Topologicaly protected abelian Josephson qubits: theory and experiment.

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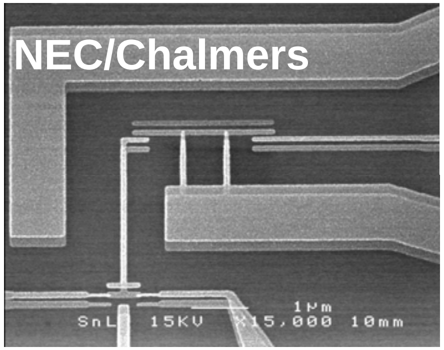
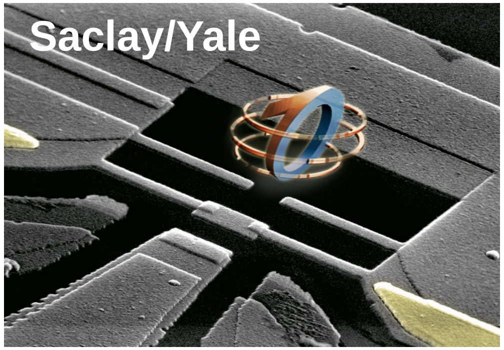
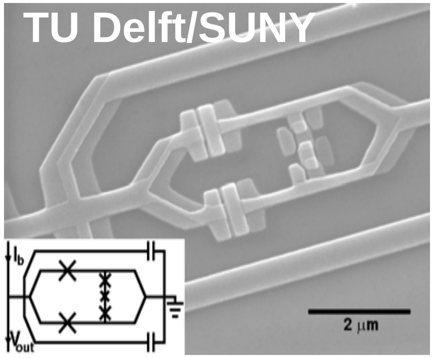
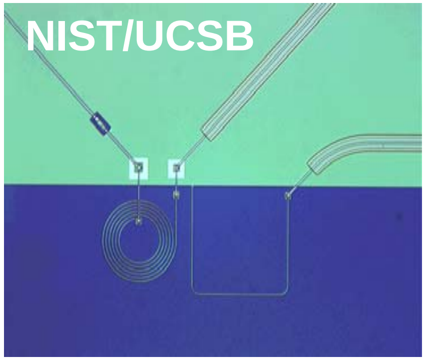
Plan

- Honest (pessimistic) review of the state of the field of superconducting qubits.
- Theoretical models of protected qubits and their implementations in Josephson junctions.
- Experimental results
- Conclusions

State of the Art in Superconducting Qubits

Charge	Charge/Phase	Flux	Phase
			

Junction size ⟶ $E_J = E_C$

Charging versus Josephson energy

$$\text{Ideal Hamiltonian}: H = 4E_C(\hat{n} - n_0)^2 + E_J\cos(\varphi + \Phi) - I\varphi$$

$$\hat{n} = i\frac{\partial}{\partial\varphi} - \text{number of Cooper pairs}$$

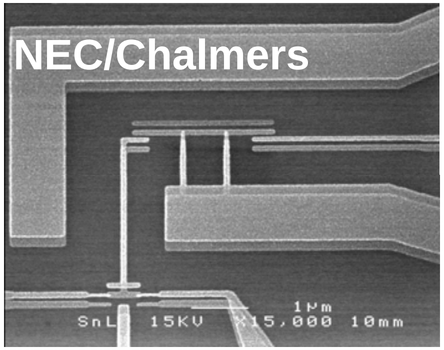
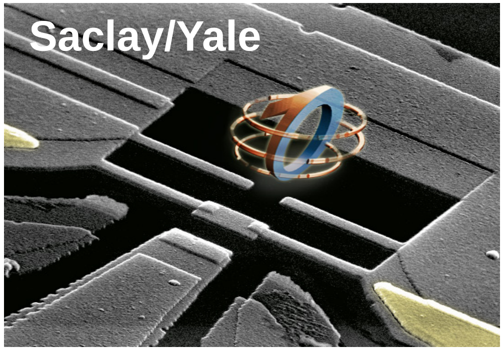
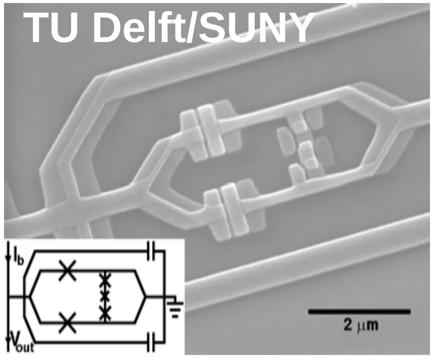
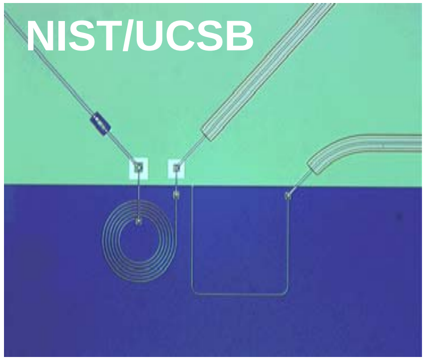
$$I - \text{ external current}$$

Two states of the qubit differ by

charge $\hat{n}$ if $E_C \gg E_J$

phase φ if $E_J \gg E_C$

State of the Art in Superconducting Qubits

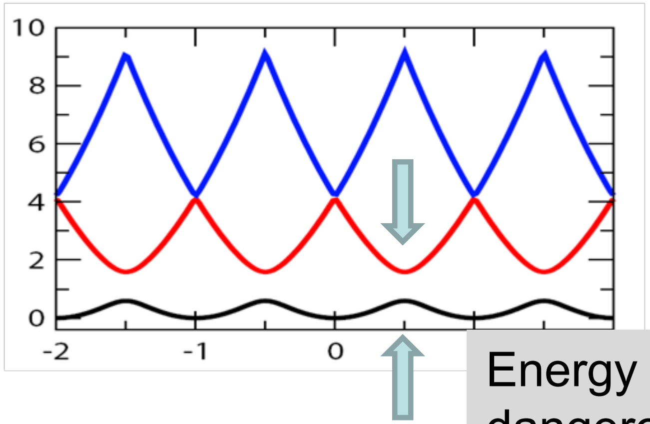
Charge	Charge/Phase	Flux	Phase
			

Junction size → $E_J = E_C$ ← # of Cooper pairs

In real life $H = 4E_C[\hat{n} - n_0 + \delta n(t)]^2 + [E_J + \delta E_J(t)]\cos(\varphi + \delta\Phi(t)) - I\varphi$

$\delta n(t)$ charge noise, $\delta E_J(t)$ critical current noise, $\delta\Phi(t)$ flux noise

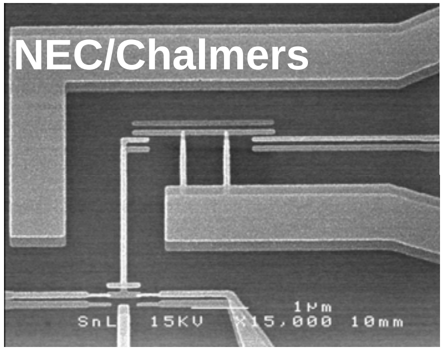
Partial remedy:
Sweet spots
(linear protection)

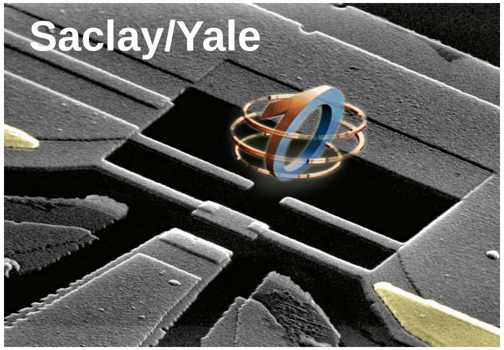


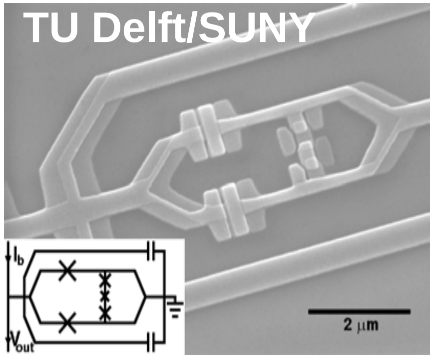
Energy as a function of the most dangerous parameter (charge, flux, etc)

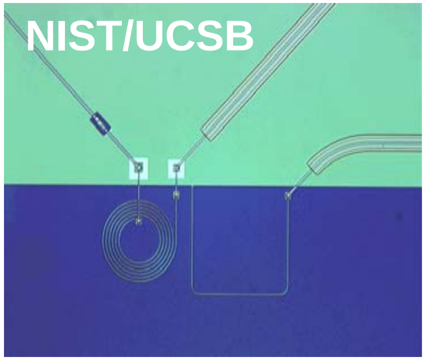
State of the Art in Superconducting Qubits

Charge	Charge/Phase	Flux	Phase









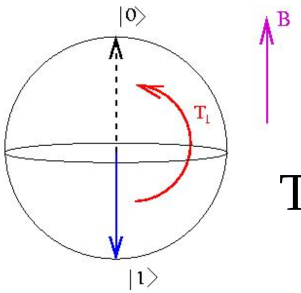
Junction size ⟶ $E_J = E_C$ ⟵ # of Cooper pairs

Charging versus Josephson energy

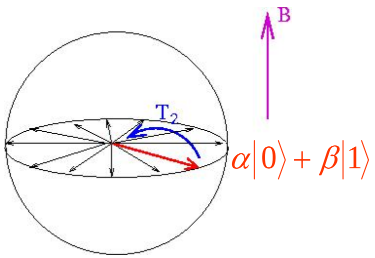
History

- 1st qubit demonstrated in 1998 (NEC Labs, Japan)
- "Long" coherence shown 2002 (Saclay/Yale)
- Several experiments with two degrees of freedom
- C-NOT gate (2003 NEC, 2006 Delft and UCSB)
- Bell inequality tests being attempted (2006, UCSB) [failure due to low readout visibility!]
- 1st time domain tunable coupling of two flux qubits (2007, NEC Labs, Japan)
- Coupling superconducting qubits via a cavity bus (2007 Yale and NIST)

