

MAE 559

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Final Project: Ballpoint pen

Paul Reverdy

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1 Introduction

The ballpoint pen is one of the most simple, ubiquitous devices whose function depends on surface tension. The history of its invention is complicated: during several decades many inventors tried to improve on the fountain pen without much practical success. In the years leading up to World War 2 the Bíró brothers from Hungary developed the general design that has become standard.¹ Several years after the war Michel Bich developed a cheaper, mass market version of the Bíró design and the Bic Crystal pen was born.

Modern ballpoint pens are much improved compared to fountain pens, but still do not always perform as desired. As anyone who has spent enough time writing will recognize, the pens will occasionally skip or smudge. In general, there are conditions under which the pen will not lay down a consistent layer of ink. In the same spirit as studies of the printer's instability, I intend to study the quantitative conditions under which ballpoint pens fail to perform as desired. This analysis would be useful for designing better pens, and as we will see below is related to a variety of other interesting problems.

¹This is why in British English ballpoint pens are often referred to as Biro's.

In Figure 1 I show a sketch of the standard B  ro design. The ball bearing that applies the ink to paper has diameter of approximately 1 mm, and so is of a size where capillary effects dominate. Clearly gravity is not completely negligible, as any normal ballpoint pen will begin to fail if used upside down for an extended period of time.² However, in normal operating environments the ink flow is slow and primarily driven by capillary forces.

In Figure 2 I show a close-up photo of the ball bearing that is the point of the pen for illustrative purposes. In particular, one can see that the ball is indeed coated by a thin film of ink, that its surface is at least somewhat rough, and that there is a buildup of inky grime around the edge of the socket.

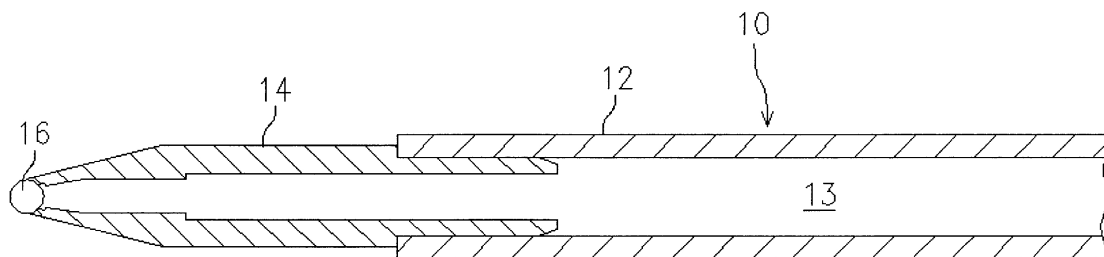


Figure 1: Diagram of a standard ballpoint pen design. The ballpoint (16) is held in the socket piece (14), fed ink (13) which is all held by the body (12). From United States Patent 6443648.

²An exception is the Fisher Space Pen, which uses a sealed, pressurized ink cartridge to ensure a continuous supply of ink. However, this is an unusual design, and capillarity still dominates the ink application process between ball and paper.



Figure 2: A close-up photo of the ball bearing at the heart of a ballpoint pen. Note the thin film of ink on the ball and the amount of buildup around the edge of the socket. From Wikimedia Commons.

2 Questions

Based on the above observations, we can break down the functioning of the pen into a variety of smaller problems for analysis.

2.1 Lubricated ball rotating in a socket

The point of the pen is essentially a lubricated ball bearing. It is driven by friction between itself and the paper or other writing surface. Thus, one potential mode of failure for the pen is for the ball to become jammed in the socket or stop spinning because of a lack of friction between the ball and the paper. This last hypothesis is supported by the difficulty of writing on slick surfaces, such as wax-coated paper, glass or many plastics.

There is a literature on lubrication theory in ball bearings which is relevant here. The two papers by Savage [5],[6] are a good beginning. There are a number of qualitative questions that one can ask, such as whether or not such lubrication films are stable. More quantitative questions include whether there is a critical speed above which the film is unstable, or whether the ball will tend to jam depending on the amount of pressure with which the pen is put to paper.

2.2 Ink film on a ball

As my group showed in the project for Homework 3, a thin film on a sphere is unstable even in the absence of gravity. In the presence of gravity, one would expect the film to be even more unstable.

Purely considering the instability of the film when the pen is out of use, this could cause failure in at least two ways: first, if the film goes unstable and ruptures in some places, the lack of ink could cause the pen to skip on beginning writing. Second, the instability will cause the film to thicken and pool in some some locations, which could then either be deposited directly on the paper or pushed towards the edge of the socket. This

second hypothesis provides a potential explanation for the grime seen around the edge of the socket in the photo.

