of consumption indices for a number of sectors (e.g. Manufacturing, Services) indexed by $s \in S$:

$$C_n = \left[\sum_{s \in S} C_{ns}^{\frac{\beta-1}{\beta}} \right]^{\frac{\beta}{\beta-1}}, \tag{2}$$

where β is the elasticity of substitution between goods. Sectors can be either substitutes ($\beta > 1$) or complements in goods consumption ($0 < \beta < 1$), where the standard assumption in the literature on structural transformation in macroeconomics is complements (e.g. Ngai and Pissarides 2007, Yi and Zhang 2010).

The consumption index for each sector is in turn a CES function of consumption of a continuum of goods (e.g. Motor Vehicles, Drugs and Medicines) indexed by $j \in [0, 1]$:

$$C_{ns} = \left[\int_0^1 c_{ns}(j)^{\frac{\sigma_s - 1}{\sigma_s}} dj \right]^{\frac{\sigma_s}{\sigma_s - 1}}, \tag{3}$$

where the elasticity of substitution between goods σ_s varies across sectors. While in the data we observe a finite number of goods within sectors, we adopt the theoretical assumption of a continuum of goods within sectors for reasons of tractability, because it enables us to make use of law of large numbers results in determining specialization at the sectoral level. Goods can be either substitutes ($\sigma_s > 1$) or complements ($0 < \sigma_s < 1$) and we can allow any ranking of the elasticities of substitution between goods and sectors, although the conventional

[1] For empirical evidence using U.S. data in support of the constant expenditure share implied by the Cobb-Douglas functional form, see Davis and Ortalo-Magne (2011).

assumption in such a nested CES structure is a higher elasticity of substitution at the more disaggregated level ($\sigma_s > \beta$).

Since the upper tier of utility is Cobb-Douglas, utility maximization implies that workers allocate constant shares of aggregate income to goods consumption and residential land use:

$$P_n C_n = \alpha v_n L_n, \tag{4}$$

$$r_n H_n = (1-\alpha) v_n L_n. \tag{5}$$

where P_n is the price index dual to the goods consumption index (C_n); v_n is income per worker; and L_n is the population of location n.

Expenditure on residential land in each location is assumed to be redistributed lump-sum to residents of that location, as in Helpman (1998). Therefore aggregate income in each location equals payments to labor used in production plus expenditure on residential land:

$$v_n L_n = w_n L_n + (1-\alpha) v_n L_n = \frac{w_n L_n}{\alpha}, \tag{6}$$

where w_n is the wage. Equilibrium land rents in each location (r_n) are determined by land market clearing, which requires that total land income equals total expenditure on land. Combining equilibrium expenditure on land (5), aggregate income (6) and land market clearing ($H_n = \bar{H}_n$), we obtain equilibrium land rents as a function of wages, population, the exogenous land supply and parameters:

$$r_n = \frac{1-\alpha}{\alpha} \frac{w_n L_n}{\bar{H}_n}. \tag{7}$$

2.2 Production

Goods are homogeneous in the sense that one unit of a given good is the same as any other unit of that good. Production occurs under conditions of perfect competition and constant returns to scale. The cost to a consumer in location n of purchasing one unit of good j within sector s from location i is therefore:

$$p_{nis}(j) = \frac{d_{nis} G_{is}(j)}{z_{is}(j)}, \tag{8}$$

where d_{nis} are iceberg transport costs, such that $d_{nis} > 1$ must be shipped from location i to location n within sector s in order for one unit to arrive; no arbitrage ensures that the triangle inequality $d_{nis} \leq d_{nks} d_{kis}$ is satisfied and we assume $d_{nns} = 1$; $z_{is}(j)$ is productivity for good j within sector s in location i; and $G_{is}(j)$ is the unit cost of the composite factor of production used for good j within sector s in location i, as determined below.

Final goods productivity is stochastic and modeled as in Eaton and Kortum (2002) and Costinot, Donaldson and Komunjer (2012). Final goods productivity for each good, sector and location is assumed to be drawn independently from a Fréchet distribution:[2]

$$F_{is}(z) = e^{-T_{is} L_{is}^{\eta_s} z^{\theta_s}}, \tag{9}$$

[2] To simplify the exposition, we use i to denote locations of production and n to denote locations of consumption, except where otherwise indicated.

where the shape parameter $\theta_s > 1$ controls the dispersion of productivity across goods within each sector, which determines comparative advantage across goods. In contrast, the scale parameter ($T_{is} L_{is}^{\eta_s}$, where $\eta_s > 0$) determines average productivity within each sector for each location, which determines comparative advantage across sectors. We allow average productivity in a sector and location to be increasing in employment in that sector and location to capture agglomeration forces in the form of external economies of scale in final goods production (e.g. Ethier 1982). As these external economies are external to the firm, they are consistent with our assumption of perfect competition, since each firm takes productivity as given when making its decisions.

As in the Ricardian model of trade, our framework features comparative advantage across goods and sectors, which explains the specialization across goods and sectors observed in the data. But we also observe specialization across occupations and tasks in the data, which is not captured in the standard Ricardian framework. To account for this additional layer of specialization, we assume that each good is produced using a number of stages of production, where each stage of production within a sector is supplied by a separate occupation indexed by $o \in O_s$ (e.g. Managers, Operatives). Output of good j within sector s in location i ($Y_{is}(j)$) is a CES function of the inputs of each occupation ($X_{iso}(j)$):

$$Y_{is}(j) = \left[\sum_{o \in O_s} X_{iso}(j)^{\frac{\mu_s - 1}{\mu_s}} \right]^{\frac{\mu_s}{\mu_s - 1}}, \tag{10}$$

where μ_s is the elasticity of substitution between occupations and again we can allow occupations to be either substitutes ($\mu_s > 1$) or complements ($0 < \mu_s < 1$). Under our assumption of a CES technology, the value marginal product of each occupation becomes infinite as the input of that occupation converges towards zero. Therefore the inputs of all occupations $o \in O_s$ within each sector are used in positive amounts. But sectors can differ in their set of occupations O_s and firms within each sector can adjust the proportions in which the inputs of these occupations are used depending their cost.

Workers within each occupation perform a continuum of tasks $t \in [0, 1]$ as in Grossman and Rossi-Hansberg (2008) (e.g. Advising, Typing, Stretching, Stamping). The input for occupation o and good j within sector s and location i ($X_{iso}(j)$) is a CES function of the inputs for these tasks ($x_{iso}(j, t)$):

$$X_{iso}(j) = \left[\int_0^1 x_{iso}(j, t)^{\frac{\nu_{so} - 1}{\nu_{so}}} dt \right]^{\frac{\nu_{so}}{\nu_{so} - 1}} \tag{11}$$

where the elasticity of substitution between tasks ν_{so} varies across sectors and occupations. While in the data we observe a finite number of tasks within occupations, we adopt the theoretical assumption of a continuum of tasks within occupations for reasons of tractability, because it enables us to make use of law of large numbers results in determining specialization at the occupational level.[3] We allow tasks within occupations to be either substitutes ($\nu_{so} > 1$) or complements ($0 < \nu_{so} < 1$), and we can consider any ranking of the elasticities of substitution between tasks and occupations, although the conventional assumption in such a nested CES structure is again a higher elasticity of substitution at the more disaggregated level ($\nu_{so} > \mu_s$). Under our assumption of a CES technology, the value marginal product of each task also becomes infinite as the use of

[3]To reduce the notational burden, we assume the same $[0, 1]$ interval of tasks for all occupations, but it is straightforward to allow this interval to vary across occupations.

that task converges towards zero. Therefore all tasks within each occupation are used in positive amounts, although firms can adjust the proportions in which these tasks are used depending on their cost.[4]

Tasks are performed by labor using a constant returns to scale technology and can be traded between locations. For example, product design can be undertaken in one location, while production and assembly occur in another location. The cost to a firm in location n of sourcing a task t from location i within occupation o and sector s is:

$$g_{niso}(j,t) = \frac{\tau_{niso} w_i}{a_{iso}(j,t)}, \tag{12}$$

where w_i is the wage; τ_{niso} are iceberg communication costs, such that $\tau_{niso} > 1$ units of the task must be performed in location i in order for one unit to be completed in location n for occupation o and sector s; no arbitrage ensures that the triangle inequality $\tau_{niso} \leq \tau_{nkso}\tau_{kiso}$ is satisfied and we assume $\tau_{nnso} = 1$; $a_{iso}(j,t)$ is productivity for task t and good j within occupation o and sector s in location i.

Input productivity for each task, occupation, sector and location is also stochastic and is assumed to be drawn independently from a Fréchet distribution:

$$\mathcal{F}_{iso} = e^{-U_{iso} L_{iso}^{\chi_{so}} a^{-\epsilon_{so}}}, \tag{13}$$

where the shape parameter $\epsilon_{so} > 1$ controls the dispersion of productivity across tasks within occupations, which determines comparative advantage across tasks. In contrast, the scale parameter ($U_{iso} L_{iso}^{\chi_{so}} > 0$, where $\chi_{so} > 0$) controls average productivity within each occupation, which determines comparative advantage across occupations. We allow average productivity in an occupation, sector and location to be increasing in employment in that occupation, sector and location ($\chi_{so} > 0$) to capture external economies of scale in task production (e.g. Grossman and Rossi-Hansberg 2012). As these increasing returns to scale are again external to the firm, they are consistent with our assumption of perfect competition, since each firm takes productivity as given when making its decisions.

2.3 Trade in Tasks and Input Costs

2.3.1 Locations' Shares of Costs within an Occupation and Sector

Firms within a given location n source each task t within an occupation o, good j and sector s from the lowest cost source of supply for that task:

$$g_{nso}(j,t) = \min\left\{g_{niso}(j,t); i \in N\right\}.$$

Using task prices (12) and the Fréchet distribution of input productivities (13), the distribution of task prices in country n for goods sourced from country i in occupation o within sector s is:

$$\mathcal{F}_{niso}(g) = \Pr[g_{niso} \leq g] = 1 - \mathcal{F}_{iso}\left(\frac{w_i \tau_{niso}}{g}\right),$$

$$\mathcal{F}_{niso}(g) = 1 - e^{-U_{iso} L_{iso}^{\chi_{so}} (\tau_{niso} w_i)^{-\varepsilon_{so}} g^{\varepsilon_{so}}}. \tag{14}$$

[4] While we interpret production as being undertaken by workers in occupations that perform many tasks, an equivalent interpretation is that each occupation corresponds to a stage of production and each task corresponds to an intermediate input within that stage of production.