Relaxation and dephasing times



$T_1 \equiv$ Relaxation time



$T_2 \equiv$ Dephasing time

$\omega_0 / 2\pi = 5 \div 10$ GHz

Design	Group	T_2	T_1	Visibility	Operation time	Logical quality factor
Phase qubit	UCSB	~ 85 nsec	110 nsec	> 90%		
Flux qubit	NEC	~ 0.8 μsec	1 μsec	~ 20/30%	~ 0.02 μsec	10 – 100*
Transmon (CPB in a cavity)	Yale	~ 2 μsec	~ 1.5 μsec	> 95 %	~ 0.05 μsec	10 - 200*

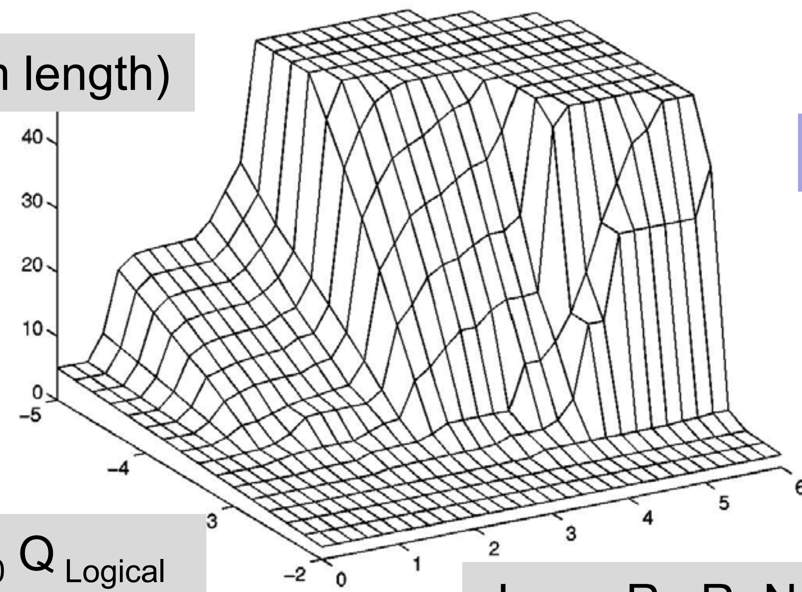
*Projected values

$$Q_{\text{Physical}} \cong \nu_0 T_2 \approx 10^3 - 10^4$$

$Q_{\text{Logical}} < 0.1\, Q_{\text{Physical}} = 10^2 - 10^3$

What do we need?

Error rates that we need for quantum computation.



Steane PRA (2003)

$Q_{Logical} < 10^2 - 10^3$

Need $Q_{Logical} \gg 10^4$ for <u>many</u> qubit system

Advantage of protection

Charge qubit: Q<100

Quantronium, (charge qubit at the optimal point): $Q=10^3-10^4$

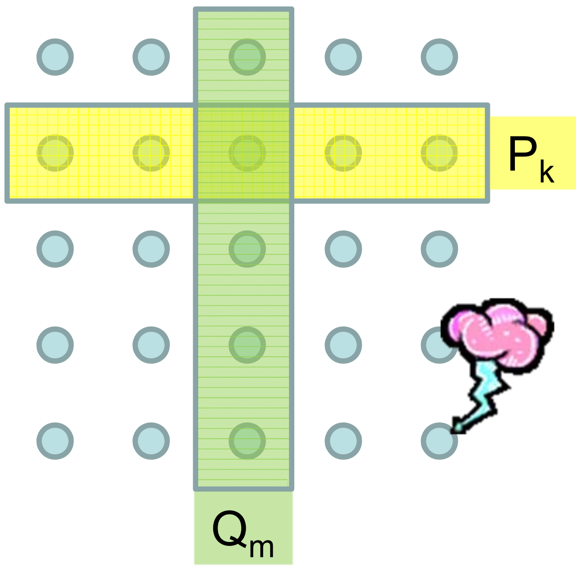
Flux qubit: away from the optimal point Q~10

At optimal point $Q=10^3-10^4$
$T_1\sim4\mu s$ $T_2\sim3\mu s$ $T_2^{echo}\sim4\mu s$

Devices decoupled from the leading source of noise in the linear order.

? **Devices decoupled in higher orders** ?

Protected Qubit (General)



Protected Doublet:
Special Spin Hamiltonians H with a large number of (non-local) integrals of motion P, Q:
$[H,P_k]=0$, $[H,Q_m]=0$, $[P_k,Q_m]\neq 0$

Any physical (local) noise term $\delta H(t)$ commutes with all P_k and Q_m except a O(1) number of each.
Effect of noise appears in N order of the perturbation theory:

$$\delta E \sim (\delta H(t) / \Delta)^{N-1} \, \delta H(t)$$

Simplest Spin Hamiltonian

$$H=\Sigma_{kl} J^{x}_{kl} \sigma^{x}_{k} \sigma^{x}_{l} + \Sigma_{kl} J^{z}_{kl} \sigma^{z}_{k} \sigma^{z}_{l}$$

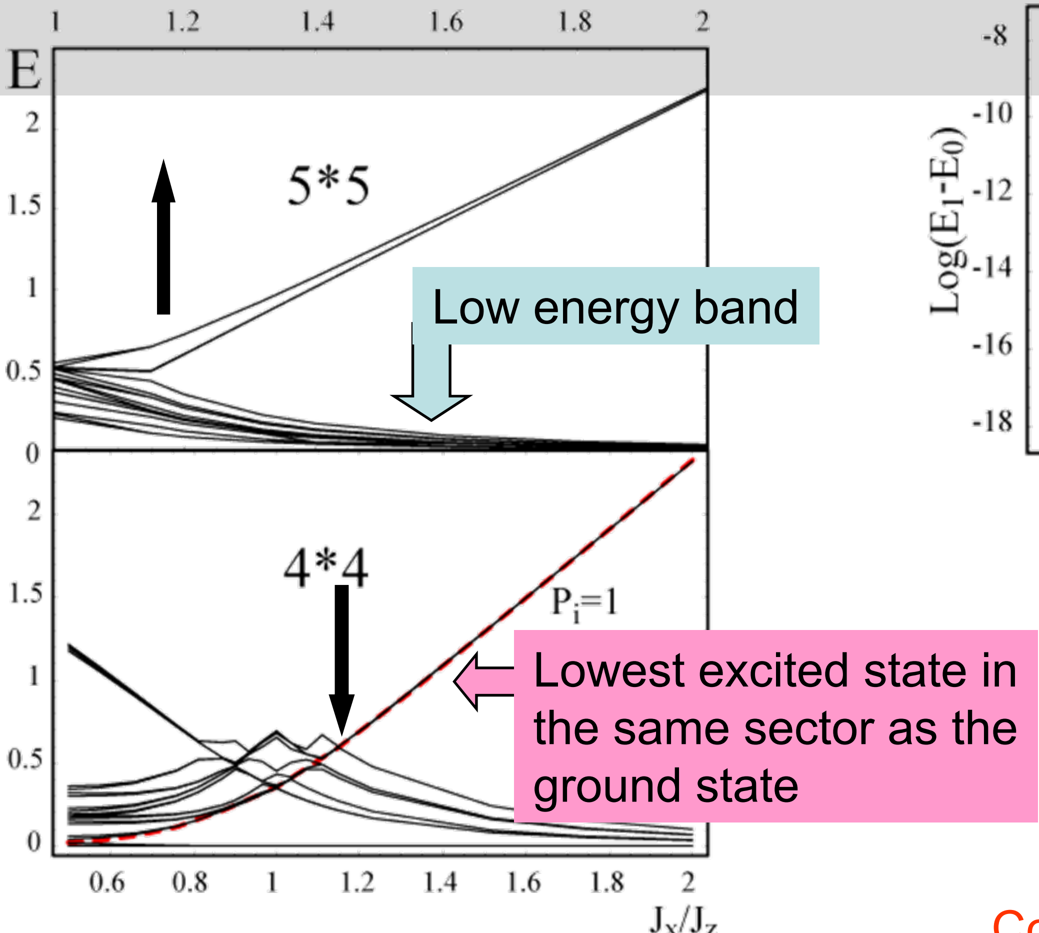
Rows	Columns
$P_k=\prod_l \sigma^z_l$	$Q_k=\prod_l \sigma^x_l$

Crucial issues:

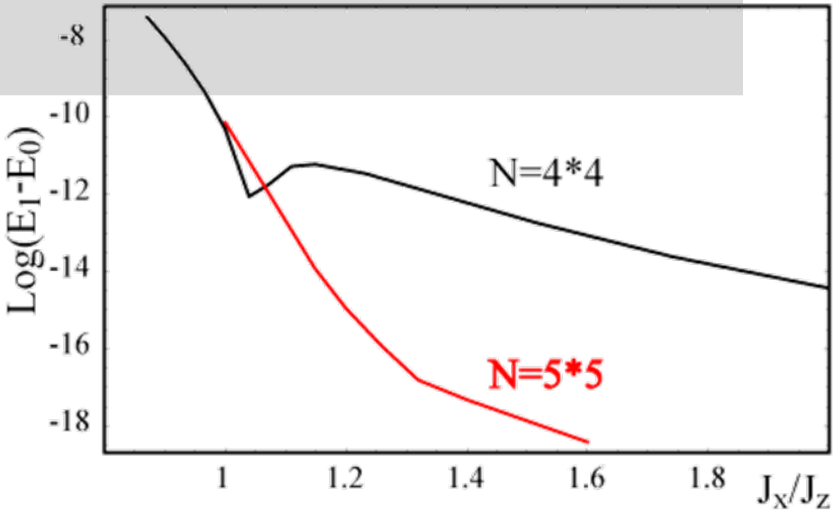
1. Which spin model has a large gap Δ?
2. Which spin model is easiest to realize in Josephson junction arrays?

