INTERNATIONAL ECONOMIC REVIEW
Vol. 43, No. 4, November 2002

# PATH DEPENDENCE, ENDOGENOUS INNOVATION, AND GROWTH*

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The article presents a model of endogenous innovation and growth, in which technological change is path dependent. The historical pattern of technological development plays a central role in determining the pace of future technological change. Path dependence is explained using a distinction between fundamental and secondary knowledge. The economy moves endogenously between periods of drastic and nondrastic innovation. Technological lock-in is shown to be a special case of path dependence. The model provides a rationale for cycles in technological leadership. This rationale exists in equilibria with positive levels of fundamental research and in a world with no imitation.

## 1. INTRODUCTION

A variety of authors in the literatures on the history and microeconomics of technology have argued that innovation is "path dependent." That is, the historical pattern of technological development is thought to play a central role in determining the pace of future technological change.[2] An extreme example is "technological lock-in," where agents continue to employ an existing technology even though potentially more productive technologies could be found. Historical examples include the replacement of "mule-spinning" by "ring-spinning" in the eighteenth century cotton industry, the switch from the "Leblanc" to the "Solvay" soda production process, and, more recently, the continued use of the QWERTY keyboard.[3] This article presents a tractable model of endogenous innovation and growth, in which technological change is path dependent and in which technological lock-in may occur.

The analysis combines four features of technological change emphasized in empirical work and discussed further below. First, innovation is shaped by the

* Manuscript received February 4, 2001.

[1] I am especially grateful to Philippe Aghion, Steve Nickell, Jon Temple, and two anonymous referees for their very helpful comments and suggestions. I would also like to thank Anthony Atkinson, Christopher Bliss, Gavin Cameron, Cecilia Garcia-Penalosa, Peter Howitt, Louise Keely, Tim Leunig, Dan Maldoom, James Mirrlees, Michele Piccione, James Proudman, Danny Quah, and seminar participants at Nuffield College, Oxford, and the Royal Economic Society Conference for their helpful comments and suggestions. The usual disclaimer applies. Please address correspondence to: Stephen Redding, Department of Economics, London School of Economics, Houghton Street, London. WC2A 2AE. Tel: +44 171 955 7483. Fax: +44 171 831 1840. E-mail: *s.j.redding@lse.ac.uk*. Web: http://econ.lse.ac.uk/~sredding.

[2] See, in particular, Arthur (1994), David (1975, 1988), Dosi (1988), Mokyr (1990), and Rosenberg (1994).

[3] See, respectively, Broadberry (1998) and Sandberg (1969), Lindert and Trace (1971), and David (1985).

intentional choices of profit-seeking agents. Second, the discovery of new technologies is an intrinsically uncertain process. Third, technological progress is the result of a combination of "fundamental innovations," which open up whole new areas for technological development, and "secondary innovations," which are the incremental improvements that realize the potential in each fundamental innovation. Fourth, the secondary knowledge acquired for one fundamental technology (e.g., mule-spinning) is often of limited relevance for the next (e.g., ring-spinning).

The main findings of the article are as follows: First, these four features of technological change provide a microeconomic rationale for path dependence. In particular, if spillovers of secondary knowledge across fundamental technologies are incomplete, an increase in the stock of secondary knowledge relating to one fundamental technology $m$ reduces agents' incentives to engage in research directed at the discovery of fundamental technology $m+1$. Second, depending on the (random) interval of time between fundamental innovations, the economy moves endogenously through periods of "drastic" and "nondrastic" innovation. That is, depending on the (random) interval of time between fundamental innovations, a new fundamental technology either may or may not face competition from existing technologies at the profit-maximizing monopoly price.

Third, in periods of nondrastic innovation, equilibrium employment in fundamental research is monotonically decreasing in the stock of secondary knowledge accumulated for an existing fundamental technology. Fourth, technological lock-in is a special case, where the stock of accumulated secondary knowledge becomes so large that equilibrium employment in fundamental research falls to zero. Fifth, a model of endogenous technological change with path dependence provides a rationale for cycles in technological leadership, where an initially backward country catches up with and overtakes an initially more advanced country. This rationale applies in equilibria with positive levels of fundamental research and is not limited to the special case of technological lock-in. Cycles in technological leadership may occur even in a world with no international knowledge spillovers and no imitation. Once international knowledge spillovers are allowed, the extent of technological catch-up and leapfrogging depends on the relative magnitude of spillovers of fundamental and secondary knowledge.

The model's tractability enables us to consider a very general specification of secondary knowledge spillovers across fundamental technologies, which encompasses the special cases of no spillovers and perfect spillovers. It also enables us to extend the analysis in a variety of directions. First, we introduce uncertainty over the magnitude of secondary knowledge spillovers. This further generalizes the dynamics of technological change in the model. A distinction emerges between *temporary technological lock-in* (where it is not profitable to employ a new fundamental technology, once discovered, for small *realizations* of secondary knowledge spillovers) and *permanent technological lock-in* (where it is no longer profitable to search for new fundamental technologies given the *expected* magnitude of secondary knowledge spillovers). Second, we introduce multiple intermediate goods sectors. Technological change remains path dependent, and individual sectors may experience technological lock-in, whereas the economy as a whole exhibits endogenous growth as a result of both fundamental research and secondary development. Third, we show that the article's results are robust to the introduction of diminishing returns in the process of secondary development.

The article is related to three main strands of existing work. First, there is a literature that considers technological lock-in in models where the arrival of new technologies is exogenous and agents decide whether or not to adopt these technologies (see, for example, Arthur, 1989; Brezis et al., 1993; Chari and Hopenhayn, 1991; Parente, 1994; Jovanovic and Nyarko, 1996; Solow, 1997). Following a wide range of empirical evidence, this article models the arrival of new technologies as an *endogenous* process. This is not only in accord with the empirical evidence, but also yields new insights. For example, technological lock-in is a special case of a more general phenomenon (path-dependent technological change), path dependence can be explained using a distinction between fundamental and secondary knowledge, and the economy moves endogenously through periods of drastic and nondrastic innovation.

Second, the endogenous growth literature contains a number of models with one or more of the four features of technological change listed above. Thus, Aghion and Howitt (1992) and Grossman and Helpman (1991) present models of endogenous quality-augmenting innovation, in which the outcome of costly investments in R&D is uncertain, but in which there is no distinction between fundamental and secondary innovation. Aghion and Howitt (1996, 1997), Helpman and Trajtenberg (1998), Jovanovic and Rob (1990), and Young (1993) all present models in which a distinction between different types of technological change exists. However, there is no analysis of the idea that technological change is path dependent or that technological lock-in may occur.

Third, there is a literature concerned with "cycles in technological leadership," where an initially backward country catches up with and eventually leapfrogs an initially more advanced country. This hypothesis has received considerable attention in the economic history literature, including, for example, Broadberry (1994, 1998), Kindleberger (1995), and Nelson and Wright (1992). Thus, Broadberry (1994, 1998) notes that Britain's early industrial development was largely based upon low throughput, craft-based, skilled labor-intensive methods of manufacture. These techniques were progressively refined and developed during the nineteenth century, and it is argued that this provides part of the explanation for Britain's slow adoption of more modern methods of manufacture, first introduced in the United States and involving high throughput, machine-intensive, mass production of standardized products. The choice and development of these two alternative methods of manufacture are seen as a key determinant of the evolution of relative levels of productivity and income per capita.

Cycles in technological leadership have been formalized in models of technology adoption with exogenous arrival of new technologies (e.g., Brezis et al., 1993) and in models of Northern innovation and Southern imitation (e.g., Barro and Sala-i-Martin, 1995, 1997). In models of technology adoption, cycles may occur if an initially advanced country becomes locked into one technology, whereas it remains profitable for an initially backward country to adopt a more sophisticated technology. This explanation continues to exist in the present article. However, modeling the arrival of new technologies as an endogenous process yields new insights. In particular, cycles in technological leadership are a more general feature of path-dependent technological change. They exist in equilibria with positive levels of fundamental research and are not restricted to the case of technological lock-in. Cycles in technological leadership are explained by the distinction between fundamental and secondary innovation, and occur even in a world with no international knowledge spillovers and no imitation.

The article is structured as follows: Section 2 examines the empirical motivation for the four features of technological change listed above and for the findings of path dependence and technological lock-in. Section 3 introduces the basic model of economic growth with a single intermediate goods sector. Section 4 solves for general equilibrium, establishes the path-dependent nature of technological change, and proves that technological lock-in may occur. Section 5 explores the implications for final output growth. Section 6 introduces uncertainty over the magnitude of secondary knowledge spillovers. Section 7 generalizes the analysis to allow for a large number of intermediate input sectors. Section 8 summarizes the article's conclusions. An Appendix shows that the article's results are robust to introducing diminishing returns in the process of secondary development.

## 2. TECHNOLOGICAL CHANGE

Of the four features of technological change listed above, the role of intentional choices in determining the rate of innovation and the pervasive uncertainty of the innovative process are well documented (see, for example, Schmookler, 1966, Mansfield et al., 1971, and the discussion in Aghion and Howitt, 1992, and Grossman and Helpman, 1991). There is also substantial support for the idea that technological progress is achieved through a combination of fundamental innovations and a much larger number of secondary developments that realize the potential of each fundamental innovation (see, for example, Rosenberg, 1982, Mokyr, 1990, and the discussion in Aghion and Howitt, 1996, and Young, 1993).

The fourth feature of technological change is also supported by a substantial body of empirical evidence. This is the idea that the secondary knowledge acquired for one fundamental technology is often of limited relevance to the next. One example from economic history is the replacement of the "mule" (invented by Crompton in 1779) by "ring"-spinning (discovered by Thorp in 1828) in the cotton industry during the early nineteenth century.[4] Each of these technologies required a distinct set of skills and physical machinery. Mule-spinning in particular required specialized skills and considerable strength (hence operatives were predominantly male), whereas these skills were of little relevance in ring-spinning, which could in fact be carried out by a largely unskilled female labor force.

The British cotton industry had grown to be the largest in the world through the use of the mule and the earlier inventions of the spinning jenny and water frame. However, even once the technology for ring-spinning was known, the British industry was very slow to adopt the technology compared to all other major cotton producers. One of the main reasons cited by economic historians is the abundant

[4] The discussion here draws on Broadberry (1998), Mokyr (1990), and Sandberg (1969).

supply of skilled mule spinners in the United Kingdom, whose specialized skills would be rendered obsolete by the new technology.[5] Other examples include the historical switch from the Leblanc to the Solvay soda production process and the contemporary example of the acquisition of QWERTY-specific skills by touch typists.[6]

Together, these four features of technological change will provide a microeconomic explanation for path dependence. A fundamental innovation is "both . . . an artifact to be developed and improved (such as a car, an integrated circuit, a lathe, each with its particular technoeconomic characteristics)—and *a set of heuristics* (e.g., Where do we go from here? Where should we search? What sort of knowledge should we draw on?)" (Dosi, 1988, p. 1127). Each fundamental technology provides a particular set of opportunities for secondary development, and this historical path of technological development will, in general, have implications for agents' incentives to search for new fundamental technologies. In this way, an economy's or a sector's particular history of incremental development will influence endogenous rates of innovation and long-run growth.

An extreme example of path dependence is when the process of secondary development proceeds so far that there is no incentive to invest in the discovery of a new fundamental technology, and the economy becomes *locked into* an existing fundamental technology. The most frequently quoted example is the QWERTY keyboard referred to above (see David, 1985). However, there are a wide range of other examples, both from economic history (see the discussion above and Frankel, 1955), and from contemporary experience (see Shapiro and Varian, 1998, Chap. 5, for a number of IT-related examples).

