Web-Based Technical Appendix: Multi-Product Firms and Trade Liberalization

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This appendix contains additional technical derivations and supplementary material for the main paper. Section 1 presents model solutions for the special case where firm productivity and consumer tastes are Pareto distributed. Section 2 contains the complete proof of Proposition 3 in the paper. Section 3 shows that it is straightforward to extend the model to introduce dependence in consumer tastes. Section 4 augments the model to include stochastic variation in firm productivity and consumer tastes over time, which induces steady-state adding and dropping of products within firms, but does not change the model's cross-sectional predictions that are the focus of the paper. Section 5 shows that the model can be extended to incorporate comparative advantage based on cross-country differences in factor endowments and cross-industry differences in factor intensity.

1. Solutions for the Special Case of Pareto Distributions

In this section of the appendix, we solve the model for the special case when firm productivity is drawn from the Pareto distribution $g(\varphi) = a\varphi_{\min}^{a}\varphi^{-(a+1)}$ and consumer tastes are drawn from the Pareto distribution $z(\lambda) = z\lambda_{\min}^{z}\lambda^{-(z+1)}$. We assume $\varphi_{\min} > 0$, $\lambda_{\min} > 0$ and $a > z > \sigma > 1$, which ensures that firm revenue has a finite mean.

1.1. *General Equilibrium with Pareto Distributions*

We begin by solving for the equilibrium sextuple $\{\varphi_d^*,\ \varphi_x^*,\ \lambda_d^*(\varphi_d^*),\ \lambda_x^*(\varphi_x^*),\ P,\ R\}$. Using the Pareto distribution of consumer tastes in the zero-profit productivity cutoff condition (equation (14) in the paper) and the exporting productivity cutoff condition (equation (16) in the paper), we have:

$$\lambda_d^*(\varphi_d^*) = \left[\frac{\sigma - 1}{z - (\sigma - 1)} \frac{f_d}{F_d} \right]^{\frac{1}{z}} \lambda_{\min}. \tag{1}$$

$$\lambda_x^*(\varphi_x^*) = \left[\frac{\sigma - 1}{z - (\sigma - 1)} \frac{f_x}{F_x} \right]^{\frac{1}{z}} \lambda_{\min}. \tag{2}$$

where we focus on parameter values for which we have an interior equilibrium and selection into export markets: $\lambda_x^*(\varphi_x^*) > \lambda_d^*(\varphi_d^*) > \lambda_{\min}$. Combining (1) and (2) with the relationship between the exporting and zero-profit cutoff abilities (equation (17) in the paper), we obtain:

$$\varphi_x^* = \tau \left(\frac{f_x}{f_d} \right)^{\frac{z-(\sigma-1)}{z(\sigma-1)}} \left(\frac{F_d}{F_x} \right)^{\frac{1}{z}} \varphi_d^*. \tag{3}$$

where we again focus on parameter values for which we have an interior equilibrium and selection into export markets: $\varphi_x^* > \varphi_d^*$. Using the Pareto distributions of productivity and consumer tastes in the free entry condition (equation (19) in the paper) yields:

$$v_e = \frac{1}{\delta} \frac{z}{a - z} \left[F_d \left(\frac{\varphi_{\min}}{\varphi_d^*} \right)^a + n F_x \left(\frac{\varphi_{\min}}{\varphi_x^*} \right)^a \right] = f_e, \tag{4}$$

which together with (3) uniquely determines φ_d^* as a function of parameters alone. With a Pareto distribution of productivity and consumer tastes, average firm revenue (equation (36) in the Appendix of the paper) is:

$$\bar{r} = \left(\frac{za}{a - z} \right) \left(\frac{\sigma}{\sigma - 1} \right) \left[F_d + n F_x \left(\frac{\varphi_d^*}{\varphi_x^*} \right)^a \right],$$

where from (3) $(\varphi_d^*/\varphi_x^*)$ is a function of parameters alone, which implies that we have determined $\bar{r}$ as a function of parameters alone.

From the steady-state stability condition (equation (20) in the paper), free entry condition (equation (19) in the paper) and labor market clearing condition (equation (21) in the paper), we have $R = L$. The mass of firms follows immediately from aggregate revenue and average revenue: $M = R/\bar{r}$. Using the Pareto distribution of productivity and consumer tastes in equations (23) and (24) in the paper, the masses of firms supplying each product to the domestic and export markets are:

$$M_d = \frac{\left(\frac{a}{a-z}\right)}{\frac{\sigma-1}{z-(\sigma-1)}\frac{f_d}{F_d}} M, \qquad M_x = \frac{\left(\frac{a}{a-z}\right)}{\frac{\sigma-1}{z-(\sigma-1)}\frac{f_x}{F_x}} M$$

where again we focus on an interior equilibrium for which $1 < \left(\frac{a}{a-z}\right) < \frac{\sigma-1}{z-(\sigma-1)}\frac{f_d}{F_d} < \frac{\sigma-1}{z-(\sigma-1)}\frac{f_x}{F_x}$.

Finally, weighted-average productivities in the domestic and export markets are:

$$\tilde{\varphi}_d = \left(\frac{z}{z-(\sigma-1)}\right)^{\frac{1}{\sigma-1}} \varphi_d^* \lambda_d^* \left(\varphi_d^*\right), \qquad \tilde{\varphi}_x = \left(\frac{z}{z-(\sigma-1)}\right)^{\frac{1}{\sigma-1}} \varphi_x^* \lambda_x^* \left(\varphi_x^*\right).$$

With the mass of firms and weighted average productivity in both the domestic and export markets determined, the price index for each product, P, follows immediately from equation (25) in the paper. Revenue for each product follows immediately from the CES revenue function, the product price index and aggregate revenue. This completes the characterization of the equilibrium sextuple $\{\varphi_d^*, \varphi_x^*, \lambda_d^*(\varphi_d^*), \lambda_x^*(\varphi_x^*), P, R\}$.

1.2. Pareto Distributions and the Margins of Trade

Consider first the extensive margins of products and countries (the *within-firm extensive margins*). With a Pareto distribution of consumer tastes, the share of products exported to a given country by a firm with productivity φ and the share of countries to which a firm with productivity φ exports a given product are both given by:

$$\left[1 - Z\left(\lambda_x^*(\varphi)\right)\right] = \left(\frac{\lambda_{\min}}{\lambda_x^*(\varphi)}\right)^z = \left(\frac{\lambda_{\min}}{\lambda_x^*(\varphi_x^*)}\frac{\varphi}{\varphi_x^*}\right)^z, \tag{5}$$

where we have used equation (11) in the paper; $\lambda_x^*(\varphi_x^*)$ is given by (2); and φ_x^* is determined by (3) and (4). From the above expression, it follows immediately that the extensive margins of products and destinations are both increasing in firm productivity. To determine their relationship with variable trade costs, note that from (2) $\lambda_x^*(\varphi_x^*)$ is independent of variable trade costs τ, while from (3) and (4) φ_x^* is increasing in variable trade costs. Therefore reductions in variable trade costs increase the extensive margins of both products and destinations.

Consider next average exports per firm-product-country (the *intensive margin*). With a Pareto distribution of consumer tastes, we have:

$$\begin{aligned} \bar{r}_x(\varphi) &= \frac{1}{1 - Z(\lambda_x^*(\varphi))} \int_{\lambda_x^*(\varphi)}^{\infty} \left(\frac{\lambda_x}{\lambda_x^*(\varphi)} \right)^{\sigma - 1} \sigma f_x z(\lambda_x) \, d\lambda_x, \\ &= \frac{z}{z - (\sigma - 1)} \sigma f_x, \end{aligned} \tag{6}$$

which is *independent* of both variable trade costs and firm productivity.

