Goods Trade, Factor Mobility and Welfare*

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Abstract

This paper extends a recent class of quantitative models of international trade to incorporate factor mobility within countries. We present a model-based decomposition of the variance of economic activity into the contributions of locational fundamentals, market access and their covariance. We show how the standard framework for undertaking model-based counterfactuals in trade can be augmented to obtain predictions for endogenous changes in the distribution of economic activity across regions within countries. A region's trade share with itself is no longer a sufficient statistic for the welfare gains from trade, which also depend on endogenous changes in the distribution of mobile factors.

KEYWORDS: international trade, factor mobility, welfare gains from trade

JEL CLASSIFICATION: F11, F12, F16

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1 Introduction

This paper extends a recent class of quantitative models of international trade to incorporate factor mobility across regions within countries. We show that the standard conditions for general equilibrium in these models are augmented by an additional relationship that determines the distribution of economic activity within countries. Using the structure of the model and observed data on mobile factor supplies and trade shares, we show that the dispersion of mobile factors can be decomposed into the contributions of locational fundamentals, market access and their covariance. We show how the model's system of equations for general equilibrium can be used to undertake counterfactuals for the impact of changes in trade costs or other comparative statics. A key feature of these counterfactuals is that external trade liberalization leads to endogenous changes in the internal organization of economic activity across regions within countries. We show that these endogenous internal reallocations have important implications for the welfare gains from trade. A location's trade share with itself is no longer a sufficient statistic for the welfare gains from trade, which also depend on endogenous changes in the distribution of mobile factors.

Our analysis proceeds as follows. We begin by developing our results in a simple neoclassical framework following Eaton and Kortum (2002), in which an economy of many regions is linked through both goods trade and factor mobility. Our results are, however, more general and hold within a wider class of models that has many features in common with the class of models considered by Arkolakis et al. (2012). These features include Dixit-Stiglitz preferences, perfect or monopolistic competition, balanced trade, aggregate profits that are a constant share of aggregate revenues (possibly zero), and CES import demand. The key difference in our analysis is the presence of both mobile and immobile factors of production, which introduces reallocations of mobile factors as a new channel through which international trade in goods can affect welfare. This new channel influences the overall magnitude of the welfare gains from trade, which can be no longer inferred from the change in a location's trade share with itself. To illustrate the broader applicability of our results, we show that they also hold within the Helpman (1998) model of new economic geography, which introduces agglomeration forces through the combination of love of variety preferences, transport costs and increasing returns to scale.

To develop our argument as clearly as possible, we initially consider settings in which there is a single immobile factor (land) that enters consumption (residential land use), a single mobile factor of production (labor), and all regions are linked through goods trade and factor mobility. We next generalize our analysis to allow the immobile factor to enter production (commercial land use), to introduce intermediate inputs, and to consider an economy of multiple countries in which factors are mobile across regions within countries but immobile between countries. In this setting, external trade liberalization between countries induces internal reallocations of mobile factors within countries that affect trade and welfare. We use this augmented framework as the basis for an empirical analysis of trade between Canadian provinces and U.S. states. Using data from Anderson and Van Wincoop

(2003), we show that allowing for factor mobility within countries leads to quite different estimates of the welfare effects of changes in barriers to goods trade.

Our paper is most closely related to recent quantitative models of international trade following Eaton and Kortum (2002), including Alvarez and Lucas (2007), Arkolakis et al. (2012), Caliendo and Parro (2011), Costinot et al. (2011), Donaldson (2011), Eaton et al. (2011a, 2011b), Fieler (2011), Hsieh and Ossa (2011), Ossa (2011) and Simonovska and Waugh (2011). In contrast to these studies, we introduce factor mobility across regions, and show how it influences the predictions of these models for the general equilibrium effects of goods trade on wages, prices and welfare. We show how the approach to undertaking model-based counterfactuals pioneered by Dekle, Eaton and Kortum (2005) can be generalized to incorporate this factor mobility and how this extended approach yields additional predictions for changes in the distribution of economic activity within countries.

Our research is also related to the economic geography literature following Krugman (1991a,b), including in particular research based on the Helpman (1998) model such as Michaels et al. (2012), Hanson (2005) and Redding and Sturm (2011).[1] While the theoretical literature on economic geography has uncovered the mechanisms underlying the agglomeration of economic activity, there has been little research developing quantitative versions of these models, partly because of their complexity which often restricts the analysis to a small number of symmetric regions. Our key contribution relative to this literature is to develop a tractable quantitative model of economic geography that incorporates goods trade and factor mobility between a large number of asymmetric regions connected by a rich geography of trade costs. This framework permits a counterfactual analysis of the impact of international trade between countries on the distribution of economic activity within countries and yields simple sufficient statistics for the effects of trade on welfare.

Finally, our focus on factor mobility builds on a wider literature in international trade that has examined the extent to which goods and factor movements are complements or substitutes, as in Markusen (1983), Mundell (1957) and Jones (1967), as well as work that has considered the role of the geography within countries in influencing international trade, such as Courant and Deardorff (1992, 1993) and Rauch (1991). We introduce factor mobility into a recent class of quantitative models of international trade and examine its implications for model-based counterfactuals and the welfare gains from trade.

The remainder of the paper is structured as follows. Section 2 introduces the baseline model that starkly illustrates the role of factor mobility. Section 3 shows that these results hold in a wider class of models, including those featuring endogenous agglomeration forces. Section 4 considers a more general setting with factor mobility within but not across countries. Section 5 illustrates the quantitative relevance of factor mobility for the welfare effects of changes in trade costs using data on U.S. states and Canadian Provinces. Section 6 concludes.

[1]See also Davis and Weinstein (2002), Desmet and Rossi-Hansberg (2011), Fujita et al. (1999), Hanson (1996, 1997), Head and Ries (2001), Redding and Venables (2004), and Rossi-Hansberg (2005).

2 Theoretical Framework

We consider an economy consisting of a set N of regions indexed by n. Each region is endowed with an exogenous quality-adjusted supply of land (H_i). The economy as a whole is endowed with a measure $\bar{L}$ of workers, where each worker has one unit of labor that is supplied inelastically with zero disutility. Workers are perfectly mobile across regions and hence in equilibrium real wages are equalized across all populated regions. Regions are connected by a bilateral transport network that can be used to ship goods such that $d_{ni} \geq 1$ units must be shipped from region i in order for one unit to arrive in region n.[2]

2.1 Consumer Preferences

Preferences are defined over goods consumption (C_n) and residential land use (H_{Un}) and are assumed to take the Cobb-Douglas form:[3]

$$U_n = C_n^{\alpha} H_{Un}^{1-\alpha}, \qquad 0 < \alpha < 1. \tag{1}$$

The goods consumption index is defined over consumption of a fixed continuum of goods $j \in [0, 1]$:

$$C_n = \left[\int_0^1 c_{nj}^{\rho} dj \right]^{1/\rho}, \qquad \sigma = \frac{1}{1-\rho}. \tag{2}$$

2.2 Production

Each location draws an idiosyncratic productivity z_j for each good j. Productivity is independently drawn across goods and locations from a Fréchet distribution:

$$F_i(z) = e^{-T_i z^{-\theta}}, \tag{3}$$

where the scale parameter T_i determines average productivity for location i and the shape parameter θ controls the dispersion of productivity across goods.

Goods are homogeneous in the sense one unit of a given good is the same as any other unit of that good. Each good is produced with labor under conditions of perfect competition according to a linear technology.[4] The cost to a consumer in location n of purchasing one unit of good j from location i is therefore:

$$p_{ni}(j) = \frac{d_{ni} w_i}{z_i(j)}, \tag{4}$$

where w_i denotes the wage in location i.

[2]Since our analysis uses patterns of bilateral trade to reveal information about unobserved bilateral trade frictions, we can allow for general variation in $d_{ni} \geq 1$ between pairs of locations n and i and are not required to impose $d_{ii} = 1$.

[3]For empirical evidence using U.S. data in support of the constant housing expenditure share implied by the Cobb-Douglas functional form, see Davis and Ortalo-Magne (2011).

[4]While to simplify the exposition we begin by assuming that land is only used residentially, this assumption is straightforward to relax, as shown in Section 4 below.

2.3 Expenditure Shares and Price Indices

The representative consumer in location n sources each good from the lowest-cost supplier to that location:

$$p_n(j) = \min\{p_i(j); i \in N\}. \tag{5}$$

Using equilibrium prices (4) and the properties of the Fréchet distribution following Eaton and Kortum (2002), the share of expenditure of region n on goods produced by region i is:

$$\pi_{ni} = \frac{T_i\left(d_{ni}w_i\right)^{-\theta}}{\sum_{s\in N} T_s\left(d_{ns}w_s\right)^{-\theta}}, \tag{6}$$

while the price index dual to (2) can be expressed as:

$$P_n = \gamma\left[\sum_{i\in N} T_i\left(d_{ni}w_i\right)^{-\theta}\right]^{-1/\theta}, \tag{7}$$

where $\gamma = \left[\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)\right]^{\frac{1}{1-\sigma}}$ and $\Gamma(\cdot)$ denotes the Gamma function.

Using the expenditure share (6), the price index (7) can be also written as:

$$P_n = \gamma\left(\frac{T_n}{\pi_{nn}}\right)^{-\frac{1}{\theta}} w_n, \tag{8}$$

so that locations with higher wages and higher trade shares with themselves have higher consumer goods price indices.

2.4 Income and Population Mobility

Expenditure on land in each location is redistributed lump sum to the workers residing in that location, as in Helpman (1998). Therefore total income in each location (v_n) equals labor income plus expenditure on residential land:

$$v_nL_n = w_nL_n + (1-\alpha)v_nL_n = \frac{w_nL_n}{\alpha}. \tag{9}$$

Labor income in each location equals expenditure on goods produced in that location:

$$w_iL_i = \sum_{n\in N} \pi_{ni}w_nL_n. \tag{10}$$

Land market clearing implies that the equilibrium land rent can be determined from the equality of land income and expenditure:

$$r_n = \frac{(1-\alpha)v_nL_n}{H_n} = \frac{1-\alpha}{\alpha}\frac{w_nL_n}{H_n}. \tag{11}$$

Population mobility implies that workers receive the same real income in all populated locations:

$$V_n = \frac{v_n}{P_n^{\alpha}r_n^{1-\alpha}} = \bar{V}, \tag{12}$$

where r_n denotes the land rent in location n and we have absorbed the constants $\alpha^{-\alpha}$ and $(1-\alpha)^{-(1-\alpha)}$ into the definition of $\bar{V}$.