## 3. THE MODEL

3.1. *Introduction.* We consider an economy populated by a sequence of overlapping generations indexed by $t \in [1, \infty)$. Each generation consists of a large number of consumer-workers ($H$) who live for two periods. Each worker is endowed with one unit of labor per period and an exogenous quantity of land ($L/H$). Time is indexed by $\tau$, and units for time are chosen such that each period of a generation's life lasts for one unit of time.[7]

The economy consists of four sectors: fundamental research, secondary development, intermediate input production, and final goods production. Intermediate inputs are indexed by their quality or productivity, and this depends upon a stock of fundamental knowledge and a stock of secondary knowledge. Fundamental knowledge is modeled as a sequence of potentially more productive blueprints for intermediate input production. The realization of the productive potential of these blueprints depends on the accumulation of secondary knowledge. Secondary knowledge is modeled as the acquisition of human capital that is specific to a particular fundamental technology, in the sense that it is more productive when used with that technology than when used with any other fundamental technology.

[5] See, in particular, Broadberry (1998, Chap. 10) and Sandberg (1969).

[6] See, respectively, Lindert and Trace (1971) and David (1985).

[7] Thus, generation $t$ is born at some time $\tau$ and dies at time $\tau + 2$. To simplify notation, we suppress the implicit dependence on time, except where important.

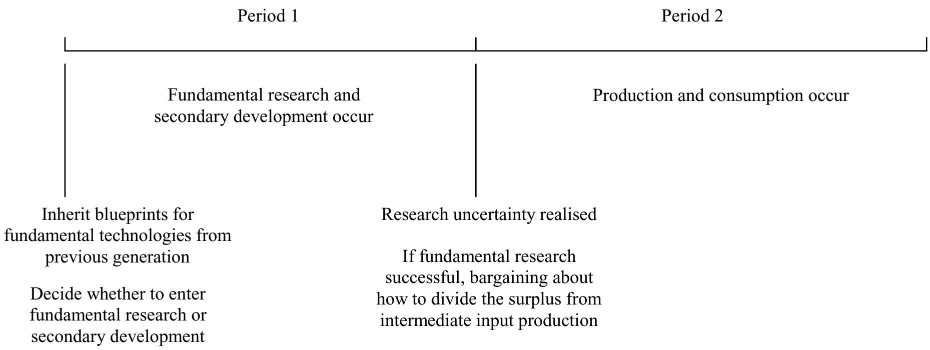


FIGURE 1

THE TIMING OF DECISIONS

The timing of decisions is summarized in Figure 1, and is as follows. At the beginning of period 1, workers inherit a stock of blueprints for fundamental technologies from the previous generation, and decide whether to engage in fundamental research or secondary development. Secondary developers spend period 1 assimilating and augmenting a body of existing secondary knowledge inherited from the previous generation. Fundamental researchers spend period 1 engaged in (uncertain) research directed at the discovery of a new, potentially more productive, fundamental technology. All research uncertainty is realized at the end of period 1.

Production and consumption take place in period 2 of workers' lives. Some secondary knowledge (fundamental technology-specific human capital) is required to produce intermediate inputs, and these are therefore produced by secondary developers in period 2. Final goods are produced with intermediate inputs and land.[8] If a worker is successful in fundamental research in period 1, she receives a one-period patent for the new fundamental technology. Bargaining with secondary developers takes place at the beginning of period 2 about how to divide the surplus from intermediate input production. If research is unsuccessful in period 1, intermediate inputs are produced with an existing fundamental technology in period 2. Since fundamental knowledge spills over across generations, all individuals have access to existing fundamental technologies, and production of intermediate inputs occurs under conditions of perfect competition.[9]

3.2. *Consumer Preferences.* Workers are endowed with one unit of labor per period. At the beginning of period 1, they decide whether to engage in fundamental research or secondary development. Since some secondary knowledge is

[8] Land is a specific factor, used only in final goods production, and could also be interpreted as physical capital.

[9] It is also possible to consider patents of more than one period in length (which requires patent rights to be enforced across generations). In this case, bargaining with secondary developers takes place both when fundamental research is successful and when it is unsuccessful. This substantially complicates the analysis, without adding any insight.

required to produce intermediate inputs in period 2, this corresponds to a decision about their lifetime labor supply. We denote the number of workers entering fundamental research by $H_t^F$ and the corresponding number entering secondary development by $H_t^S$. There is no disutility from supplying labor, and preferences are defined over consumption of the final good. Workers are assumed to be risk neutral, and the lifetime utility of a representative consumer-worker in generation $t$ is thus a linear function of second-period consumption of the final good,

$$U_t = c_{2t} \tag{1}$$

3.3. *Production and Technology.* Following Aghion and Howitt (1992), final goods output ($y$) is produced from intermediate inputs ($x$) and a sector-specific factor of production that is interpreted as land ($l$). For simplicity, we begin by considering a single intermediate goods sector. Section 7 extends the analysis to allow for multiple intermediate goods sectors. Production of final goods occurs under conditions of perfect competition and with a Cobb–Douglas technology,

$$y_{2t} = A_{2t} \cdot x_{2t}^{\alpha} l_{2t}^{1-\alpha}, \qquad 0 < \alpha < 1 \tag{2}$$

where $A_{2t}$ denotes the productivity or quality of intermediate inputs, and final goods output is chosen for the numeraire so that $p_{2t} = 1$ for all $t$.

The key departure from the standard quality ladder model is the assumption that technological progress takes the form of a sequence of fundamental technologies, each of which may be improved through a process of secondary development. Fundamental technologies are indexed by $k \in \{0, 1, ..., m\}$; $k$ denotes the interval starting with the $k$th innovation and ending with the $k + 1$st, whereas $m$ is the most advanced fundamental technology currently available. Each fundamental technology is potentially more productive than the previous one, but the realization of this productive potential is dependent on a process of secondary development. Conditional on the same level of secondary development, each successive fundamental technology has a quality or productivity of $\gamma > 1$ times the last. The stock of fundamental knowledge available to generation $t$ is determined by the most advanced fundamental technology currently available, and is thus $F_m = \gamma^m \cdot F(0)$, where $F(0)$ is normalized to 1.

The productivity of each fundamental technology may be increased through a process of secondary development. If the stock of secondary knowledge that can be employed with a fundamental technology $k$ is denoted by $\tilde{S}_k$ (referred to as the "effective stock of secondary knowledge"), then the quality or productivity of intermediate inputs produced with fundamental technology $k$ is as follows:

$$A_{2t} = A_{2tk} = (F_{2tk})^{\nu} \cdot \tilde{S}_{2tk}, \qquad \nu > 0 \tag{3}$$

That is, the quality or productivity of intermediate inputs is assumed to be a constant elasticity function of the stock of fundamental knowledge and the effective stock of secondary knowledge. The structure of knowledge is as illustrated in Figure 2.

The effective stock of secondary knowledge captures the idea that secondary skills are fundamental technology-specific and spill over imperfectly across

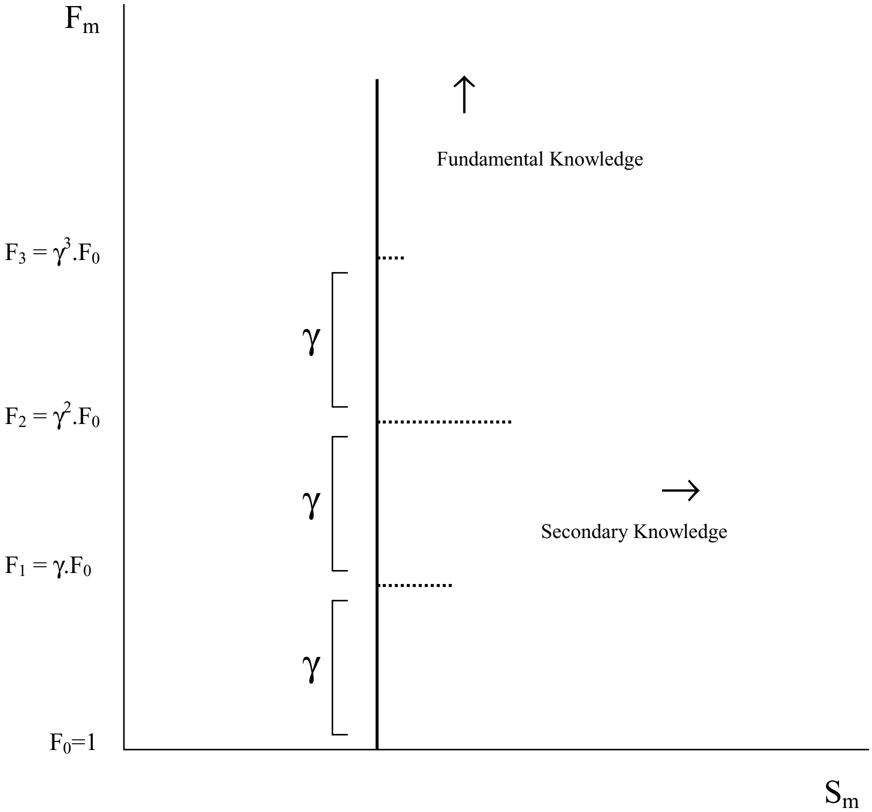


FIGURE 2

THE STRUCTURE OF KNOWLEDGE

fundamental technologies. If a worker has accumulated a quantity $S_j$ of secondary skills specific to a particular fundamental technology $j$, it is assumed that the effective stock of secondary knowledge for all fundamental technologies $k$ ($\tilde{S}_k$) is as follows:

$$\tilde{S}_k = \begin{array}{lll} \phi^z(S_j) & \text{if} & k = j - z \\ \vdots & & \\ \phi(S_j) & \text{if} & k = j - 1 \\ S_j & \text{if} & k = j \\ \phi(S_j) & \text{if} & k = j + 1 \\ \vdots & & \\ \phi^z(S_j) & \text{if} & k = j + z \end{array}, \qquad \begin{array}{c} z \geq 0, \quad j + z \leq m \\ \phi : \Re^+ \rightarrow \Re^+ \end{array} \tag{4}$$

The stock of secondary knowledge is assumed to be bounded below by 1 ($S_j \geq 1$ and $\phi(1) = 1$), and this may be interpreted as a minimum level of secondary knowledge available to all fundamental technologies.

The specification in Equation (4) is very general and encompasses a whole range of different assumptions concerning the nature of secondary knowledge spillovers. Each of these assumptions corresponds to a different set of restrictions placed upon the function $\phi(\cdot)$. For example, we may consider the two special cases of no spillovers of secondary knowledge ($\tilde{S}_{j+1} = \phi(S_j) = 1$ for all $S_j \geq 1$) and perfect (or complete) spillovers ($\tilde{S}_{j+1} = \phi(S_j) = S_j$ for all $S_j \geq 1$). The empirical discussion above suggests that secondary knowledge spillovers are in fact imperfect, and, therefore, the following restriction is placed on the function $\phi(\cdot)$:

$$0 < \frac{d(\tilde{S}_{j+1})}{dS_j}\frac{S_j}{\tilde{S}_{j+1}} = \frac{\phi'(S_j)\cdot S_j}{\phi(S_j)} < 1 \tag{5}$$

That is, the accumulation of secondary knowledge for one fundamental technology $j$ has a positive effect on the productivity of other fundamental technologies $k \neq j$ ($\phi'(S_j) > 0$). However, the accumulation of secondary knowledge specific to fundamental technology $j$ raises the productivity of $j$ by a greater proportion than it raises the productivity of all other fundamental technologies $k \neq j$.[10]

The assumption of imperfect knowledge spillovers is plausible, and is supported by the empirical discussion above. We begin by considering the properties of the model when the assumption in (5) is made; it is straightforward to consider how the results of the analysis change if the assumption is relaxed. Since $\phi(1) = 1$, Equation (5) implies that $\phi(S_j) < S_j$ for all $S_j > 1$. That is, only some of the secondary knowledge acquired for fundamental technology $j$ can be transferred to other technologies $k \neq j$. The arrival of each fundamental innovation results in secondary knowledge obsolescence, and this is one respect in which growth is a process of creative destruction.