Consider finally the share of firms that export (the *across-firm extensive margin*). With a Pareto distribution of consumer tastes, we have:

$$\chi \equiv \frac{[1 - G(\varphi_x^*)]}{[1 - G(\varphi_d^*)]} = \left(\frac{\varphi_d^*}{\varphi_x^*} \right)^a = \frac{1}{\tau} \left(\frac{f_d}{f_x} \right)^{\frac{z - (\sigma - 1)}{z(\sigma - 1)}} \left(\frac{F_x}{F_d} \right)^{\frac{1}{z}}, \tag{7}$$

which is decreasing in variable trade costs. Hence reductions in variable trade costs increase the share of firms that export.

Note that the *within-firm* and *across-firm* extensive margins are also decreasing in product fixed exporting costs, f_x (from (5) and (7) using (2), (3) and (4)). In contrast, the *intensive margin* is *increasing* in f_x (from (6)).

1.3. Pareto Distributions and Heterogeneous Fixed Costs

The result that the intensive margin is independent of firm productivity requires both a Pareto distribution of consumer tastes and a product fixed exporting cost that is independent of consumer tastes. Suppose instead that product fixed exporting costs vary with consumer tastes: $f_x = \delta_0 \lambda_x^{\delta_1}$. In this case, we obtain:

$$\bar{r}_x(\varphi) = \frac{z}{z - (\sigma - 1)} \sigma \delta_0 (\lambda_x^*(\varphi))^{\delta_1},$$

$$\lambda_x^*(\varphi) = \left(\frac{\varphi_x^*}{\varphi} \right)^{\frac{\sigma - 1}{\sigma - 1 - \delta_1}} \lambda_x^*(\varphi^*),$$

$$\int_{\lambda_x^*(\varphi_x^*)}^{\infty} \left[\left(\frac{\lambda_x}{\lambda_x^*(\varphi_x^*)} \right)^{\sigma - 1} (\lambda_x^*(\varphi_x^*))^{\delta_1} - \lambda_x^{\delta_1} \right] \delta_0 z(\lambda_x) \, d\lambda_x = F_x.$$

Therefore if higher consumer taste products have higher fixed exporting costs, $\delta_1 > 0$, the intensive margin is decreasing in firm productivity, φ. In contrast, if higher consumer taste products have lower fixed exporting costs, $\delta_1 < 0$, the intensive margin is increasing in firm productivity, φ. If λ is interpreted as a component of firm productivity that is specific to individual products and countries (see Footnote 17 in the paper), $\delta_1 < 0$ is perhaps more natural, since it implies that lower values of λ are associated with higher variable and fixed costs.

2. Complete Proof of Proposition 3

Proof. We first characterize $d\varphi_d^*/d\tau$. From the free entry condition (equation (19) in the paper), define $\Upsilon = V - f_e$. By the implicit function theorem, $d\varphi_d^*/d\tau = -\left(d\Upsilon/d\tau\right)/\left(d\Upsilon/d\varphi_d^*\right)$. Substituting for φ_x^*, $\lambda_d^*\left(\varphi\right)$ and $\lambda_x^*\left(\varphi\right)$ in equation (19) in the paper using equations (9), (14), (11), (16), and (17) in the paper, we obtain $dV/d\tau < 0$ and $dV/d\varphi_d^* < 0$. Therefore, we have established that $d\varphi_d^*/d\tau < 0$.

We next characterize $d\varphi_x^*/d\tau$. Differentiating with respect to τ in equation (17) in the paper, we obtain:

$$\left(\frac{d\varphi_x^*}{d\tau}\frac{\tau}{\varphi_x^*}\right) = 1 + \left(\frac{d\varphi_d^*}{d\tau}\frac{\tau}{\varphi_d^*}\right), \tag{8}$$

It follows that to establish $d\varphi_x^*/d\tau > 0$, it suffices to show that $\left(d\varphi_d^*/d\tau\right)/\left(\tau/\varphi_d^*\right) > -1$. To do so, we again use the implicit function theorem to evaluate $d\varphi_d^*/d\tau = -\left(d\Upsilon/d\tau\right)/\left(d\Upsilon/d\varphi_d^*\right)$. Additionally, equations (9), (14), (11), (16), and (17) in the paper imply the following: $d\lambda_x^*\left(\varphi\right)/d\tau = \lambda_x^*\left(\varphi\right)/\tau$, $d\lambda_x^*\left(\varphi\right)/d\varphi_d^* = \lambda_x^*\left(\varphi\right)/\varphi^*$, $d\varphi_x^*/d\tau = \varphi_x^*/\tau$ and $d\varphi_x^*/d\varphi_d^* = \varphi_x^*/\varphi_d^*$. Combining these results with $d\varphi_d^*/d\tau = -\left(d\Upsilon/d\tau\right)/\left(d\Upsilon/d\varphi_d^*\right)$, we obtain $\left(d\varphi_d^*/d\tau\right)/\left(\tau/\varphi_d^*\right) > -1$. Therefore we have established that $d\varphi_x^*/d\tau > 0$.

Since $\lambda_d^*\left(\varphi\right) = \left(\varphi_d^*/\varphi\right)\lambda_d^*\left(\varphi_d^*\right)$, where $\lambda_d^*\left(\varphi_d^*\right)$ is invariant to τ, and since $d\varphi_d^*/d\tau < 0$, we have established that $d\lambda_d^*\left(\varphi\right)/d\tau < 0$. Additionally, since $\lambda_x^*\left(\varphi\right) = \left(\varphi_x^*/\varphi\right)\lambda_x^*\left(\varphi_x^*\right)$, where $\lambda_x^*\left(\varphi_x^*\right)$ is invariant to τ, and since $d\varphi_x^*/d\tau > 0$, we have established that $d\lambda_x^*\left(\varphi\right)/d\tau > 0$.

To evaluate the impact of the reduction in variable trade costs on measured firm productivity, denote the values of variables before the reduction by the superscript T and the values of variables after the reduction by the superscript TT. From the above: $\lambda_d^{*TT}(\varphi) > \lambda_d^{*T}(\varphi)$ and $\lambda_x^{*TT}(\varphi) < \lambda_x^{*T}(\varphi)$.

(a) For domestic firms, products with consumer tastes $\lambda \in [\lambda_d^{*T}(\varphi), \lambda_d^{*TT}(\varphi))$ are dropped from the domestic market and therefore experience a decline in their share of firm revenue. In contrast, products with consumer tastes $\lambda \in [\lambda_d^{*TT}(\varphi), \infty)$ experience a rise in their share of firm revenue. Therefore the distribution $\tilde{r}^{TT}(\varphi, \lambda)$ first-order stochastically dominates the distribution $\tilde{r}^{T}(\varphi, \lambda)$, where $\tilde{r}(\varphi, \lambda) = r(\varphi, \lambda) z(\lambda) / r(\varphi)$. Hence the reduction in variable trade costs raises measured firm productivity (equation (33) in the Appendix of the paper) for domestic firms. To characterize the magnitude of the rise, note that before the change in variable trade costs, the share of products in firm revenue for domestic firms is:

$$\tilde{r}_D^T(\varphi, \lambda) = \frac{(\varphi\lambda)^{\sigma-1} z(\lambda)}{\int_{\lambda_d^{*T}(\varphi)}^{\infty} (\varphi\lambda)^{\sigma-1} z(\lambda) d\lambda} \equiv \frac{(\varphi\lambda)^{\sigma-1} z(\lambda)}{AA}, \quad \lambda \in [\lambda_d^{*T}(\varphi), \infty). \tag{9}$$