Numerics for short range model (nearest neighbor interactions)

$$H=\Sigma_{(kl)} J^{x}_{kl} \sigma^{x}_{k} \sigma^{x}_{l} + \Sigma_{(kl)} J^{z}_{kl} \sigma^{z}_{k} \sigma^{z}_{l}$$



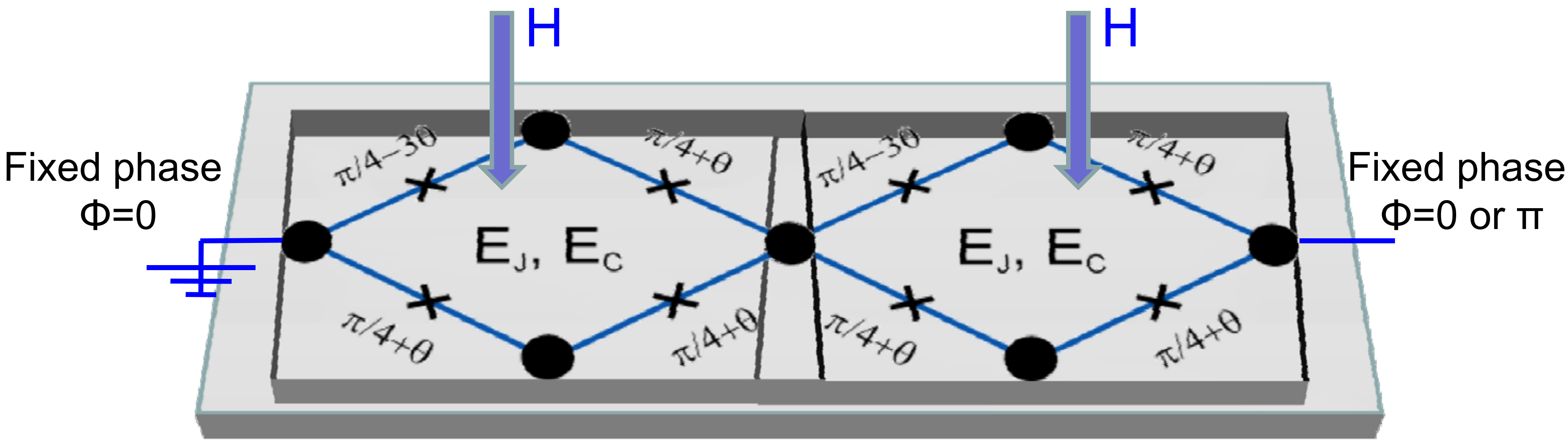
Low energy states for 4*4 and 5*5 spin array. Low energy band contains 2^4 and 2^5 states



Splitting of two lowest (degenerate) levels by random field applied to each spin and distributed in interval (-0.05, 0.05).

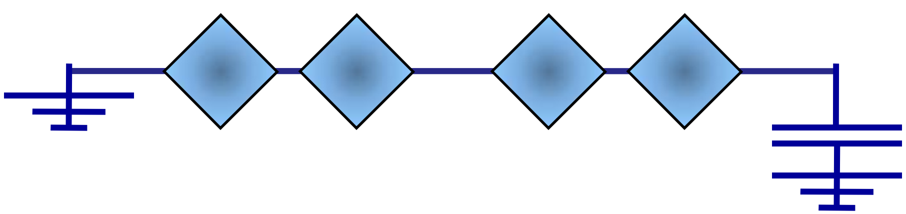
Conclusion: relatively small arrays provide very good protection, especially in one channel!

Realization of individual spins and their interaction by Josephson Junction Arrays



Only simultaneous flips are possible: $H=t\,\sigma^x_k\,\sigma^x_l$

Longer chains: $H=t\,\Sigma_{k,m}\,\sigma^x_k\,\sigma^x_m$ + constraint $\prod_k \sigma^z_k=\text{const}$



Large capacitor preventing phase changes of the end point.

Where is the catch?

- Josephson elements are not discrete.

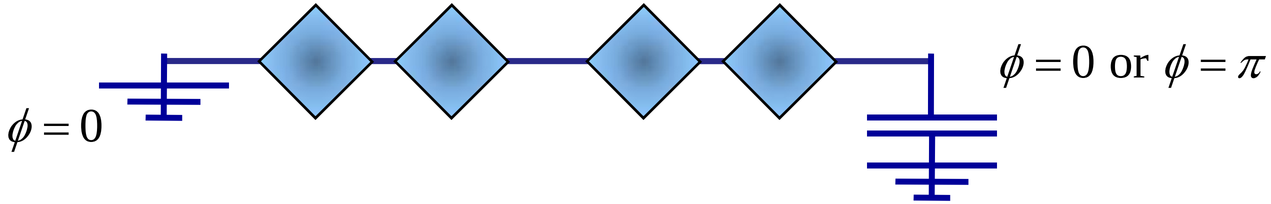
Noise suppression contains

$$\left(\frac{\delta\Phi}{\Phi_0}\frac{E_J}{r}\right)^{k-1} \quad \left(\gamma\frac{\delta E_J}{r}\right)^{k-1}$$

(r ~ t – transition amplitude)

→ we need large quantum fluctuations.

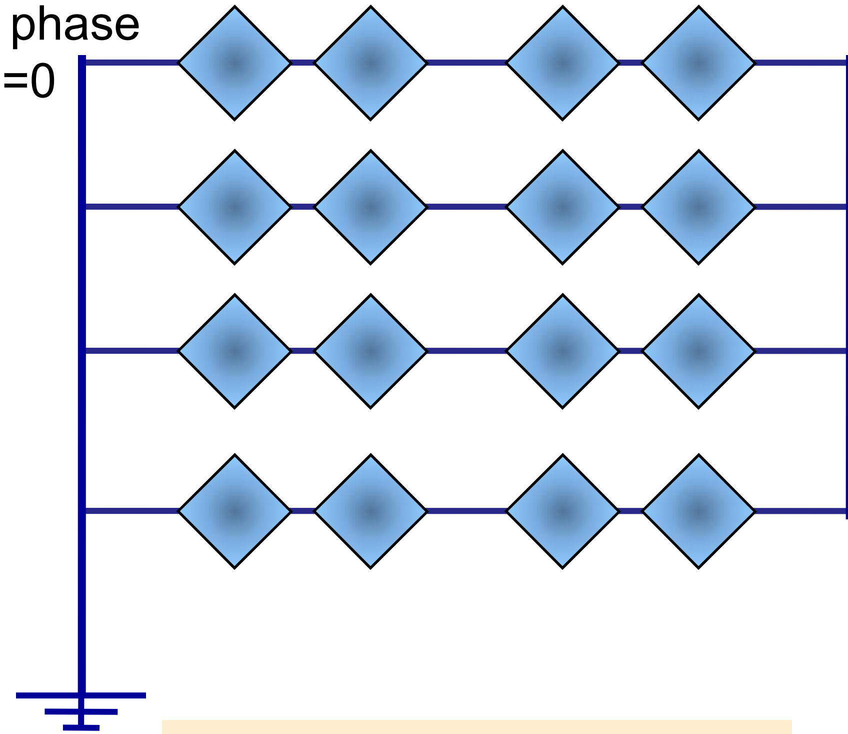
But large quantum fluctuations → low phase rigidity across the chain



→ no distinction between $\phi = 0$ and $\phi = \pi$ states

Resolution(s)