In the specification in Equation (4), the size of secondary knowledge spillovers depends on the distance in technology space between fundamental technologies $k$ and $j$. This is consistent with the empirical modeling of knowledge spillovers in, for example, Jaffe et al. (1993). However, it does treat less sophisticated ($k < j$) and more sophisticated ($k > j$) fundamental technologies symmetrically. One might want to allow secondary knowledge spillovers to be larger for less sophisticated technologies. To capture this, a special case where secondary knowledge spills over perfectly to all fundamental technologies $k < j$ ($\tilde{S}_k = S_j$ for all $k \leq j$), but imperfectly to all fundamental technologies $k > j$ ($\tilde{S}_k = \phi^{k-j}(S_j)$ for all $k > j$), is also considered.

All that remains to complete the specification of production is to consider the technology for intermediate inputs. It is assumed that these are produced by secondary developers according to a constant returns to scale technology,

$$x_{2t} = h_{2t} \tag{6}$$

where $h_{2t}$ denotes the number of secondary developers employed in intermediate input production in period 2.

[10] Where, from (3) and (4), the proportion by which an increase in $S_j$ raises the productivity parameter $A$ for fundamental technology $j$ is simply 1.

3.4. *Fundamental Research and Secondary Development.* Fundamental researchers spend period 1 engaged in research directed at the discovery of fundamental technology $m+1$. Following Aghion and Howitt (1992), and in line with the second characteristic of technology emphasized in the introduction, we assume that fundamental research is uncertain. Each of the $H_t^F$ individuals entering fundamental research innovates with probability $\lambda$, where $0 < \lambda < 1$. If more than one individual innovates, a one-period patent to the new fundamental technology $(m+1)$ is allocated randomly among the $H_t^F$ researchers. The probability that any one individual obtains the patent is thus,

$$\Lambda\left(H_t^F\right) \equiv \frac{1}{H_t^F}\left[1-(1-\lambda)^{H_t^F}\right] \tag{7}$$

Taking logarithms in (7) and differentiating with respect to $H_t^F$, it is clear that the probability of an individual receiving the patent $(\Lambda(H_t^F))$ is monotonically decreasing in the number of researchers $H_t^F$.[11] The aggregate probability that a new fundamental technology is discovered is simply $\Lambda(H_t^F) \cdot H_t^F$.

If workers enter secondary development, they spend period 1 assimilating and augmenting the body of secondary knowledge inherited from the previous generation. This takes the form of a distribution of effective secondary knowledge, $\tilde{S}_{2(t-1)k}$, across all known fundamental technologies $k \leq m$. Secondary developers choose endogenously for which fundamental technology $k \leq m$ to acquire secondary knowledge. If they choose to acquire secondary knowledge for a particular fundamental technology $j$, they augment the stock of secondary knowledge for this technology, $S_j$, by a constant proportion $\mu > 1$. The impact on the effective stock of secondary knowledge for all other fundamental technologies $k \neq j$ is determined using Equation (4),[12]

$$\tilde{S}_{2tk} = \begin{cases} \phi^z\left(\mu \cdot S_{2(t-1)j}\right) & \text{for} \quad k = j-z \\ \vdots & \\ \phi\left(\mu \cdot S_{2(t-1)j}\right) & \text{for} \quad k = j-1 \\ \mu \cdot S_{2(t-1)j} & \text{for} \quad k = j \\ \phi\left(\mu \cdot S_{2(t-1)j}\right) & \text{for} \quad k = j+1 \\ \vdots & \\ \phi^z\left(\mu \cdot S_{2(t-1)j}\right) & \text{for} \quad k = j+z \end{cases}, \qquad \begin{matrix} \mu > 1, \quad z \geq 0, \quad j+z \leq m \\ \phi : \Re^+ \to \Re^+ \end{matrix} \tag{8}$$

[11] An alternative would be to assume that the probability an individual researcher receives the patent is independent of $H_t^F$. For example, suppose that, if $H_t^F$ workers enter fundamental research, there is a probability $\lambda$ (where $0 < \lambda < 1$) that one researcher obtains the patent to fundamental technology $m+1$. All of the article's results are robust to this alternative specification. In particular, direct analogues of Propositions 1 and 2 exist.

[12] In the specification in (8), secondary development is *technologically* unbounded (though, in an equilibrium with positive fundamental research, it is *economically* bounded by the secondary knowledge obsolescence induced by the arrival of each fundamental technology). The Appendix extends the analysis to introduce diminishing returns to secondary development and an upper bound to the stock of secondary knowledge that may be accumulated. All of the article's results are robust to this extension.

## 4. GENERAL EQUILIBRIUM

4.1. *Definition of Equilibrium.* General equilibrium is a set of prices for final goods output, intermediate inputs of quality $k \leq m$, secondary developers working with intermediate inputs of quality $k \leq m$, and land $\{\hat{p}_{2t}, \hat{q}_{2tk}, \hat{w}_{2tk}, \hat{r}_{2t}\}$; a set of expected lifetime returns to fundamental research and secondary development $\{\hat{V}_t^F, \hat{V}_t^S\}$; an allocation of consumption, final goods production, intermediate production, employment in fundamental research, employment in secondary development, and usage of land $\{\hat{c}_{2t}, \hat{y}_{2t}, \hat{x}_{2tk}, \hat{H}_t^F, \hat{H}_t^S, \hat{l}_{2t}\}$; together with a choice of fundamental technology for secondary development ($j$, where $j \leq m$).

Given the structure of decision making in Figure 1, general equilibrium can be solved for in two stages. First, we solve for equilibrium in the final goods, intermediate inputs, secondary developers, and land markets in period 2, for a given number of individuals entering fundamental research and secondary development in period 1 ($H_t^F$ and $H_t^S$, respectively), a choice of technology for secondary development ($j$), and for each of the two possible states of the world (successful and unsuccessful research). Second, having determined the equilibrium period 2 payoffs in each state of the world as a function of $H_t^F$, $H_t^S$, and $j$, we solve for the equilibrium number of individuals entering fundamental research and secondary development in period 1 and the equilibrium choice of technology for secondary development in period 1.

Equilibrium in the final goods, intermediate inputs, secondary developers, and land markets in period 2 requires that the following set of conditions are satisfied. First, consumers choose second period consumption to maximize utility taking prices $\{\hat{p}_{2t}, \hat{q}_{2tk}, \hat{w}_{2tk}, \hat{r}_{2t}\}$ as given and subject to their budget constraints. Second, final goods producers choose output, usage of intermediate inputs, and usage of land to maximize profits taking prices $\{\hat{p}_{2t}, \hat{q}_{2tk}, \hat{w}_{2tk}, \hat{r}_{2t}\}$ as given and subject to the production technology. Perfect competition and constant returns to scale imply that, in an equilibrium with positive final goods output, there are zero equilibrium profits in the final goods sector.

Third, equilibrium in the intermediate input and secondary developer markets depends upon whether or not a fundamental innovation occurs. If research is unsuccessful, intermediate inputs are produced with an existing fundamental technology $k \leq m$ under conditions of perfect competition. If research is successful, the owner of the patent to the new fundamental technology $m+1$ bargains with secondary developers at the beginning of period 2 about how to divide the surplus from intermediate input production.

The objective of the fundamental researcher with technology $m+1$ is to maximize the profit from intermediate input production ($\pi_{2t(m+1)} = q_{2t(m+1)} \cdot x_{2t(m+1)} - w_{2t(m+1)}h_{2t(m+1)}$), whereas the objective of secondary developers is to maximize the surplus from working with technology $m+1$ rather than some other technology $k \leq m$ ($[w_{2t(m+1)} - \sup_{k \leq m}(w_{2tk})] \cdot h_{2t(m+1)}$). In bargaining over employment and wages, the fundamental researcher and secondary developers take as given the derived demand curve for intermediate inputs produced with technology $m+1$ ($q_{2t(m+1)} = \alpha F_{2t(m+1)}^{\nu} \tilde{S}_{2t(m+1)} x_{2t(m+1)}^{\alpha-1} l_{2t}^{1-\alpha}$), the price of intermediate inputs produced with other technologies ($q_{2tk}$ for all $k \leq m$), the wages offered by other technologies ($w_{2tk}$ for all $k \leq m$), and the rental rate for land ($r_{2t}$). All bargaining power is assumed to reside with the fundamental researcher, who therefore makes a take-it-or-leave it offer to secondary developers.

The fourth condition for equilibrium in period 2 is that demand equals supply for secondary developers, intermediate inputs, land, and final goods. Having determined period 2 equilibrium as a function of the state of the world, $H_t^F$, $H_t^S$, and $j$, the final conditions for general equilibrium are that (a) the expected lifetime return to fundamental research equals the expected lifetime return to secondary development for positive levels of fundamental research ($\hat{V}_t^F = \hat{V}_t^S$ for $\hat{H}_t^F > 0$), (b) the fundamental technology chosen for secondary development $j$ offers the highest period 2 equilibrium wage ($w_{2tj} \geq w_{2tk}$ for all $k \leq m$), and (c) $\hat{H}_t^S = H - \hat{H}_t^F$.

### 4.2. *Period 2 Equilibrium*

4.2.1. *Unsuccessful fundamental research* If research is unsuccessful, intermediate inputs are produced with an existing fundamental technology $k \leq m$ under conditions of perfect competition. The fundamental technology used in equilibrium to produce intermediate inputs will be the one with the highest period 2 productivity or quality (technology $n$),

$$A_{2tn} = \sup_{k \leq m} \left\{ A_{2tk} = F_{2tk}^{\nu} \cdot \tilde{S}_{2tk} \right\} \tag{9}$$

Secondary developers receive a wage equal to their value marginal product (VMP) using technology $n$,

$$\begin{aligned} \underline{\hat{w}_{2tn}} = \underline{\hat{q}_{2tn}} &= \alpha F_{2tn}^{\nu} \tilde{S}_{2tn} \left( \frac{\hat{x}_{2tn}}{\hat{l}_{2t}} \right)^{\alpha - 1} \\ &= \alpha F_{2tn}^{\nu} \tilde{S}_{2tn} \left( \frac{\hat{h}_{2tn}}{\hat{l}_{2t}} \right)^{\alpha - 1} \end{aligned} \tag{10}$$

where a bar underneath a variable indicates the state of the world where fundamental research is unsuccessful. There are zero equilibrium profits from intermediate input production. Period 2 demand for secondary developers must, in equilibrium, equal their supply, as endogenously determined by period 1 choices,

$$\underline{\hat{h}_{2tn}} = H_t^S \tag{11}$$

Equation (11) and the requirement that the land market clear ($\hat{l}_{2t} = L$) imply that period 2 final goods output is

$$\underline{\hat{y}_{2t}} = A_{2tn} \cdot \left( H_t^S \right)^{\alpha} L^{1-\alpha} \tag{12}$$