Tasks are sourced from the lowest-cost supplier and the distribution of minimum task prices in country n in occupation o within sector s is:

$$\mathcal{F}_{nso}\left(g\right) = 1 - \prod_{i \in N} \left[1 - \mathcal{F}_{niso}(g)\right] = 1 - e^{-\Psi_{nso} g^{\varepsilon_{so}}}, \tag{15}$$

$$\Psi_{nso} \equiv \sum_{i \in N} U_{iso} L_{iso}^{\chi_{so}} \left(\tau_{niso} w_i\right)^{-\varepsilon_{so}}, \tag{16}$$

Since tasks are sourced from the lowest-cost supplier, the probability that location n sources a task t within occupation o and sector s from location i is:

$$\begin{aligned} \lambda_{niso} &= \Pr\left[g_{niso}\left(t\right) \leq \min\left\{g_{nkso}\left(t\right)\right\}; k \neq i\right] \\ &= \int_0^\infty \prod_{k \neq i} \left[1 - \mathcal{F}_{nkso}\left(g\right)\right] d\mathcal{F}_{niso}\left(g\right). \end{aligned}$$

Using the bilateral price distribution (14), the probability that location n sources a task t from location i within occupation o and sector s is:

$$\lambda_{niso} = \frac{U_{iso} L_{iso}^{\chi_{so}} \left(\tau_{niso} w_i\right)^{-\epsilon_{so}}}{\sum_{k \in N} U_{kso} L_{kso}^{\chi_{so}} \left(\tau_{nkso} w_k\right)^{-\epsilon_{so}}}, \tag{17}$$

Another implication of the Fréchet distribution of input productivities is that the distribution of task prices in location n for tasks actually sourced from another location i is independent of the identity of the location i and equal to the distribution of minimum prices in location n. To derive this result, note that the distribution of task prices in location n conditional on sourcing tasks from location i is:

$$\frac{1}{\lambda_{niso}} \int_0^g \prod_{k \neq i} \left[1 - \mathcal{F}_{nkso}\left(g'\right)\right] d\mathcal{F}_{niso}\left(g'\right) = 1 - e^{-\Psi_{nso} g^{\varepsilon_{so}}} = \mathcal{F}_{nso}\left(g\right),$$

where we have used the bilateral and multilateral price distributions, (14) and (15) respectively. Intuitively, under the assumption of a Fréchet distribution of input productivity, a source location i with a higher scale parameter ($U_{iso} L_{iso}^{\chi_{so}}$), and hence a higher average input productivity, expands on the extensive margin of the number of tasks supplied exactly to the point at which the distribution of prices for the tasks it actually sells in market n is the same as destination n's distribution of minimum prices.

Since the distribution of prices in location n for goods actually purchased is the same across all source locations i, it follows that the share of location n's expenditure on products sourced from another location i within occupation o and sector s is equal to the probability of sourcing a task from that location (λ_{niso}). Therefore the share of location n's expenditure on tasks sourced from another location i within occupation o and sector s is given by (17).

2.3.2 Occupations' Shares of Costs

We begin by determining the cost function for occupation o and sector s in location n using the distribution of minimum task prices (15):

$$\begin{aligned} G_{nso} &= \left[\int_0^1 g_{nso}(t)^{1-\nu_{so}}\, dt \right]^{\frac{1}{1-\nu_{so}}}, \\ &= \left[\int_0^\infty g_{nso}^{1-\nu_{so}} d\mathcal{F}_{nso}(g) \right]^{\frac{1}{1-\nu_{so}}}, \\ &= \left[\int_0^\infty \varepsilon_{so} \Psi_{nso} g^{\varepsilon_{so}-\nu_{so}} e^{-\Psi_{nso} g^{\varepsilon_{so}}} dg \right]^{\frac{1}{1-\nu_{so}}}. \end{aligned}$$

Using the following change of variable:

$$\tilde{g} = \Psi_{nso} g^{\varepsilon_{so}},$$

$$\Rightarrow \qquad g = \left(\frac{\tilde{g}}{\Psi_{nso}} \right)^{\frac{1}{\varepsilon_{so}}}, \qquad dg = \frac{1}{\theta_K} \left(\frac{\tilde{g}}{\Psi_{nso}} \right)^{\frac{1-\varepsilon_{so}}{\varepsilon_{so}}} \frac{1}{\Psi_{nso}} d\tilde{g},$$

we obtain:

$$G_{nso} = \Psi_{nso}^{-1/\varepsilon_{so}} \left[\int_0^\infty \tilde{g}^{(1-\nu_{so})/\varepsilon_{so}} e^{-\tilde{g}} d\tilde{g} \right]^{\frac{1}{1-\nu_{so}}},$$

which yields the following expression for the cost function for occupation o and sector s in location n:

$$G_{nso} = \gamma_{so} \Psi_{nso}^{-1/\varepsilon_{so}} = \gamma_{so} \left[\sum_{i \in N} U_{iso} L_{iso}^{\chi_{so}} (\tau_{niso} w_i)^{-\varepsilon_{so}} \right]^{-1/\varepsilon_{so}}, \tag{18}$$

$$\text{where} \qquad \gamma_{so} \equiv \left[\Gamma \left(\frac{\varepsilon_{so} + 1 - \nu_{so}}{\varepsilon_{so}} \right) \right]^{\frac{1}{1-\nu_{so}}},$$

where $\Gamma(\cdot)$ is the gamma function.

Together the cost share (17) and cost function (18) imply that the unit cost for occupation o and sector s in location n also can be written as:

$$G_{nso} = \gamma_{so} \left(\frac{U_{nso} L_{nso}^{\chi_{so}}}{\lambda_{nnso}} \right)^{-\frac{1}{\epsilon_{so}}} w_n. \tag{19}$$

Given these unit costs for each occupation, we now solve for the overall unit cost for sector s in location n. From the CES production technology (10), the overall unit cost is:

$$G_{ns} = \left[\sum_{o \in O_s} G_{nso}^{1-\mu_s} \right]^{\frac{1}{1-\mu_s}},$$

which using the unit costs for each occupation (19) can be written as the expression in the paper:

$$G_{ns} = \left[\sum_{o \in O_s} \gamma_{so}^{1-\mu_s} \left(\frac{U_{nso} L_{nso}^{\chi_{so}}}{\lambda_{nnso}} \right)^{-\frac{1-\mu_s}{\epsilon_{so}}} \right]^{\frac{1}{1-\mu_s}} w_n. \tag{20}$$

The CES production technology (10) also implies that the share of occupation o in unit costs within sector s in location n is:

$$e_{nso} = \frac{G_{nso}^{1-\mu_s}}{\sum_{m \in O_s} G_{nsm}^{1-\mu_s}},$$

which using the unit costs for each occupation (19) can be written as the expression in the paper:

$$e_{nso} = \frac{\gamma_{so}^{1-\mu_s} \left(\frac{U_{nso} L_{nso}^{\chi_{so}}}{\lambda_{nnso}} \right)^{-\frac{1-\mu_s}{\epsilon_{so}}}}{\sum_{m \in O_s} \gamma_{sm}^{1-\mu_s} \left(\frac{U_{nsm} L_{nsm}^{\chi_{so}}}{\lambda_{nnsm}} \right)^{-\frac{1-\mu_s}{\epsilon_{sm}}}}. \tag{21}$$

2.4 Trade in Final Goods and Price Indices

2.4.1 Locations' Shares of Sectoral Expenditure

Consumers within a given location n source each final good j within a sector s from the lowest cost source of supply for that final good:

$$p_{ns}(j) = \min \left\{ p_{nis}(j); i \in N \right\}.$$

Using final goods prices (8) and the Fréchet distribution of final goods productivities (9), the distribution of goods prices in country n for goods sourced from country i within sector s is:

$$F_{nis}(p) = \Pr[p_{nis} \leq p] = 1 - F_{is} \left(\frac{d_{nis} G_{is}}{p} \right),$$

$$F_{nis}(p) = 1 - e^{-T_{is} L_{is}^{\eta_s} (d_{nis} G_{is})^{-\theta_s} p^{\theta_s}}. \tag{22}$$

Final goods are sourced from the lowest-price supplier and the distribution of minimum final goods prices in country n in within sector s is:

$$F_{ns}(p) = 1 - \prod_{i \in N} \left[1 - F_{nis}(p) \right] = 1 - e^{-\Lambda_{ns} p^{\theta_s}}, \tag{23}$$

$$\Lambda_{ns} \equiv \sum_{i \in N} T_{is} L_{is}^{\eta_s} \left(d_{nis} G_{is} \right)^{-\theta_s}. \tag{24}$$

Since final goods are sourced from the lowest-price supplier, the probability that location n sources a final good j within sector s from location i is:

$$\begin{aligned} \pi_{nis} &= \Pr \left[p_{nis}(j) \leq \min \left\{ p_{nks}(j) \right\}; k \neq i \right], \\ &= \int_0^{\infty} \prod_{k \neq i} \left[1 - F_{nks}(p) \right] dF_{nis}(p). \end{aligned}$$

Using the bilateral price distribution (22), the probability that location n sources a final good j from location i within sector s is:

$$\pi_{nis} = \frac{T_{is} L_{is}^{\eta_s} \left(d_{nis} G_{is} \right)^{-\theta_s}}{\sum_{k \in N} T_{ks} L_{ks}^{\eta_s} \left(d_{nks} G_{is} \right)^{-\theta_s}},$$

which can be in turn re-written as the following expression in the paper:

$$\pi_{nis} = \frac{T_{is} L_{is}^{\eta_s} \left(d_{nis} \Phi_{is} w_i \right)^{-\theta_s}}{\sum_{k \in N} T_{ks} L_{ks}^{\eta_s} \left(d_{nks} \Phi_{is} w_i \right)^{-\theta_s}}, \tag{25}$$

where from the previous subsection:

$$\Phi_{is} = \left[\sum_{o \in O_s} \gamma_{so}^{1-\mu_s} \left(\frac{U_{iso} L_{iso}^{\chi_{so}}}{\lambda_{iiso}} \right)^{-\frac{1-\mu_s}{\epsilon_{so}}} \right]^{\frac{1}{1-\mu_s}} .$$

Another implication of the Fréchet distribution of final goods productivities is that the distribution of final goods prices in location n for goods actually sourced from another location i is independent of the identity of the location i and equal to the distribution of minimum prices in location n. To derive this result, note that the distribution of final goods prices in location n conditional on sourcing goods from location i is:

$$\frac{1}{\pi_{nis}} \int_0^p \prod_{k \neq i} \left[1 - F_{nks}\left(p'\right) \right] dF_{nis}\left(p'\right) = 1 - e^{-\Lambda_{ns} p^{\theta_s}} = F_{ns}\left(p\right),$$

where we have used the bilateral and multilateral price distributions, (22) and (23) respectively. Intuitively, under the assumption of a Fréchet distribution of final goods productivity, a source location i with a higher scale parameter ($T_{is} L_{is}^{\eta_s}$), and hence a higher average final goods productivity, expands on the extensive margin of the number of final goods supplied exactly to the point at which the distribution of prices for the goods it actually sells in market n is the same as destination n's distribution of minimum prices.

Since the distribution of prices in location n for goods actually purchased is the same across all source locations i, it follows that the share of location n's expenditure on final goods sourced from another location i within sector s is equal to the probability of sourcing a final good from that location (π_{nis}). Therefore the share of location n's expenditure on final goods sourced from another location i within sector s is given by (25).

