Radioactivity
PHYS 312

1. Brief review of nuclear interactions

Recall that atomic nuclei consist of roughly equal numbers of protons and neutrons. Protons and neutrons, in turn, are made from three quarks, which are believed to be elementary particles. The quarks carry color charge and interact through the strong force mediated by gluons. Protons and neutrons are color-neutral but have an attractive interaction due residual strong forces, in the same way as neutral atoms are held by residual electromagnetic interactions to form solids. The details of the nuclear forces are quite complicated, they still cannot be derived from first principles and are usually described by various phenomenological models. In a popular Shell Model, it is assumed that nucleons (protons and neutrons) move in a harmonic oscillator potential created by their own attractive interactions. The Fermi exclusion principle then states that adding more neutrons or protons to the nucleus requires occupation of higher energy levels in the harmonic potential. However, since neutrons and protons are not identical, they can occupy the same energy states. Since the nuclear forces between neutrons and protons are nearly the same, this simple argument explains why stable nuclei have approximately equal numbers of protons and neutrons. For heavier nuclei, the electrostatic repulsion between protons becomes comparable to the nuclear forces, and it makes it energetically favorable to have fewer protons than neutrons.

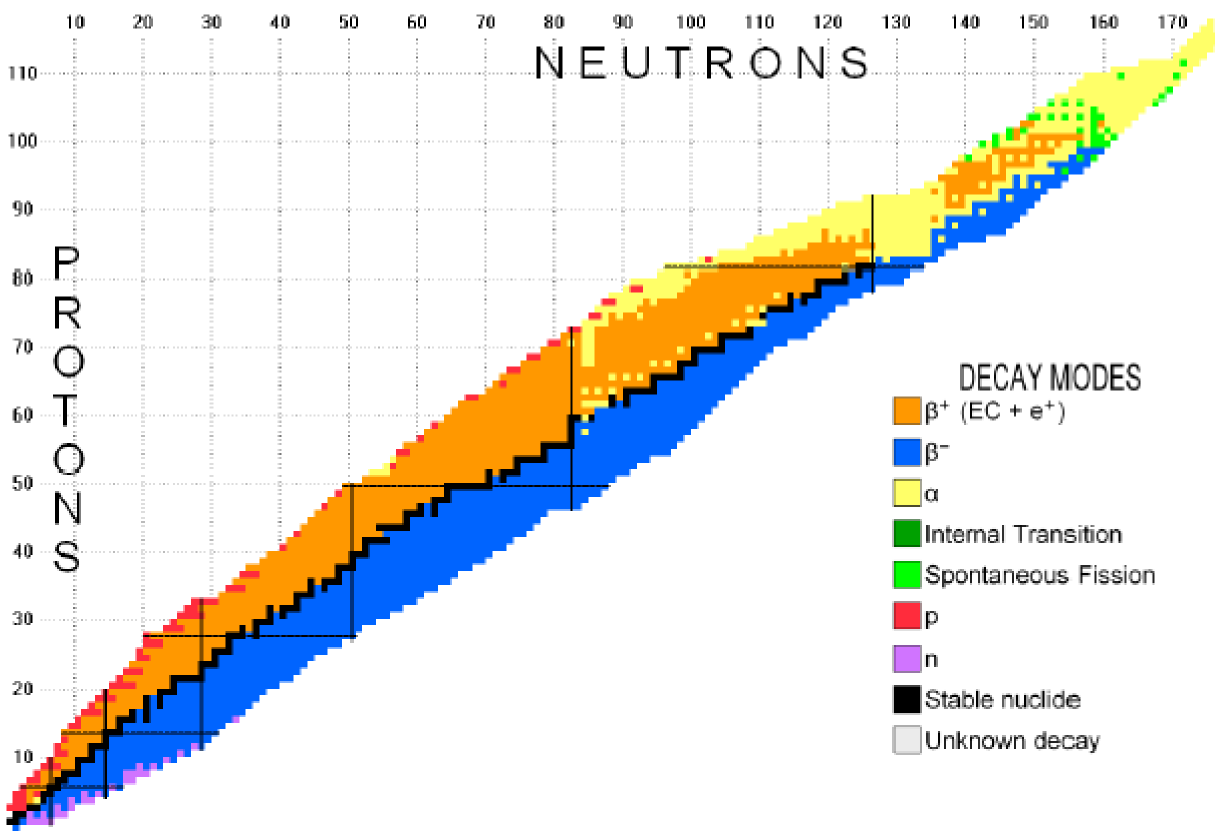

Fig. 1 Nuclear chat, showing all stable and observed unstable nuclei and their decay modes.