In equilibrium, the rental rate on land equals its VMP using fundamental technology $n$. Imposing the requirement that the final goods market clears, equilibrium period 2 consumption of the final good is obtained:

$$\underline{\hat{c}_{2t}} = \underline{\hat{y}_{2t}} \tag{13}$$

4.2.2. *Successful fundamental research.* A successful researcher receives a patent for the new fundamental technology $m+1$, and is the monopoly supplier of intermediate inputs produced using that technology. All bargaining power is assumed to reside with the fundamental researcher. She therefore chooses output and wages to maximize profits, subject to the derived demand curve for intermediate inputs, the production technology, the constraint that the wage offered to secondary developers is greater than or equal to the wage received with technologies $k \leq m$, and the constraint that final goods production using intermediate inputs produced with fundamental technology $m+1$ is no more expensive than using those produced with other technologies $k \leq m$:

$$\max_{x_{2t(m+1)}, w_{2t(m+1)}} \left\{ q_{2t(m+1)} \cdot x_{2t(m+1)} - w_{2t(m+1)} h_{2t(m+1)} \right\} \tag{14}$$

$$\text{subject to} \quad \begin{aligned} & x_{2t(m+1)} \geq 0 \\ & x_{2t(m+1)} = h_{2t(m+1)} \\ & w_{2t(m+1)} \geq w_{2tk} && \text{for all} \quad k \leq m \\ & b_{2t(m+1)}\left[q_{2t(m+1)}, r_{2t}\right] \leq b_{2tk}[q_{2tk}, r_{2t}] && \text{for all} \quad k \leq m \\ & q_{2t(m+1)} = \alpha F^{\nu}_{2t(m+1)} \tilde{S}_{2t(m+1)} \cdot x^{\alpha-1}_{2t(m+1)} l^{1-\alpha}_{2t} \end{aligned}$$

where $b_{2tk}(\,\cdot\,)$ is the unit cost of producing final goods output using intermediate inputs of fundamental technology $k$, as a function of the price of intermediate inputs ($q_{2tk}$) and the rental rate for land ($r_{2t}$). This constrained optimization problem may be written as

$$\begin{aligned} \max_{h_{2t(m+1)}, w_{2t(m+1)}} \mathcal{L} = {} & \alpha F^{\nu}_{2t(m+1)} \tilde{S}_{2t(m+1)} l^{1-\alpha}_{2t} \cdot h^{\alpha}_{2t(m+1)} - w_{2t(m+1)} h_{2t(m+1)} \\ & - \zeta_1 \Big[ \sup_{k \leq m}(w_{2tk}) - w_{2t(m+1)} \Big] \\ & - \zeta_2 \Big[ b_{2t(m+1)}\big[q_{2t(m+1)}, r_{2t}\big] - \min_{k \leq m} b_{2tk}\big[q_{2tk}, r_{2t}\big] \Big] \\ & - \zeta_3 [0 - h_{2t(m+1)}] \end{aligned} \tag{15}$$

The first-order conditions are

(16)

$$\alpha^2 F^{\nu}_{2t(m+1)} \tilde{S}_{2t(m+1)} \cdot h^{\alpha-1}_{2t(m+1)} l^{1-\alpha}_{2t} - w_{2t(m+1)} - \zeta_2 \cdot \frac{\partial b_{2t(m+1)}(\,\cdot\,)}{\partial q_{2t(m+1)}} \cdot \frac{\partial q_{2t(m+1)}}{\partial h_{2t(m+1)}} + \zeta_3 = 0$$

$$-h_{2t(m+1)} + \zeta_1 = 0 \tag{17}$$

$$\zeta_1 \Big[ \sup_{k \leq m}(w_{2tk}) - w_{2t(m+1)} \Big] = 0 \tag{18}$$

$$\zeta_2 \Big[ b_{2t(m+1)} \left[q_{2t(m+1)}, r_{2t}\right] - \min_{k \leq m} b_{2tk} \left[q_{2tk}, r_{2t}\right] \Big] = 0 \tag{19}$$

$$\zeta_3\left[0 - h_{2t(m+1)}\right] = 0 \tag{20}$$

The successful researcher faces potential competition from all existing fundamental technologies $k \leq m$. Each individual in generation $t$ has access to the blueprints for these technologies, and the wage received from producing intermediate inputs using fundamental technology $k \leq m$ is secondary developers' VMP. Thus, the outside option of secondary developers in bargaining with the successful fundamental researcher is their VMP with the most productive of all existing fundamental technologies $k \leq m$ (technology $n$). From Equation (14), profits from intermediate input production with fundamental technology $m+1$ are monotonically decreasing in the wage paid to secondary developers. Hence, in equilibrium, the holder of the patent to fundamental technology $m+1$ will pay secondary developers a wage no higher than their outside option,

$$\bar{\bar{w}}_{2t(m+1)} = \bar{\bar{w}}_{2tn} = \bar{\bar{q}}_{2tn} = \alpha \cdot A_{2tn} \cdot \left(\bar{\bar{h}}_{2t(m+1)}\right)^{\alpha-1} (\bar{\bar{l}}_{2t})^{1-\alpha} \tag{21}$$

where a bar above a variable indicates the state of the world where fundamental research is successful.

If equilibrium output of intermediate inputs is positive ($\zeta_3 = 0$ in Equation (16)), there are two possible equilibrium values for the price of intermediate inputs produced with fundamental technology $m+1$. First, if it is cheaper for final goods producers to employ fundamental technology $m+1$ at the profit-maximizing monopoly price rather than the most productive existing technology $n$, fundamental technology $m+1$ constitutes a "drastic" innovation. In this case, the fourth constraint in Equation (14) fails to bind, and $\zeta_2 = 0$ in Equation (16). Equilibrium output of intermediate inputs produced with technology $m+1$ and equilibrium employment of secondary developers are

$$\bar{\bar{x}}_{2t(m+1)} = \bar{\bar{h}}_{2t(m+1)} = \left(\frac{\bar{\bar{w}}_{2t(m+1)}}{\alpha^2 A_{2t(m+1)} \bar{\bar{l}}_{2t}^{1-\alpha}}\right)^{1/(\alpha-1)} \tag{22}$$

Using Equation (22) in the derived demand curve for intermediate inputs, the profit-maximizing monopoly price may be written as follows:

$$\bar{\bar{q}}_{2t(m+1)} = \frac{1}{\alpha} \cdot \bar{\bar{w}}_{2t(m+1)} = \frac{1}{\alpha} \cdot \bar{\bar{w}}_{2tn} \tag{23}$$

and equilibrium profits from intermediate input production are

$$\bar{\bar{\pi}}_{2t(m+1)} = \left(\frac{1}{\alpha} - 1\right) \cdot \bar{\bar{w}}_{2tn} \cdot \bar{\bar{h}}_{2t(m+1)} \tag{24}$$

The condition for fundamental technology $m+1$ to constitute a drastic innovation is,

$$b_{2t(m+1)}((1/\alpha \cdot \bar{\bar{w}}_{2tn}), \bar{\bar{r}}_{2t}) < b_{2tn}(\bar{\bar{w}}_{2tn}, \bar{\bar{r}}_{2t})$$

where we use Equations (23) and (21) to substitute for the equilibrium price of intermediate inputs of quality $m+1$ ($q_{2t(m+1)}$), the equilibrium wage ($w_{2t(m+1)}$), and the equilibrium price of intermediate inputs of quality $n$ ($q_{2tn}$). Since the unit cost function for final goods production is Cobb–Douglas, this corresponds to the condition

$$\frac{(\gamma^{\nu})^{m+1-n} \cdot \phi^{m+1-n}(S_{2tn})}{S_{2tn}} > \left(\frac{1}{\alpha}\right)^{\alpha} \tag{25}$$

Second, if it is not cheaper for final goods producers to employ fundamental technology $m+1$ at the profit-maximizing monopoly price rather than the most productive existing technology $n$, fundamental technology $m+1$ constitutes a "nondrastic" innovation. In this case, the fourth constraint in Equation (14) binds, and $\zeta_2 > 0$ in Equation (16). The equilibrium price of intermediate inputs produced with technology $m+1$ ($q_{2t(m+1)}$) is determined by the requirement that

$$b_{2t(m+1)}\left(\bar{\bar{q}}_{2t(m+1)}, \bar{\bar{r}}_{2t}\right) = b_{2tn}(\bar{\bar{w}}_{2tn}, \bar{\bar{r}}_{2t})$$

where we use Equation (21) to substitute for the equilibrium price of intermediate inputs of quality $n$ ($q_{2tn}$). The successful researcher charges a "limit price" that leaves final goods producers indifferent between employing the new fundamental technology $m+1$ and the most productive existing technology $n$. Using the fact that the final goods unit cost function is Cobb–Douglas, the equilibrium price for a nondrastic fundamental innovation is thus

$$\bar{\bar{q}}_{2t(m+1)} = \Gamma_{2t(m+1)} \cdot \bar{\bar{w}}_{2tn} \tag{26}$$

where

$$\Gamma_{2t(m+1)} \equiv \left[\frac{(\gamma^{\nu})^{m+1-n} \cdot \phi^{m+1-n}(S_{2tn})}{S_{2tn}}\right]^{\frac{1}{\alpha}}$$

and equilibrium profits from intermediate input production are

$$\bar{\bar{\pi}}_{2t(m+1)} = \left[\Gamma_{2t(m+1)} - 1\right] \cdot \bar{\bar{w}}_{2tn} \cdot \bar{\bar{h}}_{2t(m+1)} \tag{27}$$

Thus, equilibrium profits from intermediate input production are given by Equation (24) if a fundamental innovation is drastic and Equation (27) if a fundamental innovation is nondrastic. Since $0 < \alpha < 1$, equilibrium profits are necessarily positive for a drastic fundamental innovation, and hence equilibrium output of intermediate inputs produced with fundamental technology $m+1$ will be strictly positive ($\zeta_3 = 0$ in Equation (16)). For a nondrastic fundamental innovation, equilibrium profits will only be positive if $\Gamma_{2t(m+1)} > 1$ or $(\gamma^{\nu})^{m+1-n} \cdot \phi^{m+1-n}(S_{2tn}) > S_{2tn}$. This corresponds to a requirement that the new fundamental technology

$m+1$, when discovered, is more productive than the currently most productive technology $n$. If this condition is satisfied, the limit price charged by a successful researcher yields positive equilibrium profits from intermediate input production. Thus, equilibrium output of intermediate inputs produced with fundamental technology $m+1$ will be strictly positive ($\zeta_3 = 0$ in Equation (16)).

Both (a) the condition for a fundamental innovation to be drastic and (b) the condition for positive equilibrium profits with a nondrastic fundamental innovation are functions of the secondary knowledge obsolescence induced by the discovery of the new fundamental technology ($\phi^{m+1-n}(S_{2tn}) < S_{2tn}$). Since secondary knowledge spillovers are imperfect (Equation (5)), the accumulation of secondary knowledge specific to an existing fundamental technology $n$ raises the productivity of that technology by more than it raises the productivity of fundamental technology $m+1$. An increase in the stock of accumulated secondary knowledge reduces the left-hand side of the inequality in (25) and the value of the limit price in (26). In economic terms, an increase in the stock of accumulated secondary knowledge makes it less likely that a new fundamental technology, when discovered, will constitute a drastic innovation, and reduces the value of the equilibrium limit price charged in the case of nondrastic innovation. Each of these implications receives further consideration below.

For both drastic and nondrastic fundamental innovations, equilibrium period 2 demand for secondary developers is required to equal their supply,

$$\bar{\hat{h}}_{2t(m+1)} = H_t^S \tag{28}$$

Equation (28) and the requirement that the land market clear ($\hat{l}_{2t} = L$) imply that period 2 final goods output is

$$\bar{\hat{y}}_{2t} = A_{2t(m+1)} \cdot \left(H_t^S\right)^{\alpha} L^{1-\alpha} \tag{29}$$

In equilibrium, the rental rate on land equals its VMP using fundamental technology $m+1$. Imposing the requirement that the final goods market clears, equilibrium period 2 consumption of the final good is obtained:

$$\bar{\hat{c}}_{2t} = \bar{\hat{y}}_{2t} \tag{30}$$

4.3. *Period 1 Choice of Technology for Secondary Development.* Equations (10), (11), (21), and (28) imply that the equilibrium wage of secondary developers is the same in the case of successful and unsuccessful research, and equals their VMP with the most productive of the existing fundamental technologies $k \leq m$ in period 2 (technology $n$). Accumulating secondary knowledge raises the productivity of the fundamental technology for which it is acquired by a constant proportion $\mu > 1$ and the productivity of all other fundamental technologies by a smaller proportion (Equation (8)). In period 1, secondary developers will therefore choose to acquire skills for the technology $j$ that was the most productive of all existing technologies $k \leq m$ in period 2 of the previous generation $t-1$. After further secondary development, this technology will remain the most productive in period

2 of generation $t$, and will yield the highest period 2 wage for secondary developers. In terms of the notation above, $j = n$,

$$A_{2tj} = A_{2tn} = \sup_{k \leq m} \left\{ A_{2tk} = F_{2tk}^{\nu} \cdot \tilde{S}_{2tk} \right\}$$

In principle, any of the existing fundamental technologies $k \leq m$ may be the most productive. However, if we begin with an initial distribution of effective secondary knowledge such that all fundamental technologies have the same stock of effective secondary knowledge ($\tilde{S}_{20k} = \tilde{S}_{20}$ for all $k$), the most advanced fundamental technology $m$ will always be chosen for secondary development in each subsequent generation. This can be seen clearly for generation $t = 1$. The most advanced fundamental technology $m(1)$ has a higher stock of fundamental knowledge and the same stock of secondary knowledge as all other technologies, and will therefore be chosen for secondary development. Secondary development will lead to an increase in the productivity of this technology relative to all existing technologies $k \leq m(1)$. If no fundamental innovation occurs during generation 1's lifetime, it will therefore remain optimal for generation 2 to choose for the most advanced fundamental technology $m(2) = m(1)$ for secondary development. If a fundamental innovation occurs during generation 1's lifetime, it will again remain optimal for generation 2 to choose the most advanced fundamental technology $m(2) > m(1)$ for secondary development. This follows from the fact that, for equilibrium fundamental research to be positive, we require profits from intermediate input production with a new fundamental technology to be strictly positive. However, we saw above that a necessary and sufficient condition for profits to be strictly positive is that the new fundamental technology, when discovered, has a higher level of productivity than the most productive of all existing technologies.