2.4.2 Sectors' Shares of Expenditure

We begin by determining the price index for sector s in location n using the distribution of minimum final goods prices (23):

$$\begin{aligned} P_{ns} &= \left[\int_0^1 p_{ns}\left(j\right)^{1-\sigma_s} dj \right]^{\frac{1}{1-\sigma_s}}, \\ &= \left[\int_0^\infty p_{ns}^{1-\sigma_s} dF_{ns}\left(p\right) \right]^{\frac{1}{1-\sigma_s}}, \\ &= \left[\int_0^\infty \theta_s \Lambda_{ns} p^{\theta_s - \sigma_s} e^{-\Lambda_{ns} p^{\theta_s}} dp \right]^{\frac{1}{1-\sigma_s}} . \end{aligned}$$

Using the following change of variable:

$$\tilde{p} = \Lambda_{ns} g^{\theta_s},$$

$$\Rightarrow \qquad p = \left(\frac{\tilde{p}}{\Lambda_{ns}} \right)^{\frac{1}{\theta_s}}, \qquad dp = \frac{1}{\theta_s} \left(\frac{\tilde{p}}{\Lambda_{ns}} \right)^{\frac{1-\theta_s}{\theta_s}} \frac{1}{\Lambda_{ns}} d\tilde{p},$$

we obtain:

$$P_{ns} = \Lambda_{ns}^{-1/\theta_s} \left[\int_0^\infty \tilde{p}^{(1-\sigma_s)/\theta_s} e^{-\tilde{p}} d\tilde{p} \right]^{\frac{1}{1-\sigma_s}},$$

which yields the following expression for the price index for sector s in location n:

$$P_{ns} = \kappa_s \Lambda_{ns}^{-1/\theta_s} = \kappa_s \left[\sum_{i \in N} T_{is} L_{is}^{\eta_s} \left(d_{nis} G_{is} \right)^{-\theta_s} \right]^{-1/\theta_s},$$

$$\text{where} \qquad \kappa_s \equiv \left[\Gamma \left(\frac{\theta_s + 1 - \sigma_s}{\theta_s} \right) \right]^{\frac{1}{1-\sigma_s}},$$

where $\Gamma(\cdot)$ is the gamma function. This expression for the sectoral price index can be in turn re-written as:

$$P_{ns} = \kappa_s \left[\sum_{k \in N} T_{ks} L_{ks}^{\eta_s} \left(d_{nks} \Phi_{ks} w_k \right)^{-\theta_s} \right]^{-\frac{1}{\theta_s}}, \tag{26}$$

where Φ_{ks} is defined above.

Together the expenditure share (25) and cost function (26) imply that the price index for sector s in location n also can be written as:

$$P_{ns} = \kappa_s \left(\frac{T_{ns} L_{ns}^{\eta_s}}{\pi_{nns}} \right)^{-\frac{1}{\theta_s}} \Phi_{ns} w_n. \tag{27}$$

Given this price index for each sector, we now solve for the overall goods consumption price index in location n. From the CES goods consumption index (2), the corresponding dual price index is:

$$P_n = \left[\sum_{s \in S} P_{ns}^{1-\beta} \right]^{\frac{1}{1-\beta}},$$

which using the price index for each sector (27) can be written as the expression in the paper:

$$P_n = \left[\sum_{s \in S} \kappa_s^{1-\beta} \left(\frac{T_{ns} L_{ns}^{\eta_s}}{\pi_{nns}} \right)^{-\frac{1-\beta}{\theta_s}} \Phi_{ns}^{1-\beta} \right]^{\frac{1}{1-\beta}} w_n, \tag{28}$$

The CES goods consumption index (2) also implies that the share of sector s in aggregate goods consumption expenditure is:

$$E_{ns} = \frac{P_{ns}^{1-\beta}}{\sum_{r \in S} P_{nr}^{1-\beta}},$$

which using the price index for each sector (27) can be written as the expression in the paper:

$$E_{ns} = \frac{\kappa_s^{1-\beta} \left(\frac{T_{ns} L_{ns}^{\eta_s}}{\pi_{nns}} \right)^{-\frac{1-\beta}{\theta_s}} \Phi_{ns}^{1-\beta}}{\sum_{r \in S} \kappa_r^{1-\beta} \left(\frac{T_{nr} L_{nr}^{\eta_r}}{\pi_{nnr}} \right)^{-\frac{1-\beta}{\theta_r}} \Phi_{nr}^{1-\beta}}. \tag{29}$$

2.5 Population Mobility

Population mobility implies that workers must receive the same indirect utility in all populated locations:

$$V_n = \frac{v_n}{P_n^{\alpha} r_n^{1-\alpha}} = \bar{V}. \tag{30}$$

Using land market clearing (7), this population mobility condition becomes:

$$V_n = \frac{v_n}{P_n^{\alpha}\left(\frac{1-\alpha}{\alpha}\frac{w_n L_n}{\bar{H}_n}\right)^{1-\alpha}} = \bar{V},$$

which using the equality of income and expenditure (6) becomes:

$$V_n = \frac{w_n^{\alpha}}{\alpha P_n^{\alpha}\left(\frac{1-\alpha}{\alpha}\frac{L_n}{\bar{H}_n}\right)^{1-\alpha}} = \bar{V},$$

which using the aggregate price index (28) can be written as:

$$V_n = \frac{\left(\sum_{s\in S}\kappa_s^{-(1-\beta)}\left(\frac{T_{ns}L_{ns}^{\eta_s}}{\pi_{nns}}\right)^{\frac{1-\beta}{\theta_s}}\Phi_{ns}^{-(1-\beta)}\right)^{\frac{\alpha}{1-\beta}}\bar{H}_n^{1-\alpha}}{\alpha\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}L_n^{(1-\alpha)}} = \bar{V}. \tag{31}$$

Re-arranging the above population mobility condition, we obtain the following expression for equilibrium population:

$$L_n = \frac{\left[\sum_{s\in S}\kappa_s^{-(1-\beta)}\left(\frac{T_{ns}L_{ns}^{\eta_s}}{\pi_{nns}}\right)^{\frac{1-\beta}{\theta_s}}\left[\sum_{o\in O_s}\gamma_{so}^{-(1-\mu_s)}\left(\frac{U_{nso}L_{nso}^{\chi_{so}}}{\lambda_{nnso}}\right)^{\frac{1-\mu_s}{\epsilon_{so}}}\right]^{\frac{1-\beta}{1-\mu_s}}\right]^{\frac{\alpha}{(1-\alpha)(1-\beta)}}\bar{H}_n}{\alpha^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\bar{V}^{\frac{1}{1-\alpha}}}, \tag{32}$$

where labor market clearing requires:

$$\sum_{n\in N} L_n = \bar{L}. \tag{33}$$

2.6 Welfare Gains from Trade

Rearranging the population mobility condition (31), indirect utility can be written as:

$$V_n = \frac{\left(\sum_{s\in S}\kappa_s^{-(1-\beta)}\left(\frac{T_{ns}L_{ns}^{\eta_s}}{\pi_{nns}}\right)^{\frac{1-\beta}{\theta_s}}\left[\sum_{o\in O_s}\gamma_{so}^{-(1-\mu_s)}\left(\frac{U_{nso}L_{nso}^{\chi_{so}}}{\lambda_{nnso}}\right)^{\frac{1-\mu_s}{\epsilon_{so}}}\right]^{\frac{1-\beta}{1-\mu_s}}\right)^{\frac{\alpha}{1-\beta}}\bar{H}_n^{1-\alpha}}{\alpha\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}L_n^{(1-\alpha)}}.$$

where the special case of no trade in final goods or tasks ($\lim_{d_{nis}\to\infty}$ and $\lim_{\tau_{niso}\to\infty}$) implies $\pi_{nns} = \lambda_{nnso} = 1$ for all s, o.

Therefore the welfare gains from trade depend on three components in this model. First, there are welfare gains from trade in final goods ($0 < \pi_{nns} < 1$). Second, there are welfare gains from trade in tasks ($0 < \lambda_{nnso} < 1$). Third, population mobility equalizes indirect utility across locations in both the closed and open economy. Therefore, if the opening of trade has uneven effects on the welfare of locations, population adjusts to ensure real wage equalization. It follows that the welfare gains from trade are the same for all locations and also depend on endogenous population (L_n). To the extent that trade in tasks ($0 < \lambda_{nnso} < 1$) is not fully captured in standard data on trade in goods, the model implies that measures of the welfare gains from trade based on these standard data will understate the true magnitude of the welfare gains from trade.

2.7 Wages and Employment

Wages in each location can be determined from the equality between a location's labor income and expenditure on tasks performed in that location:

$$w_i L_i = \sum_s \sum_o \sum_n \sum_k \lambda_{kiso} e_{kso} \left[\pi_{nks} E_{ns} w_n L_n \right], \tag{34}$$

where the term inside the square parentheses on the right-hand side is market n's population (L_n) times its wage (w_n) times the fraction of income that is allocated to final goods supplied by location k in sector s ($\pi_{nks} E_{ns}$); the term outside the square parentheses is the share of final goods revenue in sector s in location k that is spent on tasks performed by location i in occupation o ($\lambda_{kiso} e_{kso}$). To obtain total labor income in location i, we sum across sectors s, occupations o, locations of final goods production k and markets n.

Similarly, employment in each sector and location satisfies the equality between payments to workers employed in that sector and location and expenditure on tasks supplied by workers in that sector and location:

$$w_i L_{is} = \sum_o \sum_n \sum_k \lambda_{kiso} e_{kso} \left[\pi_{nks} E_{ns} w_n L_n \right]. \tag{35}$$

Finally, employment in each occupation, sector and location satisfies the equality between payments to workers employed in that occupation, sector and location and expenditure on tasks supplied by workers in that occupation, sector and location:

$$w_i L_{iso} = \sum_n \sum_k \lambda_{kiso} e_{kso} \left[\pi_{nks} E_{ns} w_n L_n \right]. \tag{36}$$

2.8 General Equilibrium

The general equilibrium of the model can be referenced by the vector of wages for all locations (w_n) and the allocation of employment to each occupation, sector and location (L_{nso}). Equilibrium wages and employment allocations are determined by the following system of equations:

$$w_i L_i = \sum_{o \in O_s} \sum_{s \in S} \sum_{n \in N} \sum_{k \in N} \lambda_{kiso} e_{kso} \left[\pi_{nks} E_{ns} w_n L_n \right], \tag{37}$$

$$w_i L_{is} = \sum_{o \in O_s} \sum_{n \in N} \sum_{k \in N} \lambda_{kiso} e_{kso} \left[\pi_{nks} E_{ns} w_n L_n \right],$$

$$w_i L_{iso} = \sum_{n \in N} \sum_{k \in N} \lambda_{kiso} e_{kso} \left[\pi_{nks} E_{ns} w_n L_n \right],$$

$$\lambda_{niso} = \frac{U_{iso} L_{iso}^{\chi_{so}} \left(\tau_{niso} w_i \right)^{-\epsilon_{so}}}{\sum_{k \in N} U_{kso} L_{kso}^{\chi_{so}} \left(\tau_{nkso} w_k \right)^{-\epsilon_{so}}}, \tag{38}$$

$$e_{nso} = \frac{\gamma_{so}^{1-\mu_s} \left(\frac{U_{nso} L_{nso}^{\chi_{so}}}{\lambda_{nnso}} \right)^{-\frac{1-\mu_s}{\epsilon_{so}}}}{\sum_{m \in O_s} \gamma_{sm}^{1-\mu_s} \left(\frac{U_{nsm} L_{nsm}^{\chi_{so}}}{\lambda_{nnsm}} \right)^{-\frac{1-\mu_s}{\epsilon_{sm}}}}, \tag{39}$$

$$\pi_{nis} = \frac{T_{is} L_{is}^{\eta_s} \left(d_{nis} \Phi_{is} w_i \right)^{-\theta_s}}{\sum_{k \in N} T_{ks} L_{ks}^{\eta_s} \left(d_{nks} \Phi_{ks} w_k \right)^{-\theta_s}}, \tag{40}$$