A. Many (K>>1) Parallel Chains for k=4



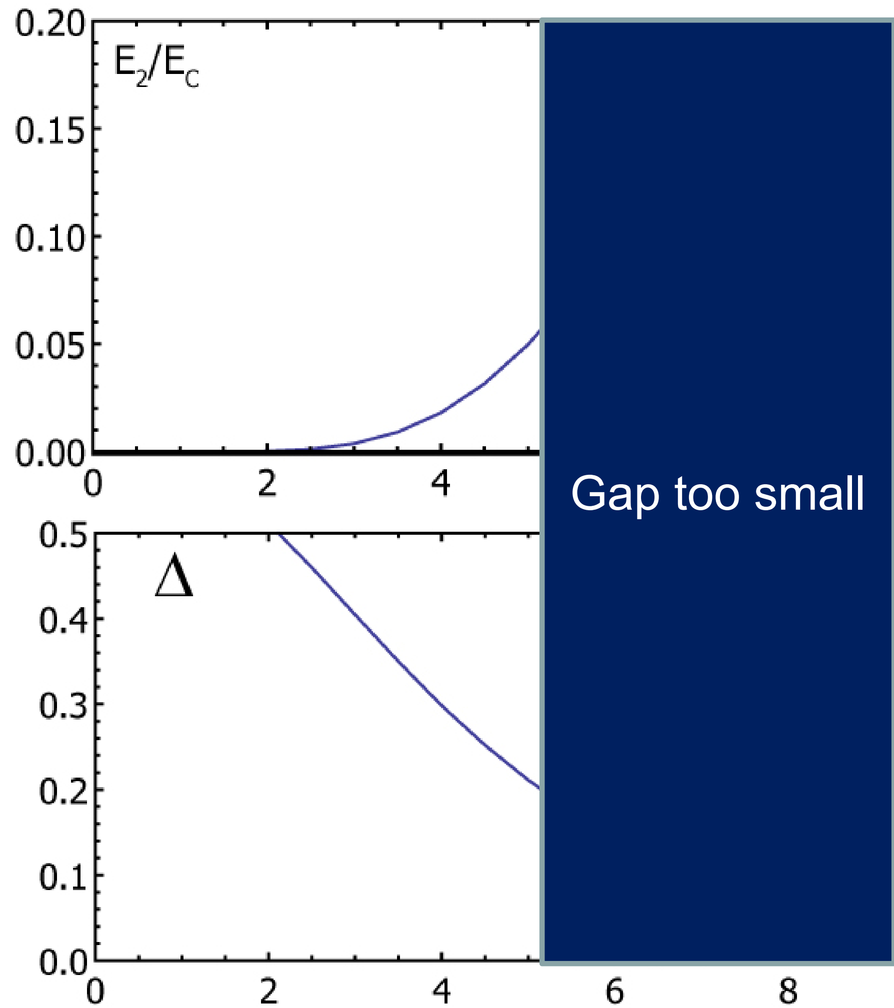
$V(\Phi) = K\, V_{chain}(\Phi)$

$C_{eff} = K\, C_{chain}$

Need $K^2\, \Delta v_{chain} / E_{c\ chain} >> 1$

$E_{c\ 4\ rhombi\ chain} \sim E_c$

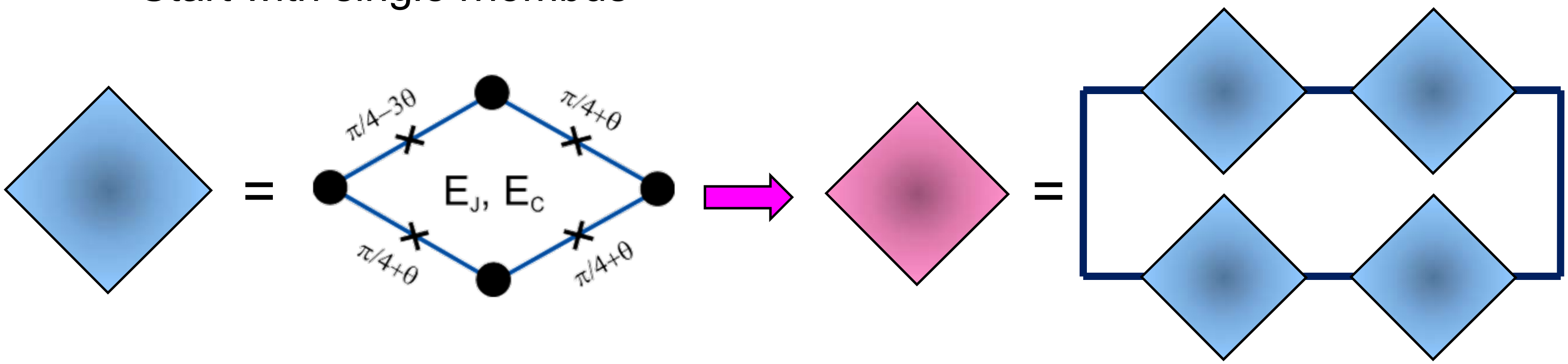
Need K~10-20



B. Hierarchical construction

Start with single rhombus

Combine 4 rhombi into super-rhombus



$$H^n = -E_2^n \cos 2\phi - E_1^n \cos\phi + 4E_C^n \hat{q}^2 \qquad \Rightarrow \qquad H^{n+1} = -E_2^{n+1} \cos 2\phi - E_1^{n+1} \cos\phi + 4E_C^{n+1} \hat{q}^2$$

Need E_2^n / E_C^n to increase (or stay constant) but E_1^n / E_C^n to decrease

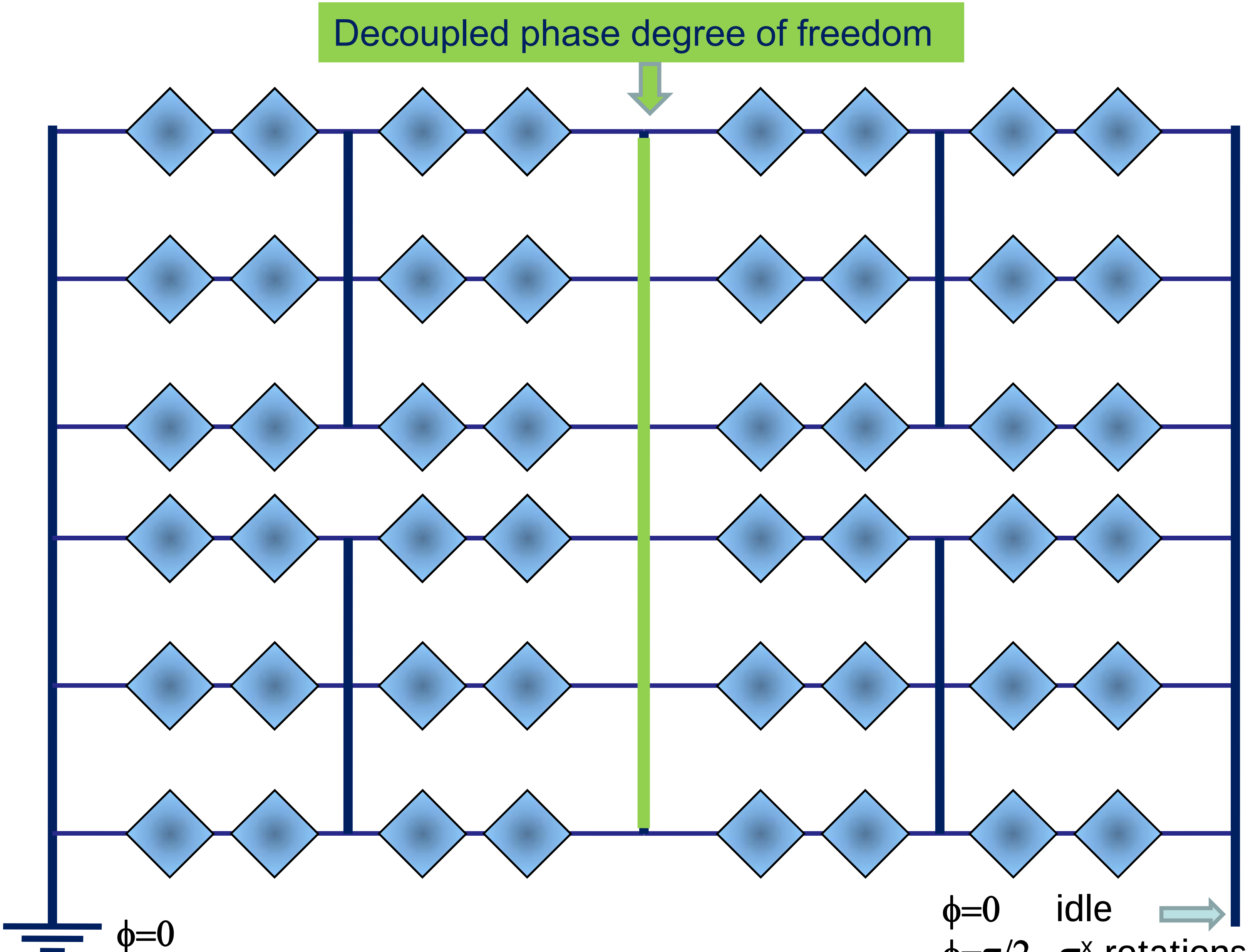
$$E_2^{n+1} / E_C^{n+1} = f(E_2^n / E_C^n) \quad \text{for small } x \; f(x) = \frac{k^2}{16} x^2$$

$$E_1^{n+1} / E_C^{n+1} = g(E_1^n / E_C^n) \quad \text{for small } x \; g(x) = \frac{k^2}{4} x^2$$

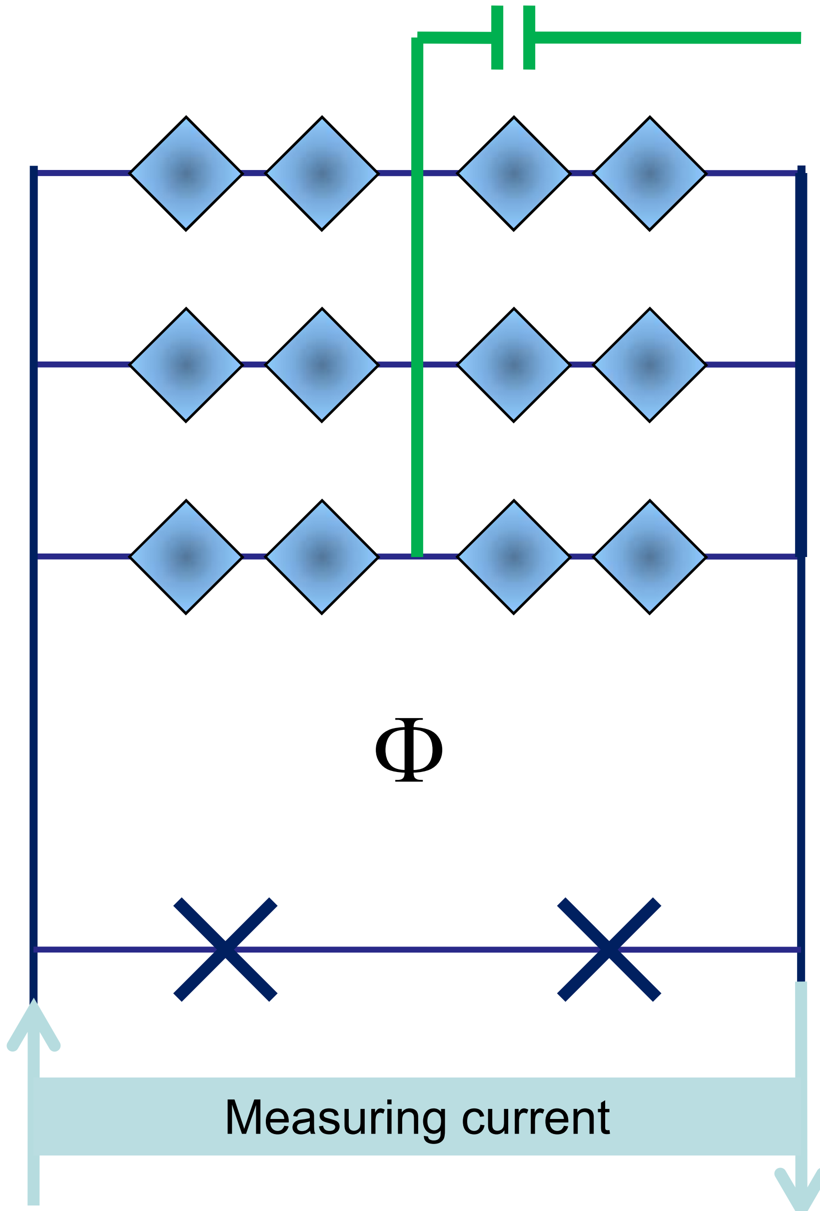
k - number of chains in parallel (k=2 above)