Except for Section 6, the remainder of the article will be concerned with equilibria where the currently most productive fundamental technology $m$ is chosen for secondary development. Hence, except where otherwise indicated, we substitute $m$ for $n$ in the analysis that follows. It is straightforward to consider other equilibria. Section 6 introduces uncertainty over the magnitude of secondary knowledge spillovers. In this case, a restriction on initial conditions no longer ensures that the most advanced fundamental technology $m(t)$ is the most productive in all subsequent generations $t$. A newly discovered fundamental technology may have a lower *realized* level of productivity than the currently most productive fundamental technology, and will not be used for intermediate input production or chosen for secondary development by subsequent generations.

4.4. *Equilibrium Levels of Fundamental Research and Secondary Development.* Having determined the period 1 choice of fundamental technology for secondary development, this subsection endogenizes the number of individuals entering fundamental research and secondary development, and solves for general equilibrium. In an equilibrium with positive levels of fundamental research, we require the expected lifetime return from fundamental research ($\hat{V}_t^F$) to equal the expected lifetime return from secondary development ($\hat{V}_t^S$),

$$\hat{V}_t^F = \hat{V}_t^S \tag{31}$$

With probability $\Lambda(H_t^F)$, an individual researcher obtains the patent to the next fundamental technology $m+1$ and enjoys an equilibrium flow of profits equal to (24) in the case of drastic innovation and (27) in the case of nondrastic innovation. With probability $(1-\Lambda(H_t^F))$, she fails to obtain the patent to fundamental technology $m+1$ and receives zero period 2 returns from fundamental research.[13] The expected lifetime return from fundamental research is thus

$$\hat{V}^F_{t(m+1)} = \Lambda\big(\hat{H}_t^F\big) \cdot \big[\Omega_{2t(m+1)} - 1\big] \cdot \hat{w}_{2tm} \hat{H}_t^S \tag{32}$$

where

$$\Omega_{2t(m+1)} = \begin{cases} \frac{1}{\alpha} & \text{if} \quad \gamma^{\nu}\phi(S_{2tm})/S_{2tm} > \left(\frac{1}{\alpha}\right)^{\alpha} \\ \Gamma_{2t(m+1)} & \text{if} \quad \gamma^{\nu}\phi(S_{2tm})/S_{2tm} \leq \left(\frac{1}{\alpha}\right)^{\alpha} \end{cases} \tag{33}$$

From the analysis above, the period 2 equilibrium wage of a secondary developer equals her VMP with the currently most productive fundamental technology ($j = n = m$). This is true irrespective of whether fundamental research is successful in period 1. The expected lifetime return from secondary development is thus

$$\hat{V}^S_{tm} = \hat{w}_{2tm} \tag{34}$$

In equilibrium, we require that the number of secondary developers equals the supply of workers minus the number of fundamental researchers,

$$\hat{H}_t^S = H - \hat{H}_t^F \tag{35}$$

Using Equations (32), (34), and (35) in the requirement that the expected lifetime return to fundamental research equals the expected lifetime return to secondary development (31), we obtain

$$1 = \Lambda\big(\hat{H}_t^F\big) \cdot \big[\Omega_{2t(m+1)} - 1\big] \cdot \big(H - \hat{H}_t^F\big) \tag{36}$$

Equation (36) determines the equilibrium allocation of workers to fundamental research and secondary development. The left- and right-hand sides of the equation may be interpreted as the private marginal cost and benefit of fundamental research, respectively. Since the terms $\Lambda(H_t^F)$ and $(H - H_t^F)$ are both monotonically decreasing in $H_t^F$, we immediately obtain Proposition 1.

PROPOSITION 1. *If $\Omega_{2t(m+1)} > 1$ and $\Lambda(1) \cdot [\Omega_{2t(m+1)} - 1] \cdot (H-1) > 1$, a unique positive equilibrium level of employment in fundamental research, $\hat{H}_t^F$, exists.*

[13] Although the researcher receives a period 2 income of $\hat{r}_{2t} \cdot (L/H)$ from her endowment of land.

PROOF. Proposition 1 follows immediately from Equation (36). ■

Proposition 1 makes clear that, for positive equilibrium levels of fundamental research to occur, two conditions must be satisfied. First, we require $\Omega_{2t(m+1)} > 1$. This is a requirement that equilibrium profits from intermediate input production with the new fundamental technology, if discovered, are strictly positive. As already discussed, this condition is necessarily satisfied for drastic innovations. For nondrastic innovations, we saw that equilibrium profits from intermediate input production will only be positive if the new fundamental technology $m+1$ is more productive than the currently most productive technology $m$. The most interesting set of parameter values are those where, in the absence of secondary development ($S_{2(t-1)m} = 1$ and $S_{2tm} = \mu \cdot 1$), a new fundamental technology would constitute a drastic innovation ($\gamma^{\nu} \cdot \phi(\mu)/\mu > \alpha^{-\alpha}$, which must hold for sufficiently large $\gamma$). In this case, the economy moves endogenously between periods of drastic and nondrastic fundamental innovation, depending on the (random) interval between the discovery of fundamental technologies. The longer the interval of time since the discovery of the last fundamental technology, the greater the accumulated stock of secondary knowledge ($S_{2(t-1)m}$) and the more likely a fundamental innovation will be nondrastic.

The second condition for positive equilibrium fundamental research is $\Lambda(1) \cdot [\Omega_{2t(m+1)} - 1] \cdot (H-1) > 1$. That is, we require the expected lifetime return to fundamental research to exceed the expected lifetime return to secondary development for the first worker entering fundamental research. Whether this condition is satisfied depends on the accumulated stock of secondary knowledge relating to fundamental technology $m$. The most interesting set of parameter values are again those where, in the absence of secondary development ($S_{2(t-1)m} = 1$ and $S_{2tm} = \mu \cdot 1$), the condition is satisfied. That is, in the absence of secondary development, the expected lifetime return to fundamental research exceeds the expected lifetime return to secondary development for the first worker entering fundamental research, and equilibrium employment in fundamental research is strictly positive. This must be the case for sufficiently large values of $\lambda$ and $\gamma$, sufficiently small values of $\alpha$, and sufficiently large values of $H$. The interior equilibrium is the interesting case, and therefore the remainder of the article concentrates upon it. However, it is also possible to analyze the corner equilibrium, where, even in the absence of secondary development, equilibrium employment in fundamental research is zero.

The determination of equilibrium research employment is shown diagrammatically in Figure 3. Many of the comparative statics of the model are as expected from the quality ladder model without a distinction between fundamental research and secondary development. For example, equilibrium fundamental research is monotonically increasing in the probability of fundamental innovation, $\lambda$, and the supply of labor, $H$. However, unlike the conventional quality ladder model, the stock of secondary knowledge accumulated for fundamental technology $m$ ($S_{2(t-1)m}$) plays a central role in determining the equilibrium amount of fundamental research directed at the discovery of fundamental technology $m+1$.

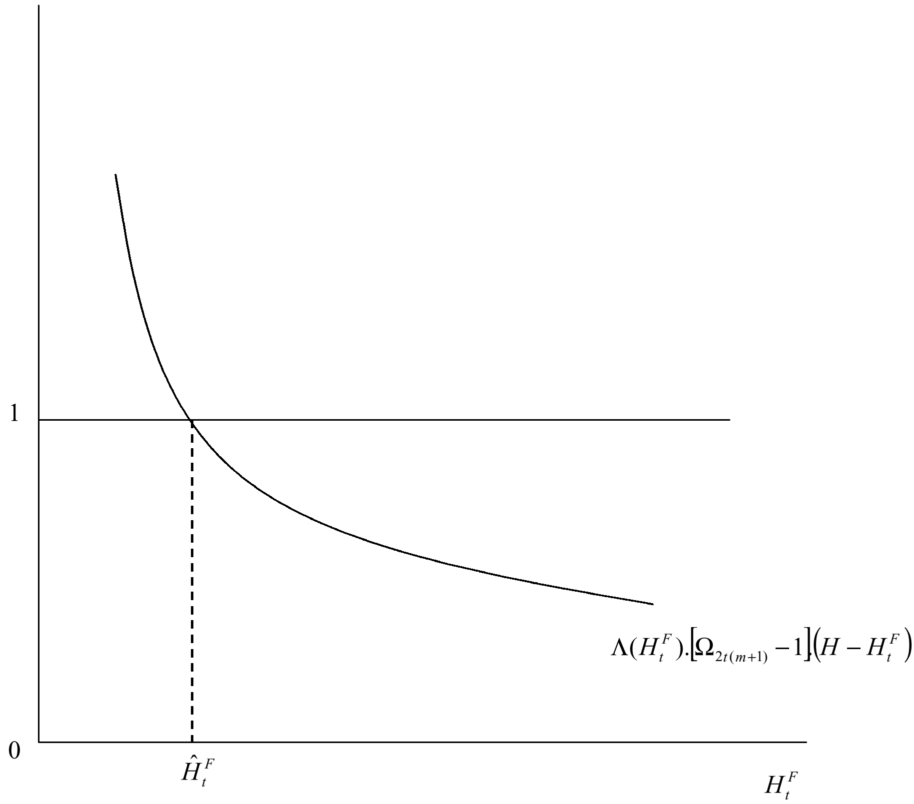


FIGURE 3

EQUILIBRIUM EMPLOYMENT IN FUNDAMENTAL RESEARCH

PROPOSITION 2. *Under the assumption of imperfect secondary knowledge spillovers* $(0 < \phi'(S_m) \cdot S_m/\phi(S_m) < 1)$,

(a) *There is a critical value for the accumulated stock of secondary knowledge,* $S^0_{2(t-1)m} \geq 1$, *such that, when* $S^0_{2(t-1)m}$ *is attained, fundamental technology* $m + 1$ *becomes a nondrastic innovation.*
(b) *There is a second critical value for the accumulated stock of secondary knowledge,* $S^1_{2(t-1)m} > 1$, *such that, when* $S^1_{2(t-1)m}$ *is attained, equilibrium employment in fundamental research is zero and technological lock-in occurs.*
(c) *For the range of values for the stock of accumulated secondary knowledge,* $S_{2(t-1)m}$, *in which fundamental innovation is nondrastic* $(S^0_{2(t-1)m} \leq S_{2(t-1)m} \leq S^1_{2(t-1)m})$, *equilibrium employment in fundamental research is monotonically decreasing in the stock of accumulated secondary knowledge.*

PROOF. See Appendix.