$$\Phi_{is} = \left[\sum_{o \in O_s} \gamma_{so}^{1-\mu_s} \left(\frac{U_{iso} L_{iso}^{\chi_{so}}}{\lambda_{iiso}} \right)^{-\frac{1-\mu_s}{\epsilon_{so}}} \right]^{\frac{1}{1-\mu_s}}, \tag{41}$$

$$E_{ns} = \frac{\kappa_s^{1-\beta} \left(\frac{T_{ns} L_{ns}^{\eta_s}}{\pi_{nns}} \right)^{-\frac{1-\beta}{\theta_s}} \Phi_{ns}^{1-\beta}}{\sum_{r \in S} \kappa_r^{1-\beta} \left(\frac{T_{nr} L_{nr}^{\eta_r}}{\pi_{nnr}} \right)^{-\frac{1-\beta}{\theta_r}} \Phi_{nr}^{1-\beta}}, \tag{42}$$

$$\xi_n = \frac{\left[\sum_{s \in S} \kappa_s^{-(1-\beta)} \left(\frac{T_{ns} L_{ns}^{\eta_s}}{\pi_{nns}} \right)^{\frac{1-\beta}{\theta_s}} \left[\sum_{o \in O_s} \gamma_{so}^{-(1-\mu_s)} \left(\frac{U_{nso} L_{nso}^{\chi_{so}}}{\lambda_{nnso}} \right)^{\frac{1-\mu_s}{\epsilon_{so}}} \right]^{\frac{1-\beta}{1-\mu_s}} \right]^{\frac{\alpha}{(1-\alpha)(1-\beta)}} H_n}{\sum_{k \in N} \left[\sum_{s \in S} \kappa_s^{-(1-\beta)} \left(\frac{T_{ks} L_{ks}^{\eta_s}}{\pi_{kks}} \right)^{\frac{1-\beta}{\theta_s}} \left[\sum_{o \in O_s} \gamma_{so}^{-(1-\mu_s)} \left(\frac{U_{kso} L_{kso}^{\chi_{so}}}{\lambda_{kkso}} \right)^{\frac{1-\mu_s}{\epsilon_{so}}} \right]^{\frac{1-\beta}{1-\mu_s}} \right]^{\frac{\alpha}{(1-\alpha)(1-\beta)}} H_k}, \tag{43}$$

which can be solved numerically for arbitrary numbers of locations, sectors and occupations.

2.9 Simple Special Case

One simple special case of the model that permits a particularly tractable characterization of general equilibrium is when the following conditions are satisfied: (a) there are no external economies of scale in final goods or task production ($\eta_s = \chi_{so} = 0$) so that productivity in each sector and location is determined solely by exogenous fundamentals $\{T_{is}, U_{iso}\}$, (b) one of the sectors is an outside sector that produces a homogeneous good that is costlessly traded between locations and produced under conditions of perfect competition with a deterministic labor requirement ($Y_{i0} = T_{i0} L_{i0}$ for the outside sector $s = 0$).

In this special case, the model acquires a recursive structure, in which wages can be first determined before determining all the other components of the general equilibrium as a function of wages. We choose the outside good as the numeraire ($p_{i0} = 1$) and consider an equilibrium in which all locations produce the outside good, as can be ensured by the appropriate choice of productivity in this sector for each location. Since the outside good is costlessly traded and produced in all locations, the wage in each location is pinned down by productivity in this sector alone:

$$w_i = T_{i0}.$$

Having determined wages, shares of locations in trade in tasks within all other sectors follow immediately:

$$\lambda_{niso} = \frac{U_{iso} \left(\tau_{niso} w_i \right)^{-\epsilon_{so}}}{\sum_{k \in N} U_{kso} \left(\tau_{nkso} w_k \right)^{-\epsilon_{so}}}, \qquad s \neq 0,$$

from which we obtain the share of occupations in costs for all other sectors:

$$e_{nso} = \frac{\gamma_{so}^{1-\mu_s} \left(\frac{U_{nso}}{\lambda_{nnso}} \right)^{-\frac{1-\mu_s}{\epsilon_{so}}}}{\sum_{m \in O_s} \gamma_{sm}^{1-\mu_s} \left(\frac{U_{nsm}}{\lambda_{nnsm}} \right)^{-\frac{1-\mu_s}{\epsilon_{sm}}}}, \qquad s \neq 0.$$

Having solved for wages and trade in tasks, shares of locations in trade in final goods within each sector follow immediately:

$$\pi_{nis} = \frac{T_{is} \left(d_{nis} \Phi_{is} w_i \right)^{-\theta_s}}{\sum_{k \in N} T_{ks} \left(d_{nks} \Phi_{ks} w_k \right)^{-\theta_s}}, \qquad s \neq 0,$$

$$\Phi_{is} = \left[\sum_{o \in O_s} \gamma_{so}^{1-\mu_s} \left(\frac{U_{iso}}{\lambda_{iiso}} \right)^{-\frac{1-\mu_s}{\epsilon_{so}}} \right]^{\frac{1}{1-\mu_s}}, \qquad s \neq 0,$$

from which we obtain the share of sectors in expenditure:

$$E_{ns} = \frac{\kappa_s^{1-\beta} \left(\frac{T_{ns}}{\pi_{nns}} \right)^{-\frac{1-\beta}{\theta_s}} \Phi_{ns}^{1-\beta}}{1 + \sum_{r \neq 0} \kappa_r^{1-\beta} \left(\frac{T_{nr}}{\pi_{nnr}} \right)^{-\frac{1-\beta}{\theta_r}} \Phi_{nr}^{1-\beta}}, \qquad s \neq 0.$$

Finally, having determined trade in tasks and final goods, we obtain population shares:

$$\xi_n = \frac{\left[1 + \sum_{s \neq 0} \kappa_s^{-(1-\beta)} \left(\frac{T_{ns}}{\pi_{nns}} \right)^{\frac{1-\beta}{\theta_s}} \left[\sum_{o \in O_s} \gamma_{so}^{-(1-\mu_s)} \left(\frac{U_{nso}}{\lambda_{nnso}} \right)^{\frac{1-\mu_s}{\epsilon_{so}}} \right]^{\frac{1-\beta}{1-\mu_s}} \right]^{\frac{\alpha}{(1-\alpha)(1-\beta)}} \bar{H}_n}{\sum_{k \in N} \left[1 + \sum_{s \neq 0} \kappa_s^{-(1-\beta)} \left(\frac{T_{ks}}{\pi_{kks}} \right)^{\frac{1-\beta}{\theta_s}} \left[\sum_{o \in O_s} \gamma_{so}^{-(1-\mu_s)} \left(\frac{U_{kso}}{\lambda_{kkso}} \right)^{\frac{1-\mu_s}{\epsilon_{so}}} \right]^{\frac{1-\beta}{1-\mu_s}} \right]^{\frac{\alpha}{(1-\alpha)(1-\beta)}} \bar{H}_k}.$$

2.10 Secular Reallocation towards Interactive Occupations

One key feature of our data is a reallocation of employment towards more interactive occupations over time, which occurs across all locations and both between and within sectors. As in the macroeconomics literature on structural transformation, the model accounts for such secular reallocation in terms of differences in productivity growth across sectors and inelastic demand between sectors (e.g. Ngai and Pissarides 2007, Yi and Zhang 2010). To show this formally, partition final goods productivity in a sector-location into a sector component ($\tilde{T}_s$), a location component ($\tilde{T}_n$) and a residual ($\tilde{T}_{ns}$): $T_{ns} = \tilde{T}_s \tilde{T}_n \tilde{T}_{ns}$. Since the sector component $\tilde{T}_s$ is common to all locations, it cancels from the numerator and denominator of the location trade share (π_{nis}) and hence does not directly effect π_{nis} for given wages. In contrast, the sector component $\tilde{T}_s$ directly affects the share of sectors in aggregate expenditure (E_{ns}) for all locations. Taking the partial derivative of this expenditure share (29) with respect to $\tilde{T}_s$ at the initial equilibrium vectors of wages ($\mathbf{w}$) and employment ($\mathbf{L_{so}}$), faster productivity growth in sector s reduces the share of that sector in expenditure and increases the share of all other sectors in expenditure when sectors are complements and has the reverse effect when sectors are substitutes:

$$\begin{aligned} \left. \frac{\partial E_{ns}}{\partial \tilde{T}_s} \frac{\tilde{T}_s}{E_{ns}} \right|_{\mathbf{w}, \mathbf{L_{so}}} &= -\left(\frac{1-\beta}{\theta_s} \right) (1 - E_{ns}) < 0, && 0 < \beta < 1, \\ \left. \frac{\partial E_{nr}}{\partial \tilde{T}_s} \frac{\tilde{T}_s}{E_{nr}} \right|_{\mathbf{w}, \mathbf{L_{so}}} &= \left(\frac{1-\beta}{\theta_s} \right) E_{nr} > 0, && r \neq s, \ 0 < \beta < 1, \end{aligned}$$

where this productivity growth and the changes in expenditure shares it induces in turn have general equilibrium effects for wages and employment allocations.

While the macroeconomics literature on structural transformation typically focuses on sectors, secular changes in the shares of occupations in employment across all locations can be explained in the model by an analogous process of differences in productivity growth across occupations and inelastic demand between occupations. To show this formally, partition average input productivity in an occupation-sector-location into an occupation component ($\tilde{U}_o$), sector component ($\tilde{U}_s$), location component ($\tilde{U}_n$) and a residual ($\tilde{U}_{nso}$):

$U_{nso} = \tilde{U}_o \tilde{U}_s \tilde{U}_n \tilde{U}_{nso}$. Since the occupation component $\tilde{U}_o$ is common to all locations, it cancels from the numerator and denominator of the location trade share (λ_{niso}) and hence does not directly effect λ_{niso} for given wages. In contrast, the occupation component $\tilde{U}_o$ directly affects the share of occupations in sectoral expenditure (e_{nso}) for all locations. Taking the partial derivative of this expenditure share (21) with respect to $\tilde{U}_o$ at the initial equilibrium vectors of wages ($\mathbf{w}$) and employment ($\mathbf{L_{so}}$), faster productivity growth in occupation o reduces the share of that occupation in costs and increases the share of all other occupations in costs when occupations are complements and has the reverse effect when occupations are substitutes:

$$\begin{aligned}
\left.\frac{\partial e_{nso}}{\partial \tilde{U}_o}\frac{\tilde{U}_o}{e_{nso}}\right|_{\mathbf{w},\mathbf{L_{so}}} &= -\left(\frac{1-\mu_s}{\epsilon_{so}}\right)(1-e_{nso}) < 0, && 0 < \mu_s < 1, \\
\left.\frac{\partial e_{nsm}}{\partial \tilde{U}_o}\frac{\tilde{U}_o}{e_{nsm}}\right|_{\mathbf{w},\mathbf{L_{so}}} &= \left(\frac{1-\mu_s}{\epsilon_{so}}\right) e_{nsm} > 0, && m \neq o,\ 0 < \mu_s < 1,
\end{aligned}$$

where this productivity growth and the changes in expenditure shares it induces in turn have general equilibrium effects for wages and employment allocations.

2.11 Increased Relative Interactiveness of more Densely-populated Locations

Another key feature of our data is that the reallocation of employment towards more interactive occupations over time is more pronounced in more densely-populated locations. The model explains these differential changes in employment across occupations by specialization according to comparative advantage.

