Topologically protected abelian Josephson qubits: theory and experiment.

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Plan

• Honest (pessimistic) review of the state of the field of superconducting qubits.
• Theoretical models of protected qubits and their implementations in Josephson junctions.
• Experimental results
• Conclusions
State of the Art in Superconducting Qubits

**Charge**

**Charge/Phase**

**Flux**

**Phase**

Junction size $\rightarrow$ $E_J = E_C$

Charging versus Josephson energy

Ideal Hamiltonian: $H = 4E_C (\hat{n} - n_0)^2 + E_J \cos(\phi + \Phi) - I\phi$

$\hat{n} = i \frac{\partial}{\partial \phi}$ – number of Cooper pairs

$I$ – external current

Two states of the qubit differ by

charge $\hat{n}$ if $E_C \not\square E_J$

phase $\phi$ if $E_J \not\square E_C$
State of the Art in Superconducting Qubits

Charge

Charge/Phase

Flux

Phase

Junction size → $E_J = E_C$ → # of Cooper pairs

In real life $H = 4E_C[\hat{n} - n_0 + \delta n(t)]^2 + [E_J + \delta E_J(t)]\cos(\varphi + \delta \Phi(t)) - I\varphi$

$\delta n(t)$ charge noise, $\delta E_J(t)$ critical current noise, $\delta \Phi(t)$ flux noise

Partial remedy: Sweet spots (linear protection)

Energy as a function of the most dangerous parameter (charge, flux, etc)
State of the Art in Superconducting Qubits

History

- 1st qubit demonstrated in 1998 (NEC Labs, Japan)
- “Long” coherence shown 2002 (Saclay/Yale)
- Several experiments with two degrees of freedom
- C-NOT gate (2003 NEC, 2006 Delft and UCSB)
- Bell inequality tests being attempted (2006, UCSB) [failure due to low readout visibility!]
- 1st time domain tunable coupling of two flux qubits (2007, NEC Labs, Japan)
- Coupling superconducting qubits via a cavity bus (2007 Yale and NIST)
Relaxation and dephasing times

\[ \omega_0 / 2\pi = 5 \div 10 \text{ GHz} \]

<table>
<thead>
<tr>
<th>Design</th>
<th>Group</th>
<th>( T_2 )</th>
<th>( T_1 )</th>
<th>Visibility</th>
<th>Operation time</th>
<th>Logical quality factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase qubit</td>
<td>UCSB</td>
<td>(~ 85 ) nsec</td>
<td>110 nsec</td>
<td>&gt; 90%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flux qubit</td>
<td>NEC</td>
<td>(~ 0.8 ) ( \mu \text{sec} )</td>
<td>1 ( \mu \text{sec} )</td>
<td>(~ 20/30% )</td>
<td>(~ 0.02 ) ( \mu \text{sec} )</td>
<td>10 – 100*</td>
</tr>
<tr>
<td>Transmon (CPB in a cavity)</td>
<td>Yale</td>
<td>(~ 2 ) ( \mu \text{sec} )</td>
<td>(~ 1.5 ) ( \mu \text{sec} )</td>
<td>&gt; 95 %</td>
<td>(~ 0.05 ) ( \mu \text{sec} )</td>
<td>10 – 200*</td>
</tr>
</tbody>
</table>

\[ Q_{\text{Logical}} < 0.1 \quad Q_{\text{Physical}} = 10^2 - 10^3 \]

What do we need?
Error rates that we need for quantum computation.

\[ \log_{10}(Q_{\text{Logical}}) \]

\[ \log_{10}(R) \quad R = N/K - \text{redundancy} \]

\[ Q_{\text{Logical}} < 10^2 - 10^3 \]

Need \( Q_{\text{Logical}} \gg 10^4 \) for many qubit system

Steane PRA (2003)
Advantage of protection

Charge qubit: $Q < 100$

Flow qubit: away from the optimal point $Q \sim 10$

Quantronium, (charge qubit at the optimal point): $Q = 10^3 - 10^4$

At optimal point $Q = 10^3 - 10^4$
$T_1 \sim 4\mu s$  $T_2 \sim 3\mu s$  $T_2^{echo} \sim 4\mu s$

Devices decoupled from the leading source of noise in the linear order.