After the change in variable trade costs, the share of products in firm revenue for domestic firms can be written as:

$$\begin{aligned} \tilde{r}_D^{TT}(\varphi, \lambda) &= \frac{(\varphi\lambda)^{\sigma-1} z(\lambda)}{BB}, \qquad \lambda \in [\lambda_d^{*TT}(\varphi), \infty), \\ BB &\equiv AA - \Theta_3, \\ \Theta_3 &\equiv \int_{\lambda_d^{*T}(\varphi)}^{\lambda_d^{*TT}(\varphi)} (\varphi\lambda)^{\sigma-1} z(\lambda) d\lambda > 0. \end{aligned} \tag{10}$$

Therefore, from (9) and (10), the ratio of firm revenue shares after and before the change in variable trade costs for domestic firms is:

$$\frac{\tilde{r}_D^{TT}(\varphi, \lambda)}{\tilde{r}_D^{T}(\varphi, \lambda)} = \frac{AA}{BB} > 1, \qquad \lambda \in [\lambda_d^{*TT}(\varphi), \infty). \tag{11}$$

(b) For new exporters, the share of products in firm revenue before the reduction in variable trade costs, $\tilde{r}_{NE}^{T}(\varphi, \lambda)$, is the same as for domestic firms in (9) for all $\lambda \in [\lambda_d^{*T}(\varphi), \infty)$. After the reduction in variable trade costs, products with consumer tastes $\lambda \in [\lambda_d^{*T}(\varphi), \lambda_d^{*TT}(\varphi))$ are dropped from the domestic market and therefore experience a decline in their share of firm revenue. On the other hand, products with consumer tastes $\lambda \in [\lambda_d^{*TT}(\varphi), \lambda_x^{*TT}(\varphi))$ experience an ambiguous change in their share of firm revenue:

$$\begin{aligned}
\tilde{r}_{NE}^{TT}(\varphi,\lambda) &= \frac{(\varphi\lambda)^{\sigma-1} z(\lambda)}{CC}, \qquad \lambda \in [\lambda_d^{*TT}(\varphi), \lambda_x^{*TT}(\varphi)), \\
CC &\equiv AA - \Theta_4 > BB, \\
\Theta_4 &\equiv \int_{\lambda_d^{*T}(\varphi)}^{\lambda_d^{*TT}(\varphi)} (\varphi\lambda)^{\sigma-1} z(\lambda)\, d\lambda - \int_{\lambda_x^{*TT}(\varphi)}^{\infty} n\left(\tau^{TT}\right)^{1-\sigma} (\varphi\lambda)^{\sigma-1} z(\lambda)\, d\lambda,
\end{aligned}$$

Since Θ_4 is ambiguous in sign, we have $\tilde{r}_{NE}^{TT}(\varphi,\lambda) \gtrless \tilde{r}_{NE}^{T}(\varphi,\lambda)$ for $\lambda \in [\lambda_d^{*TT}(\varphi), \lambda_x^{*TT}(\varphi))$.
Finally, products with consumer tastes $\lambda \in [\lambda_x^{*TT}(\varphi), \infty)$ experience a rise in their share of firm revenue:

$$\tilde{r}_{NE}^{TT}(\varphi,\lambda) = \frac{\left[1 + n\left(\tau^{TT}\right)^{1-\sigma}\right] (\varphi\lambda)^{\sigma-1} z(\lambda)}{\left[1 + n\left(\tau^{TT}\right)^{1-\sigma}\right] AA - \Theta_5}, \qquad \lambda \in [\lambda_x^{*TT}(\varphi), \infty),$$

$$\Theta_5 \equiv \left[1 + n\left(\tau^{TT}\right)^{1-\sigma}\right] \int_{\lambda_d^{*T}(\varphi)}^{\lambda_d^{*TT}(\varphi)} (\varphi\lambda)^{\sigma-1} z(\lambda)\, d\lambda + n\left(\tau^{TT}\right)^{1-\sigma} \int_{\lambda_d^{*TT}(\varphi)}^{\lambda_x^{*TT}(\varphi)} (\varphi\lambda)^{\sigma-1} z(\lambda)\, d\lambda > 0,$$

where we have re-written the denominator of $\tilde{r}_{NE}^{TT}(\varphi,\lambda)$ in a different form. As $\Theta_5 > 0$, we have $\tilde{r}_{NE}^{TT}(\varphi,\lambda) > \tilde{r}_{NE}^{T}(\varphi,\lambda)$ for $\lambda \in [\lambda_x^{*TT}(\varphi), \infty)$.
Therefore, irrespective of whether the revenue share of products with consumer tastes $\lambda \in [\lambda_d^{*TT}(\varphi), \lambda_x^{*TT}(\varphi))$ rises or falls, the difference between $\tilde{r}^{TT}(\varphi,\lambda)$ and $\tilde{r}^{T}(\varphi,\lambda)$ goes from being negative at low values of λ to being positive at high values of λ. This is a sufficient condition for the distribution $\tilde{r}^{TT}(\varphi,\lambda)$ to first-order stochastically dominate the distribution $\tilde{r}^{T}(\varphi,\lambda)$. Hence the reduction in variable trade costs raises measured firm productivity (equation (33) in the Appendix of the paper) for new exporters.
Additionally, the magnitude of the change in the shares of products in firm revenue for new exporters can be characterized as:

$$\begin{aligned}
\frac{\tilde{r}_{NE}^{TT}(\varphi,\lambda)}{\tilde{r}_{NE}^{T}(\varphi,\lambda)} &= \frac{AA}{CC} < \frac{AA}{BB}, \qquad \lambda \in [\lambda_d^{*TT}(\varphi), \lambda_x^{*TT}(\varphi)), \\
\frac{\tilde{r}_{NE}^{TT}(\varphi,\lambda)}{\tilde{r}_{NE}^{T}(\varphi,\lambda)} &= \frac{\left[1 + n\left(\tau^{TT}\right)^{1-\sigma}\right] AA}{BB + DD} > \frac{AA}{BB} > 1, \quad \lambda \in [\lambda_x^{*TT}(\varphi), \infty), \\
DD &\equiv n\left(\tau^{TT}\right)^{1-\sigma} \int_{\lambda_x^{*TT}(\varphi)}^{\infty} (\varphi\lambda)^{\sigma-1} z(\lambda)\, d\lambda, \\
DD &< n\left(\tau^{TT}\right)^{1-\sigma} BB.
\end{aligned}$$

Therefore, from the above expressions and (11), the change in revenue shares of products with low consumer tastes $\lambda \in [\lambda_d^{*T}(\varphi), \lambda_d^{*TT}(\varphi))$ is the same for new exporters as for domestic firms, the change in revenue shares of products with intermediate consumer tastes $\lambda \in [\lambda_d^{*TT}(\varphi), \lambda_x^{*TT}(\varphi))$ is smaller for new exporters than for domestic firms, and the change in revenue shares for products with high consumer tastes $\lambda \in [\lambda_x^{*TT}(\varphi), \infty)$ is larger for new exporters than for domestic firms. These are sufficient conditions for the distribution $\tilde{r}_{NE}^{TT}(\varphi, \lambda)$ to first-order stochastically dominate the distribution $\tilde{r}_D^{TT}(\varphi, \lambda)$. Hence new exporters experience greater measured productivity growth than domestic firms following the reduction in variable trade costs.