2. Radioactive decays

There are several modes of radioactive nuclear decays, as indicated in Fig. 1. A bare neutron is unstable and decays to a proton, electron and an anti-neutrino in about 900 seconds. Such "β" decays can also occur in heavier nuclei if they are energetically allowed, i.e. if the mass of the resulting "daughter" nucleus is smaller than the mass of the parent nucleus plus the mass of the electron. There are also several other possible decay modes involving conversion of neutron to proton or vise versa, for example:

$^{40}\mathrm{K} \rightarrow {}^{40}\mathrm{Ca} + e^{-} + \bar{\nu}_e \quad \beta - \text{decay}$

$^{40}\mathrm{K} + e^{-} \rightarrow {}^{40}\mathrm{Ar} + \nu_e \quad \text{Electron capture}$

$^{40}\mathrm{K} \rightarrow {}^{40}\mathrm{Ar} + e^{+} + \nu_e \quad \text{Inverse } \beta - \text{decay}$

In the first decay a neutron is converted to a proton inside the nucleus, while in the other two a proton is converted to a neutron. The lifetimes for β decays vary widely from milliseconds to billions of years.

Heavier nuclei can also decay by emission of α particles, which are ^{4}He nuclei. An example of a decay chain involving many α and β decays is shown in Fig. 2. The decay of ^{238}U in the Earth generates ^{222}Rn as one of the daughters, which diffuses from the soil and can be dangerous if inhaled.

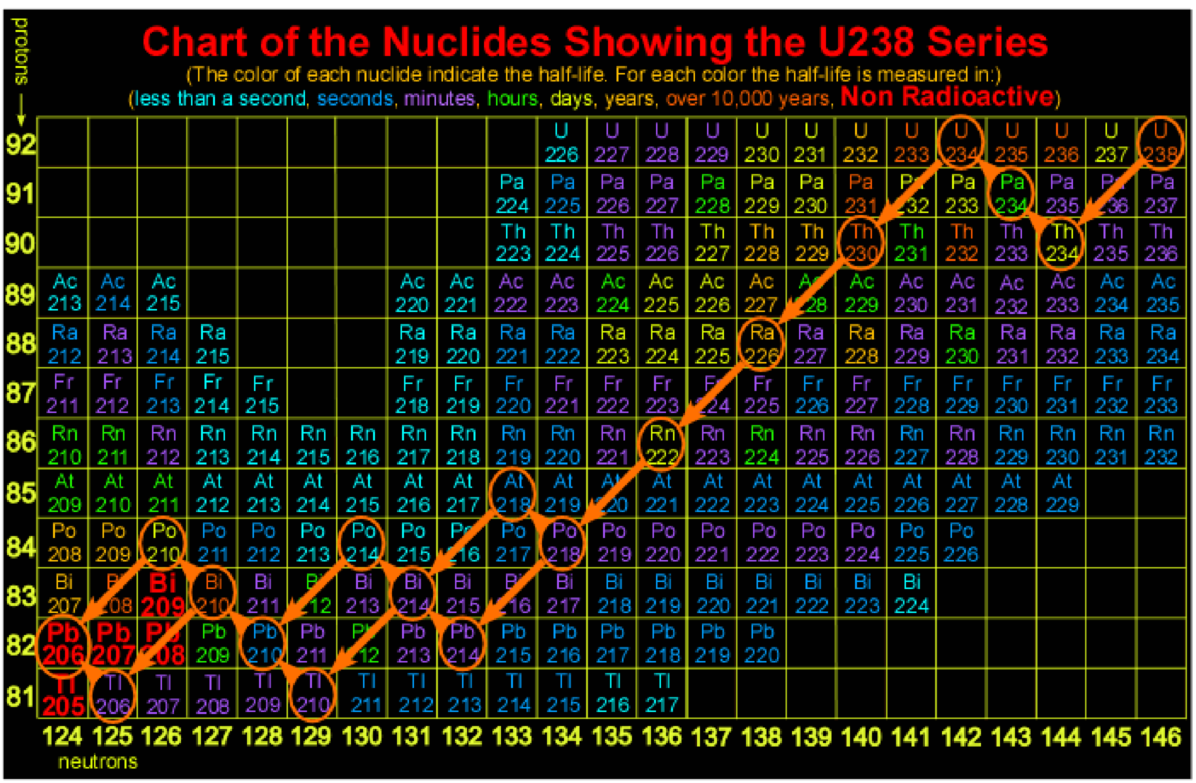

Figure 2: Decay chain of ^{238}U. Long downward arrows indicate α decay (loss of 2 protons and 2 neutrons), short upward arrows indicate β decay (conversion of a neutron to a proton). Note that the lifetimes of the nuclei (indicated in color) range from less than a second to more than 10,000 years. The decay chain ends in ^{206}Pb, which is stable.