Using land market clearing (11), the equality of income and expenditure (9) and the price index (8), the above population mobility condition can be used to solve for the equilibrium population of each location as a function of locational fundamentals (productivity (T_n) and land quality (H_n)), the location's trade share with itself (π_{nn}) and the common level of real income ($\bar{V}$):

$$L_n = \frac{\left(\frac{T_n}{\pi_{nn}}\right)^{\frac{\alpha}{\theta(1-\alpha)}} H_n}{\alpha^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha}\right) \gamma^{\frac{\alpha}{1-\alpha}} \bar{V}^{\frac{1}{1-\alpha}}}, \tag{13}$$

where labor market clearing requires:

$$\sum_{n \in N} L_n = \bar{L}. \tag{14}$$

The expression for equilibrium population (13) has an intuitive interpretation. Locations with higher productivity (T_n) pay higher wages (w_n), which attracts population and bids up the price of land until real wages are equalized. Locations with higher land quality (H_n) have lower quality-adjusted prices of land, which again attracts population and bids up the price of land until real wages are equalized. Finally, locations that have low equilibrium trade shares with themselves (π_{nn}) for given values of productivity (T_n) and land quality (H_n) have low trade costs to other locations (high market access). These low trade costs to other locations imply low price indices for tradeable goods (P_n), which must be offset in equilibrium by a higher population that bids up the price of land to achieve real wage equalization. Note that locations with high productivity (T_n) have high trade shares with themselves (π_{nn}), other things equal, both because higher productivity directly increases a location's trade share with itself in (6), and also because higher productivity increases a location's population, which reduces its wage relative to a weighted average of wages in other locations in (10).

2.5 General Equilibrium

The general equilibrium of the model can be represented by the share of workers in each location ($\lambda_n = L_n/\bar{L}$), the share of each location's expenditure on goods produced in other locations (π_{ni}) and the wage in each location (w_n). Using labor income (10), the trade share (6), population mobility (13) and labor market clearing (14), this equilibrium triple $\{\lambda_n, \pi_{ni}, w_n\}$ solves the following system of equations for all $i, n \in N$:

$$w_i \lambda_i = \sum_{n \in N} \pi_{ni} w_n \lambda_n, \tag{15}$$

$$\pi_{ni} = \frac{T_i \left(d_{ni} w_i\right)^{-\theta}}{\sum_{k \in N} T_k \left(d_{nk} w_k\right)^{-\theta}}, \tag{16}$$

$$\lambda_n = \frac{\left(\frac{T_n}{\pi_{nn}}\right)^{\frac{\alpha}{\theta(1-\alpha)}} H_n}{\sum_{k \in N} \left(\frac{T_k}{\pi_{kk}}\right)^{\frac{\alpha}{\theta(1-\alpha)}} H_k}. \tag{17}$$

Proposition 1 *Given locational fundamentals (productivity (T_n), land quality (H_n) and bilateral trade frictions (d_{ni})), there exist equilibrium population shares (λ_n), trade shares (π_{ni}) and wages (w_n) that solve the wage equation (15), trade equation (16) and population mobility condition (17).*

Proof. See the appendix. ■

The proposition establishes the existence of equilibrium as follows. First, for given population shares (λ_i), the wage equation (15) and trade equation (16) determine equilibrium wages (w_i) and trade shares (π_{ni}) for each location. Second, the existence of equilibrium population shares can be established by using the population mobility condition (17) together with the wage equation (15) and trade equation (16), as shown formally in the proof of the proposition.

In Figure 1, we illustrate the determination of equilibrium graphically for the special case of two regions $\{A, B\}$, which is also considered more formally in the appendix. In this case, $\lambda_B = 1 - \lambda_A$ and we choose the wage in location A as the numeraire. The figure shows the left and right-hand sides of the population mobility condition (17) on the y-axis against population share (λ_A) on the x-axis. The left-hand side of (17) for region A is shown by the ray from the origin with slope 45 degrees, while the right-hand side corresponds to the downward-sloping curve. As location A's population share converges towards zero ($\lim_{\lambda_A \to 0}$), the wage equation (15) and trade equation (16) imply that its trade share with itself converges towards zero ($\pi_{AA} \to 0$), which in turn implies that the right-hand side of the population mobility condition (17) converges towards infinity. Conversely, as location A's population share converges towards one ($\lim_{\lambda_A \to 1}$), the wage equation (15) and trade equation (16) imply that the trade share of location B with itself converges towards zero ($\pi_{BB} \to 0$), which in turn implies that the right-hand side of the population mobility condition (17) for region A converges towards zero. Finally, the wage equation (15), trade equation (16) and population mobility condition (17) are continuous in λ_A, and hence there exist equilibrium population shares $\{\hat{\lambda}_A, 1 - \hat{\lambda}_A\}$. Having established the existence of these equilibrium population shares, equilibrium wages $\{1, \hat{w}_B\}$ and trade shares $\{\hat{\pi}_{AA}, \hat{\pi}_{AB}, \hat{\pi}_{BB}, \hat{\pi}_{BA}\}$ can be determined from the wage equation (15) and trade equation (16).

Proposition 2 *Given observed data on population (L_n) and trade shares (π_{ni}), the population mobility condition (13) can be used to determine a composite measure of fundamentals for each location that incorporates productivity and land quality ($H_n T_n^{\alpha/\theta(1-\alpha)}$). Using this measure, the variance of population across locations can be decomposed into the contributions of fundamentals ($H_n T_n^{\alpha/\theta(1-\alpha)}$), market access ($\pi_{nn}$) and their covariance.*

Proof. The proposition follows immediately from the population mobility condition (13) given observed data on population (L_n) and trade shares (π_{ni}):

$$\begin{aligned}\operatorname{var}\{\log L_n\} = {} & \operatorname{var}\left\{\log\left(T_n^{\alpha/\theta(1-\alpha)} H_n\right)\right\} + \left(\tfrac{\alpha}{\theta(1-\alpha)}\right)^2 \operatorname{var}\{\log \pi_{nn}\} \\ & -2\left(\tfrac{\alpha}{\theta(1-\alpha)}\right) \operatorname{covar}\left\{\log\left(T_n^{\alpha/\theta(1-\alpha)} H_n\right), \log\left(\pi_{nn}\right)\right\}.\end{aligned}$$

■

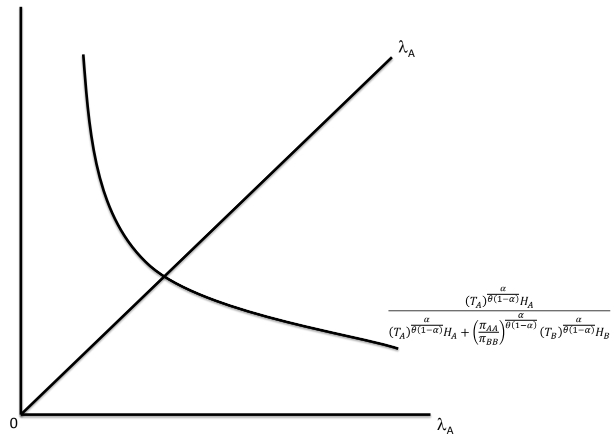

Figure 1: Equilibrium Population Shares $\{\lambda_A, 1 - \lambda_A\}$ for the Case of Two Regions

As summarized in the above proposition, the model yields a decomposition of the variance of population across locations into the contributions of locational fundamentals, market access and their covariance. Locational fundamentals are captured by a composite measure that incorporates productivity (T_n) and land quality (H_n). Market access is summarized by a location's trade share with itself (π_{nn}), where a lower own trade share implies lower relative trade costs to other locations (greater market access). One key implication of the population mobility condition (13) is that a composite measure of unobserved locational fundamentals ($H_n T_n^{\alpha/\theta(1-\alpha)}$) can be obtained from the model using only data on population (L_n) and trade shares (π_{nn}) and given a normalization of the common level of utility $\bar{V}$. To measure these locational fundamentals, we are not required to make assumptions about the form of bilateral trade costs (d_{ni}), which are instead implicitly revealed by observed bilateral trade flows, as reflected in each location's trade share with itself (π_{nn}).

2.6 Welfare Gains from Trade

Having solved for the equilibrium triple $\{\lambda_n, \pi_{ni}, w_n\}$, the price index (7) follows immediately from equilibrium wages. Using population mobility (13), welfare in each location can be expressed as:

$$V_n = \frac{\left(\frac{T_n}{\pi_{nn}}\right)^{\frac{\alpha}{\theta}} H_n^{1-\alpha}}{\alpha \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} \gamma^\alpha L_n^{1-\alpha}} = \bar{V}, \tag{18}$$

where terms in wages (w_n) have canceled and land quality (H_n) is a parameter.

Proposition 3 *The change in a location's trade share with itself and the change in its population are sufficient statistics for the welfare gains from goods trade.*

Proof. The proposition follows immediately from (18) since:

$$\frac{V_n^{\text{Trade}}}{V_n^{\text{No Trade}}} = \left(\pi_{nn}^{\text{Trade}}\right)^{-\frac{\alpha}{\theta}} \left(\frac{L_n^{\text{Trade}}}{L_n^{\text{No Trade}}}\right)^{-(1-\alpha)} = \frac{\bar{V}^{\text{Trade}}}{\bar{V}^{\text{No Trade}}},$$

where $\pi_{nn}^{\text{No Trade}} = 1$. ■

In Proposition 3, we consider a reduction in barriers to trade in goods between locations that are linked through factor mobility. As a result of this factor mobility, the welfare gains from trade cannot be inferred simply from the change in each location's trade share with itself, but also depend on the change in each location's population. The reason is that a larger population raises the demand for land, which in turn increases the price of land and hence reduces welfare. Indeed, the model implies that the changes in population and trade shares induced by goods market integration are systematically related to one another. Suppose that we start from an equilibrium in which population mobility has equalized real wages and that there is a reduction in barriers to trade in goods that is uneven across locations (e.g. coastal locations with lower geographic barriers to trade benefit disproportionately from the opening of closed economies to trade). Those locations that experience larger reductions in their trade share with themselves as a result of this goods market integration become more attractive locations, and hence experience larger increases in their population until real wages are again equalized in the new equilibrium following the reduction in trade barriers.

Our framework satisfies many of the conditions specified in Arkolakis et al. (2012) for a location's trade share with itself to be a sufficient statistic for the welfare gains from trade. In particular, our framework features Dixit-Stiglitz preferences, linear cost functions, perfect or monopolistic competition, balanced trade, aggregate profits are a constant share of aggregate revenues (zero), and CES import demand. The one condition violated is a single factor of production. Indeed, it is the combination of a mobile factor (labor) and immobile factor (land) that is central to our result in Proposition 3. To the extent that some factors of production are mobile across locations, our analysis highlights an interaction between trade in goods and factor allocations. This interaction will also prove important below when we consider the effects of goods market integration in a setting where factors are mobile across regions within countries but immobile across countries.

While the combination of mobile and immobile factors opens up a new channel through which goods market integration affects welfare, the presence of an immobile factor also mechanically dampens the overall magnitude of the welfare gains from trade in goods, because these welfare gains are only realized for tradable consumption goods. As a result, the exponent on a location's trade share with itself (π_{nn}) in welfare (18) is the share of tradeable goods in consumption (α) divided by the elasticity of trade flows with respect to trade costs (θ).