(c) For continuing exporters, products with consumer tastes $\lambda \in [\lambda_d^{*T}(\varphi), \lambda_d^{*TT}(\varphi))$ are dropped from the domestic market and therefore experience a decline in their share of firm revenue. On the other hand, products with consumer tastes $\lambda \in [\lambda_d^{*TT}(\varphi), \lambda_x^{*TT}(\varphi))$ and $\lambda \in [\lambda_x^{*TT}(\varphi), \lambda_x^{*T}(\varphi))$ experience an ambiguous change in their share of firm revenue:

$$\begin{aligned}
\tilde{r}_E^T(\varphi, \lambda) &= \frac{(\varphi\lambda)^{\sigma-1} z(\lambda)}{AA + EE}, & \lambda \in [\lambda_d^{*T}(\varphi), \lambda_x^{*T}(\varphi)), \\
EE &\equiv n\left(\tau^T\right)^{1-\sigma} \int_{\lambda_x^{*T}(\varphi)}^{\infty} (\varphi\lambda)^{\sigma-1} z(\lambda)\, d\lambda, & \\
\tilde{r}_E^{TT}(\varphi, \lambda) &= \frac{(\varphi\lambda)^{\sigma-1} z(\lambda)}{AA - FF}, & \lambda \in [\lambda_d^{*TT}(\varphi), \lambda_x^{*TT}(\varphi)), \\
\tilde{r}_E^{TT}(\varphi, \lambda) &= \frac{\left[1 + n\left(\tau^{TT}\right)^{1-\sigma}\right] (\varphi\lambda)^{\sigma-1} z(\lambda)}{AA - FF}, & \lambda \in [\lambda_x^{*TT}(\varphi), \lambda_x^{*T}(\varphi)),
\end{aligned}$$

$$FF \equiv \int_{\lambda_d^{*T}(\varphi)}^{\lambda_d^{*TT}(\varphi)} (\varphi\lambda)^{\sigma-1} z(\lambda)\, d\lambda - \left(\left(\frac{\tau^{TT}}{\tau^T}\right)^{1-\sigma} EE + n\left(\tau^{TT}\right)^{1-\sigma} \int_{\lambda_x^{*TT}(\varphi)}^{\lambda_x^{*T}(\varphi)} (\varphi\lambda)^{\sigma-1} z(\lambda)\, d\lambda\right).$$

$$\frac{1}{AA - FF} \gtrless \frac{1}{AA + EE}, \qquad \text{and} \qquad \frac{1 + n\left(\tau^{TT}\right)^{1-\sigma}}{AA - FF} \gtrless \frac{1}{AA + EE}.$$

Therefore we have $\tilde{r}_{NE}^{TT}(\varphi, \lambda) \gtrless \tilde{r}_E^T(\varphi, \lambda)$ for $\lambda \in [\lambda_d^{*TT}(\varphi), \lambda_x^{*TT}(\varphi))$ and $\lambda \in [\lambda_x^{*TT}(\varphi), \lambda_x^{*T}(\varphi))$. Finally, products with consumer tastes $\lambda \in [\lambda_x^{*T}(\varphi), \infty)$ also experience an ambiguous change in their share of firm revenue:

$$\tilde{r}_E^T(\varphi, \lambda) = \frac{\left[1 + n\left(\tau^T\right)^{1-\sigma}\right] (\lambda\varphi)^{\sigma-1} z(\lambda)}{AA + EE}, \qquad \lambda \in [\lambda_x^{*T}(\varphi), \infty),$$

$$\tilde{r}_E^{TT}(\varphi,\lambda) = \frac{\left[1+n\left(\tau^{TT}\right)^{1-\sigma}\right](\lambda\varphi)^{\sigma-1} z(\lambda)}{AA-FF}, \qquad \lambda \in [\lambda_x^{*T}(\varphi), \infty).$$

As $1/(AA-FF) \gtrless 1/(AA+EE)$, we have $\tilde{r}_{NE}^{TT}(\varphi,\lambda) \gtrless \tilde{r}_E^{T}(\varphi,\lambda)$ for $\lambda \in [\lambda_x^{*T}(\varphi), \infty)$. To characterize the magnitude of the change in revenue shares, consider the ratio of product firm revenue shares after and before the reduction in variable trade costs:

$$\frac{\tilde{r}_E^{TT}(\varphi,\lambda)}{\tilde{r}_E^{T}(\varphi,\lambda)} = \frac{AA+EE}{AA-FF}, \qquad \lambda \in [\lambda_d^{*TT}(\varphi), \lambda_x^{*TT}(\varphi)),$$

$$\frac{\tilde{r}_E^{TT}(\varphi,\lambda)}{\tilde{r}_E^{T}(\varphi,\lambda)} = \frac{\left[1+n\left(\tau^{TT}\right)^{1-\sigma}\right](AA+EE)}{AA-FF}, \qquad \lambda \in [\lambda_x^{*TT}(\varphi), \lambda_x^{*T}(\varphi)),$$

$$\frac{\tilde{r}_E^{TT}(\varphi,\lambda)}{\tilde{r}_E^{T}(\varphi,\lambda)} = \frac{\left[1+n\left(\tau^{TT}\right)^{1-\sigma}\right](AA+EE)}{\left[1+n\left(\tau^{T}\right)^{1-\sigma}\right](AA-FF)}, \qquad \lambda \in [\lambda_x^{*T}(\varphi), \infty).$$

As $1+n\left(\tau^{TT}\right)^{1-\sigma} > 1+n\left(\tau^{T}\right)^{1-\sigma} > 1$, the change in product firm revenue shares for continuing exporters is smallest for $\lambda \in [\lambda_d^{*TT}(\varphi), \lambda_x^{*TT}(\varphi))$, largest for $\lambda \in [\lambda_x^{*TT}(\varphi), \lambda_x^{*T}(\varphi))$, and of intermediate size for $\lambda \in [\lambda_x^{*T}(\varphi), \infty)$. With this ordering of changes in the shares of products in firm revenue, the change in measured productivity (equation (33) in the Appendix of the paper) for continuing exporters is in general ambiguous. Additionally, from the above expressions and (11), the change in measured productivity (equation (33) in the Appendix of the paper) for continuing exporters can be higher or lower than for domestic firms. ■

3. Dependence in Consumer Tastes

While the model's simplifying assumption that the consumer tastes distributions are independently distributed provides a tractable way to introduce heterogeneity across products and countries within firms, it is straightforward to extend the analysis to introduce dependence in consumer tastes. For example, suppose that consumer tastes for a firm's variety of a product include a common component, λ_k, which is the same across countries j for a given product k, and an idiosyncratic component, λ_{jk}, which varies across both countries j and products k:

$$\lambda = \lambda_k^{\varsigma} \lambda_{jk}^{1-\varsigma}, \qquad 0 \leq \varsigma \leq 1.$$

The common component of consumer tastes plays a similar role to firm productivity which, as in the paper, is common across products and countries within firms. Once the sunk entry cost is paid, a firm observes the common component of consumer tastes for each product, λ_k, and the idiosyncratic component of consumer tastes, λ_{jk}, for each product and country. The common component of consumer tastes is drawn separately for each product from a continuous distribution $z_k(\lambda_k)$ with cumulative distribution function $Z_k(\lambda_k)$. The idiosyncratic component of consumer tastes is drawn separately for each product and country from a continuous distribution $z_{jk}(\lambda_{jk})$ with cumulative distribution function $Z_{jk}(\lambda_{jk})$.