In addition to β and α decays, nuclei can also emit γ-rays, which are photons with typical energies of tens of keV to a few MeV. γ-rays are typically emitted following another decay. For example, a β-decay can leave the nucleus in an excited state, which will then decay by emitting a photon to the ground state. Since γ-ray emission does not change the number of neutrons or protons, it does not move the nucleus on the nuclear chart. The lifetime for γ-ray emission is typically very short, 10^{-9} sec or less, but a few nuclei have relatively stable excited states, called isomers, with lifetimes in the hours or even years. Another way for an excited nucleus to loose energy is through a process known as internal conversion. In this case the extra energy of the nuclear excited state is transferred to an electron from the inner shell of the nucleus. The electron is then ejected from the nucleus with a characteristic energy, roughly the same as one would get in a γ-ray.

3. Interaction of radiation with matter

There are several processes responsible for energy loss of high energy particles in matter. For electrons typically emitted in nuclear decays with energies less than a few MeV, as well as for heavier charged particles, such as α particles and muons, the dominant energy loss mechanism is due to Coulomb scattering on the electrons in matter. Rough estimate of the particle energy loss can be obtained using the classical Coulomb scattering cross-section for two charged particles. The result is

$$\frac{dE}{dx} = -\frac{4\pi n q^2 e^2}{m_e v^2} \ln(B)$$

where n is the density of the electrons in the material, q is the charge and v is the velocity of the high-energy particle, and m_e is the electron mass. The factor B takes into account binding of electrons in the material and depends on the particle energy, but the logarithm remains on the order of unity. The energy loss for various materials is shown in Figure 3. For relativistic particles with $p \geq Mc$ the energy loss is on the order of 1 MeV/cm for a material density of 1 g/cm^3. However, the energy loss becomes much larger for non-relativistic particles. Thus, α particles, which are emitted in nuclear decays with energies of a few MeV, much smaller than their rest mass of about 4 GeV, loose their energies over much smaller distance than electrons and positrons.

For γ-rays the dominant energy loss mechanisms are Compton scattering on electrons and the photo-electric effect, i.e. electron excitation associated with photon absorption. The cross-section of Compton scattering for photon energy less than or on the order of the electron rest mass can be estimated from the classical Thomson cross-section for scattering of electromagnetic radiation by a free charge

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2,$$

where the quantity in parentheses is known as the classical electron radius. The amount of energy that photons loose in each scattering event follows from relativistic kinematics for an elastic collision between the photon with energy E_0 and an electron at rest:

$$\frac{p_s}{p_0} = \frac{E_s}{E_0} = \frac{E_0}{1 + (E_0 / mc^2)(1 - \cos\theta)}; \qquad \Delta E = E_s - E_0 = -\frac{(E_0^{\ 2} / mc^2)(1 - \cos\theta)}{1 + (E_0 / mc^2)(1 - \cos\theta)}$$

Thus, even though the scattering cross-section is independent of energy, the amount of energy lost by the photon in each collision ΔE decreases as E^2 for small E_0. Therefore, Compton scattering is an effective mechanism of energy loss only for photon energy on the same order as the electron rest mass.

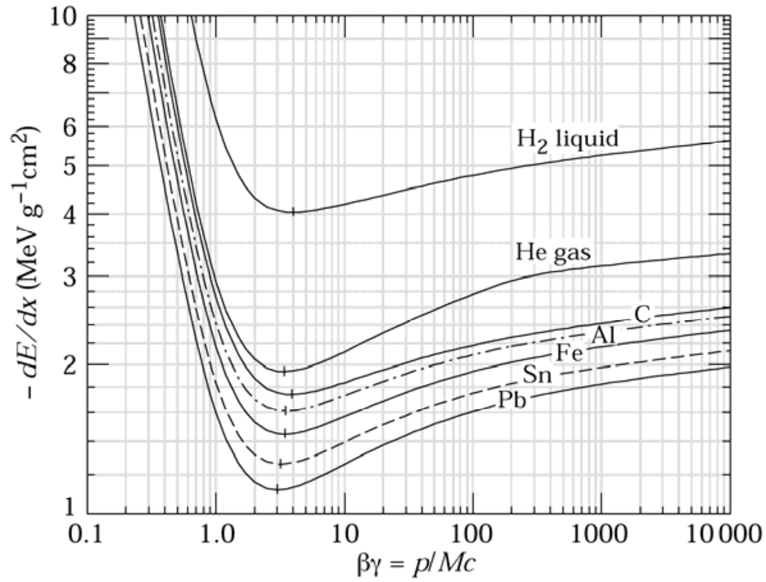

Figure 3: Energy loss for charged particles in matter due to Coulomb scattering. The energy loss per unit length is shown as a function of the relativistic momentum of the particle. The energy loss scales linearly with the density of the material.