2.7 Counterfactuals

As in the recent class of quantitative trade theories, the system of equations for general equilibrium (15)-(17) can be used to undertake model-based counterfactuals following the methodology introduced by Dekle, Eaton and Kortum (2005). In contrast to these recent quantitative trade theories, our framework yields predictions for endogenous changes in population across locations, which in turn feedback to influence wages, price indices and trade patterns between regions.

The system of equations for general equilibrium (15)-(17) must hold both before and after any counterfactual change in for example trade frictions. We denote the value of variables in the counterfactual equilibrium with a prime (x') and the relative value of variables in the counterfactual and initial equilibria by a tilde ($\tilde{x} = x'/x$). Using this notation, the system of equations for the counterfactual equilibrium (15)-(17) can be re-written as follows:

$$\tilde{w}_i \tilde{\lambda}_i Y_i = \sum_{n \in N} \pi'_{ni} \tilde{w}_n \tilde{\lambda}_n Y_n, \tag{19}$$

$$\pi'_{ni} = \frac{\pi_{ni} \left(\tilde{d}_{ni} \tilde{w}_i \right)^{-\theta}}{\sum_{k \in N} \pi_{nk} \left(\tilde{d}_{nk} \tilde{w}_k \right)^{-\theta}}, \tag{20}$$

$$\tilde{\lambda}_n = \frac{\lambda_n \tilde{\pi}_{nn}^{-\frac{\alpha}{\theta(1-\alpha)}}}{\sum_{k \in N} \lambda_k \tilde{\pi}_{kk}^{-\frac{\alpha}{\theta(1-\alpha)}}}, \tag{21}$$

where $Y_i = w_i L_i$ denotes labor income in the initial equilibrium.

This system of equations (19)-(21) can be solved for the counterfactual changes in wages ($\tilde{w}_n$), population shares ($\tilde{\lambda}_n$) and trade shares ($\tilde{\pi}_{ni}$) as a function of the exogenous change in trade costs $\tilde{d}_{ni}$ and the values of observed variables in the initial equilibrium $\{Y_n, \pi_{ni}, \lambda_n\}$ for all locations $i, n \in N$. As discussed in the previous subsection, the change in each location's population share and its trade share with itself provide sufficient statistics for the change in its welfare between the initial and counterfactual equilibrium.

3 New Economic Geography

In our baseline model, the distribution of population across locations reflects a tension between differences in productivity (which concentrate population in high productivity locations), transport costs (which favor locations with good access to other locations) and an inelastic supply of land (which tends to disperse population). In this section, we show how our analysis can incorporate an endogenous agglomeration force in the form of pecuniary externalities from consumer love of variety, increasing returns to scale and transport costs, as in the new economic geography literature following Krugman (1991a,b). Specifically, we consider the Helpman (1998) model, which combines these endogenous agglomeration forces with a congestion force in the form of an inelastic supply of land. We show that a multi-region version of this model yields a system of equations for general equilibrium that takes exactly the same form as in our baseline model in the previous section. For parameter values for which there is a non-degenerate distribution of population across locations, this system of equations can be used to undertake model-based decompositions and counterfactuals in exactly the same way as in the previous section.

3.1 Consumer Preferences

Preferences are again defined over goods consumption (C_n) and residential land use (H_{Un}) and take the same form as in (1). The goods consumption index (C_n), however, is now defined over the endogenous measures of horizontally differentiated varieties supplied by each region (M_i):

$$C_n = \left[\sum_{i \in N} \int_0^{M_i} c_{ni}\left(j\right)^{\rho} dj \right]^{\frac{1}{\rho}}, \tag{22}$$

where trade between regions i and n is again subject to iceberg variable trade costs of $d_{ni} \geq 1$.

3.2 Production

Varieties are produced under conditions of monopolistic competition. To produce a variety, a firm must incur a fixed cost of F units of labor and a constant variable cost in terms of labor that we normalize to one. Therefore the total amount of labor ($l_n(j)$) required to produce $x_n(j)$ units of a variety j in country n is:

$$l_n(j) = F + x_n(j). \tag{23}$$

Profit maximization and zero profits imply that equilibrium prices are a constant mark-up over marginal cost:

$$p_{ni}(j) = \left(\frac{\sigma}{\sigma - 1} \right) d_{ni} w_i, \tag{24}$$

and equilibrium output of each variety is equal to a constant:

$$x_n(j) = \bar{x} = F\left(\sigma - 1\right). \tag{25}$$

Given constant equilibrium output of each variety, labor market clearing implies that the total measure of varieties supplied by each location is proportional to the endogenous supply of workers choosing to locate there:

$$M_i = \frac{L_i}{\sigma F}. \tag{26}$$

3.3 Price Indices and Expenditure Shares

Using equilibrium prices (24) and labor market clearing (26), the price index dual to the consumption index (22) can be expressed as:

$$P_n = \frac{\sigma}{\sigma - 1} \left(\frac{1}{\sigma F} \right)^{\frac{1}{1-\sigma}} \left[\sum_{i \in N} L_i \left(d_{ni} w_i \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{27}$$

Using the CES expenditure function, equilibrium prices (24) and labor market clearing (26), the share of location n's expenditure on goods produced in location i is:

$$\pi_{ni} = \frac{M_i p_{ni}^{1-\sigma}}{\sum_{k \in N} M_k p_{nk}^{1-\sigma}} = \frac{L_i \left(d_{ni} w_i \right)^{1-\sigma}}{\sum_{k \in N} L_k \left(d_{nk} w_k \right)^{1-\sigma}}. \tag{28}$$

Together (27) and (28) imply that each location's price index can be again written in terms of its trade share with itself:

$$P_n = \frac{\sigma}{\sigma - 1} \left(\frac{L_n}{\sigma F \pi_{nn}} \right)^{\frac{1}{1-\sigma}} w_n. \tag{29}$$

3.4 Population Mobility and Welfare

As in the previous section, expenditure on land in each location is redistributed lump sum to the workers residing in that location, so that total income is proportional to labor income in (9). Population mobility again requires that workers receive the same real income in all populated locations as in (12). Using these two relationships together with land market clearing (11), welfare in each location can be expressed as follows:

$$V_n = \frac{\left(\frac{1}{\sigma F \pi_{nn}}\right)^{\frac{\alpha}{\sigma-1}} L_n^{-\left(\frac{\sigma(1-\alpha)-1}{\sigma-1}\right)} H_n^{1-\alpha}}{\alpha\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}\left(\frac{\sigma}{\sigma-1}\right)^{\alpha}} = \bar{V}, \tag{30}$$

where terms in wages (w_n) have again canceled and land quality (H_n) is a parameter.

Rearranging this population mobility condition, equilibrium population in each location can be expressed as a function of fundamentals (H_n), the location's trade share with itself (π_{nn}) and the common level of real income ($\bar{V}$) across locations:

$$L_n = \left[\frac{\left(\frac{1}{\sigma F \pi_{nn}}\right)^{\frac{\alpha}{\sigma-1}} H_n^{1-\alpha}}{\alpha\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}\left(\frac{\sigma}{\sigma-1}\right)^{\alpha}\bar{V}}\right]^{\frac{\sigma-1}{\sigma(1-\alpha)-1}}. \tag{31}$$

A key difference between this model and the framework in the previous section is the presence of an endogenous agglomeration force as a result of the combination of love of variety, increasing returns to scale and transport costs. If this agglomeration force is sufficiently strong relative to the congestion force from an inelastic supply of land ($0 < \sigma(1-\alpha) < 1$), each location's welfare (30) is increasing in its population for a given trade share with itself (π_{nn}), which generates the possibility of multiple equilibria and a degenerate population distribution with all population concentrated in a single region. Since we do not observe a degenerate population distribution in the data, we assume that the agglomeration force is sufficiently weak relative to the congestion force from an inelastic supply of land ($\sigma(1-\alpha) > 1$), which ensures the existence of a stable equilibrium with a dispersed population distribution, as shown formally in the next subsection.

3.5 General Equilibrium

The general equilibrium of the model can be again represented by the share of workers in each location ($\lambda_n = L_n/\bar{L}$), the share of each location's expenditure on goods produced by other locations (π_{ni}) and the wage in each location (w_n). Using labor income (10), the trade share (28), population mobility (31) and labor market clearing (14), the equilibrium triple $\{\lambda_n, \pi_{ni}, w_n\}$ solves the following system of equations for all $i, n \in N$:

$$w_i \lambda_i = \sum_{n \in N} \pi_{ni} w_n L_n, \tag{32}$$

$$\pi_{ni} = \frac{\lambda_n \left(d_{ni} w_i\right)^{1-\sigma}}{\sum_{k \in N} \lambda_k \left(d_{nk} w_k\right)^{1-\sigma}}, \tag{33}$$

$$\lambda_n = \frac{\left[H_n^{1-\alpha}\left(\frac{1}{\pi_{nn}}\right)^{\frac{\alpha}{\sigma-1}}\right]^{\frac{\sigma-1}{\sigma(1-\alpha)-1}}}{\sum_{k\in N}\left[H_k^{1-\alpha}\left(\frac{1}{\pi_{kk}}\right)^{\frac{\alpha}{\sigma-1}}\right]^{\frac{\sigma-1}{\sigma(1-\alpha)-1}}}. \tag{34}$$

Proposition 4 *Assuming $\sigma(1-\alpha) > 1$ and given locational fundamentals (land quality (H_n) and bilateral trade frictions (d_{ni})), there exist equilibrium population shares (λ_n), trade shares (π_{ni}) and wages (w_n) that solve the wage equation (32), trade equation (33) and population mobility condition (34).*

Proof. See the appendix. ■

While the above system of equations for general equilibrium takes the same form as in our baseline model in Section 2, relative population shares (λ_n) now enter the trade equation (33). The reason is consumer love of variety. Relative population shares determine the relative measures of varieties supplied by each location, and since all varieties are consumed, the relative measures of varieties supplied by each location influence trade shares.[5]

Since for $\sigma(1-\alpha) > 1$ there exists a non-degenerate stable distribution of economic activity, the system of equations for general equilibrium (32)-(34) can be used to undertake model-based counterfactuals of the same form as in equations (19)-(21) in the previous section. Again changes in barriers to trade in goods result in endogenous changes in the distribution of population across locations, which now not only affect trade shares (33) indirectly through wages (w_n) but also directly through changes in the measures of varieties supplied by each location (captured by λ_n).

Given data on only population (L_n) and trade shares (π_{nn}), the population mobility condition (31) can be again used to measure unobserved locational fundamentals, which permits a model-based decomposition of the variance of population across locations into the contributions of locational fundamentals, market access and their covariance.