This specialization occurs across occupations within sectors and can be characterized by a double difference across exporting locations and occupations within an importing location. The first difference computes the ratio of exports of tasks from two locations i and k in a third market n in a single occupation; the second difference compares this ratio of exports of tasks for two separate occupations o and m:

$$\frac{\lambda_{niso}/\lambda_{nkso}}{\lambda_{nism}/\lambda_{nksm}} = \frac{\left[U_{iso}L_{iso}^{\chi_{so}}\left(\tau_{niso}w_i\right)^{-\epsilon_{so}}\right] / \left[U_{kso}L_{kso}^{\chi_{so}}\left(\tau_{nkso}w_k\right)^{-\epsilon_{so}}\right]}{\left[U_{ism}L_{ism}^{\chi_{so}}\left(\tau_{nism}w_i\right)^{-\epsilon_{sm}}\right] / \left[U_{ksm}L_{ksm}^{\chi_{so}}\left(\tau_{nksm}w_k\right)^{-\epsilon_{sm}}\right]}.$$

From the above double difference, locations export relatively more tasks in occupations in which they have relatively lower costs of supply, where these costs of supply depend on relative productivities (which in turn depend on relative employments through the external economies of scale), relative wages and task trade costs. Each location is a net exporter of tasks in some occupations and a net importer of tasks in other occupations (inter-occupation trade in tasks). This inter-occupation trade generates differences across locations in the pattern of employment across occupations within sectors.

There is also specialization according to comparative advantage across sectors, which can be characterized by an analogous double difference across exporting locations and sectors within an importing location. The first difference computes the ratio of exports of final goods from two locations i and k in a third market n in a single sector; the second difference compares this ratio of exports of final goods for two separate sectors s and r:

$$\frac{\pi_{nis}/\pi_{nks}}{\pi_{nir}/\pi_{nkr}} = \frac{\left[T_{is}L_{is}^{\eta_s}\left(d_{nis}\Phi_{is}w_i\right)^{-\theta_s}\right] / \left[T_{ks}L_{ks}^{\eta_s}\left(d_{nks}\Phi_{ks}w_k\right)^{-\theta_s}\right]}{\left[T_{ir}L_{ir}^{\eta_r}\left(d_{nir}\Phi_{ir}w_i\right)^{-\theta_r}\right] / \left[T_{kr}L_{kr}^{\eta_r}\left(d_{nkr}\Phi_{kr}w_k\right)^{-\theta_r}\right]}.$$

From the above double difference, locations export relatively more final goods in sectors in which they have relatively lower costs of supply, where these costs of supply depend on relative productivities (which in turn depend on relative employments through the external economies of scale and on trade in tasks), relative unit costs (which depend on wages and trade in tasks), and final goods trade costs. Each location is a net exporter of final goods in some sectors and a net importer of final goods in other sectors (inter-industry trade in goods). This inter-industry trade generates differences across locations in the pattern of employment across sectors.

Reductions in final goods trade costs (d_{nis}) induce specialization across sectors according to standard theories of comparative advantage. Reductions in task trade costs (τ_{niso}) induce an analogous process of specialization across occupations within sectors. When task trade costs are prohibitively high, all tasks are performed in the location in which the final good is produced. As task trade costs fall, it becomes feasible to unbundle production across locations and trade tasks between these locations.[5] To the extent that densely-populated locations are relatively more productive in interactive tasks (e.g. because agglomeration forces χ_{so} are stronger for interactive tasks), reductions in task trade costs induce densely-populated locations to specialize in more-interactive occupations, while more sparsely-populated locations specialize in less-interactive occupations.[6] Thus the model accounts for an increase in the interactiveness of more-densely-populated locations relative to less-densely-populated locations during our sample period in terms of falling trade costs and increased specialization according to comparative advantage.

3 Additional Empirical Results

The remainder of this appendix contains the additional tables and figures discussed in the main text of the paper.

[5]For further discussion of the increased unbundling of production, see for example Baldwin (2012).

[6]While our model focuses on trade in tasks across locations within countries, a similar process of trade in tasks could also occur between countries. To the extent that there is greater offshoring of tasks in less interactive occupations from metro areas than from non-metro areas, this provides a related explanation in terms of the same mechanism for the increased concentration of employment in interactive occupations in metro areas relative to non-metro areas over time.

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Table A1: Specialization Across Narrow Occupations 1880-2000

Panel A: Top 20 occupations whose concentration in metro areas increased most from 1880-2000	**Difference in Ranks 2000-1880**	**Panel B: Top 20 occupations whose concentration in metro areas increased least from 1880-2000**	**Difference in Ranks 2000-1880**
Geologists and geophysicists	-141	Tool makers, and die makers and setters	58
Lawyers and judges	-130	Bookbinders	59
Physicians and surgeons	-128	Inspectors (nec)	61
Members of the armed services	-121	Stationary firemen	63
Biological scientists	-119	Newsboys	71
Mining-Engineers	-118	Janitors and sextons	74
Dentists	-114	Recreation and group workers	75
Plasterers	-112	Conductors, railroad	76
Officials and administratators (nec), public administration	-106	Dispatchers and starters, vehicle	78
Editors and reporters	-99	Oilers and greaser, except auto	79
Buyers and dept heads, store	-93	Filers, grinders, and polishers, metal	81
Civil-Engineers	-88	Sawyers	82
Professional, technical and kindred workers (nec)	-83	Taxicab drivers and chauffeurs	83
Dancers and dancing teachers	-82	Painters, except construction or maintenance	84
Painters, construction and maintenance	-75	Charwomen and cleaners	91
Managers and superintendants, building	-74	Welders and flame cutters	92
Tinsmiths, coppersmiths, and sheet metal workers	-74	Cement and concrete finishers	101
Teachers (n.e.c.)	-73	Weavers, textile	105
Pattern and model makers, except paper	-71	Upholsterers	126
Metallurgical, metallurgists-Engineers	-68	Veterinarians	131

Notes: Coefficients estimated from a regression of the share of employment in metro areas in an occupation-sector-year on occupation-year and sector-year fixed effects (regression (11) in the paper). Occupation-year and sector-year fixed effects are each normalized to sum to zero. A separate regression is estimated for each year. Standard errors are clustered by occupation.

Table A2: Verbs Most and Least Strongly Correlated with Metro Area Employment Shares (1939 DOTs)

Panel A: Verbs Most Strongly Correlated with Metro Area Employment Shares							
Rank	**1880**	**1900**	**1920**	**1940**	**1960**	**1980**	**2000**
1	Thread	Prevent	Detail	Keep	Interview	Interview	Advise
2	Slot	Flower	Interview	Accompany	Issue	Devise	Audit
3	Flower	Need	Keep	Prevent	Accompany	Take	Question
4	Help	Keep	Issue	Interview	Detail	Issue	Present
5	Stitch	Accompany	Address	Issue	Keep	Detail	Specialize
6	Layer	Detail	Devise	Address	Devise	Address	Report
7	Straighten	Address	Prevent	Attend	Attend	Judge	Devise
8	Address	Thread	License	Detail	Address	Need	Promote
9	Observe	Issue	Validate	Wage	Take	Tell	Formulate
10	Unscrew	Interview	Tell	Live	Judge	Validate	Implement
Panel B: Verbs Least Strongly Correlated with Metro Area Employment Shares							
Rank	**1880**	**1900**	**1920**	**1940**	**1960**	**1980**	**2000**
1079	Fund	Promote	Promote	Proceed	Pack	Hole	Grease
1080	Sign	Formulate	Advance	Allow	Skin	Splash	Wheel
1081	Side	Lead	Fund	Formulate	Size	Employ	Jack
1082	Program	Fund	Drum	Turn	Angle	Grease	Edge
1083	Build	Sign	Screw	Drum	Bundle	Lift	Employ
1084	Bore	Bore	Mark	Discharge	Shoot	Relieve	Lift
1085	Work	Determine	Bore	Program	Work	Jack	Move
1086	Drill	Program	Hang	Hang	Piece	Move	Remove
1087	Part	Part	Weigh	Direct	Pile	Put	Advance
1088	Mark	Mark	Formulate	Advance	Advance	Advance	Put

Notes: Coefficients estimated from a regression of the share of occupation-sector employment in metro areas on the frequency with which a verb is used for an occupation and verb-sector-year fixed effects (regression (12) in the paper). A separate regression is estimated for each verb and verbs are sorted by their estimated coefficients. Verbs are from the time-invariant occupational descriptions from the 1939 Dictionary of Occupations (DOTs).

Table A3: Correlations With Independent Measures of Interactiveness

Panel A: Unweighted Correlations		Panel B: Weighted Correlations	
	Interactiveness		Interactiveness
Interactiveness	1	Interactiveness	1
Assisting and caring for others	0.23***	Assisting and caring for others	0.27***
Coaching and developing others	0.41***	Coaching and developing others	0.49***
Communicating with persons outside organization	0.51***	Communicating with persons outside organization	0.60***
Communicating with Supervisors, Peers, or Subordinates	0.46***	Communicating with Supervisors, Peers, or Subordinates	0.43***
Coordinating the work and activities of others	0.33***	Coordinating the work and activities of others	0.40***
Developing and building teams	0.41***	Developing and building teams	0.45***
Establishing and maintaining interpersonal relationships	0.63***	Establishing and maintaining interpersonal relationships	0.59***
Guiding, directing and motivating subordinates	0.35***	Guiding, directing and motivating subordinates	0.40***
Interpreting the meaning of information for others	0.51***	Interpreting the meaning of information for others	0.50***
Monitoring and controlling resources	0.35***	Monitoring and controlling resources	0.31***
Performing administrative activities	0.68***	Performing administrative activities	0.59***
Performing for or working directly with the public	0.29***	Performing for or working directly with the public	0.21***
Provide consultation and advice to others	0.55***	Provide consultation and advice to others	0.51***
Resolving conflict and negotiating with others	0.55***	Resolving conflict and negotiating with others	0.54***
Selling or influencing others	0.43***	Selling or influencing others	0.38***
Staffing organizational units	0.50***	Staffing organizational units	0.53***
Training and teaching others	0.39***	Training and teaching others	0.50***

Note: Table reports correlations between our baseline measure of interactiveness using the occupational descriptions from the 1991 DOTs and independent measures of interactiveness from ONET. We consider all 17 subcategories of "Work Activities - Interacting with Others" from ONET. Correlations reported across the sample of occupations in 2000. Weighted correlations are weighted by occupation employment in 2000. * significant at 10%; ** significant at 5%; *** significant at 1%.