If one rhombus is good enough to produce $E_1^1 / E_2^1 < 1/4$ the unwanted term will decrease for the optimal $E_2^{(1)} / E_C^{(1)} \approx 1-2$

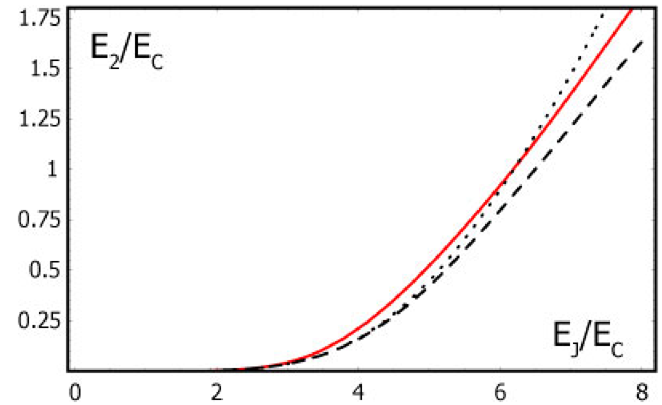
Protected qubit (3rd level)



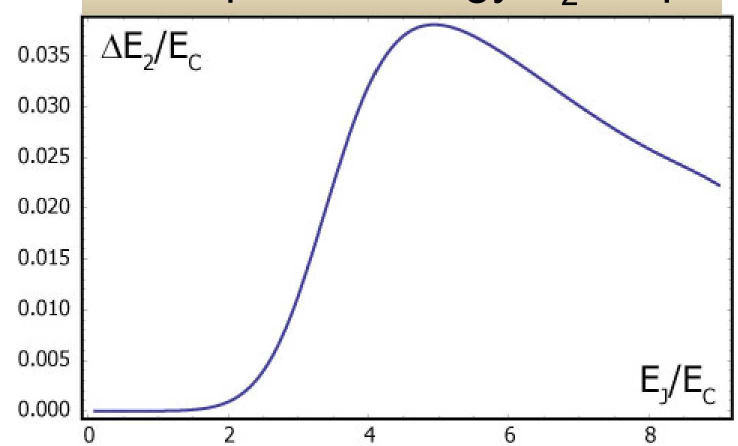
Minimalistic protected system



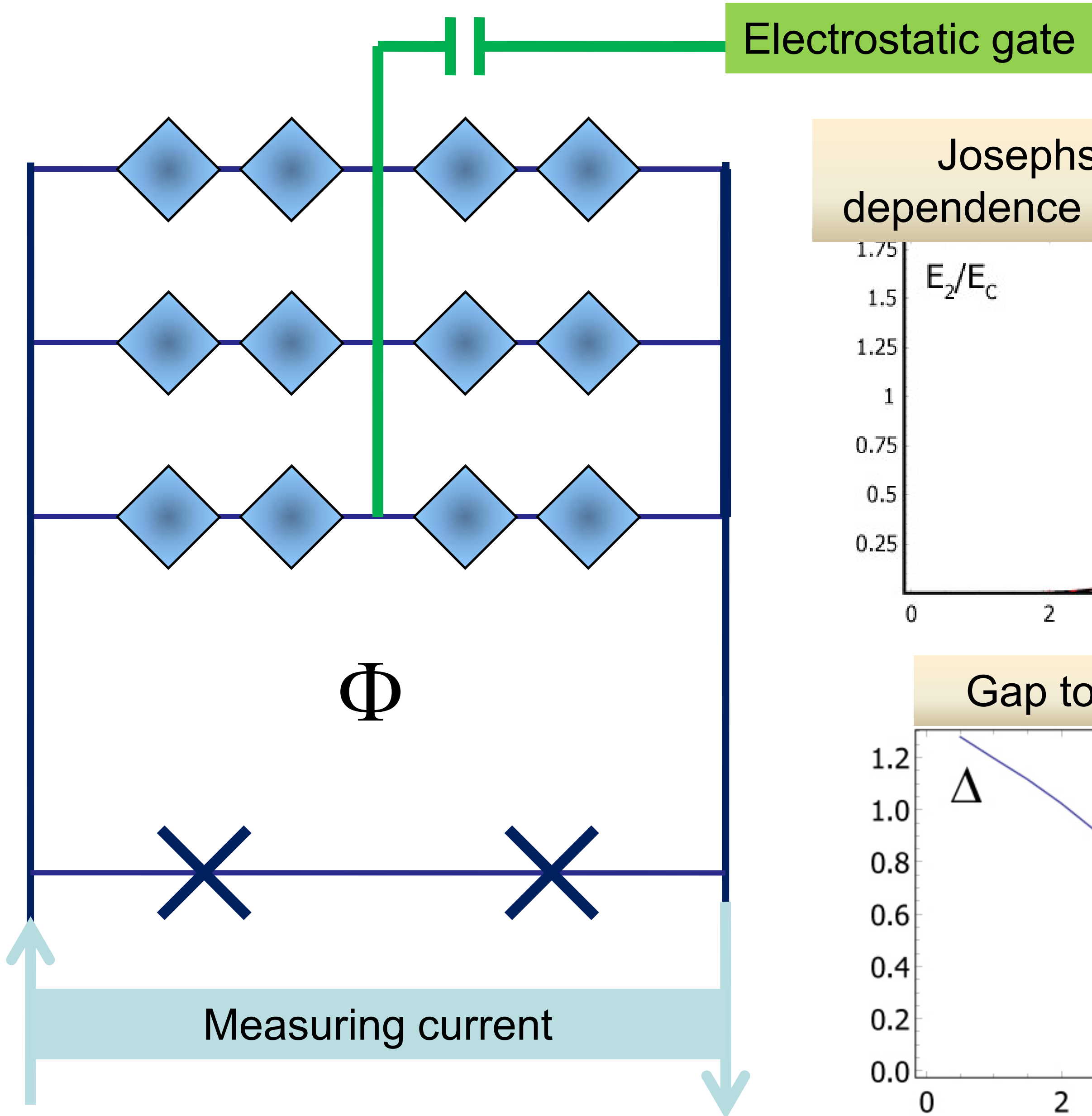
Josephson energy $E_2\cos\phi$ dependence on junction parameters

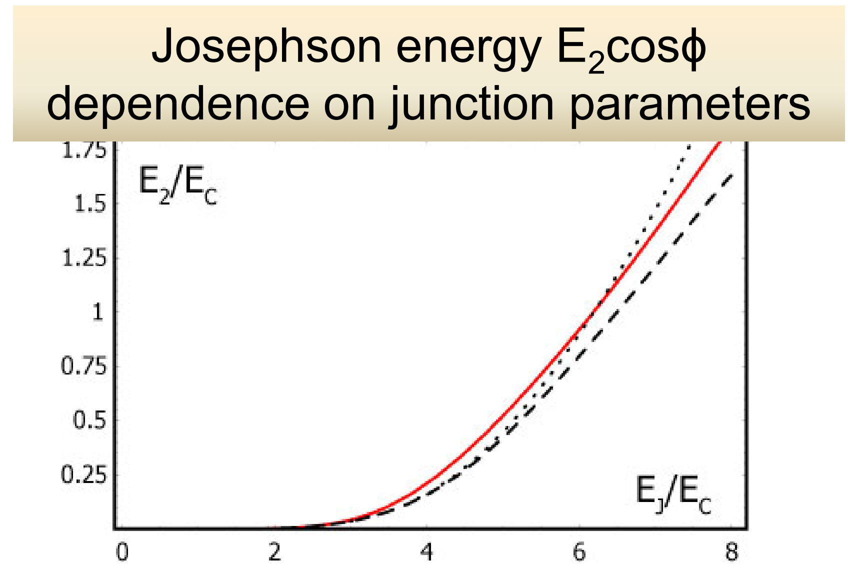


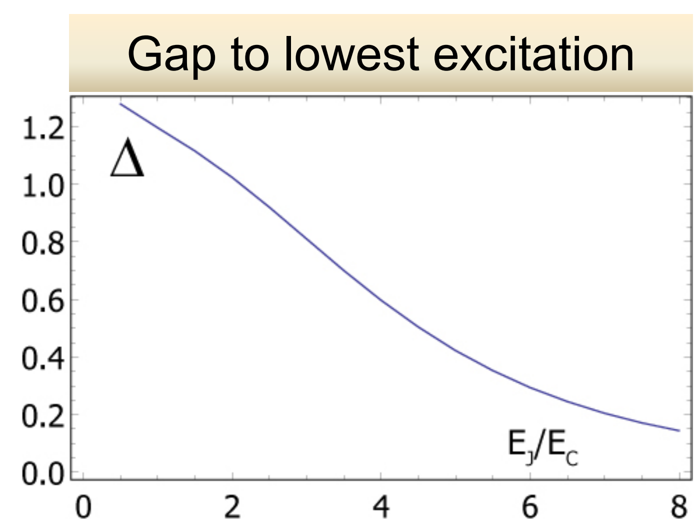
Effect of the gate on the Josephson energy $E_2\cos\phi$