Proposition 5 *Assuming $\sigma(1-\alpha) > 1$ and given observed data on population (L_n) and trade shares (π_{ni}), the population mobility condition (31) can be used to determine a measure of locational fundamentals in the form of land quality (H_n). Using this measure, the variance of population across locations can be decomposed into the contributions of fundamentals $\left(H_n^{\frac{(1-\alpha)(\sigma-1)}{\sigma(1-\alpha)-1}}\right)$, market access $\left(\pi_{nn}^{\frac{-\alpha(\sigma-1)}{(\sigma-1)(\sigma(1-\alpha)-1)}}\right)$ and their covariance.*

Proof. The proposition follows immediately from the population mobility condition (31) given observed data on population (L_n) and trade shares (π_{ni}):

$$\begin{aligned}\operatorname{var}\{\log L_n\} = \left(\tfrac{(1-\alpha)(\sigma-1)}{\sigma(1-\alpha)-1}\right)^2 \operatorname{var}\{\log H_n\} + \left(\tfrac{\alpha(\sigma-1)}{(\sigma-1)(\sigma(1-\alpha)-1)}\right)^2 \operatorname{var}\{\log \pi_{nn}\} \\ -2\left(\tfrac{(1-\alpha)(\sigma-1)}{\sigma(1-\alpha)-1}\right)\left(\tfrac{\alpha(\sigma-1)}{(\sigma-1)(\sigma(1-\alpha)-1)}\right) \operatorname{covar}\{\log H_n, \log \pi_{nn}\}.\end{aligned}$$

[5]If the neoclassical model in the previous section were augmented to allow the productivity of each location to depend on its population through external economies of scale ($T_n = G_n L_n^{\eta}$ where $G_n, \eta > 0$), relative population shares would also enter the trade equation in that model. In that case, the existence of a non-degenerate population distribution would again impose a restriction on parameters that the agglomeration force created by these external economies of scale is not too strong relative to the congestion force from an inelastic supply of land.

■

As in our baseline model in the previous section, a location's trade share with itself is no longer a sufficient statistic for the welfare gains from goods trade, which also depend on changes in the population distribution. In contrast to a model in which all factors of production are geographically immobile, these changes in the population distribution ensure that all locations experience the same welfare gains from trade.

Proposition 6 *The change in a location's trade share with itself and the change in its population are sufficient statistics for the welfare gains from goods trade.*

Proof. The proposition follows immediately from (30) since:

$$\frac{V_n^{\text{Trade}}}{V_n^{\text{No Trade}}} = \left(\pi_{nn}^{\text{Trade}}\right)^{-\frac{\alpha}{\sigma-1}} \left(\frac{L_n^{\text{Trade}}}{L_n^{\text{No Trade}}}\right)^{-\left(\frac{\sigma(1-\alpha)-1}{\sigma-1}\right)} = \frac{\bar{V}^{\text{Trade}}}{\bar{V}^{\text{No Trade}}}, \tag{35}$$

where $\pi_{nn}^{\text{No Trade}} = 1$ and $\sigma(1-\alpha) > 1$ is required for a non-degenerate population distribution. ■

Therefore our result that reallocations of mobile factors of production provide a new channel through which goods trade affects welfare is not specific to the neoclassical model of the previous section, but also holds in other models within the class considered by Arkolakis et al. (2012) when augmented to include mobile and immobile factors. While we demonstrate this result in this section using a model of love of variety preferences, the same result holds in a model of national product differentiation following Armington (1969).

4 Internal Versus External Trade

To simplify the exposition of our argument, we have so far considered contexts in which the mobile factor of production is mobile across all locations. In this section, we consider a more realistic setting of an economy containing distinct countries each of which consists of many regions. Labor is mobile across regions within countries but immobile between countries. The resulting extension of our baseline model from Section 2 provides a natural platform for examining the role of reallocations of resources across regions within countries in shaping the welfare gains from reductions in trade costs between countries. With a view to taking the model to the data, we also generalize our previous analysis to incorporate commercial land use and traded intermediate inputs. We show that the augmented model yields an analogous system of equations for general equilibrium that can be again used to undertake model-based decompositions and counterfactuals.

4.1 Consumer Preferences and Production

The world economy consists of a set N of regions indexed by n. A subset $N^A \subset N$ of these regions are in a home country and a subset $N^B \subset N$ are in a foreign country, where $N^A \cap N^B = \emptyset$. While for simplicity we focus on the case of two countries, it is straightforward to accommodate an arbitrary number of countries that each consist of many regions.

Preferences are again defined over goods consumption (C_n) and residential land use (H_{Un}) and take the same form as in (1). The goods consumption index (C_n) is defined over consumption of a fixed continuum of goods $j \in [0,1]$ as in (2).

Each region draws an idiosyncratic productivity z_j for each good j as in (3) and goods are again homogeneous in the sense that one unit of a given good is the same as any other unit of that good. Goods are produced with labor, land and intermediate inputs under conditions of perfect competition according to a Cobb-Douglas production technology. The cost to a consumer in region n of purchasing one unit of good j from region i is therefore:

$$p_{ni}(j) = \frac{d_{ni} w_i^{\beta} r_i^{\eta} P_i^{1-\beta-\eta}}{z_i(j)}, \qquad 0 < \beta < 1,\ 0 < \eta < 1, \tag{36}$$

where P_i is the price index for tradeable goods.

4.2 Expenditure Shares and Price Indices

The representative consumer in region n sources each good from the lowest cost supplier to that region. Using equilibrium prices (36) and the properties of the Fréchet distribution, the share of expenditure of region n on goods produced by region i is:

$$\pi_{ni} = \frac{T_i \left(d_{ni} w_i^{\beta} r_i^{\eta} P_i^{1-\beta-\eta} \right)^{-\theta}}{\sum_{j \in \{A,B\}} \sum_{k \in N^j} T_k \left(d_{nk} w_k^{\beta} r_k^{\eta} P_k^{1-\beta-\eta} \right)^{-\theta}}, \tag{37}$$

while the price index for tradeable goods is:

$$P_n = \gamma \left[\sum_{j \in \{A,B\}} \sum_{i \in N^j} T_i \left(d_{ni} w_i^{\beta} r_i^{\eta} P_i^{1-\beta-\eta} \right)^{-\theta} \right]^{1/\theta}, \tag{38}$$

where for clarity we separate out the regions located in each country in the summations.

4.3 Income and Population Mobility

Expenditure on land in each region is again redistributed lump sum to the workers residing in that region. Total income in each region is therefore:

$$v_n L_n = w_n L_n + (1-\alpha)\, v_n L_n + \eta R_n,$$

where R_n denotes the total revenue from production; the first term on the right-hand side is income from labor used in production; the second term is income from expenditure on residential land; and the third term is income from expenditure on commercial land.

Noting that payments to labor used in production are a constant share of revenue, $w_n L_n = \beta R_n$, total income in each region can be re-written as:

$$v_n L_n = \left(\frac{\beta+\eta}{\alpha\beta}\right) w_n L_n. \tag{39}$$

Goods market clearing requires that total revenue in each region equals expenditure on goods produced in that region:

$$R_i = \sum_{j\in\{A,B\}} \sum_{n\in N^j} \pi_{ni} \left[\alpha v_n L_n + (1-\beta-\eta) R_n\right],$$

where the first term inside the square parentheses is expenditure from final goods consumption and the second term is expenditure from intermediate inputs use. Using total income (39) and $w_n L_n = \beta R_n$, this goods market clearing condition can be written in exactly the same form as (10).

Land market clearing implies that the equilibrium land rent can be determined from the equality of land income and expenditure:

$$r_n = \frac{(1-\alpha) v_n L_n + \eta R_n}{H_n} = \left(\frac{(1-\alpha)\beta+\eta}{\alpha\beta}\right) \frac{w_n L_n}{H_n}, \tag{40}$$

where we have again used $w_n L_n = \beta R_n$.

Intra-national population mobility ensures that real incomes are equalized across regions within each country, while international population immobility implies that real incomes can differ between countries. Using land market clearing (40), the equality of income and expenditure (39), the expenditure share (37) and the price index (38) in the population mobility condition (12), welfare in each location within a given country can be expressed as follows:

$$V_n = \frac{\left(\frac{T_n}{\pi_{nn}}\right)^{\frac{\alpha}{\theta(\beta+\eta)}} L_n^{-\frac{\eta+(1-\alpha)\beta}{\beta+\eta}} H_n^{\frac{\eta+(1-\alpha)\beta}{\beta+\eta}}}{\gamma^{\frac{\alpha}{\beta+\eta}} \frac{\alpha\beta}{\beta+\eta} \left(\frac{\eta+(1-\alpha)\beta}{\alpha\beta}\right)^{\frac{\eta+(1-\alpha)\beta}{\beta+\eta}}} = \bar{V}^j, \qquad n \in N^j,\ j \in \{A, B\}, \tag{41}$$

where terms in wages (w_n) have again canceled; land quality (H_n) is a parameter; and $\bar{V}^j$ is the common level of utility across regions within country j.

Rearranging this population mobility condition, equilibrium population in each region within a country can be expressed in terms of fundamentals (productivity (T_n) and land quality (H_n)), the region's trade share with itself (π_{nn}) and the common level of real income across regions within that country ($\bar{V}^j$ for $j \in \{A, B\}$):

$$L_n = \frac{\left(\frac{T_n}{\pi_{nn}}\right)^{\frac{\alpha}{\theta(\eta+(1-\alpha)\beta)}} H_n}{\left(\frac{\alpha\beta}{\beta+\eta}\right)^{\frac{\beta+\eta}{\eta+(1-\alpha)\beta}} \left(\frac{\eta+(1-\alpha)\beta}{\alpha\beta}\right) \gamma^{\frac{\alpha}{\eta+(1-\alpha)\beta}} \left(\bar{V}^j\right)^{\frac{\beta+\eta}{\eta+(1-\alpha)\beta}}}, \qquad n \in N^j,\ j \in \{A, B\}, \tag{42}$$

where the labor market clearing condition (14) holds for each country separately.