To make use of law of large numbers results, we assume that the productivity and consumer tastes distributions are independent of one another and independently distributed across firms. Similarly, we assume that the common consumer tastes distribution, $z_k(\lambda_k)$, is independently distributed across products, while the idiosyncratic consumer tastes distribution, $z_{jk}(\lambda_{jk})$, is independently distributed across products and countries. With these assumptions, a firm's profitability is now correlated across products and countries for two reasons. First, higher productivity (φ) raises a firm's profitability across all products and countries. Second, a higher common component of consumer tastes for a product (λ_k) raises a firm's profitability across all countries for that product. These correlations are however imperfect because of stochastic variation in the idiosyncratic component of consumer tastes across both products and countries. As $\varsigma \to 0$, the extended model considered here reduces to the model in the paper, where consumer tastes are independently distributed across products and countries. As $\varsigma \to 1$, the extended model considered here reduces to the special case discussed in footnote 12 in the paper, where there is perfect correlation of consumer tastes across countries.

4. Steady-state Product Adding and Dropping

As the focus of our paper is the cross-section distribution of exports across firms, countries and products, we follow much of the literature on firm heterogeneity in international trade in abstracting from dynamics. In this section of the appendix, we show that the model can be extended to incorporate stochastic variation in firm productivity and consumer tastes over time, which induces steady-state adding and dropping of products within firms. In this extension, we embed a simplified version of the dynamics from the closed economy model of Bernard, Redding and Schott (2010) in the open economy model considered in the paper.

The specification of entry, production and demand is similar to that considered in the paper. Once a firm incurs the sunk entry cost, f_e, productivity and consumer tastes are drawn from the continuous distributions $g(\varphi)$ and $z(\lambda)$ respectively, with cumulative distributions $G(\varphi)$ and $Z(\lambda)$. After a firm observes its initial values of productivity and consumer tastes, it decides whether to produce or exit. If the firm exits, its production knowledge is lost, and the sunk cost must be incurred again in order for the firm to re-enter. If the firm enters, it faces a Poisson probability $\theta > 0$ of a shock to productivity φ, in which case a new value for productivity φ' is re-drawn from the same distribution as upon entry $g(\varphi')$. Similarly, the firm faces a Poisson probability $\varepsilon > 0$ of an idiosyncratic shock to consumer tastes λ for its variety of a given product in a given country, in which case a new value for consumer tastes λ' is re-drawn from the same distribution as upon entry $z(\lambda')$.[1]

As consumer tastes for a firm's variety of each product change over time, previously profitable products and markets become unprofitable and are dropped when λ falls below the zero-profit cutoff for a market $\{\lambda_d^*(\varphi),\ \lambda_x^*(\varphi)\}$. Similarly, previously unprofitable products become viable and are added when λ rises above the zero-profit cutoff for a market $\{\lambda_d^*(\varphi),\ \lambda_x^*(\varphi)\}$. Since the distribution from which consumer tastes are drawn following a stochastic shock is the same as the distributions from which they are drawn upon entry, the stationary distribution for consumer tastes, $\gamma_z(\lambda)$, takes a particularly

[1] The assumption that following a stochastic shock productivity and consumer tastes are re-drawn from the same distributions as upon entry is a simplifying device, which enables us to introduce dynamics in as tractable a way as possible. In the closed economy model of Bernard, Redding and Schott (2010), we consider richer forms of dynamics that allow for serial correlation in productivity and consumer tastes. While the introduction of serial correlation complicates the model's dynamics, it does not change the cross-sectional predictions of the model on which the paper is focused.

simple form:

$$\gamma_z(\lambda) = z(\lambda), \tag{12}$$

where the stationary distribution for consumer tastes conditional on a product being supplied to a market is a truncation of $z(\lambda)$ at the zero-profit cutoff for consumer tastes for a market $\{\lambda_d^*(\varphi), \lambda_x^*(\varphi)\}$.

As the distribution from which productivity is drawn following a stochastic shock is the same as the distribution from which it is drawn upon entry, the stationary distribution for firm productivity, $\gamma_g(\varphi)$, also takes a particularly simple form. The stationary distribution for firm productivity conditional on supplying a market is a truncation of the distribution $g(\varphi)$ at the cutoff productivity above which firms serve the market:

$$\gamma_g(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi_d^*)} & \text{for } \varphi \geq \varphi_d^* \text{ in the domestic market} \\ \frac{g(\varphi)}{1-G(\varphi_x^*)} & \text{for } \varphi \geq \varphi_x^* \text{ in each export market} \\ 0 & \text{otherwise} \end{cases}. \tag{13}$$

The determination of general equilibrium remains largely the same as in the paper. The only exception is the determination of the value of the firm. In the dynamic extension considered here, the combination of a sunk entry cost and stochastic shocks to firm productivity generates an option value to firm entry. If a firm chooses to exit, it forgoes both the net present value of its instantaneous flow of profits and also the option of experiencing stochastic productivity shocks. Therefore, with this option value to firm entry, the threshold productivity for firm entry and exit will in general lie below the productivity at which the instantaneous flow of total firm profits is equal to zero.[2] The value of a firm with productivity φ is determined according to the following Bellman equation:

$$v(\varphi) = \frac{\pi(\varphi)}{\delta+\theta} + \left(\frac{\theta}{\delta+\theta}\right) \int_{\varphi_d^*}^{\infty} v(\varphi')\, g(\varphi')\, d\varphi', \tag{14}$$

[2] In contrast, fixed and marginal production costs for individual products have no sunk component, and consumer tastes for a firm's variety of each product evolve independently of whether or not the variety is supplied. Therefore, once a firm has decided to enter, the firm's decision whether or not to supply each product reduces to a period-by-period comparison of contemporaneous revenue and production costs.

As the distribution from which a new value of productivity is drawn following a stochastic shock is the same as upon entry and is independent of a firm's existing value of productivity, the solution to this Bellman equation takes a particularly simple form. Substituting for $v(\varphi')$ on the right-hand side of (14) using the trial solution $v(\varphi) = \alpha\pi(\varphi) + \beta\int_{\varphi_d^*}^{\infty}\pi(\varphi')g(\varphi')d\varphi'$, and solving for α and β, yields the equilibrium value of a firm:

$$v(\varphi) = \frac{\pi(\varphi)}{\delta+\theta} + \left(\frac{\theta}{\delta+\theta}\right)\left(\frac{1}{\delta+\theta G(\varphi_d^*)}\right)\left[\int_{\varphi_d^*}^{\infty}\pi(\varphi')g(\varphi')d\varphi'\right], \tag{15}$$

Therefore the equilibrium value of a firm with productivity φ is a weighted average of the current flow of firm profits and the expected flow of firm profits following a stochastic productivity shock, where the weights depend on the probability of firm death, the probability of a productivity shock and the probability that a firm remains active following a productivity shock. Substituting the expression for the equilibrium value of a firm into the free entry condition, and rearranging, the free entry condition can be re-written as follows:

$$V = \int_{\varphi_d^*}^{\infty}\left[\frac{\pi(\varphi) + \left(\frac{\theta}{\delta+\theta}\right)\int_{\varphi_d^*}^{\infty}\left[\pi(\varphi') - \pi(\varphi)\right]g(\varphi')d\varphi'}{\delta+\theta G(\varphi_d^*)}\right]g(\varphi)d\varphi = f_e, \tag{16}$$

which can be simplified to yield the following expressions:

$$V = \int_{\varphi_d^*}^{\infty}\left(\frac{\pi(\varphi)}{\delta+\theta G(\varphi_d^*)}\right)g(\varphi)d\varphi = f_e, \tag{17}$$

$$V = \int_{\varphi_d^*}^{\infty}\left(\frac{\pi_d(\varphi)}{\delta+\theta G(\varphi_d^*)}\right)g(\varphi)d\varphi + n\int_{\varphi_x^*}^{\infty}\left(\frac{\pi_x(\varphi)}{\delta+\theta G(\varphi_d^*)}\right)g(\varphi)d\varphi = f_e$$