Minimalistic protected system







Relaxation and decay rates of realistic hierarchical structures

Theory (+simulations):
Optimal regime $E_J \approx 6\text{-}8\ E_C$
K=3 hierarchy (k=4)

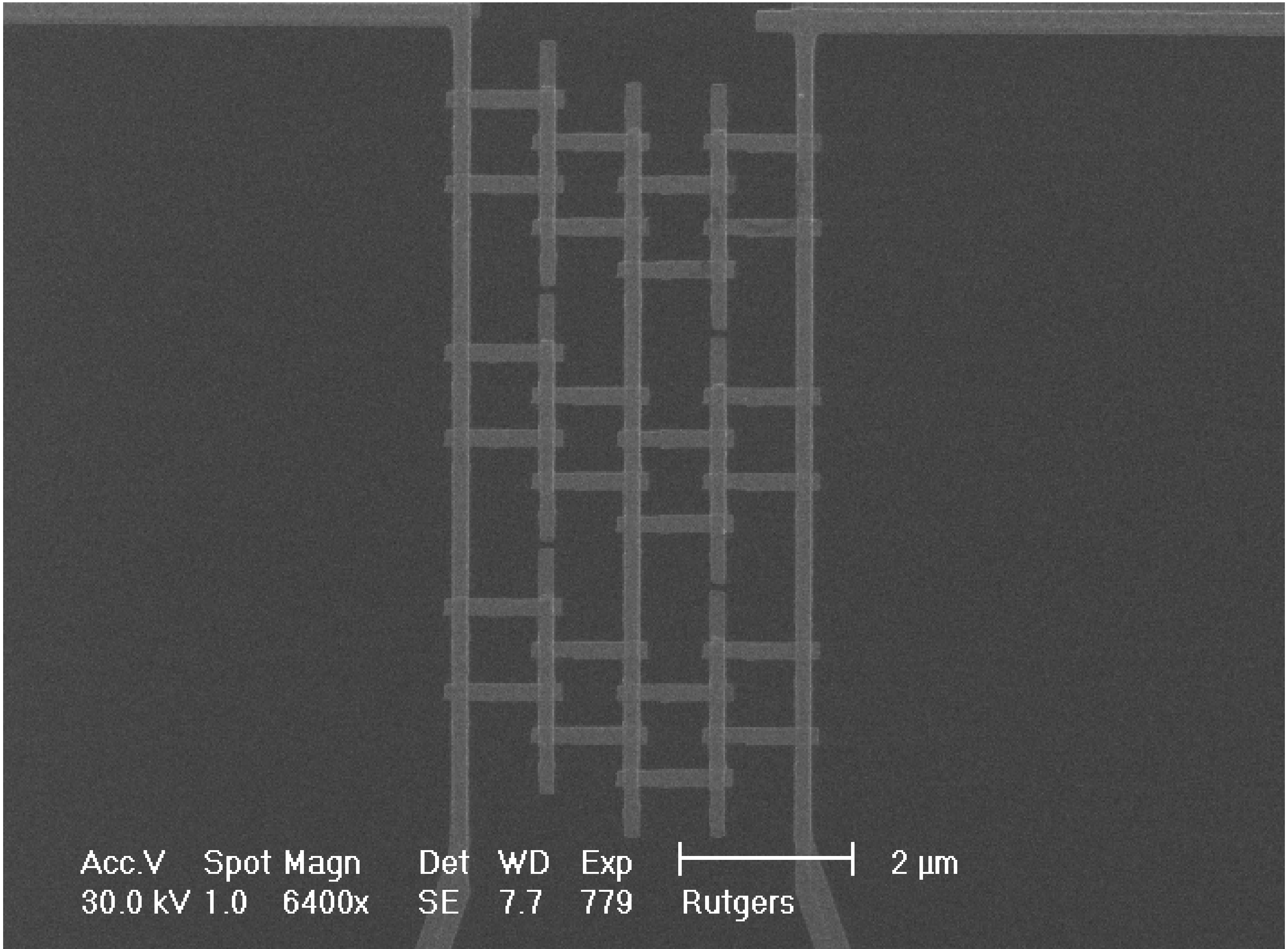
$$\Gamma_2^{hier} = \Gamma_2 \left(\gamma \frac{\delta\Phi}{\Phi_0} \frac{E_J}{r} \right)^{k-1} \approx \Gamma_2 \left(10 \frac{\delta\Phi}{\Phi_0} \right)^{k-1}$$

$$\Gamma_2^{hier} = \Gamma_2 \left(\gamma' \frac{\delta E_J}{r} \right)^{k-1} \approx \Gamma_2 \left(\frac{\delta E_J}{E_J} \right)^{k-1}$$

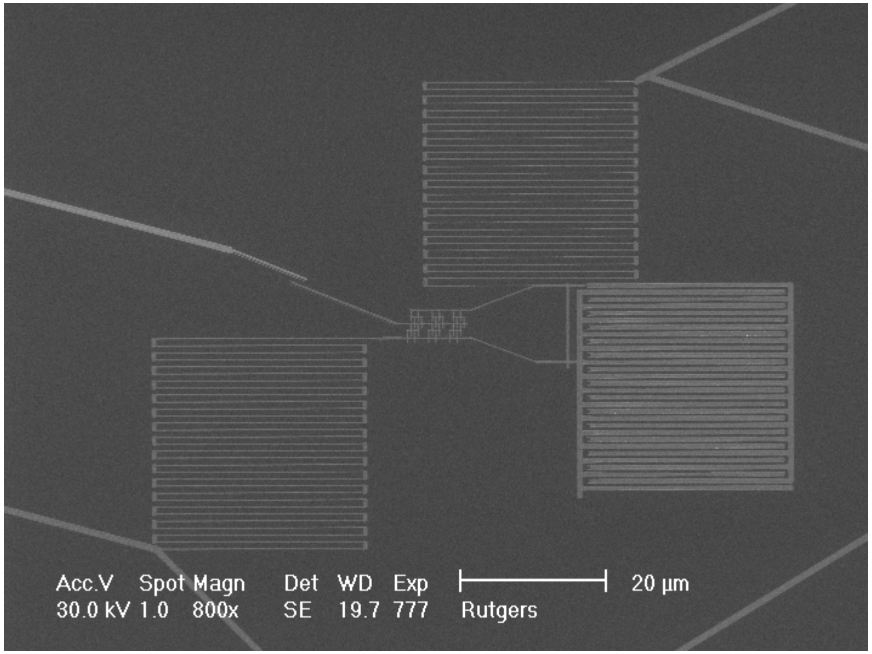
Contributions from
- flux (area) variations between the loops

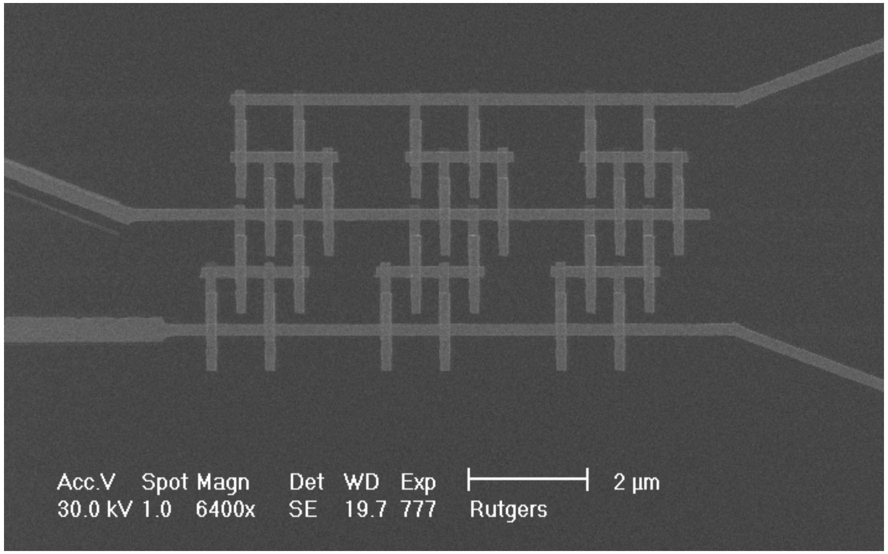
- Josephson junction variations in the same loop