4.4 General Equilibrium

The general equilibrium of the model can be represented by the share of each country's workers located in each region ($\lambda_n^j = L_n/\bar{L}^j$ for $j \in \{A, B\}$), the share of each location's expenditure on goods produced in other locations (π_{ni}), the wage in each location (w_n), and the price index in each location (P_n). Using labor income (39), the trade share (37), the price index (38), land market clearing (40), labor market clearing (14) and population mobility (42), the equilibrium quintuple $\{\lambda_n^A, \lambda_n^B, \pi_{ni}, w_n, P_n\}$ solves the following system of equations for the two countries:

$$w_i \lambda_i^s \bar{L}^s = \sum_{j \in \{A,B\}} \sum_{n \in N^j} \pi_{ni} w_n \lambda_n^j \bar{L}^j, \quad i \in N^s,\ s \in \{A, B\} \tag{43}$$

$$\pi_{ni} = \frac{T_i \left(d_{ni} w_i^{\beta+\eta} P_i^{1-\beta-\eta} \left(\lambda_i^s\right)^\eta \left(\frac{\bar{L}^s}{H_i}\right)^\eta \right)^{-\theta}}{\sum_{j \in \{A,B\}} \sum_{k \in N^j} T_k \left(d_{nk} w_k^{\beta+\eta} P_k^{1-\beta-\eta} \left(\lambda_k^j\right)^\eta \left(\frac{\bar{L}^j}{H_k}\right)^\eta \right)^{-\theta}}, \quad i \in N^s,\ s \in \{A, B\}, \tag{44}$$

$$P_n = \gamma \left[\sum_{j \in \{A,B\}} \sum_{k \in N^j} T_k \left(\kappa_P d_{nk} w_k^{\beta+\eta} P_k^{1-\beta-\eta} \left(\lambda_k^j\right)^\eta \left(\frac{\bar{L}^j}{H_k}\right)^\eta \right)^{-\theta} \right]^{-1/\theta}, \quad n \in N^s,\ s \in \{A, B\}, \tag{45}$$

$$\lambda_n^s = \frac{\left(\frac{T_n}{\pi_{nn}}\right)^{\frac{\alpha}{\theta(\beta+\eta-\alpha\beta)}} H_n}{\sum_{k \in N^s} \left(\frac{T_k}{\pi_{kk}}\right)^{\frac{\alpha}{\theta(\beta+\eta-\alpha\beta)}} H_k}, \qquad n \in N^s,\ s \in \{A, B\} \tag{46}$$

where $\bar{L}^s$ is the exogenous endowment of labor for country s and H_n is the exogenous endowment of land for each region n and

$$\kappa_P \equiv \left(\frac{(1-\alpha)\beta + \eta}{\alpha\beta} \right)^\eta. \tag{47}$$

Proposition 7 *Given locational fundamentals (productivity (T_n), land quality (H_n) and bilateral trade frictions (d_{ni})), there exist equilibrium population shares (λ_n), trade shares (π_{ni}), wages (w_n) and price indices (P_n) that solve the wage equation (43), trade equation (44), price equation (45) and population mobility condition (46).*

Proof. See the appendix. ■

While the wage equation (43), trade equation (44) and price index (45) take a similar form as in previous sections, there are also some differences. First, the population mobility condition (46) now only holds within each country. Second, population shares (λ_n) now enter the trade equation (44), because land is used in production and a higher population implies a higher price for immobile land and hence higher production costs. Third, in the previous two sections, equilibrium wages, trade shares and population shares $\{w_n, \pi_{ni}, \lambda_n\}$ could be determined before price indices (P_n), because the absence of intermediate inputs from production gave rise to a recursive structure of the model. In contrast, in this section, price indices (P_n) enter the trade equation because of intermediate input use in production. As a result, price indices (P_n) must be jointly determined with wages (w_n), trade shares (λ_n) and population shares (λ_n) in the system of equations for general equilibrium.

In this more general setting, the population mobility condition (42) together with observed data on population and trade shares can be still used to decompose the variance of population across locations into the contributions of locational fundamentals and market access.

Proposition 8 *Given observed data on population (L_n) and trade shares (π_{ni}), the population mobility condition (42) can be used to determine a composite measure of fundamentals for each location that incorporates productivity and land quality $\left(H_n T_n^{\frac{\alpha}{\theta(\beta+\eta-\alpha\beta)}}\right)$. Using this measure, the variance of population across locations can be decomposed into the contributions of fundamentals $\left(H_n T_n^{\frac{\alpha}{\theta(\beta+\eta-\alpha\beta)}}\right)$, market access $\left(\pi_{nn}^{\frac{-\alpha}{\theta(\beta+\eta-\alpha\beta)}}\right)$ and their covariance.*

Proof. The proposition follows immediately from the population mobility condition (42) given observed data on population (L_n) and trade shares (π_{ni}):

$$\begin{aligned} \operatorname{var}\{\log L_n\} = \operatorname{var}\left\{\log\left(T_n^{\alpha/\theta(\beta+\eta-\alpha\beta)} H_n\right)\right\} + \left(\tfrac{\alpha}{\theta(\beta+\eta-\alpha\beta)}\right)^2 \operatorname{var}\{\log \pi_{nn}\} \\ -2\left(\tfrac{\alpha}{\theta(\beta+\eta-\alpha\beta)}\right) \operatorname{covar}\left\{\log\left(T_n^{\alpha/\theta(\beta+\eta-\alpha\beta)} H_n\right), \log\left(\pi_{nn}\right)\right\}. \end{aligned}$$

■

The system of equations for general equilibrium (43)-(46) can be used to undertake model-based counterfactuals of the same form as in equations (19)-(21) in the previous section. These counterfactuals now yield predictions for the internal distribution of economic activity across regions within countries. As in our baseline model, a region's trade share with itself is no longer a sufficient statistic for the welfare gains from market integration, which also depend on population changes.

Proposition 9 *The change in a location's trade share with itself and the change in its population are sufficient statistics for the welfare gains from goods trade.*

Proof. The proposition follows immediately from (41) since:

$$\frac{V_n^{\text{Trade}}}{V_n^{\text{No Trade}}} = \left(\pi_{nn}^{\text{Trade}}\right)^{-\frac{\alpha}{\theta(\beta+\eta)}} \left(\frac{L_n^{\text{Trade}}}{L_n^{\text{No Trade}}}\right)^{-\left(1-\frac{\alpha\beta}{\beta+\eta}\right)}, \tag{48}$$

where $\pi_{nn}^{\text{No Trade}} = 1$. ■

Therefore the effect of changes in trade costs between countries on regional welfare depends on whether or not population is mobile within countries. If population is geographically immobile, the change in each region's trade share with itself is a sufficient statistic for the welfare effects of international goods market integration. Furthermore, border regions that have higher trade shares with the other country in the open economy equilibrium experience greater welfare gains from international goods market integration than interior regions that have lower trade shares with the other country.

In contrast, if population is mobile across regions within countries, the change in each region's welfare also depends on the change in its population, which affects welfare through the price of land. External trade liberalization leads to an endogenous internal reallocation of population such that all regions experience the same welfare gains from international goods market integration.

5 Quantitative Application

In this section, we use the model developed in the previous section together with data on trade between Canadian and U.S. regions to illustrate the quantitative relevance of factor mobility within countries for the welfare effects of goods trade. We begin by discussing the data before using the structure of the model to evaluate the welfare implications of counterfactual changes in barriers to goods trade. We examine how these welfare implications depend on whether population is mobile or immobile across regions within each country.

5.1 Data Sources and Definitions

Our main source of data is the information on trade between Canadian provinces and U.S. states for 1993 from Anderson and Van Wincoop (2003). A key feature of these data is that they contain information on internal trade between regions within each country and external trade between regions in the two different countries. We combine these data with information on the population of Canadian provinces and U.S. states from the closest Census year, which is 1990.

To take the model to the data, a number of choices must be made. To simplify the exposition of the model, we assumed a continuum of products, which implies that each location trades with all other locations.[6] To ensure that the data are consistent with this feature of the model, we aggregate Canadian provinces and U.S. states to the level of statistical regions.[7] For expositional convenience, we also focused in the model on two countries and assumed that trade is balanced (income equals expenditure). Since the goal of our empirical analysis is merely to illustrate the implications of factor mobility for the welfare gains from goods trade, we make analogous assumptions in the data. We use actual bilateral trade flows between Canadian and U.S. regions to construct a matrix of bilateral import shares for each region that we use as an input into the model. In using this matrix of bilateral import shares, we assume that the world consists solely of Canadian and U.S. regions and that trade is balanced for each region separately and hence for the two countries as a whole. Introducing the rest of the world as a third country and allowing for trade imbalance is straightforward, and would alter the specific numerical predictions under population immobility and mobility, but it would not change the basic contrast between these two cases that we illustrate in this section.

In Table 1, we report the matrix of bilateral import shares for Canadian and U.S. regions in 1993,

[6] In principle, the model in the previous section can be used to consider a finite number of products. In this case, each region draws a finite number of productivities, for which some pairs of regions may not trade, as in Eaton et al. (2012).

[7] See the data appendix for the definition of statistical regions. To ensure positive bilateral trade flows between all pairs of regions, we exclude Northern Canada, which accounts for a small share of Canadian trade and population.

where the importer appears in the columns and the exporter appears in the rows, such that the rows sum to one for each column. Unsurprisingly, the largest share of a region's trade is with itself. But these own trade shares for Canadian and U.S. regions are smaller than for Canada and the U.S. as a whole, which reflects the regions' smaller economic size and implies greater welfare gains from trade with other regions. Within Canada, the largest trade flows between pairs of distinct regions are imports of Central Canada from Atlantic Canada and imports of Western Canada from Central Canada. Within the U.S., the largest trade flows between pairs of distinct regions are imports of the South from the Mid West and imports of the North East from the South. Between the two countries, the most intense trade relationship by far is imports of Central Canada from the Mid West, followed by imports of the other Canadian regions from the Mid West, and by imports of the North East from Central Canada. Reflecting the larger economic size of the U.S. relative to Canada, U.S. regions together account for 15-25 percent of Canadian regions' imports, whereas Canadian regions together account for 5 percent or less of U.S. regions' imports.

Table 1: Import Shares of U.S. and Canadian Regions

		Importer						
		Atl. Can.	Cent. Can.	West Can.	Mid West	North East	South	West
Exporter	Atl. Can.	0.6006	0.0093	0.0044	0.0009	0.0044	0.0008	0.0001
	Cent. Can.	0.2161	0.6966	0.1205	0.0385	0.0395	0.0070	0.0027
	West. Can.	0.0266	0.0385	0.7305	0.0097	0.0058	0.0026	0.0041
	All Can.	0.8434	0.7445	0.8554	0.0490	0.0497	0.0105	0.0069
Exporter	Mid West	0.0574	0.1322	0.0547	0.4656	0.1441	0.3082	0.2046
	North East	0.0465	0.0550	0.0181	0.1263	0.3174	0.2006	0.1285
	South	0.0392	0.0503	0.0308	0.2948	0.2972	0.3681	0.3641
	West	0.0136	0.0180	0.0410	0.0643	0.1916	0.1127	0.2960
	All U.S.	0.1566	0.2555	0.1446	0.9510	0.9503	0.9895	0.9931
	All regions	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Note: Data source is Anderson and van Wincoop (2003). See the appendix for definitions of U.S. and Canadian regions. Importers appear in columns and exporters appear in rows; sub-totals for all Canadian regions and all U.S. regions sum to the total for all regions; figures need not exactly sum to one due to rounding.

In Table 2, we report the share of each region in the population sum for the country as a whole. Central Canada accounts for by far the largest share of the Canadian population, while the South is the most populous U.S. region. In general, population is more equally distributed across U.S. regions than across Canadian regions, with Atlantic Canada having the smallest population share of all regions. Taking the import and population shares reported in Tables 1 and 2 together with the population sum for Canada and the U.S., we use the wage equation (43) to solve for the implied wages (w_i) and hence labor income ($Y_i = w_i \lambda_i^s \bar{L}^s$) of each region i in the initial equilibrium of the model prior to the counterfactual change in trade costs considered below.