The effect of the accumulated stock of secondary knowledge ($S_{2(t-1)m}$) on equilibrium incentives to engage in fundamental research provides the sense in which technological change is path dependent. As the (random) interval of time since the discovery of fundamental technology $m$ increases, the accumulation of secondary knowledge relating to fundamental technology $m$ affects workers' incentives to engage in research directed at the discovery of fundamental technology $m+1$. In this way, the historical path of secondary development influences current incentives to engage in fundamental research.

There are three implications of path dependence for endogenous rates of innovation. First, as the stock of accumulated secondary knowledge increases, the secondary knowledge obsolescence that would be induced by the discovery of fundamental technology $m+1$ means this technology (when discovered at a future point in time) is less likely to be a drastic innovation (Proposition 2(a)). Second, once the stock of accumulated secondary knowledge ($S_{2(t-1)m}$) becomes sufficiently large that fundamental innovation $m+1$ is nondrastic, further secondary knowledge accumulation reduces the equilibrium value of the limit price that can be charged by the researcher who discovers fundamental technology $m+1$. In this way, secondary knowledge accumulation reduces the equilibrium profits to be made from intermediate input production using fundamental technology $m+1$ and reduces current incentives to engage in fundamental research. Thus, for the range of values for the stock of accumulated secondary knowledge ($S_{2(t-1)m}$) in which fundamental innovation is nondrastic, equilibrium employment in fundamental research is monotonically decreasing in $S_{2(t-1)m}$ (Proposition 2(c)).

Third, if sufficient further secondary knowledge accumulation occurs, the equilibrium limit price that can be charged by a successful researcher and the corresponding profits from intermediate input production may fall to such an extent that the expected lifetime return for the first worker entering fundamental research no longer exceeds the expected lifetime return from secondary development (Proposition 2(b)). In this case, the economy becomes *locked into* the existing fundamental technology, even though fundamental research would be profitable in the absence of secondary development. Despite the fact that potentially more productive fundamental technologies could be discovered (more productive after the same level of secondary development), the secondary development of the existing fundamental technology has proceeded to such an extent that it is no longer profitable to search for these alternative fundamental technologies.

In a model with endogenous fundamental innovation, technological lock-in is a special case of a more general phenomenon: path-dependent technological change. How far secondary development must proceed before technological lock-in will occur is endogenously determined by the values of the parameters $\lambda$, $\gamma$, $\nu$, $\mu$, $H$ and the form of the function $\phi(\cdot)$. Since the interval between two fundamental innovations is a random variable, it follows that, for any fundamental technology $m$, there is a finite probability that technological lock-in will occur. The time path of equilibrium research employment as a function of the stock of accumulated secondary knowledge is plotted in Figure 4.

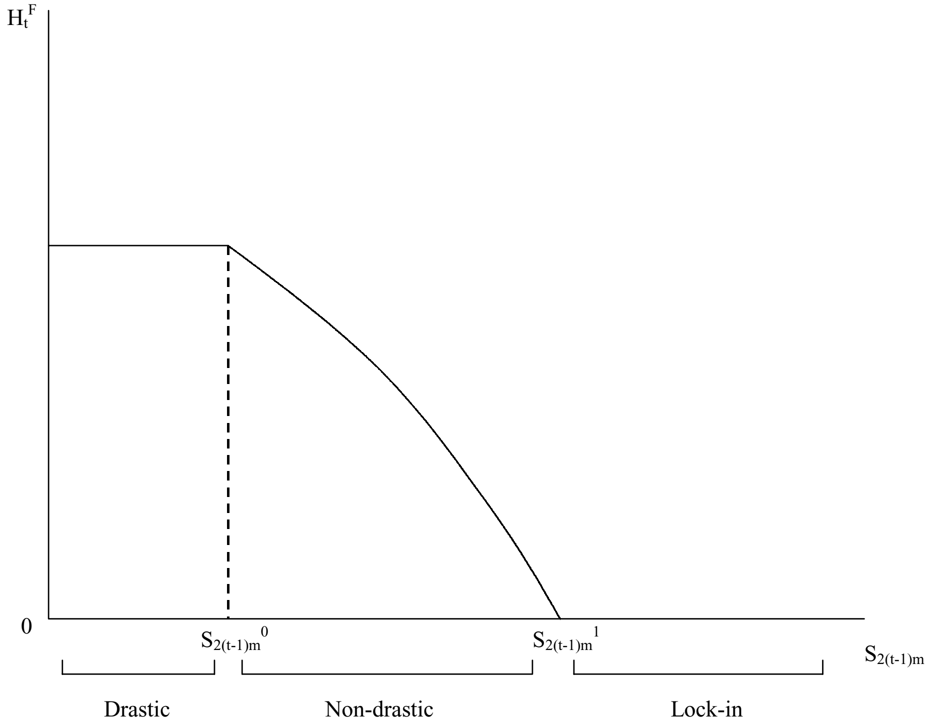


FIGURE 4

EQUILIBRIUM EMPLOYMENT IN FUNDAMENTAL RESEARCH AS A FUNCTION OF THE ACCUMULATED STOCK OF SECONDARY KNOWLEDGE

In Section 7, we extend the analysis to allow for many intermediate goods sectors. Technological lock-in may occur in individual sectors of the economy, whereas others continue to exhibit positive levels of both fundamental research and secondary development. First, Section 5 considers the implications of path-dependent technological change for output growth with a single intermediate goods sector. Section 6 introduces uncertainty over the magnitude of secondary knowledge spillovers.

## 5. FINAL OUTPUT GROWTH

From Equation (2), the rate of growth of final goods output between any two generations $t-1$ and $t$ will be a function of three sets of influences: (a) fundamental knowledge accumulation, (b) secondary knowledge accumulation, and (c) changes in employment in the intermediate goods sector. In particular, the rate of output growth will depend upon whether or not a fundamental innovation occurs in generation $t$. The discovery of a new fundamental technology $m+1$ in generation $t$ has two offsetting effects on final goods output. On the one hand, it increases the stock of fundamental knowledge, which raises final goods output ($F_{2t(m+1)} = \gamma \cdot F_{2(t-1)m}$). On the other hand, it induces secondary knowledge obsolescence, which reduces final goods output ($\tilde{S}_{2t(m+1)} = \phi(S_{2(t-1)m}) < S_{2(t-1)m}$) . If a fundamental innovation occurs in generation $t$, the rate of growth of final goods output is, from Equation (2),

$$(37) \qquad \bar{\zeta}_t = \ln\left(\frac{y_{2t}}{y_{2(t-1)}}\right) = \nu \cdot \ln\gamma + \ln\left(\frac{\phi(\mu \cdot S_{2(t-1)m})}{S_{2(t-1)m}}\right) + \alpha \cdot \ln\left(\frac{H - \hat{H}_t^F}{H - \hat{H}_{t-1}^F}\right)$$

In an equilibrium with positive fundamental research, we saw that the new fundamental technology $m+1$ must have a higher level of productivity than the currently most productive technology $m$: $\Gamma_{2t(m+1)} > 1$ or $\gamma^\nu \phi(S_{2tm}) > S_{2tm}$. Therefore, the sum of the first two terms in Equation (37) must be strictly positive. In economic terms, the positive direct effect from increased fundamental knowledge exceeds the negative indirect effect from secondary knowledge obsolescence.

The presence of the third term in Equation (37) is the result of the path-dependent nature of technological change. Proposition 2 established that, if a fundamental innovation is nondrastic, equilibrium employment in fundamental research is monotonically decreasing in the stock of accumulated secondary knowledge, $S_{2(t-1)m}$ (as illustrated in Figure 4). Therefore, for nondrastic fundamental innovations, equilibrium employment in fundamental research in generation $t$ ($\hat{H}_t^F$) will differ from that in generation $t-1$ ($\hat{H}_{t-1}^F$), depending upon the stock of accumulated secondary knowledge inherited by each generation. If generation $t$ inherits a larger stock of accumulated secondary knowledge than generation $t-1$ (which depends upon whether a fundamental innovation occurred in generation $t-1$), equilibrium employment in fundamental research will be lower ($\hat{H}_t^F < \hat{H}_{t-1}^F$). Generation $t$ allocates more labor to secondary development and intermediate input production. This constitutes an additional source of growth in final goods output, and is reflected in the third term in Equation (37).

If a fundamental innovation does not occur in generation $t$, the rate of growth of final goods output is, from Equation (2),

$$(38) \qquad \underline{\zeta}_t = \ln\left(\frac{y_{2t}}{y_{2(t-1)}}\right) = \ln\mu + \alpha \cdot \ln\left(\frac{H - \hat{H}_t^F}{H - \hat{H}_{t-1}^F}\right)$$

The accumulation of further secondary knowledge relating to fundamental technology $m$ raises final goods output by the proportion $\mu > 1$. The presence of the second term in Equation (38) reflects the path-dependent nature of technological change in exactly the same way as above.

The *expected* rate of growth of final goods output between generations $t$ and $t-1$ at the beginning of generation $t$ ($\mathrm{E}[\ln(y_t/y_{t-1})]$) is a weighted average of $\bar{\zeta}_t$ and $\underline{\zeta}_t$, with weights $\Lambda(\hat{H}_t^F) \cdot \hat{H}_t^F$ and $(1 - \Lambda(\hat{H}_t^F) \cdot \hat{H}_t^F)$, respectively. In principle, $\bar{\zeta}_t$ may be either higher or lower than $\underline{\zeta}_t$, depending upon the size of fundamental innovations and the extent to which secondary knowledge spills over across fundamental technologies (depending upon the relative size of the first two terms in Equation (37) and the first term in Equation (38)). However, if fundamental innovations are sufficiently large (large $\gamma$), $\bar{\zeta}_t$ must necessarily exceed $\underline{\zeta}_t$. In this case, the expected rate of growth of final goods output is monotonically increasing in the probability of fundamental innovation $\Lambda(\hat{H}_t^F) \cdot \hat{H}_t^F = [1 - (1-\lambda)^{\hat{H}_t^F}]$.

The probability of fundamental innovation $\Lambda(\hat{H}_t) \cdot \hat{H}_t$ is monotonically increasing in equilibrium employment in fundamental research. The effect of the probability of fundamental innovation on the economy's expected growth rate provides one way in which path-dependent technological change influences economic growth. Proposition 2(c) established that, for nondrastic fundamental innovations, equilibrium employment in fundamental research is monotonically decreasing in the accumulated stock of secondary knowledge ($S_{2(t-1)m}$). Thus, as the secondary development of an existing fundamental technology $m$ proceeds, equilibrium research employment and hence the probability of fundamental innovation fall over time. The secondary development of one fundamental technology reduces employment in fundamental research and decreases the economy's expected rate growth.

For both drastic and nondrastic fundamental innovations, the accumulation of secondary knowledge relating to an existing fundamental technology $m$ has a negative direct effect upon the economy's expected rate of growth. With imperfect knowledge spillovers ($\phi'(S_m) \cdot S_m/\phi(S_m) < 1$), the accumulation of secondary knowledge relating to fundamental technology $m$ raises the productivity of fundamental technology $m$ by a greater proportion than technology $m+1$. Hence, the larger an economy's accumulated stock of secondary knowledge ($S_{2(t-1)m}$), the greater the fall in the quality or productivity of intermediate inputs as a result of secondary knowledge obsolescence when fundamental technology $m+1$ is discovered. A larger stock of accumulated secondary knowledge thus reduces the rate of growth of final goods output when a fundamental innovation occurs ($\bar{\zeta}_t$) and hence the economy's expected rate of growth (more formally, the second term in Equation (37) is monotonically decreasing in $S_{2(t-1)m}$).