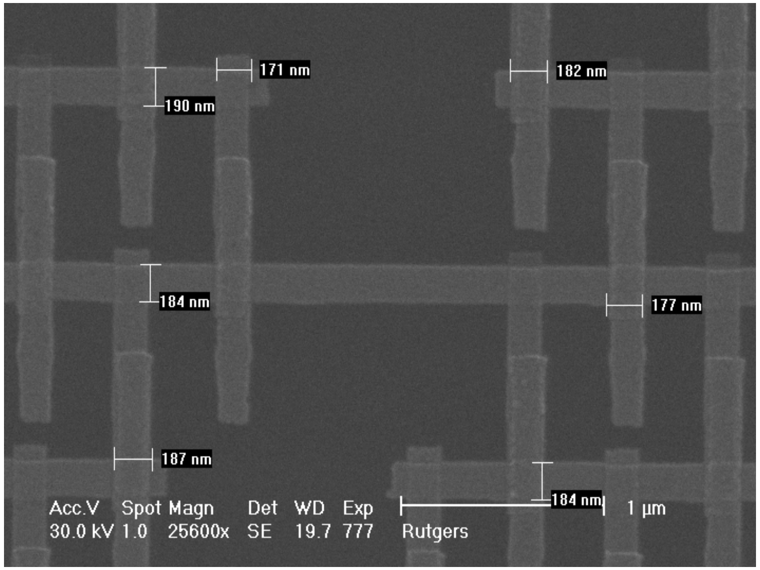
First Device



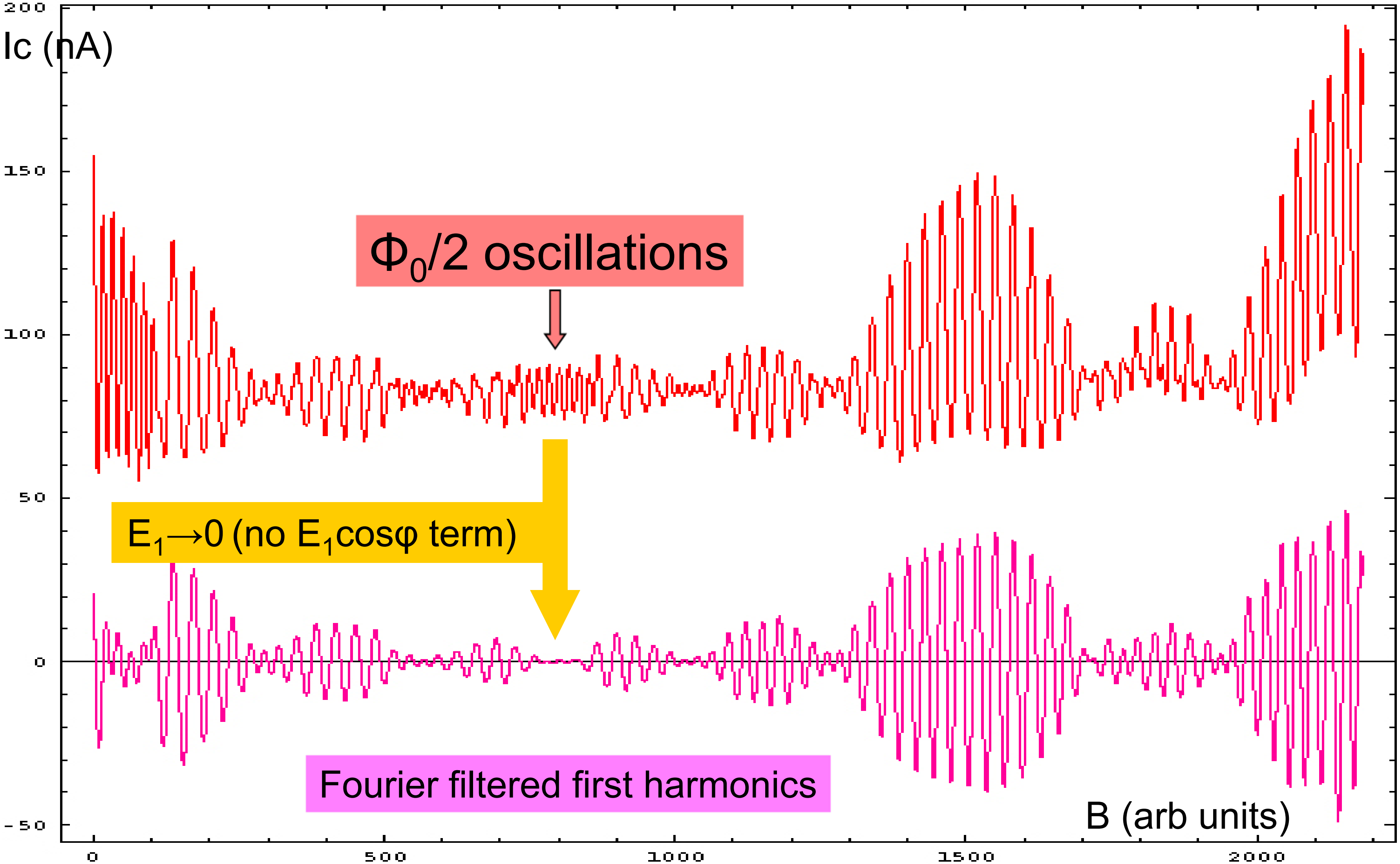
Improved design



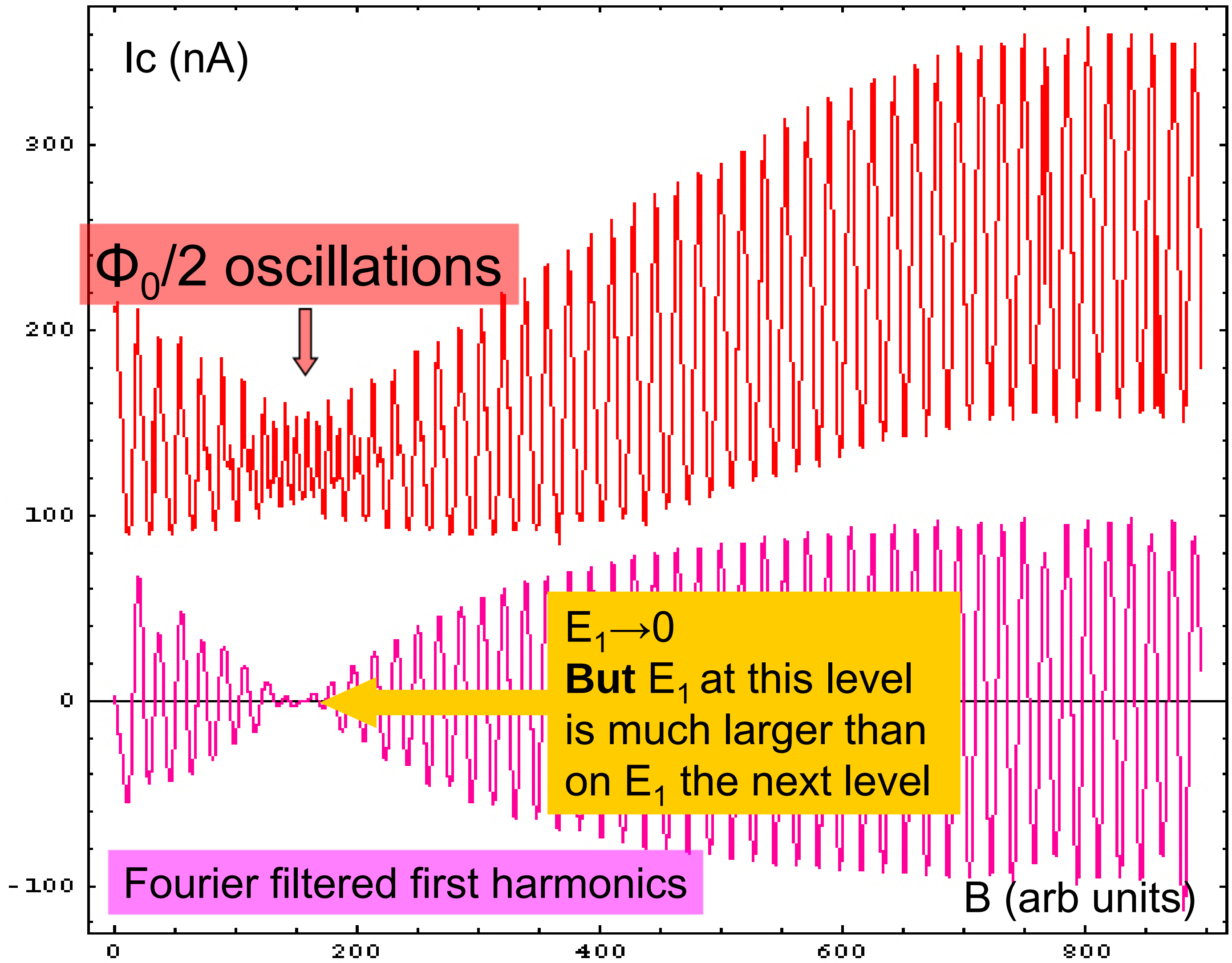




Critical current of the second level hierarchy device (12 rhombi)



Compare with 2 rhombi (first hierarchy level)



Value of critical currents: Theory versus Experiment

Theory: direct numerical diagonalization of Hamiltonian in charge basis is impossible: 91 charge degree of freedom!

$$H = -\frac{1}{2}\sum_{i;j} J_{ij}\hat{q}_i^{+}\hat{q}_j^{-} + 4E_C\sum_{i;j}(C^{-1})_{ij}q_i q_j$$

Approximate alternatives:

1.Replace actual system by 4 rhombi chain with additional capacitance in the middle and scale the result by a factor of 3^2=9. Should work well for small E_J/E_C

2. Use effective coupling produced by two rhombi chain and replace the two rhombi structure by effective Josephson element. Should work well in $K\to\infty$ limit.