Devices decoupled in higher orders
Protected Qubit (General)

Protected Doublet:
Special Spin Hamiltonians $H$ with a large number of (non-local) integrals of motion $P, Q$:
$[H, P_k] = 0, [H, Q_m] = 0, [P_k, Q_m] \neq 0$

Simplest Spin Hamiltonian
$H = \sum_{kl} J^{x}_{kl} \sigma^x_k \sigma^x_l + \sum_{kl} J^{z}_{kl} \sigma^z_k \sigma^z_l$

Rows
$P_k = \prod_{l} \sigma^z_l$

Columns
$Q_k = \prod_{l} \sigma^x_l$

Crucial issues:
1. Which spin model has a large gap $\Delta$?
2. Which spin model is easiest to realize in Josephson junction arrays?

Any physical (local) noise term $\delta H(t)$ commutes with all $P_k$ and $Q_m$ except a $O(1)$ number of each. Effect of noise appears in $N$ order of the perturbation theory:

$$\delta E \sim (\delta H(t) / \Delta)^{N-1} \delta H(t)$$
Numerics for short range model (nearest neighbor interactions)

\[ H = \sum_{(kl)} J^x_{kl} \sigma^x_k \sigma^x_l + \sum_{(kl)} J^z_{kl} \sigma^z_k \sigma^z_l \]

Low energy states for 4*4 and 5*5 spin array. Low energy band contains \(2^4\) and \(2^5\) states.

Lowest excited state in the same sector as the ground state.

Splitting of two lowest (degenerate) levels by random field applied to each spin and distributed in interval (-0.05, 0.05).

Conclusion: relatively small arrays provide very good protection, especially in one channel!
Realization of individual spins and their interaction by Josephson Junction Arrays

Fixed phase $\Phi = 0$ or $\pi$

Only simultaneous flips are possible: $H = t \sigma^x_k \sigma^x_l$

Longer chains: $H = t \sum_{k,m} \sigma^x_k \sigma^x_m + \text{constraint } \prod_k \sigma^z_k = \text{const}$

Large capacitor preventing phase changes of the end point.
Where is the catch?

- Josephson elements are not discrete.

Noise suppression contains

\[
\left( \frac{\delta \Phi}{\Phi_0} \frac{E_J}{r} \right)^{k-1} \left( \frac{\gamma \delta E_J}{r} \right)^{k-1}
\]

(r ~ t – transition amplitude)

→ we need large quantum fluctuations.

But large quantum fluctuations → low phase rigidity across the chain

→ no distinction between \( \phi = 0 \) and \( \phi = \pi \) states
Resolution(s)

A. Many (K>>1) Parallel Chains for k=4

\[ V(\Phi) = K \ V_{\text{chain}}(\Phi) \]
\[ C_{\text{eff}} = K \ C_{\text{chain}} \]

Need \( K^2 \Delta v_{\text{chain}} / E_{\text{c chain}} \gg 1 \)

\( E_{\text{c 4 rhombi chain}} \sim E_{\text{c}} \)

Need \( K \sim 10-20 \)

Gap too small
B. Hierarchical construction

Start with single rhombus

\[ H^n = -E_2^n \cos 2\phi - E_1^n \cos \phi + 4E_C^n q^2 \]

Combine 4 rhombi into super-rhombus

\[ H^{n+1} = -E_2^{n+1} \cos 2\phi - E_1^{n+1} \cos \phi + 4E_C^{n+1} q^2 \]

Need \( E_2^n / E_C^n \) to increase (or stay constant) but \( E_1^n / E_C^n \) to decrease

\[ E_2^{n+1} / E_C^{n+1} = f\left( E_2^n / E_C^n \right) \quad \text{for small } x \quad f(x) = \frac{k^2}{16} x^2 \]

\[ E_1^{n+1} / E_C^{n+1} = g\left( E_1^n / E_C^n \right) \quad \text{for small } x \quad g(x) = \frac{k^2}{4} x^2 \]

If one rhombus is good enough to produce the unwanted term will decrease for the optimal \( E_2^{(1)} / E_C^{(1)} \approx 1 - 2 \)

\( E_1^{(1)} / E_2^{(1)} < 1/4 \) the unwanted term
Protected qubit (3rd level)

Decoupled phase degree of freedom

φ=0

φ=π/2

σ^x rotations

ϕ=0 idle
Minimalistic protected system

Electrostatic gate

Josephson energy $E_2 \cos \phi$
dependence on junction parameters

Effect of the gate on the
Josephson energy $E_2 \cos \phi$

Measuring current
Minimalistic protected system

Electrostatic gate

Josephson energy $E_2 \cos \phi$
dependence on junction parameters

Gap to lowest excitation

Measuring current
Relaxation and decay rates of realistic hierarchical structures