The import shares (π_{ni}), population shares (λ_n) and labor income (Y_n) in the initial equilibrium are inputs into our counterfactual analysis. For the model's parameters, we assume central values

Table 2: Population Shares of U.S. and Canadian Regions

Canada	Population Shares	U.S.	Population Shares
Atlantic Canada	0.0854	Mid West	0.2400
Central Canada	0.6265	North East	0.2043
Western Canada	0.2881	South	0.3436
		West	0.2121
Total	1.0000	Total	1.0000

Note: Data sources are the U.S. Census Bureau and Statistics Canada. See the appendix for definitions of U.S. and Canadian regions. The population sum for Canada excludes Northern Canada. Population shares sum to one for each country separately up to rounding.

from the existing literature. We set the share of residential land in consumer expenditure $(1-\alpha)$ equal to 0.25, which corresponds to the estimated housing expenditure share using U.S. data in Davis and Ortalo-Magne (2011). We set the share of intermediate inputs in production costs $(1-\beta-\eta)$ equal to 0.50, which is consistent with empirical findings for the manufacturing sector. We set the share of land in production costs (η) equal to 0.10 and the share of labor in production costs (β) equal to 0.40. These values imply a share of immobile factors in value added of around 20 percent, which is in line with the estimates in Valentinyi and Herrendorf (2008). Finally, we set the Fréchet shape parameter (θ) equal to 4, which lies in the centre of the range of estimates in Donaldson (2010), Eaton and Kortum (2002) and Simonovska and Waugh (2011).

5.2 Counterfactuals under Population Immobility

To illustrate the implications of population mobility for the welfare gains from goods trade, we consider a change in bilateral trade frictions that reduces international trade costs between Canadian and U.S. regions by 25 percent while leaving intra-national trade costs between regions within each country unchanged. In this subsection, we solve for the counterfactual implications of this change in trade costs for the special case of the model in which labor is immobile across regions. In this case, wages (w_n), trade shares (π_{ni}) and price indices (P_n) are determined by the wage equation (43), trade equation (44) and price index equation (45), while population shares are exogenously determined by endowments. Denoting the value of variables in the initial equilibrium by a prime (x') and the relative value of variables in the counterfactual and initial equilibria by a tilde $(\tilde{x}=x'/x)$, the wage, trade and price index equations in the counterfactual equilibrium can be re-written as follows:

$$\tilde{w}_i Y_i = \sum_{j\in\{A,B\}}\sum_{n\in N^j}\pi'_{ni}\tilde{w}_n Y_n, \qquad i\in N^j,\ j\in\{A,B\} \tag{49}$$

$$\pi'_{ni} = \frac{\pi_{ni}\left(\tilde{d}_{ni}\tilde{w}_i^{\beta+\eta}\tilde{P}_i^{1-\beta-\eta}\right)^{-\theta}}{\sum_{j\in\{A,B\}}\sum_{k\in N^j}\pi_{nk}\left(\tilde{d}_{nk}\tilde{w}_k^{\beta+\eta}\tilde{P}_k^{1-\beta-\eta}\right)^{-\theta}}, \quad i,n\in N, \tag{50}$$

$$\tilde{P}_n = \gamma \left[\sum_{j \in \{A,B\}} \sum_{k \in N^j} \pi_{nk} \left(\tilde{d}_{nk} \tilde{w}_k^{\beta+\eta} \tilde{P}_k^{1-\beta-\eta} \right)^{-\theta} \right]^{-1/\theta}, \quad n \in N, \tag{51}$$

where $Y_i = w_i \lambda_i^s \bar{L}^s$ denotes labor income in the initial equilibrium; the comparative static considered in the counterfactual corresponds to $\tilde{d}_{ni} = 1$ when regions n and i are located in the same country and $\tilde{d}_{ni} = 0.75$ when regions n and i are located in different countries.

Together (49)-(51) provide a system of equations for the N locations that can be solved for the counterfactual changes in wages, trade shares and price indices $\{\tilde{w}_n, \tilde{\pi}_{ni}, \tilde{P}_n\}$ for each region. Given these solutions, the counterfactual change in welfare for each region under factor immobility follows immediately from the change in each region's trade share with itself:

$$\tilde{V}_n = \frac{V_n'}{V_n} = \left(\frac{\pi_{nn}'}{\pi_{nn}} \right)^{-\frac{\alpha}{\theta(\beta+\eta)}} = \left(\tilde{\pi}_{nn} \right)^{-\frac{\alpha}{\theta(\beta+\eta)}}. \tag{52}$$

In Table 3, we report the counterfactual change in own trade shares and welfare for each region. Consistent with imports from the U.S. being more important for Canadian regions than imports from Canada are for U.S. regions, the counterfactual changes in own trade shares and welfare are larger for Canadian regions than U.S. regions. Across Canadian regions, Central Canada benefits by more than Western Canada, because of its greater trade intensity with U.S. regions including in particular the Mid West. Across U.S. regions, the Mid West and North East benefit by more than the South and West because of their greater trade integration with Canada.

Table 3: Counterfactuals for Welfare under Population Immobility

Region	Relative Own Trade Shares π_{nn}'/π_{nn}	Relative Welfare V_n'/V_n
Atlantic Canada	0.6833	1.1535
Central Canada	0.5995	1.2115
Western Canada	0.7340	1.1230
Mid West	0.9130	1.0347
North East	0.9241	1.0301
South	0.9878	1.0046
West	0.9848	1.0057

Note: The table reports the results from a counterfactual in which international trade costs between Canadian and U.S. regions are reduced by 25 percent while intra-national trade costs between regions within each country remain unchanged; variables in the counterfactual equilibrium are denoted by a prime; variables in the initial equilibrium are denoted without a prime. See the appendix for definitions of U.S. and Canadian regions.

5.3 Counterfactuals under Population Mobility

We now examine the counterfactual implications of the same change in barriers to goods trade under population mobility. Wages (w_n), trade shares (π_{ni}), price indices (P_n) and population shares (λ_n)

are determined by the wage equation (43), trade equation (44), price index equation (45) and population mobility condition (46). Using the same notation as above, this system of equations in the counterfactual equilibrium can be re-written as follows:

$$\tilde{w}_i \tilde{\lambda}_i^s Y_i = \sum_{j \in \{A,B\}} \sum_{n \in N^j} \pi'_{ni} \tilde{w}_n \tilde{\lambda}_n^j Y_n, \qquad i \in N^j,\ s, j \in \{A, B\} \tag{53}$$

$$\pi'_{ni} = \frac{\pi_{ni} \left(\tilde{d}_{ni} \tilde{w}_i^{\beta+\eta} \tilde{P}_i^{1-\beta-\eta} \left(\tilde{\lambda}_i^A \right)^{\eta} \right)^{-\theta}}{\sum_{j \in \{A,B\}} \sum_{k \in N^j} \pi_{nk} \left(\tilde{d}_{nk} \tilde{w}_k^{\beta+\eta} \tilde{P}_k^{1-\beta-\eta} \left(\tilde{\lambda}_k^j \right)^{\eta} \right)^{-\theta}}, \quad i, n \in N, \tag{54}$$

$$\tilde{P}_n = \gamma \left[\sum_{j \in \{A,B\}} \sum_{k \in N^j} \pi_{nk} \left(\tilde{d}_{nk} \tilde{w}_k^{\beta+\eta} \tilde{P}_k^{1-\beta-\eta} \left(\tilde{\lambda}_k^j \right)^{\eta} \right)^{-\theta} \right]^{-1/\theta}, \quad n \in N, \tag{55}$$

$$\lambda_n^{s\prime} = \frac{\lambda_n^s \tilde{\pi}_{nn}^{-\frac{\alpha}{\theta(\beta+\eta-\alpha\beta)}}}{\sum_{k \in N^s} \lambda_k^s \tilde{\pi}_{kk}^{-\frac{\alpha}{\theta(\beta+\eta-\alpha\beta)}}}, \qquad n \in N^s,\ s \in \{A, B\}, \tag{56}$$

where $Y_i = w_i \lambda_i^s \bar{L}^s$ again denotes labor income in the initial equilibrium; the comparative static considered in the counterfactual again corresponds to $\tilde{d}_{ni} = 1$ when regions n and i are located in the same country and $\tilde{d}_{ni} = 0.75$ when regions n and i are located in different countries.

The general equilibrium system (53)-(56) can be solved for the counterfactual changes in wages, trade shares, price indices and population shares $\{\tilde{w}_n, \tilde{\pi}_{ni}, \tilde{P}_n, \tilde{\lambda}_n^s\}$ for each region $n \in N^s$ for each country s. Given these solutions, the counterfactual change in each region's welfare under population mobility follows immediately from (48) and depends on both the change in each region's trade share with itself and the change in its population.

In Table 4, we report the counterfactual changes in own trade shares, welfare and population shares for each region. Consistent with imports from the U.S. being more important for Canadian regions than imports from Canada are for U.S. regions, the counterfactual changes in own trade shares and welfare are again larger for Canadian regions than U.S. regions. In the results in the previous subsection under population immobility, the counterfactual changes in welfare differed across regions within each country depending on their initial level of trade integration with regions in the other country. In contrast under population mobility, internal reallocations of population ensure that the external trade liberalization has the same effect on welfare in each region, raising welfare by around 18 percent for Canadian regions and by around 2 percent for U.S. regions.

Central Canada's higher initial import shares from the U.S. than other Canadian regions imply that the reduction in bilateral trade frictions reduces the consumer price index in Central Canada relative to the other Canadian regions. As a result, Central Canada becomes a more attractive place to live, which induces a population inflow from the other Canadian regions. The mechanisms that restore equilibrium are changes in wages and the price of land across regions. As the population of Central Canada rises relative to other Canadian regions, this bids up the price of land in Central Canada relative to other Canadian regions, until real wages are equalized across all regions in the new equilibrium. As shown in Table 4, Central Canada experiences an increase in its share of the Canadian population by around five percent at the expense of Western and Atlantic Canada. Although the smaller size of Canada relative to the U.S. implies that the effects are smaller for U.S. regions, the Mid West and South experience an increase and decrease in their population shares by around three and two percent respectively.

Table 4: Counterfactuals for Welfare under Population Mobility

Region	Relative Own Trade Share π'_{nn}/π_{nn}	Relative Welfare V'_n/V_n	Relative Pop. Share λ'_n/λ_n
Atlantic Canada	0.6714	1.1828	0.9548
Central Canada	0.6051	1.1828	1.0526
Western Canada	0.7160	1.1828	0.8990
Mid West	0.9225	1.0179	1.0317
North East	0.9324	1.0179	1.0214
South	0.9756	1.0179	0.9790
West	0.9771	1.0179	0.9776

Note: The table reports the results from a counterfactual in which international trade costs between Canadian and U.S. regions are reduced by 25 percent while intra-national trade costs between regions within each country remain unchanged; variables in the counterfactual equilibrium are denoted by a prime; variables in the initial equilibrium are denoted without a prime. See the appendix for definitions of U.S. and Canadian regions.