Table A4: Correlations Between Occupational Characteristics

Panel A: Unweighted Correlations

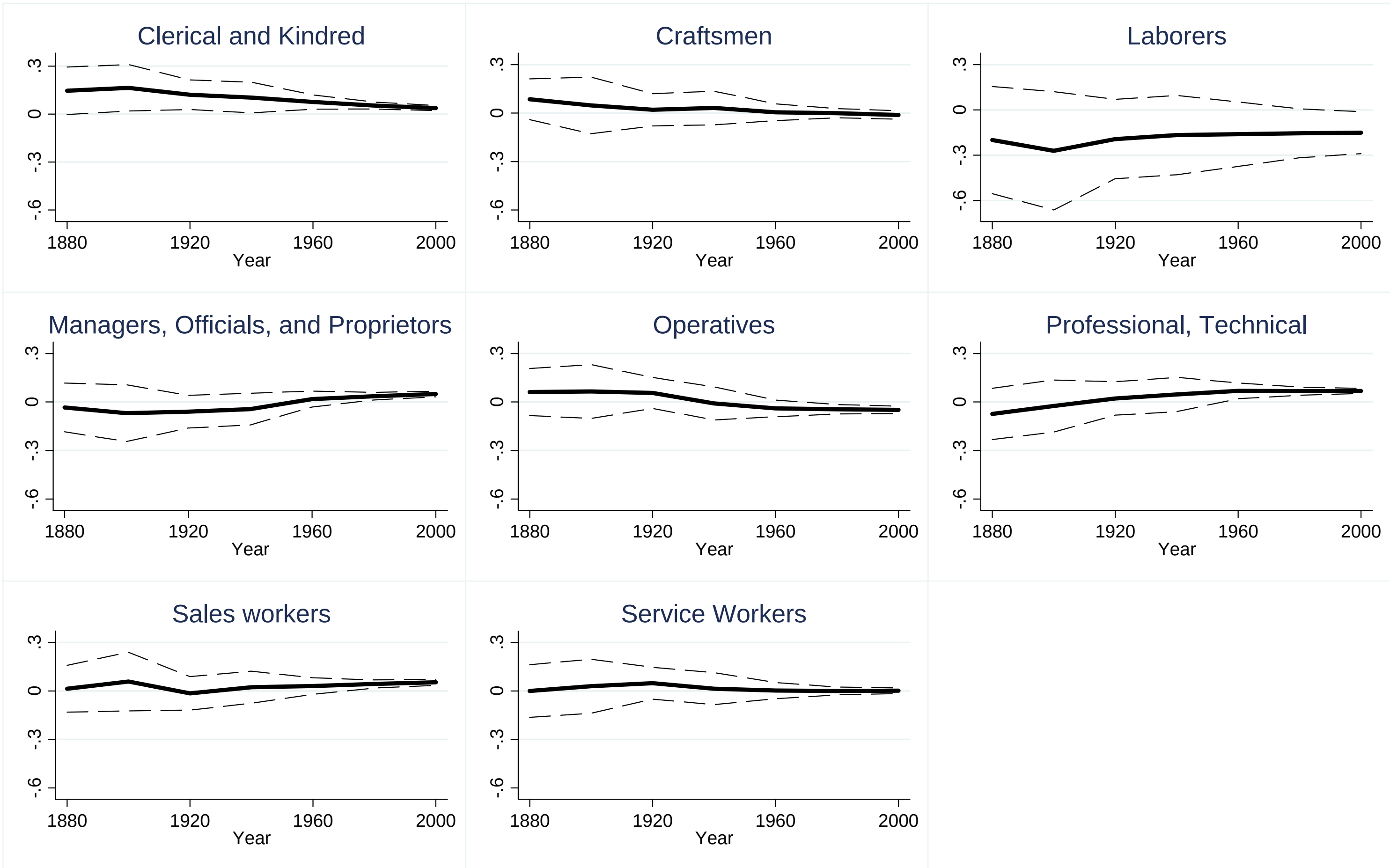
	Interactiveness	Interactiveness 1939	Nonroutine analytic (math)	Nonroutine interactive (dcp)	Routine cognitive (sts)	Routine manual (finger)	Nonroutine manual (ehf)
Interactiveness	1						
Interactiveness 1939	0.62***	1					
Nonroutine analytic (math)	0.55***	0.48***	1				
Nonroutine interactive (dcp)	0.47***	0.44***	0.54***	1			
Routine cognitive (sts)	-0.33***	-0.27***	0.25***	-0.18**	1		
Routine manual (finger)	-0.09	-0.11	0.27***	-0.08	0.52***	1	
Nonroutine manual (ehf)	-0.39***	-0.19**	-0.31***	-0.17**	0.003	-0.09	1

Panel B: Weighted Correlations

	Interactiveness	Interactiveness 1939	Nonroutine analytic (math)	Nonroutine interactive (dcp)	Routine cognitive (sts)	Routine manual (finger)	Nonroutine manual (ehf)
Interactiveness	1						
Interactiveness 1939	0.52***	1					
Nonroutine analytic (math)	0.54***	0.32***	1				
Nonroutine interactive (dcp)	0.47***	0.12	0.59***	1			
Routine cognitive (sts)	-0.25***	-0.03	0.22***	-0.33***	1		
Routine manual (finger)	-0.14*	-0.04	0.07	-0.30***	0.38***	1	
Nonroutine manual (ehf)	-0.47***	-0.22***	-0.31***	-0.21***	0.05	-0.09	1

Note: Table reports correlations between occupational characteristics across the sample of occupations in 2000. Interactiveness is our baseline measure using occupational descriptions from the 1991 DOTs. Interactiveness 1939 is our robustness measure using occupational descriptions from the 1939 DOTs. Nonroutine analytic, Nonroutine interactive, Routine cognitive, Routine manual and Nonroutine manual are measures based on the numerical scores in the 1991 DOTs as used in Autor, Levy and Murnane (2003). Weighted correlations are weighted by occupation employment in 2000. * significant at 10%; ** significant at 5%; *** significant at 1%.

Figure A1: Metro Area Specialization for Aggregate Occupations



Notes: Coefficients estimated from a regression of an indicator variable for whether a worker is located in a metro area on occupation-year and sector-year fixed effects (regression (11) in the paper). Occupation-year and sector-year fixed effects are each normalized to sum to zero. A separate regression is estimated for each year.

Figure A2: Metro Area Specialization for Aggregate Sectors

Notes: Coefficients estimated from a regression of an indicator variable for whether a worker is located in a metro area on occupation-year and sector-year fixed effects (regression (11) in the paper). Occupation-year and sector-year fixed effects are each normalized to sum to zero. A separate regression is estimated for each year.