Theory (+simulations):
Optimal regime $E_J \approx 6-8 E_C$
K=3 hierarchy (k=4)

Contributions from
- flux (area) variations between the loops
- Josephson junction variations in the same loop
First Device
Improved design
Critical current of the second level hierarchy device (12 rhombi)

$\Phi_0/2$ oscillations

$E_1 \rightarrow 0$ (no $E_1 \cos \varphi$ term)

Fourier filtered first harmonics
Compare with 2 rhombi (first hierarchy level)

$\Phi_0/2$ oscillations

But $E_1$ at this level is much larger than on $E_1$ the next level

Fourier filtered first harmonics
Value of critical currents: Theory versus Experiment

Theory: direct numerical diagonalization of Hamiltonian in charge basis is impossible: 91 charge degree of freedom!

$$H = -\frac{1}{2} \sum_{i,j} J_{ij} \hat{q}_i \hat{q}_j + 4E_C \sum_{i,j} (C^{-1})_{ij} q_i q_j$$

Approximate alternatives:
1. Replace actual system by 4 rhombi chain with additional capacitance in the middle and scale the result by a factor of $3^2=9$. Should work well for small $E_J/E_C$
2. Use effective coupling produced by two rhombi chain and replace the two rhombi structure by effective Josephson element. Should work well in $K \rightarrow \infty$ limit.

Results for 12 rhombi samples:

<table>
<thead>
<tr>
<th>L (contact)</th>
<th>$E_C$ (Geom)</th>
<th>$E_J$ (Am-B)</th>
<th>$E_2$ (Exp)</th>
<th>$E_2$ (Theor)</th>
</tr>
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<tbody>
<tr>
<td>0.17</td>
<td>0.62</td>
<td>2.9</td>
<td>0.15</td>
<td>0.05</td>
</tr>
<tr>
<td>0.20</td>
<td>0.46</td>
<td>5.9</td>
<td>0.3</td>
<td>0.4</td>
</tr>
</tbody>
</table>
Value of critical currents:
Theory vs. Experiment

Theory: direct numerical diagonalization of Hamiltonian in charge basis

$$
H = -\frac{1}{2} \sum_{i,j} J_{ij} \hat{q}_i \hat{q}_j + 4E_C \sum_{i,j} (C^{-1})_{ij} q_i q_j
$$

Accuracy of numerics can be verified for 2 rhombi systems for $E_J/E_C < 10$.

Results for 2 rhombi samples:

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<th>$L$ (contact)</th>
<th>$E_C$ (Geom)</th>
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<tbody>
<tr>
<td>0.17</td>
<td>0.6</td>
<td>2.2</td>
<td>0.12-0.15</td>
<td>0.10</td>
</tr>
<tr>
<td>0.21</td>
<td>0.42</td>
<td>3.3</td>
<td>0.3</td>
<td>0.4</td>
</tr>
<tr>
<td>0.27</td>
<td>0.26</td>
<td>5.3</td>
<td>0.6?</td>
<td>1.2?</td>
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</table>
Improved design
Effect of the gate potential

Oscillations of critical current \( \sim 1-2 \text{ nA} \), which correspond to \( \Delta E_2 \sim 0.02-0.04 \) \( E_C \), in agreement with numerical simulations.

- Magnetic flux away from \( \Phi_0/2 \)
- Quasiparticle Peak
- Magnetic flux equals \( \Phi_0/2 \)
Conclusions

• Parallel chains of approximately $\pi$-periodic discrete Josephson elements should provide ‘topological’ protection from the noise: decoupling in higher orders or suppressed linear order.

• Problem of soft phase fluctuations in long chains can be solved by hierarchical construction.

• Experimental realization shows appearance of $\pi$-periodicity which magnitude is in (rough) agreement with theoretical predictions and suppression of $2\pi$-periodicity.

• Observed gate periodicity is in agreement with theoretical expectations.
Next steps

- We need to confirm the quantum nature of the fluctuations. For this we shall try to
  
  A. To measure the gap in the spectrum directly by microwave spectroscopy
  
  B. We need to optimize the parameters to find the values that produce largest ratio of the second harmonics to the first

- Measurements of the qubit coherence