The effects of external trade liberalization between countries are therefore shaped to a quantitatively relevant extent by internal reallocations of resources across regions within countries. These intra-national reallocations of resources provide a distinct mechanism through which trade affects welfare that is often neglected in studies of international integration. Incorporating this mechanism changes the welfare gains from trade, which can no longer be inferred from the change in each region's trade share with itself but also depend on changes in the allocation of the mobile factor. While geographical mobility ensures that the welfare gains from trade for the mobile factor are equalized across regions, the resulting changes in the distribution of the mobile factor imply uneven changes in the price of the immobile factor.

6 Conclusions

Economic activity is highly unevenly distributed across regions within countries, and this inequality is in large part the result of factor mobility, as reflected for example in the increased concentration of economic activity in China's coastal regions in recent decades. This paper has extended a recent class of quantitative models of international trade to incorporate factor mobility within countries and shown that it has important general equilibrium implications for the effects of goods market integration. This class of models includes models of constant returns to scale with no agglomeration forces (such as an extension of Eaton and Kortum 2002) and models featuring increasing returns to scale and agglomeration forces (such as a multi-region version of Helpman 1998). Internal reallocations of resources across regions within countries provide a new channel through which external trade liberalization between countries affects welfare.

Although we model factor mobility between an arbitrary number of regions connected by an arbitrary pattern of geographical trade frictions, the analysis remains highly tractable. We use the structure of the model and observed data on population and trade shares to extract a composite measure of locational fundamentals that includes productivity and land quality. The dispersion of population across locations is driven by differences in this composite measure of locational fundamentals and variation in each location's trade share with itself, which provides an inverse summary measure of market access to other locations. Using data on trade shares, population and income in the initial equilibrium, the system of equations for general equilibrium that determines wages, prices, trade shares and population shares can be used to undertake model-based counterfactuals that yield predictions for changes in the internal distribution of economic activity within countries.

Population mobility ensures that the welfare gains from trade are equally shared across regions within countries and sufficient statistics for the welfare gains from trade are provided by the change in each region's trade share with itself and the change in its population.

A Appendix

A.1 Proof of Proposition 1

To establish the proposition, we proceed in two steps. First, we show that given the population share for each location (λ_n) the wage equation (15) and trade share equation (16) determine a unique equilibrium wage in each location (w_n) and hence a unique equilibrium trade share (π_{ni}). Second, we use the population mobility condition (17) together with the wage equation (15) and trade share equation (16) to establish the existence of an equilibrium population share (λ_n).

In our first step, we use the trade share (16) in the wage equation (15) to obtain:

$$w_i \lambda_i = \sum_{n \in N} \frac{T_i \left(d_{ni} w_i\right)^{-\theta}}{\sum_{k \in N} T_k \left(d_{nk} w_k\right)^{-\theta}} w_n \lambda_n. \tag{57}$$

Given population shares in each location (λ_n), the above wage equation provides a system of N equations in the N unknown wages for each location $n = 1, \ldots, N$. Note that this wage equation (57) takes the same form as equation (3.14) on page 1734 of Alvarez and Lucas (2007). Using (57), we can define the following excess demand system:

$$\Xi(\mathbf{w}) = \frac{1}{w_i} \left[\sum_{n \in N} \frac{T_i \left(d_{ni} w_i\right)^{-\theta}}{\sum_{k \in N} T_k \left(d_{nk} w_k\right)^{-\theta}} w_n \lambda_n - w_i \lambda_i \right] = 0, \tag{58}$$

where $\mathbf{w}$ denotes the vector of wages across locations.

Lemma 1 *Given population shares in each location (λ_n), there exists a unique wage vector $\mathbf{w} \in \Re^n_{++}$ such that $\Xi(\mathbf{w}) = 0$.*

Proof. Note that $\Xi(\mathbf{w})$ has the following properties:
(i) $\Xi(\mathbf{w})$ is continuous.
(ii) $\Xi(\mathbf{w})$ is homogeneous of degree zero.
(iii) $\mathbf{w} \cdot \Xi(\mathbf{w}) = 0$ for all $\mathbf{w} \in \Re^n_{++}$ (Walras Law)
(iv) There exists a $s > 0$ such that $\Xi_i(\mathbf{w}) > -s$ for each location i and all $\mathbf{w} \in \Re^n_{++}$
(v) If $\mathbf{w}^m \to \mathbf{w}^0$ where $\mathbf{w}^0 \neq 0$ and $w_i^0 = 0$ for some country i, then

$$\max_j \left\{ \Xi_j \left(\mathbf{w}^m\right) \right\} \to \infty.$$

(vi) Gross substitutes:

$$\frac{\partial \Xi_i(\mathbf{w})}{\partial w_j} > 0 \quad \text{for all } i, j, i \neq j, \text{ for all } \mathbf{w} \in \Re^n_{++},$$

$$\frac{\partial \Xi_i(\mathbf{w})}{\partial w_i} < 0 \quad \text{for all } i, \text{ for all } \mathbf{w} \in \Re^n_{++},$$

from which the proposition follows. See Propositions 17.B.2, 17.C.1 and 17.F.3 of Mas-Colell, Whinston and Green (1995) and Theorems 1-3 of Alvarez and Lucas (2007). ■

Our second step uses the population mobility condition together with the comparative statics of the wage and trade equations. The population mobility condition (17) can be re-written as follows:

$$\lambda_n = \frac{(T_n)^{\frac{\alpha}{\theta(1-\alpha)}} H_n}{(T_n)^{\frac{\alpha}{\theta(1-\alpha)}} H_n + \sum_{k \neq n} \left(\frac{\pi_{nn}}{\pi_{kk}}\right)^{\frac{\alpha}{\theta(1-\alpha)}} (T_k)^{\frac{\alpha}{\theta(1-\alpha)}} H_k}. \tag{59}$$

Suppose that $\lambda_n \to 0$ while $\lambda_k > 0$ for $k \neq n$. As $\lambda_n \to 0$, the left-hand side of the population mobility condition (59) converges towards zero. Additionally, as $\lambda_n \to 0$ and $\lambda_k > 0$ for $k \neq n$, the wage equation (15) and trade equation (16) imply that $w_n = \infty$ and $\pi_{nn} = 0$, while w_k and π_{kk} take finite positive values for $k \neq n$. Using these results in the population mobility condition (59), the right-hand side of this condition converges towards infinity as $\lambda_n \to 0$ and $\lambda_k > 0$ for $k \neq n$.

In contrast, suppose that $\lambda_n \to 1$ and $\lambda_k \to 0$ for $k \neq n$. As $\lambda_n \to 1$, the left-hand side of the population mobility condition (59) takes a finite positive value. Additionally, as $\lambda_n \to 1$ and $\lambda_k \to 0$ for $k \neq n$, the wage equation (15) and trade equation (16) imply that w_n and π_{nn} take finite positive values, while $w_k = \infty$ and $\pi_{kk} = 0$ for $k \neq n$. Using these results in the population mobility condition (59), the right-hand side of this condition converges towards zero as $\lambda_n \to 1$ and $\lambda_k \to 0$ for $k \neq n$.

Combining these two sets of results, the left-hand side of the population mobility condition (59) is strictly less than the right-hand side as $\lambda_n \to 0$ and is strictly greater than the right-hand side as $\lambda_n \to 1$. Furthermore, this result holds for each location $n \in N$. Noting that the population mobility condition (59), wage equation (15) and trade equation (16) are continuous in $\{\lambda_n, w_n, \pi_{nn}\}$ for all $n \in N$, it follows that there exists an equilibrium allocation of population shares across locations $\{\lambda_n\}$ for which the left and right-hand sides of the population mobility condition (59) are equalized for each location $n \in N$. This allocation involves strictly positive population shares $\lambda_n > 0$ for all locations for which $T_n, H_n > 0$. Given this equilibrium allocation of population shares $\hat{\lambda}_n$, the wage equation (15) and trade equation (16) determine equilibrium wages for each location ($\hat{w}_n$) and hence equilibrium trade shares ($\hat{\pi}_{ni}$) as established in Lemma 1.

A.2 Special case of Two Regions

In this section of the appendix, we provide a formal analysis of the special case of two regions shown in Figure 1. In this case, the wage equation (15) becomes:

$$\begin{aligned} \lambda_A &= \pi_{AA}\lambda_A + \pi_{BA} w_B (1-\lambda_A), \\ w_B (1-\lambda_A) &= \pi_{AB}\lambda_A + \pi_{BA} w_B (1-\lambda_A), \end{aligned} \tag{60}$$

where have chosen the wage in region A as the numeraire ($w_A = 1$). The trade equation (16) is now:

$$\pi_{ni} = \frac{T_i (d_{ni} w_i)^{-\theta}}{\sum_{k \in \{A,B\}} T_k (d_{nk} w_k)^{-\theta}}, \qquad n, i \in \{A, B\}, \tag{61}$$

while the population mobility condition (17) can be written as:

$$\lambda_A = \frac{(T_A)^{\frac{\alpha}{\theta(1-\alpha)}} H_A}{(T_A)^{\frac{\alpha}{\theta(1-\alpha)}} H_A + \left(\frac{\pi_{AA}}{\pi_{BB}}\right)^{\frac{\alpha}{\theta(1-\alpha)}} (T_B)^{\frac{\alpha}{\theta(1-\alpha)}} H_B}. \tag{62}$$

The existence of equilibrium can be established by exactly the same line of reasoning as for an arbitrary number of regions in the previous section. First, for given population shares in each location $\{\lambda_A, 1-\lambda_A\}$, there exist unique wages $\{1, w_B\}$ that solve the wage equation (60) and trade equation (61). Second, equilibrium population shares are determined by combining the population mobility condition (62) with the wage equation (60) and trade equation (61).

Following the same argument as in the previous section, as $L_A \to 0$, the left-hand side of the population mobility condition (62) is strictly less than the right-hand side. In contrast, as $\lambda_A \to 1$, the left-hand side of the population mobility condition (62) is strictly greater than the right-hand side. Noting that the population mobility condition (62), wage equation (15) and trade equation (16) are continuous in $\{\lambda_n, w_n, \pi_{nn}\}$ for $n \in \{A, B\}$, it follows that there exists an equilibrium allocation of population shares across the two locations $\{\lambda_A, 1-\lambda_A\}$ for which the left and right-hand sides of the population mobility condition (62) are equalized. Finally, with two regions, the wage equation (60) and trade equation (61) imply that w_B and π_{AA} are monotonically increasing in λ_A, while π_{BB} is monotonically decreasing in λ_A. It follows that the right-hand side of the population mobility condition (62) is monotonically decreasing in λ_A, as shown graphically in Figure 1.

A.3 Proof of Proposition 4

The proof of the proposition again proceeds in two steps. First, we show that given the population share for each location (λ_n) the wage equation (32) and trade share equation (33) determine a unique equilibrium wage in each location (w_n) and hence a unique equilibrium trade share (π_{ni}). Second, we use the population mobility condition (34) together with the wage equation (32) and trade share equation (33) to establish the existence of an equilibrium population share (λ_n).