Results for 12 rhombi samples:

L (contact)	E_C (Geom)	E_J (Am-B)	E_2 (Exp)	E_2 (Theor)
0.17	0.62	2.9	0.15	0.05
0.20	0.46	5.9	0.3	0.4

Value of critical currents: Theory vs. Experiment

Theory: direct numerical diagonalization of Hamiltonian in charge basis

$$H = -\frac{1}{2}\sum_{i;j} J_{ij}\hat{q}_i^{+}\hat{q}_j^{-} + 4E_C\sum_{i;j}(C^{-1})_{ij} q_i q_j$$

Charge increase/decrease operators

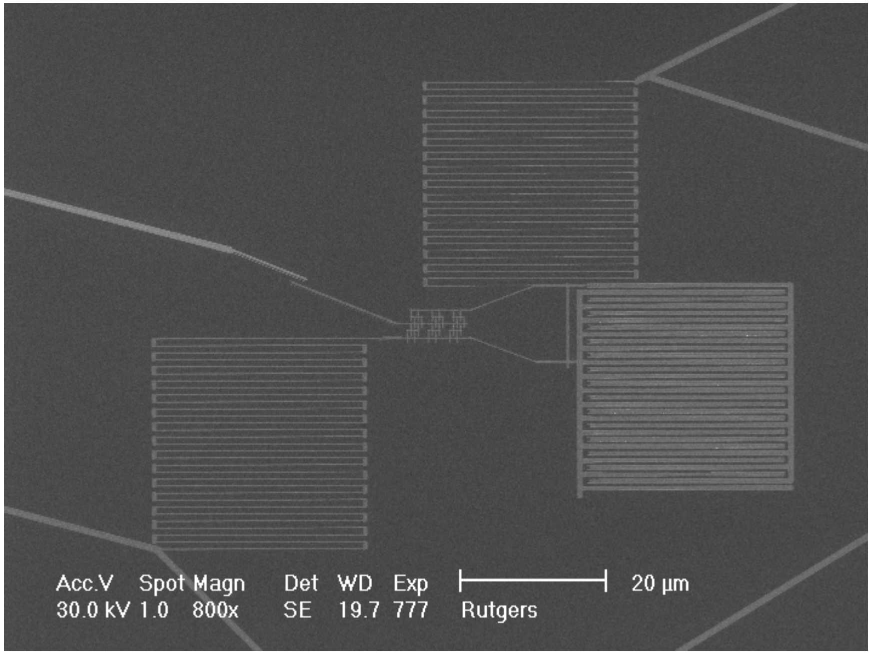
Capacitance Matrix

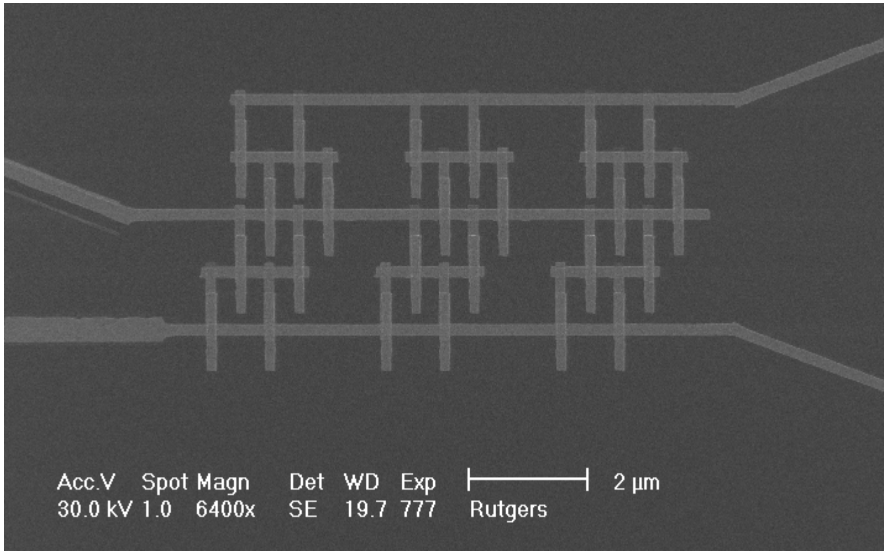
Accuracy of numerics can be verified for 2 rhombi systems for $E_J/E_C<10$.

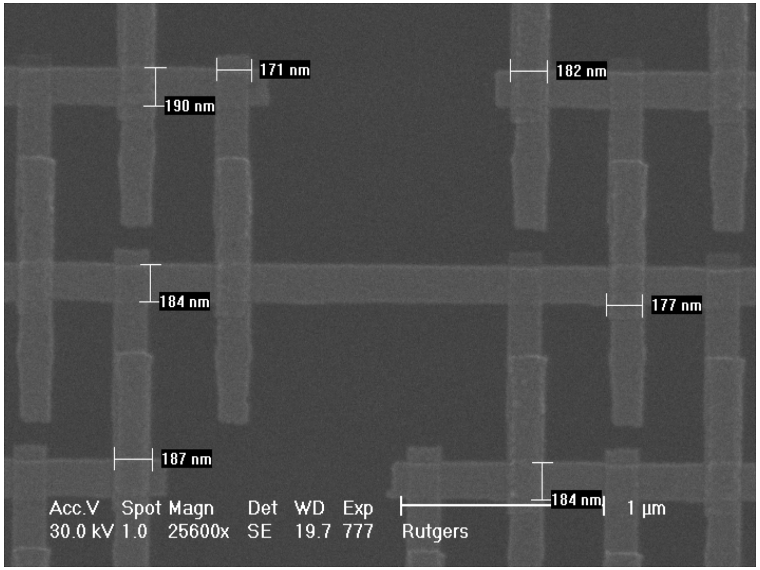
Results for 2 rhombi samples:

L (contact)	E_C (Geom)	E_J (Am-B)	E_2 (Exp)	E_2 (Theor)
0.17	0.6	2.2	0.12-0.15	0.10
0.21	0.42	3.3	0.3	0.4
0.27	0.26	5.3	0.6?	1.2?

Improved design

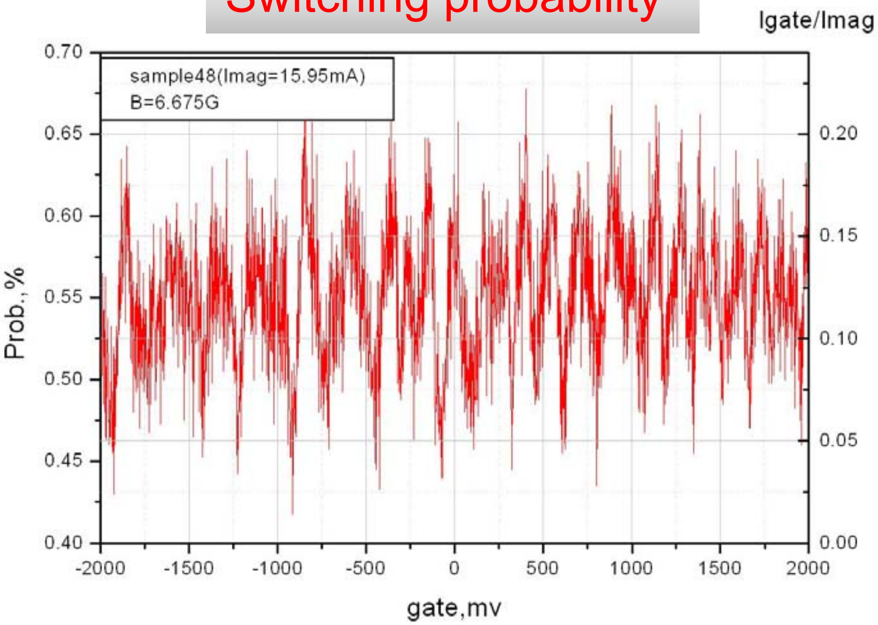




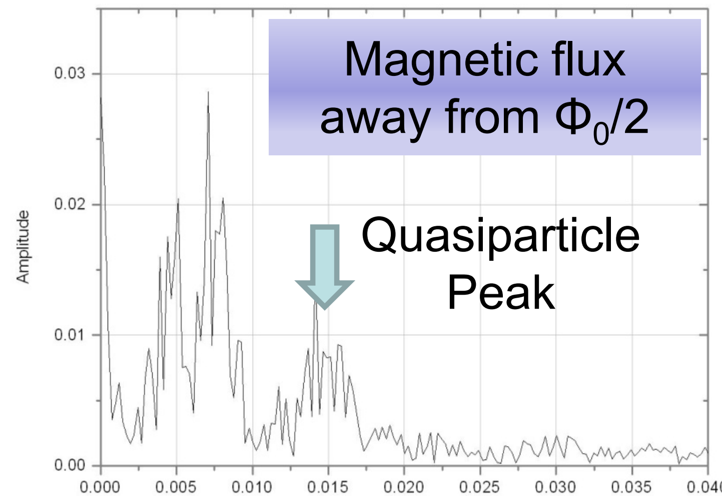


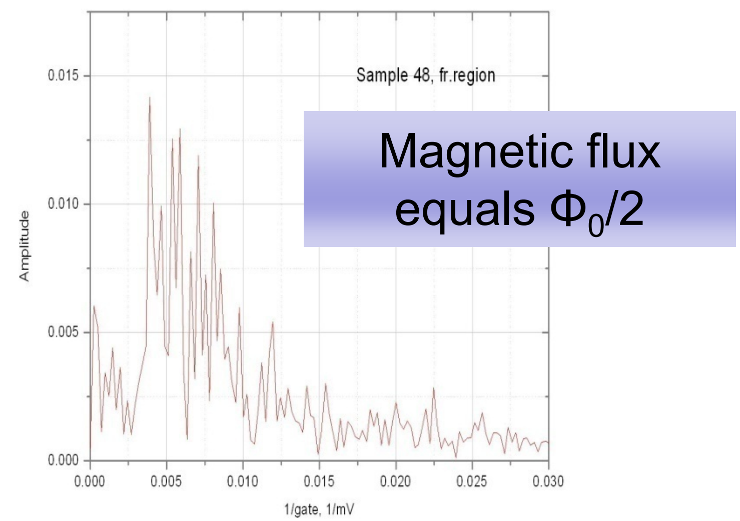
Effect of the gate potential

Switching probability



Fourier transforms of switching probability





Oscillations of critical current ~1-2 nA which correspond to $\Delta E_2 \sim 0.02\text{-}0.04\ E_C$ in agreement with numerical simulations

Conclusions

- Parallel chains of approximately π-periodic discrete Josephson elements should provide 'topological' protection from the noise: decoupling in higher orders or suppressed linear order.
- Problem of soft phase fluctuations in long chains can be solved by hierarchical construction
- Experimental realization shows appearance of π-periodicity which magnitude is in (rough) agreement with theoretical predictions and suppression of 2π-periodicity.
- Observed gate periodicity is in agreement with theoretical expectations.

Next steps

- We need to confirm the quantum nature of the fluctuations. For this we shall try to

A. To measure the gap in the spectrum directly by microwave spectroscopy

B. We need to optimize the parameters to find the values that produce largest ratio of the second harmonics to the first

- Measurements of the qubit coherence