In Proposition 2(b), it is established that, if secondary development proceeds far enough ($S_{2(t-1)m} \geq S^1_{2(t-1)m}$), the economy will become *locked into* an existing fundamental technology. In this case, the actual and expected rate of growth of final goods output will equal $\underline{\zeta}_t$ with $\hat{H}_t^F = 0$. It has already been seen that, for sufficiently large fundamental innovations (large $\gamma$) and a sufficiently small rate of secondary development (as $\mu \to 1$), $\underline{\zeta}_t < \bar{\zeta}_t$. The economy's expected rate of growth when technological lock-in occurs is thus lower than in an equilibrium characterized by positive amounts of fundamental research; the economy becomes locked into a low-growth equilibrium.

Path dependence in a model of endogenous innovation provides a rationale for "cycles in technological leadership," where an initially backward country catches up with and eventually leapfrogs an initially more advanced country. This rationale exists in equilibria characterized by positive levels of fundamental research; the point is made most clearly with an example. Consider two countries ($A$ and $B$) in generation $t$. The two countries may trade the homogenous final good, but we begin by assuming no international knowledge spillovers. The two countries have identical stocks of accumulated secondary knowledge for the current state-of-the-art fundamental technology ($m$). However, country $A$ has a more sophisticated fundamental technology ($m^A > m^B$), and this is reflected in a higher level of productivity and income per capita in country $A$. Consider the evolution of technology from generation $t$ onwards. Suppose that country $A$ experiences a series of failures in fundamental research. Secondary knowledge specific to fundamental technology $m^A$ accumulates over time. If this continues for a sufficiently long length of time, the existing fundamental technology will become so productive that fundamental technology $m^A + 1$, if discovered, would constitute a nondrastic innovation. Further secondary knowledge accumulation will reduce equilibrium employment in fundamental research and hence reduce the probability that fundamental technology $m^A + 1$ is actually discovered.

In contrast, suppose that country $B$ experiences a series of research successes. Productivity and income per capita will rise as a result of a higher stock of fundamental knowledge. Secondary knowledge obsolescence induced by the discovery of new fundamental technologies means that the successors to technology $m^B$ are either more likely to be drastic innovations or, if they remain nondrastic, will exhibit higher levels of equilibrium fundamental research employment. This increases the probability of further fundamental innovation, with consequent increases in productivity and income per capita. Thus, by generation $t' > t$, country $B$ may have overtaken or leapfrogged country $A$, both in the sense of acquiring a more sophisticated fundamental technology ($m^B > m^A$) and having a higher level of income per capita.

Once international knowledge spillovers are introduced into this framework, the potential for catch-up and leapfrogging depends upon the magnitude of international spillovers of fundamental and secondary knowledge. For example, if there are fundamental knowledge spillovers but no secondary knowledge spillovers, this increases the ability of an initially backward country to overtake or leapfrog its initially more advanced counterpart. Productivity in the backward country rises because of a higher stock of fundamental knowledge. Moreover, the obsolescence of secondary knowledge implied by the switch to a more advanced fundamental technology means that fundamental innovations are more likely to be drastic or, if they are nondrastic, will be characterized by higher equilibrium levels of fundamental research employment. Secondary knowledge spillovers with no fundamental knowledge spillovers have exactly the opposite effect. Since fundamental knowledge is modeled as a sequence of potentially more productive blueprints, whereas secondary knowledge takes the form of human capital, it seems plausible that spillovers of fundamental knowledge are larger than those of secondary knowledge.

## 6. UNCERTAIN SECONDARY KNOWLEDGE SPILLOVERS

The analysis so far has presented a model of fundamental research and secondary development, where technological change is path dependent and technological lock-in may occur. The extent of secondary development of one fundamental technology affects agents' incentives to search for more advanced fundamental technologies. The analysis is consistent with the large empirical literature emphasizing the endogeneity of technological change and the wide range of empirical studies arguing that technological change is path dependent. This section extends the analysis to allow for uncertainty in the extent of secondary knowledge spillovers. This results in more general dynamics, whereby, even if fundamental research is profitable, a newly discovered fundamental technology may not be selected for intermediate input production or chosen for secondary development by subsequent generations.

The specification of secondary development is exactly as in Sections 3.3 and 3.4, except that we allow for uncertainty over the function $\phi(\cdot)$, which determines the extent of secondary knowledge obsolescence. Secondary knowledge spillovers are always imperfect in the sense of satisfying the inequality in (5), but they are "large" with probability $\chi$ and "small" with probability $(1-\chi)$. More formally,

$$\tilde{S}_{m+1} = \chi \cdot \bar{\phi}(S_m) + (1-\chi) \cdot \underline{\phi}(S_m), \qquad 0 < \chi < 1 \tag{39}$$

where

$$\begin{cases} \text{both } \bar{\phi}(S_m) \text{ and } \underline{\phi}(S_m) \text{ satisfy the restriction in (5)} \\ \bar{\phi}(S_m) > \underline{\phi}(S_m) \ \text{ for all } S_m > 1 \end{cases}$$

Equilibrium employment in fundamental research is determined in exactly the same way as above, except that Equations (36) and (33) must be modified to take into account that there are two possible values for the function $\phi(\cdot)$. That is, equilibrium employment in fundamental research depends upon *expected* secondary knowledge spillovers, whereas the decision whether to actually produce intermediate inputs with a new fundamental technology $m+1$ depends upon the *realized* value of secondary knowledge spillovers.

There will be a critical value for the accumulated stock of secondary knowledge, such that (a) fundamental research is profitable based on expected secondary knowledge spillovers, but (b) a newly discovered fundamental technology $m+1$ has a lower level of productivity than the currently most productive technology $n$ for small realizations of secondary knowledge spillovers: $(\gamma^{\nu})^{m+1-n} \cdot \underline{\phi}^{m+1-n}(S_n) < S_n$. In this case, equilibrium profits from intermediate input production with fundamental technology $m+1$ will be strictly negative. The new technology will not be used in intermediate input production or selected for secondary development by subsequent generations.

This further generalizes the dynamics of technological change in the model. A restriction on initial conditions will no longer ensure that it is always the most advanced fundamental technology that is selected for secondary development. There are now two senses in which an economy may become locked into an existing technology. First, as before, the accumulated stock of secondary knowledge may become so large that it is no longer profitable to engage in research directed at the discovery of a new fundamental technology (*permanent lock-in*). Second, the accumulated stock of secondary knowledge may be consistent with positive equilibrium employment in fundamental research, but a new fundamental technology will not be used in intermediate input production for small realizations of secondary knowledge spillovers (*temporary lock-in*).

## 7. MANY INTERMEDIATE GOODS SECTORS

This section returns to deterministic secondary knowledge spillovers to extend the analysis in two other directions. First, many intermediate goods sectors are introduced. In each of these sectors, technological change may take the form of fundamental innovation and secondary development. Second, we allow for the possibility that the secondary development of one fundamental technology $m$ may itself play a role in the discovery of technology $m+1$; the very process of secondary development may yield insights into the shape of future fundamental technologies.

The introduction of many intermediate goods sectors follows the approach taken for the standard quality ladder model (without a distinction between fundamental research and secondary development) in Aghion and Howitt (1997). It is assumed that final goods output is produced from the output of a large number of intermediate sectors $i \in \{1, \ldots, I\}$,

$$y_{2t} = \sum_{i=1}^{I} A_{2tm(i)} \cdot x_{2ti}^{\alpha} l_{2ti}^{1-\alpha}, \qquad 0 < \alpha < 1 \tag{40}$$

In each sector $i$, the quality or productivity of intermediate inputs depends upon stocks of (sector-specific) fundamental knowledge and effective secondary knowledge,

$$A_{2tm(i)} = F_{2tm(i)}^{\nu} \cdot \tilde{S}_{2tm(i)} \tag{41}$$

where $k(i) \in \{0, 1, \ldots, m(i)\}$ denotes the interval starting with the $k$th fundamental innovation in sector $i$ and ending with the $k+1$st, and $m(i)$ is the most advanced fundamental technology currently available in sector $i$.

At the beginning of period 1, workers decide whether to engage in fundamental research or secondary development in a sector $i$. The number of individuals entering either fundamental research or secondary development in sector $i$ is denoted by $H_{ti} = H_{ti}^{F} + H_{ti}^{S}$. The process of fundamental research is modeled in exactly the same way as in the basic model with only one intermediate goods sector, and it is assumed that the probability of fundamental innovation in sector $i$ is independent of that in all other sectors. Secondary development is also as before, except in the following respect. As secondary knowledge is accumulated in sector $i$, we assume there is a probability $\delta > 0$ (however small) that this secondary knowledge accumulation will itself result in the discovery of fundamental technology $m(i)+1$. In this case, the new fundamental technology will be employed under conditions of perfect competition in sector $i$. The probability that any one fundamental researcher in sector $i$ will receive the patent to technology $m(i)+1$ is thus

$$\Lambda\left(H_{ti}^F\right) \equiv \frac{(1-\delta)}{H_{ti}^F}\left[1-(1-\lambda)^{H_{ti}^F}\right] \tag{42}$$

General equilibrium requires that the following conditions are satisfied. First, workers are indifferent between entering secondary development in sector $i$ and all other sectors $j \neq i$,

$$\hat{w}_{2tm(i)} = \hat{w}_{2tm(j)}, \qquad \forall\, i \text{ and } j \neq i$$

$$\alpha F_{2tm(i)}^{\nu} \tilde{S}_{2tm(i)} \left(\frac{\hat{H}_{ti} - \hat{H}_{ti}^F}{\hat{l}_{2ti}}\right)^{\alpha-1} = \alpha F_{2tm(j)}^{\nu} \tilde{S}_{2tm(j)} \left(\frac{\hat{H}_{tj} - \hat{H}_{tj}^F}{\hat{l}_{2tj}}\right)^{\alpha-1} \tag{43}$$

Second, the return to secondary development in all sectors $i$ is greater than or equal to the return to fundamental research (when greater than the return to fundamental research, this will be a case of technological lock-in in sector $i$),

$$1 \geq \Lambda\left(\hat{H}_{ti}^F\right) \cdot \left[\Omega_{2t(m(i)+1)} - 1\right] \cdot \left(\hat{H}_{ti} - \hat{H}_{ti}^F\right), \qquad \hat{H}_{ti}^F \geq 0 \tag{44}$$

where one of the above inequalities must hold with equality. Third, the rental rate for land is the same in sector $i$ and all other sectors $j \neq i$,

$$\hat{r}_{2ti} = \hat{r}_{2tj}, \qquad \forall\, i \text{ and } j \neq i$$

$$(1-\alpha) F_{2tm(i)}^{\nu} \tilde{S}_{2tm(i)} \left(\frac{\hat{l}_{2ti}}{\hat{H}_{ti} - \hat{H}_{ti}^F}\right)^{-\alpha} = (1-\alpha) F_{2tm(j)}^{\nu} \tilde{S}_{2tm(j)} \left(\frac{\hat{l}_{2tj}}{\hat{H}_{tj} - \hat{H}_{tj}^F}\right)^{-\alpha} \tag{45}$$

Fourth, we require that the markets for labor and land clear:

$$\sum_{i=1}^{I} \hat{H}_{ti} = H \tag{46}$$

$$\sum_{i=1}^{I} \hat{l}_{2ti} = L \tag{47}$$

Taking Equations (44), (43), and (45) for each sector $i$, and combining them with the market clearing conditions (46) and (47), a system of $3I$ independent equations in $3I$ unknowns $\{\hat{H}_{ti}, \hat{H}_{ti}^F, \hat{l}_{2ti}\}$ is obtained. Given the inherited stocks of fundamental and secondary knowledge for generation $t$ ($F_{2(t-1)m(i)}$ and $S_{2(t-1)m(i)}$, respectively), we may solve for each sector $i$ for equilibrium employment in fundamental research ($\hat{H}_{it}^F$), the equilibrium number of workers entering either fundamental research or secondary development ($\hat{H}_{it}$), and the equilibrium allocation of land ($\hat{l}_{2ti}$). It would be straightforward to simulate the model for specific parameter values. Individual intermediate goods sectors will become locked into an existing fundamental technology $m(i)$ whenever the accumulated stock of secondary knowledge exceeds the critical value $S^1_{2(t-1)m}$ derived in Section 4. Whether this critical value is attained depends upon the (random) interval between fundamental innovations in sector $i$. Thus, in any generation $t$, there will be an inflow of sectors into the state of technological lock-in, and this inflow will depend on the distribution of the accumulated stock of secondary knowledge $(S_{2(t-1)m(i)})$ across sectors $i$.