Once the photon energy drops significantly below electron rest mass, the photoelectric effect becomes the dominant energy loss mechanism. In this case the photon is absorbed in a single interaction. The cross-section for photon absorption by a hydrogen atom can be calculated as a radiative transition between a bound electron wavefunction and a plane-wave unbound wavefunction, the result is

$$\sigma_{PE} = \frac{16\sqrt{2}\pi\alpha^6\hbar^2}{3m_e^{\ 2}c^2}\left(\frac{E}{mc^2}\right)^{-7/2}$$

for $E \ll mc^2$. For other atoms such calculations require knowledge of the wavefunctions of inner electrons. Figure 4 shows a comparison between Compton scattering and photoelectric effect absorption of photons on carbon atoms. In Figure 5 the mean propagation length of photons is shown for different materials as a function of energy.

Based on these calculations we can estimate the penetration distance of 1 MeV particles in a material with density of 1 g/cm^3:

α	1 μm
β	0.5 cm
γ	20 cm

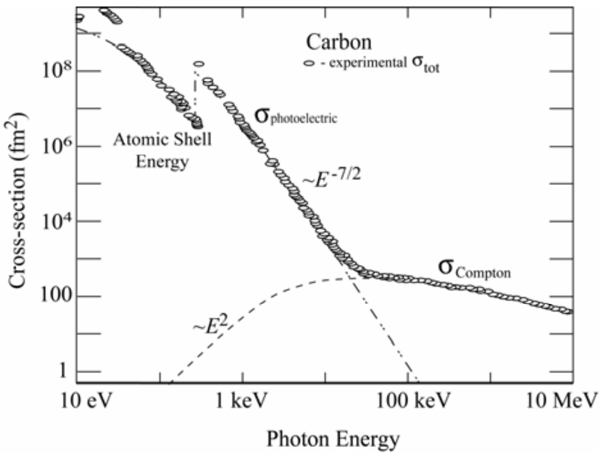

Figure 4. Cross-section for photon energy loss on carbon atoms

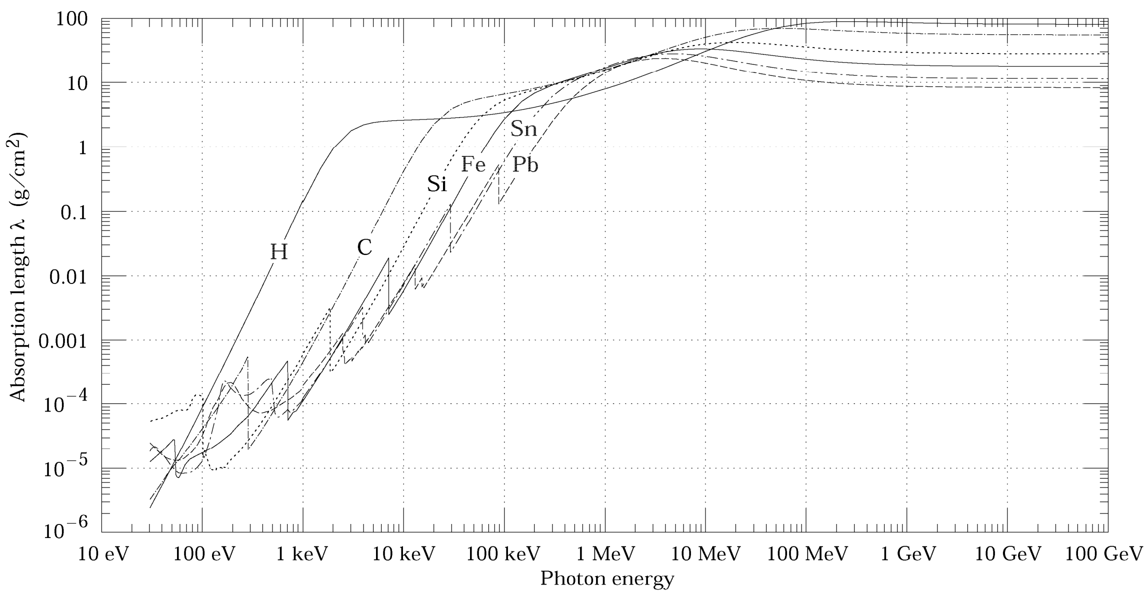

Figure 5: Absorption length for photons propagating in various materials. The photon energy flux after propagating a distance t in material with density ρ is given by $I = I_0 \exp(-t\rho/\lambda)$. For a material with density of 1 g/cm^3 the plot gives the mean propagation length in cm.

Hence, γ rays are most penetrating and require shielding by several cm or more of a high density material, such as lead. β-rays are moderately penetrating and can be shielded by a cm of material. α particles do not penetrate far, but they can cause the most biological damage because their energy deposition is very localized. That is why it is very dangerous to ingest a material emitting α radiation. In the online radiation course the relative biological impact factors of various types of radiation are discussed further.