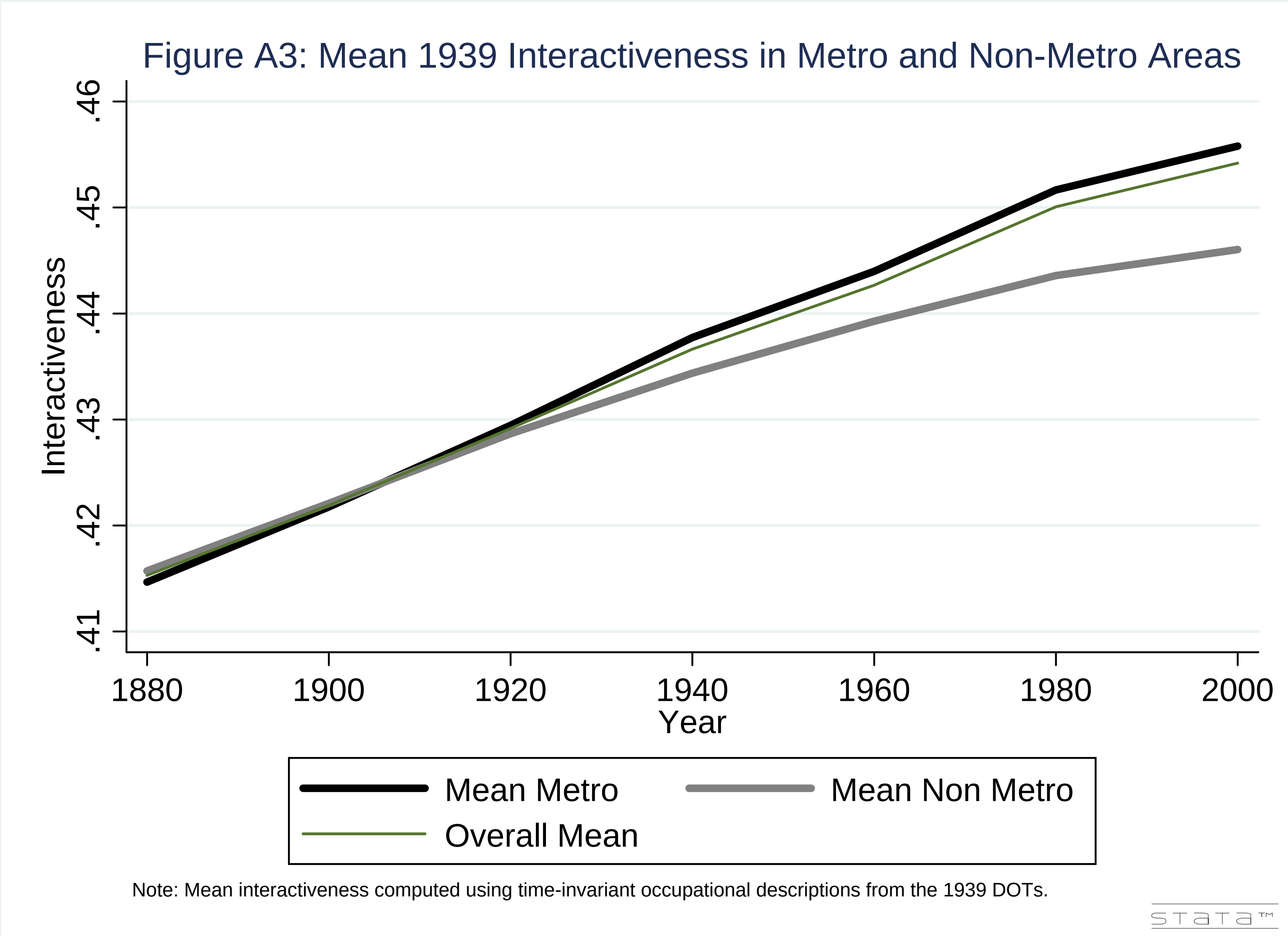
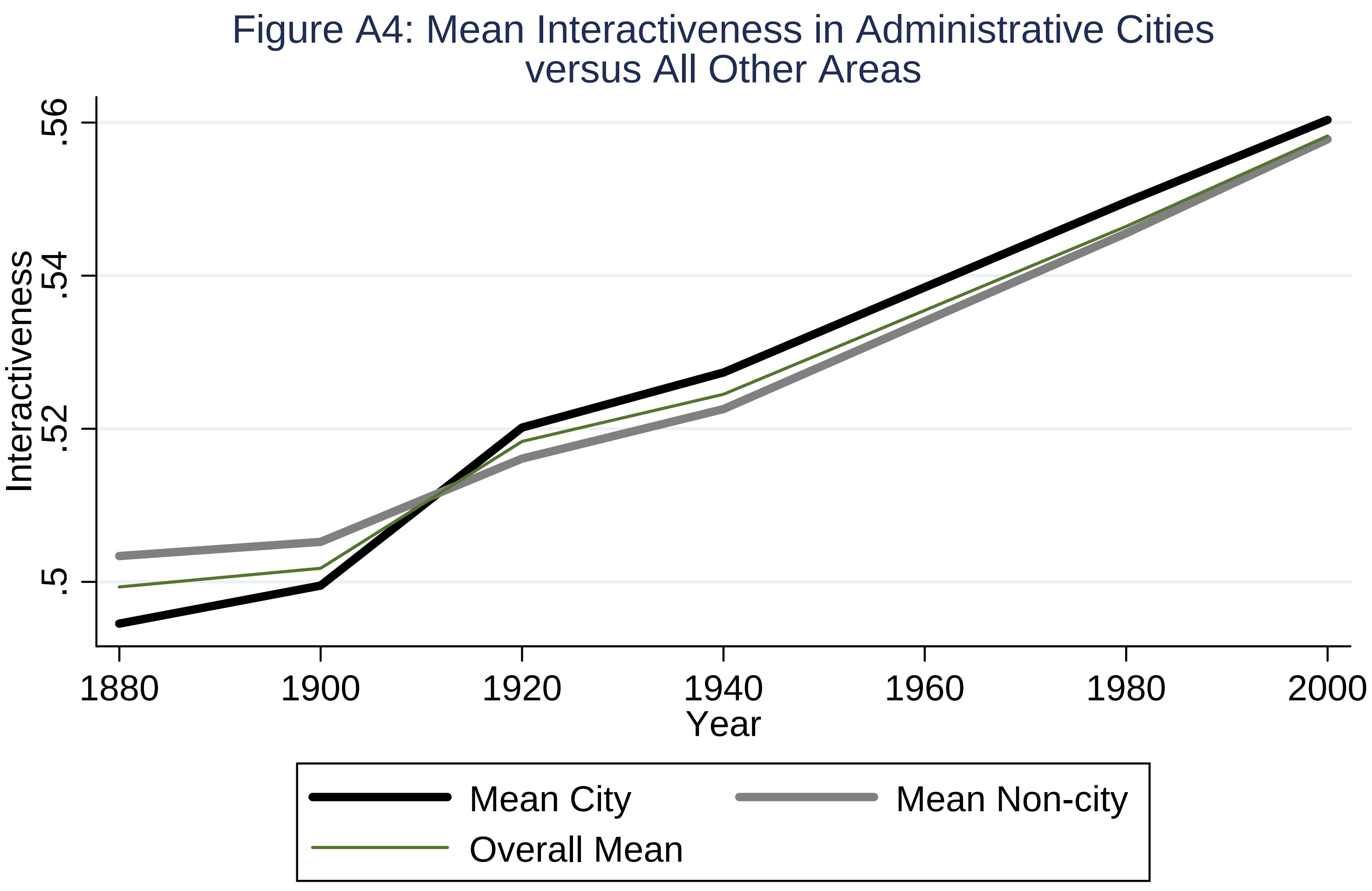
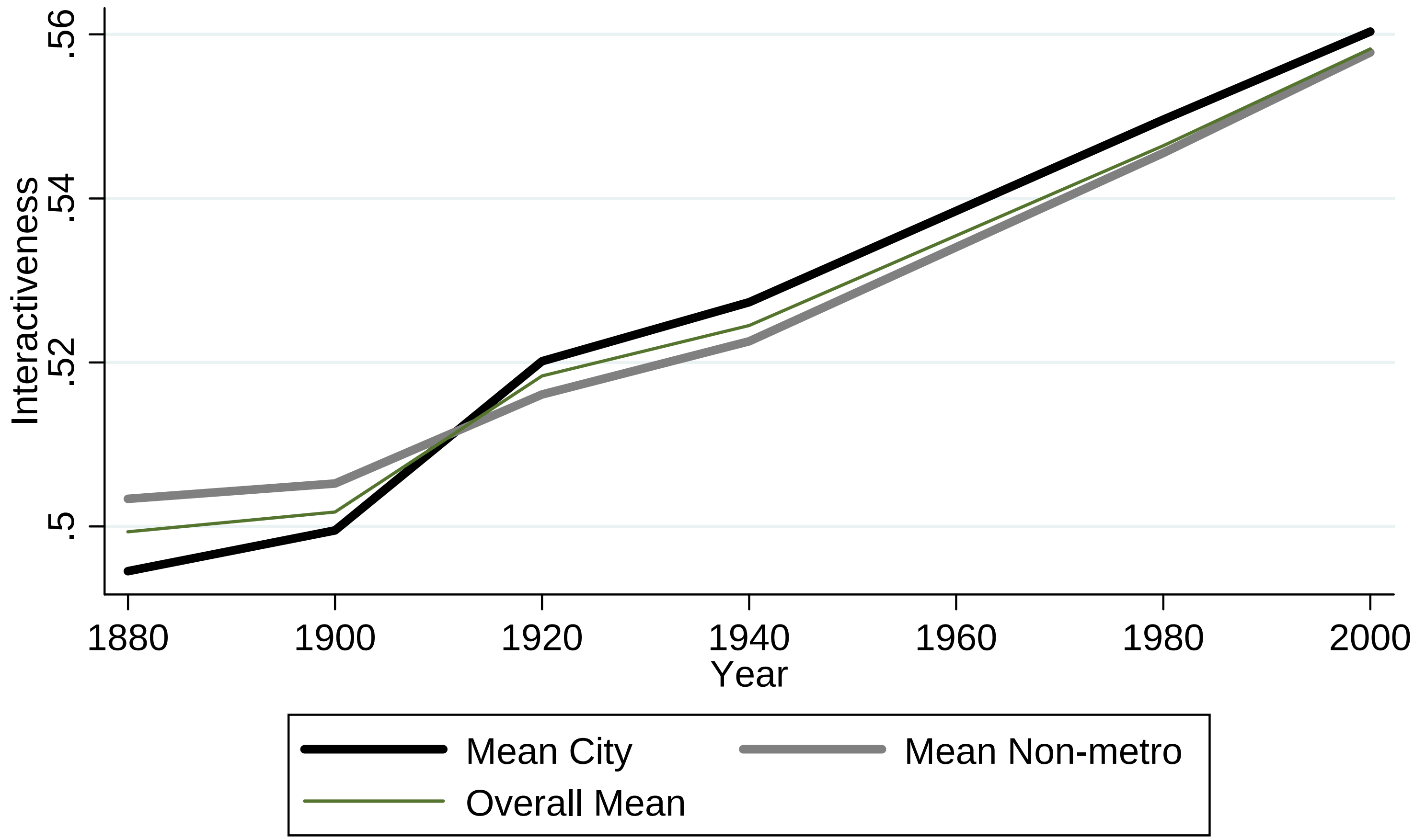


Figure A3: Mean 1939 Interactiveness in Metro and Non-Metro Areas
Interactiveness
.41
.42
.43
.44
.45
.46
1880
1900
1920
1940
1960
1980
2000
Year
Mean Metro
Mean Non Metro
Overall Mean
Note: Mean interactiveness computed using time-invariant occupational descriptions from the 1939 DOTs.



Note: Mean interactiveness computed using time-invariant occupational descriptions from the 1991 DOTs. Non-administrative cities includes both non-metro areas and the parts of metro areas outside administrative cities. The administrative cities indicator is not available in 1960 in IPUMs and hence 1960 is omitted from the figure

Figure A5: Mean Interactiveness in Administrative Cities versus Non-Metro Areas



Note: Mean interactiveness computed using time-invariant occupational descriptions from the 1991 DOTs. The administrative cities indicator is not available in 1960 in IPUMs and hence 1960 is omitted from the figure