In our first step, we use the trade share (33) in the wage equation (32) to obtain

$$w_i \lambda_i = \sum_{n \in N} \frac{\lambda_i \left(d_{ni} w_i\right)^{1-\sigma}}{\sum_{k \in N} \lambda_k \left(d_{nk} w_k\right)^{1-\sigma}} w_n \lambda_n.$$

Note that this wage equation takes the same form as equation (3.14) on page 1734 of Alvarez and Lucas (2007). Given population shares in each location (λ_n), there exists a unique wage vector $\mathbf{w} \in \Re^n_{++}$ such that $\Xi(\mathbf{w}) = 0$ by the same arguments as in the proof of Lemma 1 in Section A.1 above.

Our second step uses the population mobility condition together with the comparative statics of the wage and trade equations. The population mobility condition (34) can be re-written as follows:

$$\lambda_n = \frac{H_n^{\frac{(1-\alpha)(\sigma-1)}{\sigma(1-\alpha)-1}}}{H_n^{\frac{(1-\alpha)(\sigma-1)}{\sigma(1-\alpha)-1}} + \sum_{k \neq n} H_k^{\frac{(1-\alpha)(\sigma-1)}{\sigma(1-\alpha)-1}} \left(\frac{\pi_{nn}}{\pi_{kk}}\right)^{\frac{\alpha}{\sigma(1-\alpha)-1}}}. \tag{63}$$

Since we focus on parameter values for which $\sigma(1-\alpha) > 1$, equilibrium population shares can be determined using exactly the same line of reasoning as in Section A.1. Taking the limits $\lambda_n \to 0$ and $\lambda_n \to 1$ and using continuity establishes the existence of an equilibrium allocation of population shares across locations for which the left and right-hand side of the population mobility condition

(63) are equalized for each location $n \in N$. This equilibrium allocation involves strictly positive population shares $\lambda_n > 0$ for all locations for which $H_n > 0$. Given this equilibrium allocation of population shares $\hat{\lambda}_n$, the wage equation (32) and trade equation (33) determine equilibrium wages for each location ($\hat{w}_n$) and hence equilibrium trade shares ($\hat{\pi}_{ni}$) as discussed above.

A.4 Proof of Proposition 7

The proof of the proposition again proceeds in two steps. First, we show that given the population share for each location n in each country j (λ_n^j) the wage equation (43), price index equation (45) and trade share equation (44) determine a unique equilibrium wage (w_n) and price index (P_n) in each location and hence a unique equilibrium trade share (π_{ni}). Second, we use the population mobility condition (46) together with the wage equation (43), price index equation (45) and trade share equation (44) to establish the existence of an equilibrium population share (λ_n^j).

In our first step, we use the trade share (44) in the wage equation (43) to obtain:

$$
\begin{aligned}
w_i \lambda_i^A \bar{L}^A \quad &= \sum_{n \in N^A} \left[\frac{T_i \left(d_{ni} w_i^{\beta+\eta} P_i^{1-\beta-\eta} \left(\lambda_i^A\right)^{\eta} \left(\frac{\bar{L}^A}{H_i}\right)^{\eta} \right)^{-\theta}}{\sum_{k \in N^A} T_k \left(d_{nk} w_k^{\beta+\eta} P_k^{1-\beta-\eta} \left(\lambda_k^A\right)^{\eta} \left(\frac{\bar{L}^A}{H_k}\right)^{\eta} \right)^{-\theta} + \sum_{k \in N^B} T_k \left(d_{nk} w_k^{\beta+\eta} P_k^{1-\beta-\eta} \left(\lambda_k^B\right)^{\eta} \left(\frac{\bar{L}^B}{H_k}\right)^{\eta} \right)^{-\theta}} \right] w_n \lambda_n^A \bar{L}^A \\
&+ \sum_{n \in N^B} \left[\frac{T_i \left(d_{ni} w_i^{\beta+\eta} P_i^{1-\beta-\eta} \left(\lambda_i^A\right)^{\eta} \left(\frac{\bar{L}^A}{H_i}\right)^{\eta} \right)^{-\theta}}{\sum_{k \in N^A} T_k \left(d_{nk} w_k^{\beta+\eta} P_k^{1-\beta-\eta} \left(\lambda_k^A\right)^{\eta} \left(\frac{\bar{L}^A}{H_k}\right)^{\eta} \right)^{-\theta} + \sum_{k \in N^B} T_k \left(d_{nk} w_k^{\beta+\eta} P_k^{1-\beta-\eta} \left(\lambda_k^B\right)^{\eta} \left(\frac{\bar{L}^B}{H_k}\right)^{\eta} \right)^{-\theta}} \right] w_n \lambda_n^B \bar{L}^B,
\end{aligned}
$$

$$
\begin{aligned}
w_i \lambda_i^B \bar{L}^B \quad &= \sum_{n \in N^A} \left[\frac{T_i \left(d_{ni} w_i^{\beta+\eta} P_i^{1-\beta-\eta} \left(\lambda_i^B\right)^{\eta} \left(\frac{\bar{L}^B}{H_i}\right)^{\eta} \right)^{-\theta}}{\sum_{k \in N^A} T_k \left(d_{nk} w_k^{\beta+\eta} P_k^{1-\beta-\eta} \left(\lambda_k^A\right)^{\eta} \left(\frac{\bar{L}^A}{H_k}\right)^{\eta} \right)^{-\theta} + \sum_{k \in N^B} T_k \left(d_{nk} w_k^{\beta+\eta} P_k^{1-\beta-\eta} \left(\lambda_k^B\right)^{\eta} \left(\frac{\bar{L}^B}{H_k}\right)^{\eta} \right)^{-\theta}} \right] w_n \lambda_n^A \bar{L}^A \\
&+ \sum_{n \in N^B} \left[\frac{T_i \left(d_{ni} w_i^{\beta+\eta} P_i^{1-\beta-\eta} \left(\lambda_i^B\right)^{\eta} \left(\frac{\bar{L}^B}{H_i}\right)^{\eta} \right)^{-\theta}}{\sum_{k \in N^A} T_k \left(d_{nk} w_k^{\beta+\eta} P_k^{1-\beta-\eta} \left(\lambda_k^A\right)^{\eta} \left(\frac{\bar{L}^A}{H_k}\right)^{\eta} \right)^{-\theta} + \sum_{k \in N^B} T_k \left(d_{nk} w_k^{\beta+\eta} P_k^{1-\beta-\eta} \left(\lambda_k^B\right)^{\eta} \left(\frac{\bar{L}^B}{H_k}\right)^{\eta} \right)^{-\theta}} \right] w_n \lambda_n^B \bar{L}^B,
\end{aligned}
$$

where $\{\bar{L}_A, \bar{L}_B\}$ are the exogenous labor endowments of each country.

Note that this wage equation for each country takes the same form as equation (3.14) on page 1734 of Alvarez and Lucas (2007). Given population shares in each location (λ_n^j for $j \in \{A, B\}$), there exists a unique wage vector $\mathbf{w} \in \Re_{++}^n$ such that $\Xi(\mathbf{w}) = 0$ by the same arguments as in the proof of Lemma 1 in Section A.1 above.

Our second step uses the population mobility condition together with the comparative statics of the wage and trade equations. The population mobility condition (46) can be re-written as follows:

$$
\begin{aligned}
\lambda_n^A &= \frac{(T_n)^{\frac{\alpha}{\theta(\beta+\eta-\alpha\beta)}} H_n}{(T_n)^{\frac{\alpha}{\theta(\beta+\eta-\alpha\beta)}} H_n + \sum_{k \neq n, k \in N^A} \left(\frac{\pi_{nn}}{\pi_{kk}}\right)^{\frac{\alpha}{\theta(\beta+\eta-\alpha\beta)}} (T_k)^{\frac{\alpha}{\theta(\beta+\eta-\alpha\beta)}} H_k}, \qquad n \in N^A, \\
\lambda_n^B &= \frac{(T_n)^{\frac{\alpha}{\theta(\beta+\eta-\alpha\beta)}} H_n}{(T_n)^{\frac{\alpha}{\theta(\beta+\eta-\alpha\beta)}} H_n + \sum_{k \neq n, k \in N^B} \left(\frac{\pi_{nn}}{\pi_{kk}}\right)^{\frac{\alpha}{\theta(\beta+\eta-\alpha\beta)}} (T_k)^{\frac{\alpha}{\theta(\beta+\eta-\alpha\beta)}} H_k}, \qquad n \in N^B,
\end{aligned} \tag{64}
$$

Equilibrium population shares across locations n within each country $j \in \{A, B\}$ can be determined using exactly the same line of reasoning as in Section A.1. Taking the limits $\lambda_n^j \to 0$ and

$\lambda_n^j \rightarrow 1$ for each country $j \in \{A, B\}$ and using continuity establishes the existence of an equilibrium allocation of population shares across locations for which the left and right-hand side of the population mobility condition (64) are equalized for each location $n \in N^j$ within each country j. This equilibrium allocation involves strictly positive population shares $\lambda_n^j > 0$ for all locations for which $T_n, H_n > 0$. Given this equilibrium allocation of population shares $\hat{\lambda}_n^j$, the wage equation (43) and trade equation (44) determine equilibrium wages for each location ($\hat{w}_n$) and hence equilibrium trade shares ($\hat{\pi}_{ni}$) as discussed above.

A.5 Data on Canadian and U.S. Regions

The source for the trade data between Canadian and U.S. regions for 1993 is Anderson and van Wincoop (2003). The sources for the population data for Canadian and U.S. regions for 1990 are Statistics Canada and the U.S. Census Bureau. To ensure positive bilateral trade flows between all locations, we aggregate Canadian provinces and U.S. states to the level of statistical regions, which are defined below. We also exclude Northern Canada, which accounts for a small share of Canadian trade and population.

Table 5: Canadian provinces and regions.

Atlantic Canada	Central Canada	Western Canada
New Brunswick	Ontario	Alberta
Newfoundland and Labrador	Quebec	British Columbia
Nova Scotia		Manitoba
Prince Edward Island		Saskatchewan

Table 6: U.S. states and regions.

North East	Mid West	South	West
Connecticut	Indiana	Delaware	Arizona
Maine	Illinois	District of Columbia	Colorado
Massachusetts	Michigan	Florida	Idaho
New Hampshire	Ohio	Georgia	New Mexico
New Jersey	Wisconsin	Maryland	Montana
New York	Iowa	North Carolina	Utah
Pennsylvania	Kansas	South Carolina	Nevada
Rhode Island	Minnesota	Virginia	Wyoming
Vermont	Missouri	West Virginia	Alaska
	Nebraska	Alabama	California
	North Dakota	Kentucky	Hawaii
	South Dakota	Mississippi	Oregon
		Tennessee	Washington
		Arkansas	
		Louisiana	
		Oklahoma	
		Texas	

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