As in the paper, the model has a recursive structure, and the determination of general equilibrium is straightforward. We begin by determining $\{\varphi_d^*,\ \varphi_x^*,\ \lambda_d^*(\varphi_d^*),\ \lambda_x^*(\varphi_x^*)\}$ using four equilibrium relationships. To derive the first of these relationships, we use the expression for total firm profits in the paper as well as $\lambda_d^*(\varphi) = (\varphi_d^*/\varphi)\lambda_d^*(\varphi_d^*)$ and $\lambda_x^*(\varphi) = (\varphi_x^*/\varphi)\lambda_x^*(\varphi_x^*)$, which together imply that the free entry condition (17) can be written as:

$$V = \int_{\varphi_d^*}^{\infty} \left[\int_{(\varphi_d^*/\varphi)\lambda_d^*(\varphi_d^*)}^{\infty} \left(\left(\frac{\varphi \lambda_d}{\varphi_d^* \lambda_d^* (\varphi_d^*)} \right)^{\sigma-1} - 1 \right) f_d z(\lambda_d) \, d\lambda_d - F_d \right] \frac{g(\varphi)}{\delta + \theta G_c(\varphi_d^*)} d\varphi$$
$$+ \int_{\varphi_x^*}^{\infty} \left[\int_{(\varphi_x^*/\varphi)\lambda_x^*(\varphi_x^*)}^{\infty} \left(\left(\frac{\varphi \lambda_x}{\varphi_x^* \lambda_x^* (\varphi_x^*)} \right)^{\sigma-1} - 1 \right) f_x z(\lambda_x) \, d\lambda_x - F_x \right] \frac{n g(\varphi) \, d\varphi}{\delta + \theta G_c(\varphi_d^*)} = f_e. \tag{18}$$

Since the only sunk cost in the model is the entry cost, f_e, and we consider an equilibrium with selection into export markets, the lowest productivity firm that enters serves only the domestic market: $\varphi_d^* < \varphi_x^*$. Therefore only a firm's decision of whether or not to serve the domestic market is affected by the option value of entry. In contrast, the firm's decision whether or not to serve the export market is determined by a comparison of the instantaneous flow of revenue and the fixed exporting costs. The product and firm exporting cutoffs are therefore determined as in the paper:

$$\int_{\lambda_x^*(\varphi_x^*)}^{\infty} \left[\left(\frac{\lambda_x}{\lambda_x^*(\varphi_x^*)} \right)^{\sigma-1} - 1 \right] f_x z(\lambda_x) \, d\lambda_x = F_x. \tag{19}$$

$$\varphi_x^* = \tau \left(\frac{f_x}{f_d} \right)^{\frac{1}{\sigma-1}} \left(\frac{\lambda_d^*(\varphi_d^*)}{\lambda_x^*(\varphi_x^*)} \right) \varphi_d^*, \tag{20}$$

In deciding whether or not to enter and supply the domestic market, a firm takes into account the option value of entry. At the domestic cutoff productivity, φ_d^*, the value of the firm is equal to zero, $v(\varphi_d^*) = 0$, which from (15) implies that the flow of total firm losses exactly equals the probability of a stochastic shock to firm productivity times expected profits conditional on a stochastic shock occurring:

$$- \left[\int_{(\varphi_d^*/\varphi)\lambda_d^*(\varphi_d^*)}^{\lambda_d^*(\varphi_d^*)} \left(\left(\frac{\lambda_d}{\lambda_d^*(\varphi_d^*)} \right)^{\sigma-1} - 1 \right) f_p z(\lambda_d) \, d\lambda_d - F_d \right]$$
$$= \int_{\varphi_d^*}^{\infty} \left[\int_{(\varphi_d^*/\varphi)\lambda_d^*(\varphi_d^*)}^{\lambda_d^*(\varphi_d^*)} \left(\left(\frac{\varphi \lambda_d}{\varphi_d^* \lambda_d^*(\varphi_d^*)} \right)^{\sigma-1} - 1 \right) f_d z(\lambda_d) \, d\lambda_d - F_d \right] \frac{\theta g(\varphi)}{\delta + \theta G(\varphi_d^*)} d\varphi$$
$$+ \int_{\varphi_x^*}^{\infty} \left[\int_{(\varphi_x^*/\varphi)\lambda_x^*(\varphi_x^*)}^{\lambda_x^*(\varphi_x^*)} \left(\left(\frac{\varphi \lambda_x}{\varphi_x^* \lambda_x^*(\varphi_x^*)} \right)^{\sigma-1} - 1 \right) f_x z(\lambda_x) \, d\lambda_x - F_x \right] \frac{n\theta g(\varphi) \, d\varphi}{\delta + \theta G_c(\varphi_d^*)}. \tag{21}$$

To characterize $\{\varphi_d^*, \varphi_x^*, \lambda_d^*(\varphi_d^*), \lambda_x^*(\varphi_x^*)\}$, we first use (19) to determine $\lambda_x^*(\varphi_x^*)$ independent of the other equations of the model. Second, we use (20) to determine φ_x^* as a function of φ_d^*. Third, substituting for φ_x^* as a function of φ_d^* and using the equilibrium value of $\lambda_x^*(\varphi_x^*)$, (18) and (21) provide two equations that together determine $\{\varphi_d^*, \lambda_d^*(\varphi_d^*)\}$. Having characterized $\{\lambda_x^*(\varphi_x^*), \varphi_d^*, \lambda_d^*(\varphi_d^*)\}$, the equilibrium value of φ_x^* follows immediately from (20). Finally, having determined $\{\varphi_d^*, \varphi_x^*, \lambda_d^*(\varphi_d^*), \lambda_x^*(\varphi_x^*)\}$, the remaining elements of the equilibrium vector can be determined as in the paper.

In this extension of the model in the paper, the general equilibrium features steady-state adding and dropping of products and destinations as well as steady-state entry and exit of firms. Each period a measure of new firms incur the sunk entry cost. Of these new firms, those with a productivity draw above the domestic productivity cutoff (φ_d^*) enter, while those with a productivity draw below φ_d^* exit. Among incumbent firms, a firm with unchanged productivity supplies constant measures of products to the domestic and export markets, but the identity of these products changes as stochastic shocks to consumer tastes occur. As a result of these stochastic shocks a firm with unchanged productivity drops a measure of the products previously supplied to each market and adds an equal measure of the products not previously supplied to each market. Finally, as stochastic shocks to an incumbent firm's productivity occur, the measure of products supplied to each market expands with increases in productivity and contracts with decreases in productivity. An incumbent firm enters export markets when its productivity rises above the exporting cutoff (φ_x^*). An incumbent firm exits endogenously when its productivity falls below φ_d^* or exogenously when death occurs as a result of force majeure considerations beyond the control of the firm.

While the extended model incorporates steady-state adding and dropping of products and destinations, the model's predictions for the cross-section distribution of exports across firms, products and countries remain unchanged. As these cross-sectional predictions are the focus of our analysis, we do not pursue this dynamic extension further in the paper. Finally, although for simplicity we have assumed in this section that productivity and consumer tastes are drawn from the same distributions following a stochastic shock as upon entry, the model can be extended to introduce richer forms of dynamics, such as those considered in the closed economy framework of Bernard, Redding and Schott (2010). Again the cross-sectional predictions of the model that are the focus of our analysis remain unchanged.

5. Multi-Product Firms and Comparative Advantage

In this section of the appendix, we show that the model can be extended to incorporate comparative advantage based on cross-country differences in relative factor abundance and cross-industry differences in factor intensity. In this extension, we embed our model of multi-product firms within the two-factor, two-country and two-industry heterogeneous firm framework of Bernard, Redding and Schott (2007).