In the remaining intermediate goods sectors, fundamental research will continue to occur, and equilibrium employment in fundamental research solves $1 = \Lambda(\hat{H}^F_{it}) \cdot [\Omega_{2t(m(i)+1)} - 1] \cdot (\hat{H}_{it} - \hat{H}^F_{it})$. In these sectors, the aggregate probability that a new fundamental technology is discovered $(\Lambda(\hat{H}^F_{it}) \cdot \hat{H}^F_{it} + \delta)$ depends upon both fundamental research and the extent to which secondary development may itself result in fundamental innovation. In sectors locked into existing fundamental technologies, there is, of course, no research. However, as secondary development proceeds, there will remain a constant probability $\delta$ that a new fundamental technology is discovered. Thus in any generation $t$, there will also be a random outflow of sectors from the state of technological lock-in. The evolution of aggregate final goods output (40) over time depends upon both the number of sectors subject to technological lock-in $(S_{2(t-1)m(i)} \geq S^1_{2(t-1)m})$ and equilibrium investments in fundamental research $(\hat{H}^F_{it})$ in all other sectors.

## 8. CONCLUSION

This article has presented a model of endogenous innovation and growth, in which technological progress is path dependent and technological lock-in may occur. The analysis is motivated by the literatures concerned with the history and microeconomics of technology, in which these are central themes. The article provides a microeconomic rationale for path dependence using four features of technological change emphasized in empirical work: endogenous innovation, uncertainty, a distinction between fundamental innovation and secondary development, and imperfect spillovers of secondary knowledge across fundamental technologies.

With imperfect secondary knowledge spillovers, an increase in the stock of secondary knowledge relating to one fundamental technology $m$ reduces agents' incentives to engage in research directed at the discovery of technology $m+1$. Technological change is path dependent, in the sense that the historical path of secondary development influences current incentives to engage in fundamental research. There are a number of implications of path dependence. First, as the stock of accumulated secondary knowledge increases, the secondary knowledge obsolescence that would be induced by the discovery of fundamental technology $m+1$ means that this technology (when discovered) is less likely to constitute a drastic innovation. Thus, depending on the (random) interval between fundamental innovations, the economy moves endogenously between periods of drastic and nondrastic innovation.

Second, once fundamental innovation becomes nondrastic, further secondary development of an existing fundamental technology $m$ reduces the future limit price that can be charged when fundamental technology $m+1$ is discovered. The expected return to fundamental research falls, and the secondary development of one fundamental technology thus reduces equilibrium employment in research directed at the discovery of the next fundamental technology. Third, if secondary development proceeds sufficiently far, the economy may become *locked into* an existing fundamental technology. In such an equilibrium, secondary development has increased the productivity of the existing fundamental technology to such an extent that it is no longer profitable to search for more advanced fundamental technologies, despite the fact that these would be more productive if they had benefited from the same level of secondary development.

Fourth, a model of endogenous innovation in which technological change is path dependent provides a rationale for cycles in technological leadership. This rationale exists in equilibria with positive levels of fundamental research and is not limited to the special case of technological lock-in. Cycles in technological leadership may occur even in a world with no international knowledge spillovers and no imitation. Once international knowledge spillovers are introduced, the extent of technological catch-up and leapfrogging depends on the relative magnitude of spillovers of fundamental and secondary knowledge.

The model's tractability made possible a very general specification of secondary knowledge spillovers, and enabled us to consider a number of extensions to the basic model. Uncertainty over the magnitude of secondary knowledge spillovers leads to a distinction between *temporary technological lock-in* (where it is not profitable to employ a new fundamental technology, once discovered, for small *realizations* of secondary knowledge spillovers) and *permanent technological lock-in* (where it is no longer profitable to search for new fundamental technologies given the *expected* magnitude of secondary knowledge spillovers). Introducing multiple intermediate goods sectors allows technological lock-in to occur in individual sectors of the economy, whereas the economy as a whole experiences endogenous growth as a result of both fundamental research and secondary development.

## APPENDIX

A.1. *Proof of Proposition 2.*

(a) A fundamental innovation is nondrastic if

$$\frac{\gamma^{\nu} \cdot \phi(S_{2tm})}{S_{2tm}} < \left(\frac{1}{\alpha}\right)^{\alpha}$$

If $\phi'(S_m) \cdot S_m/\phi(S_m) < 1$, the left-hand side of this inequality is monotonically decreasing in the stock of accumulated secondary knowledge, $S_{2(t-1)m}$ (where $S_{2tm} = \mu \cdot S_{2(t-1)m}$). Therefore, there exists a critical value for the stock of accumulated secondary knowledge, $S^0_{2(t-1)m} \geq 1$, such that this condition is satisfied.

(b) For values of the stock of secondary knowledge $S_{2(t-1)m} \geq S^0_{2(t-1)m}$, fundamental innovation is nondrastic. In this case, from (36), equilibrium employment in fundamental research solves

$$1 = \Lambda(\hat{H}^F_t) \cdot \left[ \left( \frac{\gamma^\nu \phi(S_{2tm})}{S_{2tm}} \right)^{\frac{1}{\alpha}} - 1 \right] \cdot (H - \hat{H}^F_t) \tag{A.1}$$

Multiply out the term in parentheses on the right-hand side of this equation, and note that $S_{2tm} = \mu \cdot S_{2(t-1)m}$. Take logarithms of both sides of the equation, and differentiate with respect to $S_{2(t-1)m}$. The effect of $S_{2(t-1)m}$ on equilibrium research employment depends upon the sign of

$$d \log \left( \frac{\gamma^\nu \phi(\mu \cdot S_{2(t-1)m})}{\mu \cdot S_{2(t-1)m}} \right) \Big/ dS_{2(t-1)m}$$

If $\phi'(S_m) \cdot S_m / \phi(S_m) < 1$, this term is strictly negative. Therefore, the right-hand side of (A.1) is monotonically decreasing in $S_{2(t-1)m}$. There exists a critical value for the stock of accumulated secondary $S^1_{2(t-1)m} > 1$ such that

$$1 > \Lambda(1) \cdot \left[ \left( \frac{\gamma^\nu \cdot \phi(\mu \cdot S^1_{2(t-1)m})}{\mu \cdot S^1_{2(t-1)m}} \right)^{\frac{1}{\alpha}} - 1 \right] \cdot (H - 1)$$

For values of $S_{2(t-1)m} \geq S^1_{2(t-1)m}$, equilibrium employment in fundamental research is zero, and the economy becomes *locked into* the existing fundamental technology $m$.

(c) This follows immediately from (b) above. ■

A.2. *Diminishing Returns to Secondary Development.* Technological change remains path dependent and technological lock-in remains possible if we introduce diminishing returns in the process of secondary development. Consider the following modification of Equation (8) in the text,

$$\mu = \mu\left(S_{2(t-1)j}\right) \tag{A.2}$$

$$\mu(1) > 1, \qquad \frac{d\mu\left(S_{2(t-1)j}\right)}{dS_{2(t-1)j}} < 0$$

$$\lim_{S_{2(t-1)j} \to \infty} \mu\left(S_{2(t-1)j}\right) \to \kappa, \qquad 0 < \kappa < 1$$

This specification is itself very general and consistent with a wide range of functional forms for $\mu(\cdot)$. We again consider equilibria where the most advanced

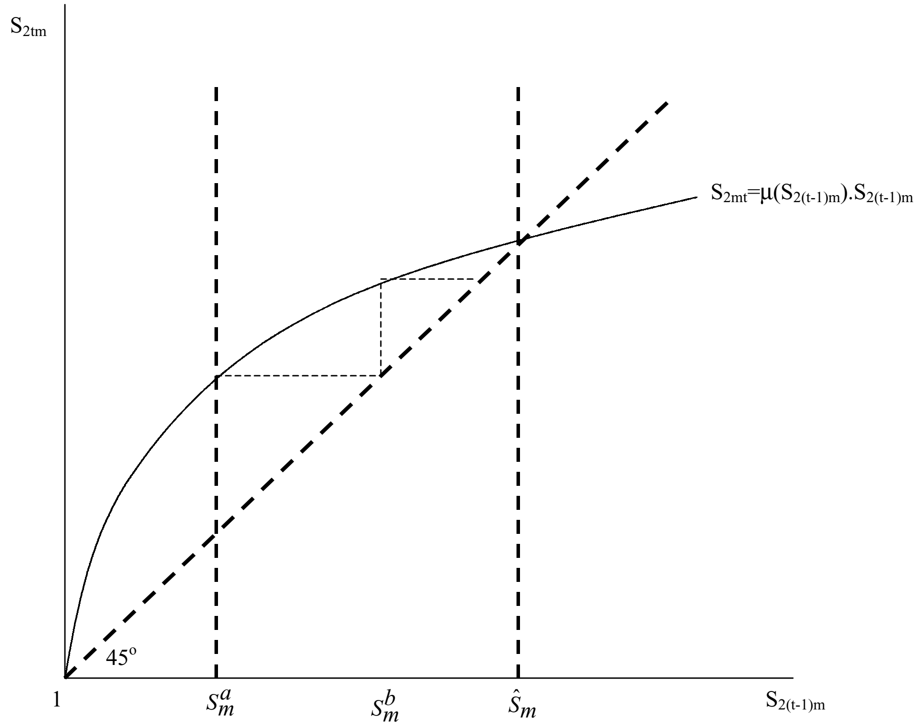


FIGURE A.1

DIMINISHING RETURNS TO SECONDARY DEVELOPMENT

fundamental technology is selected for secondary development: $j = n = m$. The main body of the article derived a sufficient condition for this to be case.

If fundamental research is unsuccessful in generation $t$, secondary knowledge relating to the existing fundamental technology $m$ will accumulate between generations $t-1$ and $t$ according to Equations (8) and (A.2). The evolution of the stock of secondary knowledge relating to fundamental technology $m$ is shown diagrammatically in Figure A.1. If $S_m^a$ denotes the initial stock of secondary knowledge when fundamental technology $m$ was discovered, secondary knowledge will, in the absence of success in fundamental research, continue to accumulate across successive generations until the steady-state value $\hat{S}_m$ is attained. This steady-state value solves $\mu(\hat{S}_m) = 1$, and provides an upper bound to the stock of secondary knowledge relating to fundamental technology $m$ that can be accumulated.

Whether the steady-state value $\hat{S}_m$ is actually achieved will depend upon the (random) interval of time between fundamental innovations. Suppose a fundamental innovation occurs in generation $t$ before the steady-state value is attained: for example, when the accumulated stock of secondary knowledge is $S_{2(t-1)m} = S_m^b$. In this case, the evolution of the stock of secondary knowledge between generations $t-1$ and $t$ depends upon the extent of secondary knowledge obsolescence: $\tilde{S}_{2t(m+1)} = \phi[\mu(S_{2(t-1)m}) \cdot S_{2(t-1)m}] = \phi[\mu(S_m^b) \cdot S_m^b]$.

The dynamics of secondary knowledge accumulation for subsequent generations are directly analogous. The determination of equilibrium employment in fundamental research is exactly the same as in the article, and technological change remains path dependent. Whether technological lock-in occurs depends upon (a) the random interval between fundamental innovations (as in the main body of the article) and (b) whether the critical value for the stock of accumulated secondary knowledge that induces technological lock-in ($S^1_{2(t-1)m}$) is lesser than or greater than the steady-state value $\hat{S}_m$.

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