Figure A6: Interactiveness and Education

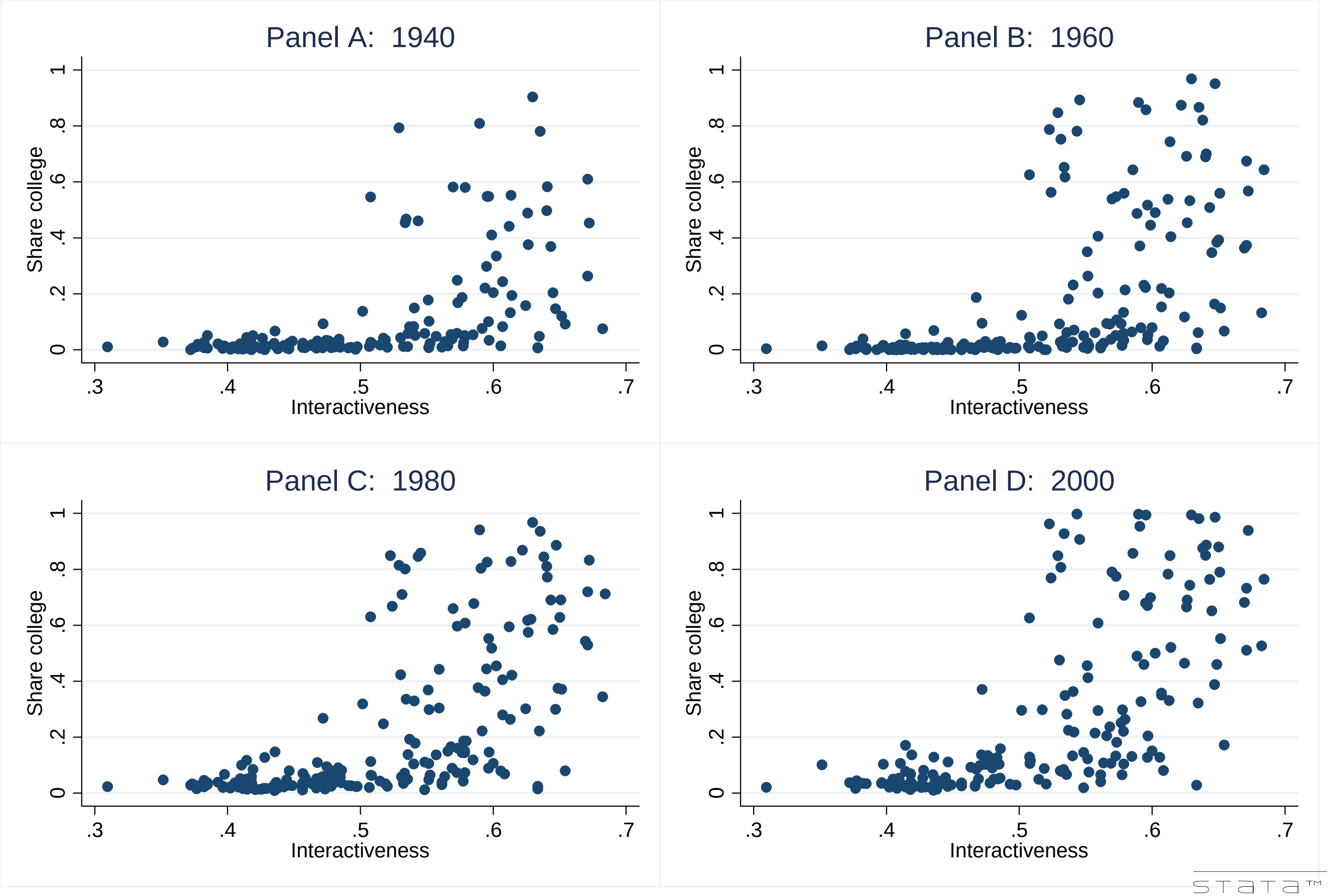


Figure A7: Mean Interactiveness in Metro and Non-Metro Areas over Time

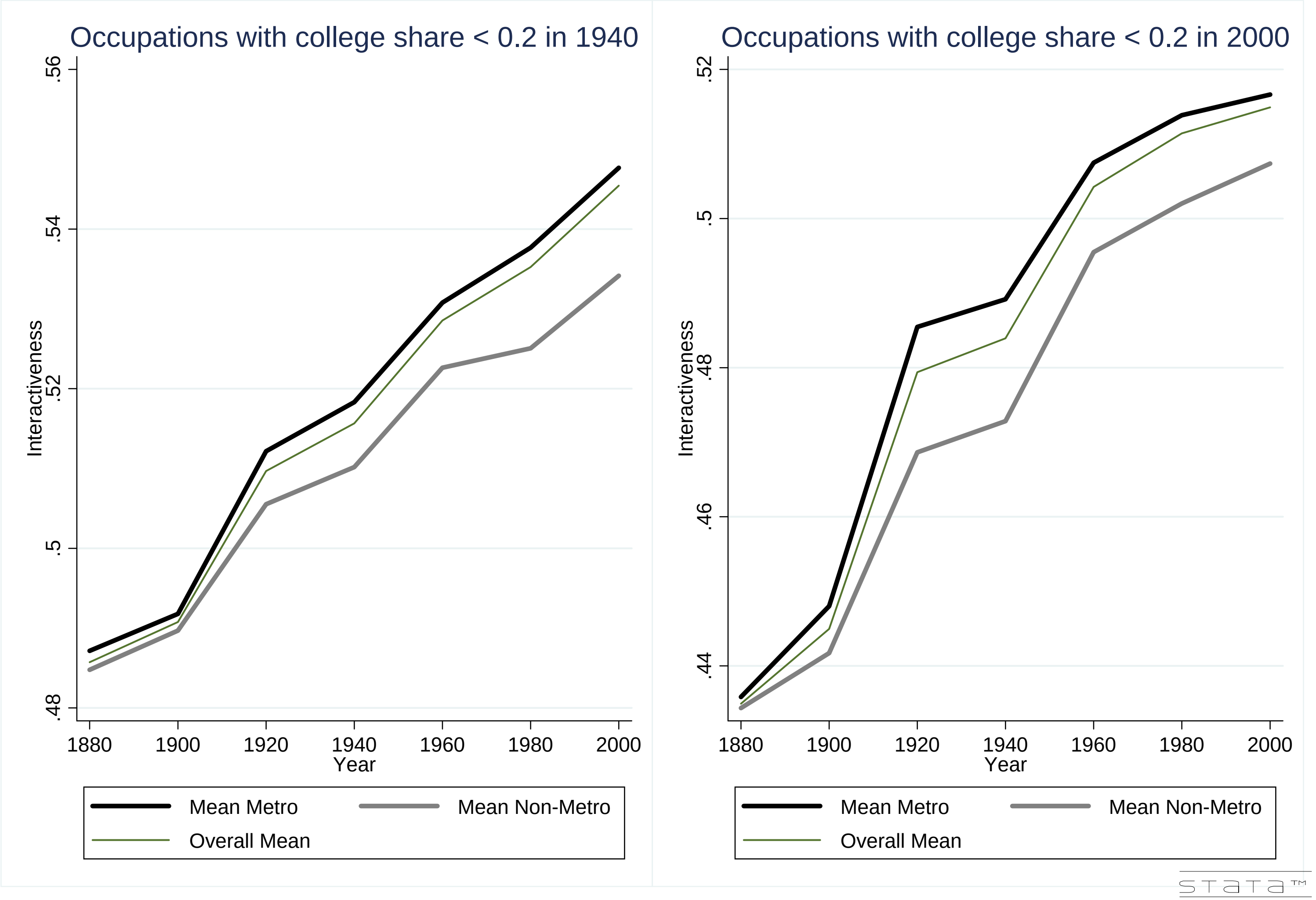
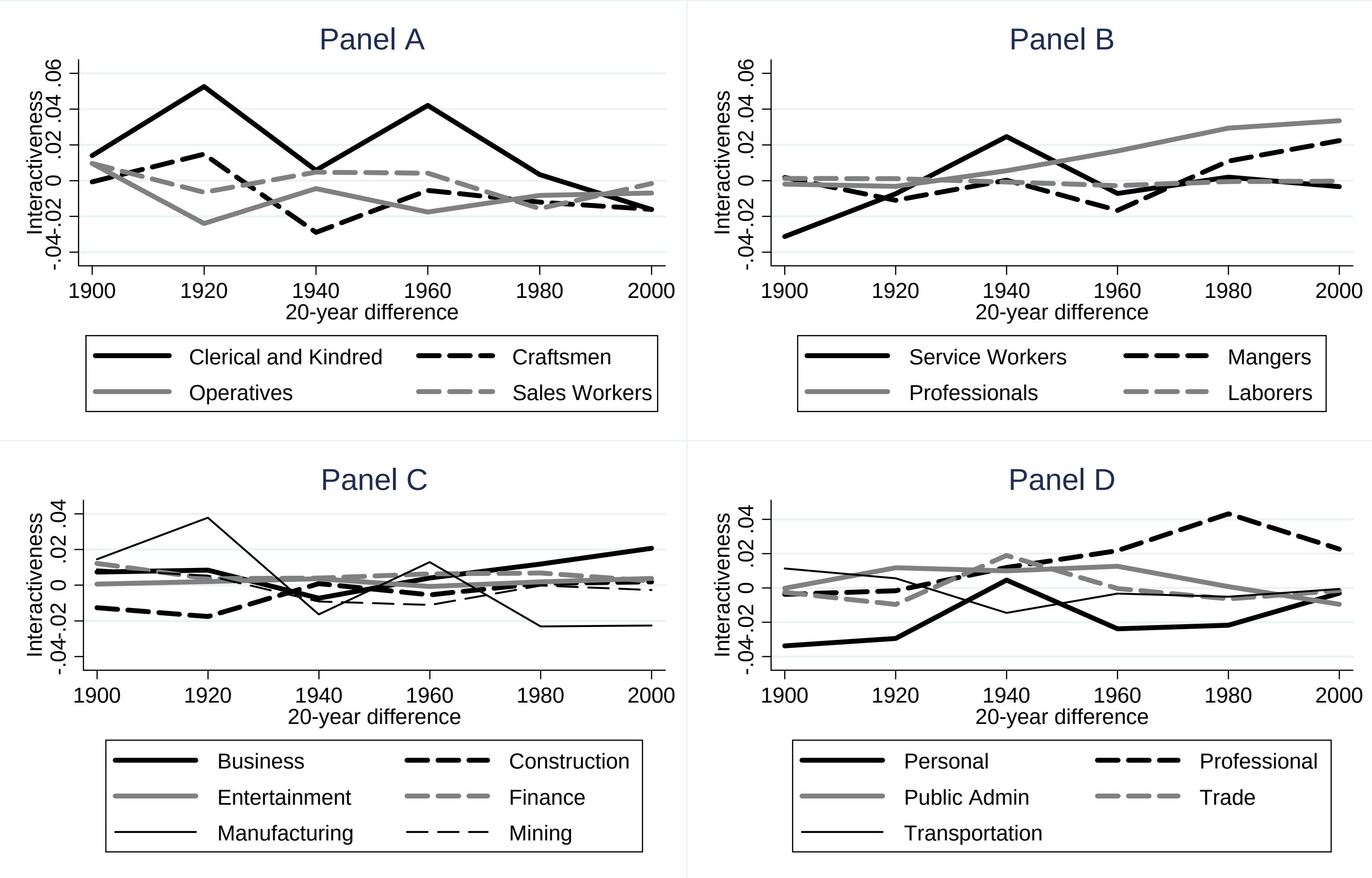
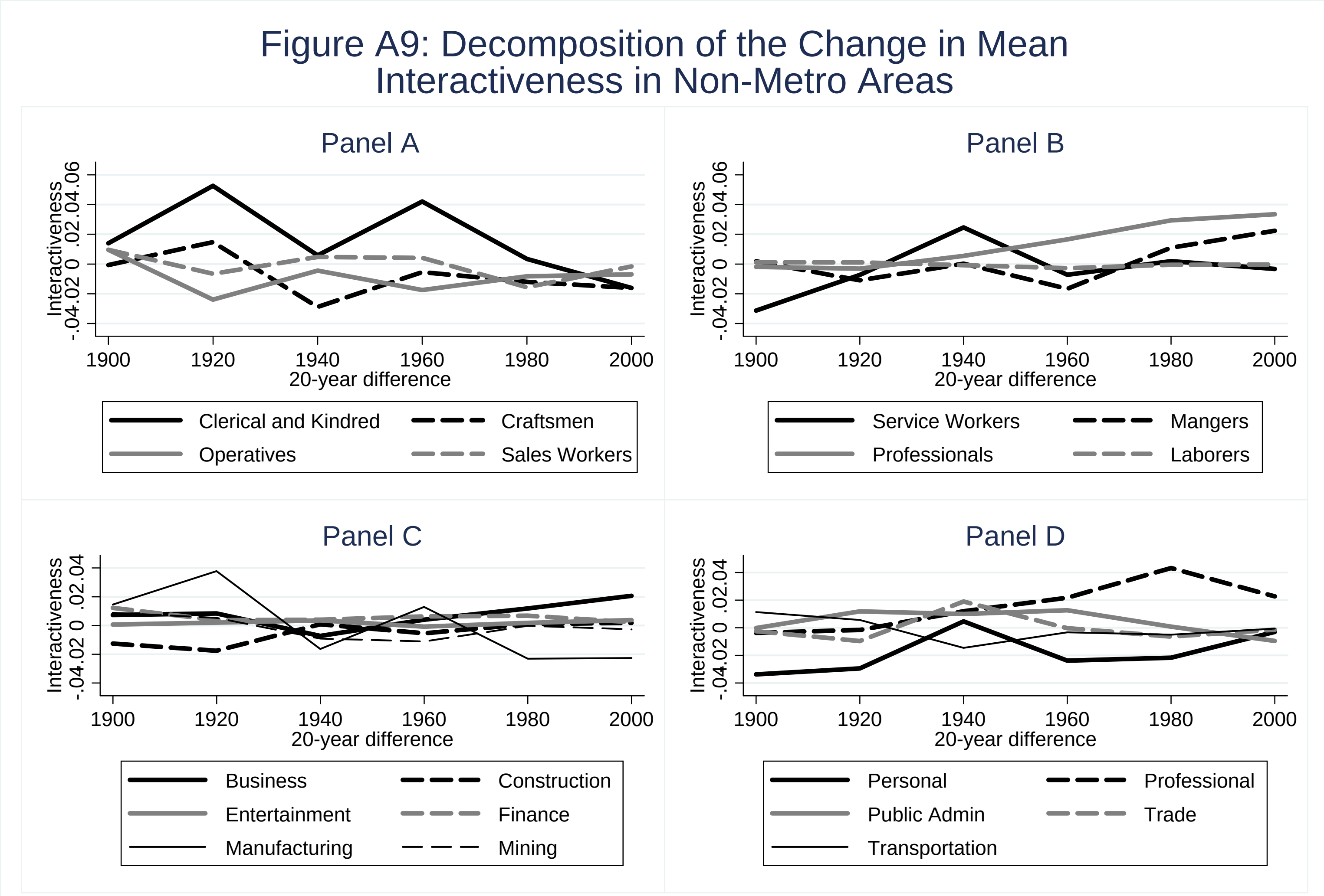


Figure A8: Decomposition of the Change in Mean Interactiveness in Metro Areas



Notes: Decomposition of the change in mean interactiveness in metro areas (equation (18) in the paper) into the contributions of two-digit occupations and sectors. Mean interactiveness based on time-invariant occupational descriptions from the 1991 DOTs.



Notes: Decomposition of the change in mean interactiveness in non-metro areas (equation (18) in the paper) into the contributions of two-digit occupations and sectors. Mean interactiveness based on time-invariant occupational descriptions from the 1991 DOTs.