The model described thus far can be viewed as capturing a single industry containing many products, with firms supplying differentiated varieties of these products. We now generalize the framework by analyzing two industries, $i = 1, 2$, each of which has this structure. The two industries enter an upper tier of the representative consumer's utility that takes the Cobb-Douglas form:

$$\mathcal{U} = U_1^{\alpha_1} U_2^{\alpha_2}, \qquad \alpha_1 + \alpha_2 = 1, \ \alpha_1 = \alpha, \tag{22}$$

where U_i is an index for industry i that is defined over the consumption C_k of a continuum of products k within the industry (as in equation (1) in the paper), and C_k is itself an index that is defined over the consumption $c_k(\omega)$ of a continuum of varieties ω within each product (as in equation (2) in the paper). "Industries" now constitute an upper tier of utility, "products" form an intermediate tier and "varieties" occupy a lower tier. While not central to the analysis, we assume for simplicity that the two industries have the same elasticity of substitution across products within industries (κ) and across varieties within products (σ).

We also assume for simplicity that there are only two countries: home and foreign. The home country is assumed to be skill-abundant relative to the foreign country and industry 1 is assumed to be skill-intensive relative to industry 2. The combination of industry-level variation in factor intensity and country-level variation in factor abundance gives rise to endowment-driven comparative advantage. The skilled wage in the home country is chosen as the numeraire.

The industries differ in the intensity with which they use two factors of production: skilled and unskilled labor. To enter an industry i, a firm from the competitive fringe must incur the sunk entry cost for that industry, which equals $f_{ei} (w_S)^{\beta_i} (w_L)^{1-\beta_i}$, where w_S denotes the skilled wage, w_L corresponds to the unskilled wage and β_i parameterizes industry factor intensity. After the sunk entry cost is paid, the firm draws its productivity and values of consumer tastes for the industry that it enters as above. The distributions of firm productivity and consumer tastes are identically and independently distributed across industries, so that information about productivity within an industry can only be obtained by incurring the sunk entry cost for that industry. The distributions of firm productivity and consumer tastes are also identically and independently distributed across countries, which ensures consistency with the Heckscher-Ohlin model's assumption of common technologies across countries.

The technology for production has the same factor intensity as for entry.[3] While the factor intensity of production varies across industries, all products within an industry are modelled symmetrically and therefore have the same factor intensity.[4] To manufacture a variety of a product, a firm must incur a fixed and variable cost of production as above. The variable cost depends upon the firm's productivity as in the paper. Fixed and variable costs use the two factors of production with the same proportions, so that the total cost of production for a firm in industry i that serves the domestic market alone is:

$$TC_i = \left(F_{di} + \int_{k \in \Psi_{di}(\varphi_i)} \left[f_{di} + \frac{q_i(\varphi_i, \lambda_{ik})}{\varphi_i} \right] dk \right) (w_S)^{\beta_i} (w_L)^{1-\beta_i}, \quad 1 > \beta_1 > \beta_2 > 0, \tag{23}$$

where $\Psi_{di}(\varphi_i)$ denotes the (endogenous) range of products supplied to the domestic

[3] Allowing factor intensity differences between entry and production introduces additional interactions with comparative advantage as discussed in Bernard, Redding and Schott (2007).

[4] The symmetry of products within industries is clearly a simplification, but is useful for the law of large numbers results that determine the fraction of products supplied by a firm with a given productivity.

market by a firm with productivity φ_i in industry i. The fixed costs of serving an export market and supplying a product to an export market are modelled analogously, and use skilled and unskilled labor with the same factor intensity as production and entry.

We determine general equilibrium using an approach very similar to that used in the paper and therefore omit the reporting of similar equations here to conserve space. The measure of products supplied to the domestic and export markets by a firm with a given productivity can be determined using expressions analogous to those in the paper. Similarly, the zero-profit cutoff and exporting cutoff for productivity can be determined using the same line of reasoning as in the paper. Entry, production and exporting costs all have the same factor intensity, and therefore terms in factor prices cancel from the relevant expressions. There is however an important general equilibrium interaction between comparative advantage and firms' product supply decisions. As countries are no longer symmetric, comparative advantage and trade costs generate cross-country differences in industry price indices. As in Bernard, Redding and Schott (2007), these differences, in turn, generate greater export opportunities in comparative-advantage industries than comparative-disadvantage industries.

The relative price indices for the two industries vary across countries because of the combination of comparative-advantage-based specialization and trade costs. Specialization leads to a larger mass of domestic firms relative to foreign firms in a country's comparative-advantage industry than in its comparative-disadvantage industry. Variable trade costs introduce a wedge between domestic and export prices for a variety. Additionally, the fixed costs of becoming an exporter imply that not all firms export, and the fixed costs for exporting individual products imply that not all the products supplied domestically are exported. Combining specialization and trade costs, the comparative-advantage industry has a greater mass of lower-price domestic varieties relative to the mass of higher-price foreign varieties than the comparative-disadvantage industry. As a result, the price index in the domestic market is lower relative to the price index in the export market in the comparative-advantage industry. Hence, the degree of competition in the domestic market is higher relative to the degree of competition in the export market in the comparative-advantage industry. These differences in the degree of competition in turn imply that variable profits in the export market are greater relative to variable profits in the domestic market in the comparative-advantage industry than in the comparative-disadvantage industry.

Proposition CA: Other things equal, the opening of trade leads to:
(a) greater focusing on core competencies in the comparative-advantage industry than the comparative-disadvantage industry ($\Delta\lambda_{d1}^{*H}(\varphi) > \Delta\lambda_{d2}^{*H}(\varphi)$ and $\Delta\lambda_{d2}^{*F}(\varphi) > \Delta\lambda_{d1}^{*F}(\varphi)$),
(b) a larger increase in the zero-profit cutoff for productivity below which firms exit in the comparative-advantage industry than the comparative-disadvantage industry ($\Delta\varphi_{1d}^{*H} > \Delta\varphi_{d2}^{*H}$ and $\Delta\varphi_{d2}^{*F} > \Delta\varphi_{d1}^{*F}$),
(c) a larger increase in weighted average productivity in the comparative-advantage industry than in the comparative-disadvantage industry ($\Delta\tilde{\varphi}_{d1}^{H} > \Delta\tilde{\varphi}_{d2}^{H}$ and $\Delta\tilde{\varphi}_{d2}^{F} > \Delta\tilde{\varphi}_{d1}^{F}$).

Proof. See the Appendix at the end of this document. ■

Following the opening of trade, countries specialize according to comparative advantage, which leads to a rise in the mass of firms in the comparative-advantage industry relative to the comparative-disadvantage industry. While the comparative-advantage industry expands and the comparative-disadvantage industry contracts, the shedding of low consumer tastes products (i.e., the focusing on "core competencies") is greater in the comparative-advantage industry. The opening of trade therefore leads to greater firm revenue-based productivity growth in the comparative-advantage industry. In addition to the greater focusing on core competencies in the comparative-advantage industry, there is also a larger rise in the zero-profit cutoff for productivity below which firms exit. Hence, the opening of trade causes a greater increase in industry revenue-based productivity in the comparative-advantage sector, because of both stronger firm revenue-based productivity growth and greater across-firm reallocations of resources.

In this extension of the model to incorporate cross-country differences in factor abundance and cross-industry differences in factor intensity, firm responses to trade liberalization vary with comparative advantage. The measured productivity of a firm depends on comparative advantage because it shapes the endogenous range of products that the firm chooses to supply to each market. The greater firm revenue-based productivity growth and across-firm reallocations of resources in the comparative-advantage industry induce Ricardian differences in revenue-based productivity, which are non-neutral across firms and industries. These non-neutral differences in revenue-based productivity magnify Heckscher-Ohlin comparative advantage and provide an additional source of welfare gains from trade.

A Appendix

A1. Proof of Proposition CA

Proof. The introduction of comparative advantage implies that countries are no longer symmetric. Therefore, the relative revenue from a product k in the domestic and export markets in industry i depends on price indices and aggregate revenue in the two countries: $r_{xik}\left(\varphi_i, \lambda_{ik}\right) = \tau_i^{1-\sigma}\left(P_{ik}^F / P_{ik}^H\right)^{\sigma-1}\left(R^F / R^H\right) r_{dik}\left(\varphi_i, \lambda_{ik}\right)$. In equilibrium, the price indices are the same for all products within an industry and country due to symmetry: $P_{ik}^F = P_i^F$ and $P_{ik}^H = P_i^H$ for all k.

The equations determining the equilibrium range of products supplied to the domestic market as a function of the zero-profit-cutoff firm productivity φ_d^* (equations (9) and (14) in the paper) and the equations determining the equilibrium range of products exported as a function of the exporting cutoff firm productivity φ_x^* (equations (11) and (16) in the paper) remain exactly the same as in the model with a single industry. These relationships compare the relative revenue of varieties of products within a given market within a given industry, and are therefore unchanged by the introduction of country asymmetries. The additional terms in factor prices due to the introduction of skilled and unskilled labor cancel from the left and right-hand sides of equations (14) and (16) in the paper.

The free entry condition also takes the same form as in the model with a single industry (equation (19) in the paper), since average profits in the domestic and export market are evaluated separately relative to the lowest consumer tastes product supplied by a firm in each market, and terms in factor prices again cancel from the left and right-hand sides of the equation.

However, the introduction of country asymmetries changes the relationship between the exporting cutoff productivity φ_{xi}^* and the zero-profit-cutoff productivity φ_{di}^*, which instead of equation (17) in the paper is given by the following expression for the home country:

$$\varphi_{xi}^{*H} = \Gamma_i^H \varphi_{di}^{*H}, \qquad \Gamma_i^H \equiv \tau_i \left(\frac{f_{xi}}{f_{di}} \frac{R^H}{R^F} \right)^{\frac{1}{\sigma-1}} \left(\frac{P_i^H}{P_i^F} \right) \left(\frac{\lambda_{di}^* \left(\varphi_{di}^*\right)}{\lambda_{xi}^* \left(\varphi_{xi}^*\right)} \right), \tag{24}$$

where an analogous expression holds for the foreign country. Since $\lambda_{xi}^* \left(\varphi_i\right) = \left(\varphi_{xi}^*/\varphi_i\right) \lambda_{xi}^* \left(\varphi_{xi}^*\right)$, the change in the relationship between φ_{xi}^* and φ_{di}^* affects the exporting cutoffs for consumer tastes for each firm productivity $\lambda_{xi}^* \left(\varphi_i\right)$ and hence average profits from the export market in the free entry condition (equation (19) in the paper).

Comparing the free entry conditions in the open and closed economies, the expected value of entry in the open economy is equal to the value for the closed economy plus an additional positive term which captures the expected profits from the export market. Since $\lambda_{xi}^* \left(\varphi_i\right) = \left(\varphi_{xi}^*/\varphi_i\right) \lambda_{xi}^* \left(\varphi_{xi}^*\right)$ and $\varphi_{xi}^* = \Gamma_i \varphi_{di}^*$, this additional positive term is larger, the smaller the value of Γ_i. Dividing equation (24) for the two industries:

$$\frac{\varphi_{x1}^{*H}/\varphi_{d1}^{*H}}{\varphi_{x2}^{*H}/\varphi_{d2}^{*H}} = \frac{\Gamma_1^H}{\Gamma_2^H} = \frac{\tau_1}{\tau_2} \left(\frac{f_{x1}/f_{d1}}{f_{x2}/f_{d2}} \right)^{\frac{1}{\sigma-1}} \left(\frac{\lambda_{d1}^* \left(\varphi_{d1}^*\right)/\lambda_{x1}^* \left(\varphi_{x1}^*\right)}{\lambda_{d2}^* \left(\varphi_{d2}^*\right)/\lambda_{x2}^* \left(\varphi_{x2}^*\right)} \right) \left(\frac{P_1^H/P_2^H}{P_1^F/P_2^F} \right)$$

The remainder of the proof follows the same structure as the proof of Proposition 4 in Bernard, Redding and Schott (2007) and we present an abbreviated version here. The price index for the skill-intensive industry relative to the labor-intensive industry is lower in the skill-abundant country than the labor-abundant country: $P_1^H/P_2^H < P_1^F/P_2^F$. Therefore, in the absence of other differences in parameters across industries except factor intensity (common values of τ_i, F_d, f_d, F_x, f_x and f_e across industries): $\Gamma_1^H < \Gamma_2^H$ and similarly $\Gamma_2^F < \Gamma_1^F$. Hence, the additional positive term in the free entry condition capturing expected profits in the export market is larger in the comparative-advantage industry than the comparative-disadvantage industry.

Noting that $\lambda_{di}^{*}(\varphi) = (\varphi_{di}^{*}/\varphi_{i})\,\lambda_{di}^{*}(\varphi_{di}^{*})$, $\lambda_{xi}^{*}(\varphi) = (\varphi_{xi}^{*}/\varphi_{i})\,\lambda_{xi}^{*}(\varphi_{xi}^{*})$, and $\varphi_{xi}^{*H} = \Gamma_{i}^{H}\varphi_{di}^{*H}$, the expected value of entry in the free entry condition (19) in the paper is monotonically decreasing in φ_{di}^{*}. Therefore, the comparative-advantage industry's larger increase in the expected value of entry following the opening of trade requires a larger rise in the zero-profit cutoff for firm productivity φ_{di}^{*} in order to restore equality between the expected value of entry and the unchanged sunk entry cost: $\Delta\varphi_{d1}^{*H} > \Delta\varphi_{d2}^{*H}$ and $\Delta\varphi_{d2}^{*F} > \Delta\varphi_{d1}^{*F}$.

Since $\lambda_{di}^{*}(\varphi) = (\varphi_{di}^{*}/\varphi_{i})\,\lambda_{di}^{*}(\varphi_{di}^{*})$ and $\lambda_{di}^{*}(\varphi_{di}^{*})$ is unchanged by the opening of trade, the comparative-advantage industry's larger rise in the zero-profit cutoff for firm productivity φ_{di}^{*} implies a greater focusing on core competencies in the comparative-advantage industry than in the comparative-disadvantage industry: $\Delta\lambda_{d1}^{*H}(\varphi) > \Delta\lambda_{d2}^{*H}(\varphi)$ and $\Delta\lambda_{d2}^{*F}(\varphi) > \Delta\lambda_{d1}^{*F}(\varphi)$.

From the definition of weighted average productivity in Section B1 of the Appendix in the paper, the comparative-advantage industry's larger rise in the zero-profit cutoff for productivity and greater focusing on core competencies implies a larger increase in weighted average productivity in the comparative-advantage industry than in the comparative-disadvantage industry: $\Delta\tilde{\varphi}_{d1}^{H} > \Delta\tilde{\varphi}_{d2}^{H}$ and $\Delta\tilde{\varphi}_{d2}^{F} > \Delta\tilde{\varphi}_{d1}^{F}$. ■

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