A relevant quantitative question here is to determine the time scale on which the instability tends to develop, to see, for example, whether it could develop quickly enough to be relevant during writing.

2.3 Transfer of ink from ball to paper

In his study of the printer's instability, Pearsons [4] has shown that, for a thin film contained by slowly diverging boundaries, there is always a critical speed above which the film is unstable. Though his work was for the quasi one-dimensional case, it is reasonable to assume that a similar instability would exist for the ballpoint pen.

In the setup of the classical printer's instability problem the roller has some rotational speed that is externally imposed. This is opposed to our case where the ball is rolling due to friction with the paper at a speed determined internally by the parameters of the problem. However, if we ignore the details of how the speed of the ball's rotation is determined a classical printer's instability calculation in this spherical geometry is relevant.

The two papers by Savage [5],[6] are again a good beginning, along with the one by Pearsons. The paper by Hakim et al. [2] is a good reference for this particular printer's instability calculation.

2.4 Imbibition problem for ink

Once the ink is on the paper there is the question of whether it can imbibe the paper or whether it will sit on the surface. If it does imbibe the paper, how quickly can it do so? Slowly imbibing ink would be more likely to get smudged and so is something to be avoided, while one would also want to avoid the ink bleeding through the paper. This is more an issue of surface and ink chemistry so I will not treat it farther beyond saying that ink formulation is important and this aspect should not be forgotten when trying to devise

new products based on the results of the other analysis.

2.5 Is the ink Newtonian?

One of the few companies to provide any details about the composition of their ballpoint pen inks is the Fisher Space Pen company, which describes the ink they use in their space pens as “thixotropic ink-semisolid until the shearing action of the rolling ball [liquefies] it.” Clearly such ink is non-Newtonian as its viscosity decreases over time under constant shear. Such ink would be stabilizing for the thin film on a sphere instability and would likely also help avoid other issues.

A quick search of the patent literature suggests that inks are often designed to be non-Newtonian. Gel inks in particular appear to have complex rheology. However, it is difficult to know the characteristics of the various inks used by other manufacturers without specifically testing them, as the companies do not generally reveal detailed information. As a first pass at analysis I will assume that the inks are Newtonian fluids.

3 Dimensional parameters

To frame the discussion that follows, I will take note of the various dimensional quantities that are relevant to the problem.

- R , Radius of ball: ≈ 1 mm.
- V_{max} , Maximum speed of the pen tip when writing: on the order of 10 cm/s, maybe up to about 50 cm/s.
- σ , Surface tension of the ink-air interface: Depends on the specific ink, but on the order of the value for the water-air interface, 70 mN/m.
- ρ , Density of the ink: again, on the order of the value for water, 1000 kg/m³.

- μ , Viscosity of the ink: Because of the non-Newtonian issues alluded to above, quoted reference values vary widely. Water at room temperature has a viscosity of approximately 1 mPa·s. I have seen reference values for ink running from about 0.6 to 7 mPa·s. The value for water at room temperature is approximately the right order of magnitude.
- h , Thickness of the film on the ball: The best guess is on the order of 10 μm .
- h_{ink} : Average thickness of the line written by the pen: Of the same order of magnitude as h , so 10 μm .

4 Ball bearing rotating in a socket

As a first step at analysis, let us attempt to estimate the amount of force required to rotate the ball in the socket. If the ball is initially at rest, the dominant effect is the surface tension forces on the edge of the socket. The force required to break the meniscus is on the order of $F = 2\pi r\gamma$, where r is the radius of the circular opening in the socket and γ is the surface tension of the ink-air interface.

Substituting in values, we see that $r \approx 1\text{mm}$ and that $\gamma \approx 70\text{mN/m}$, so we get a value of $F \approx 4.2 \times 10^{-4}$ N. How does this compare with the forces we can generate on the ball? If we have a coefficient of static friction μ_f , then the required normal force on the ball is $F_N = F/\mu_f$. This force is developed by a pressure over an area πr^2 , so the required average pressure on the cross section of the ball is

$$P = \frac{F_N}{\pi r^2} = \frac{2\gamma}{\mu_f r} \approx \frac{1}{\mu_f} 140 \text{ Pa.} \quad (1)$$

The coefficient of static friction is always less than 1, so this is actually a lower bound for the required pressure. On slick surfaces, μ_f may be as low as on the order of 10^{-2} , which would translate to a required pressure on the order of 14,000 Pa. Metals have a surface hardness on the order of a GPa, but for soft materials the hardness is on the order

of 10^5 Pa, so this is a plausible explanation for the pen to fail to begin to write on slick and soft materials.

5 Film thicknesses

In the following, it becomes important to know the thickness of the ink film on the ball and the thickness of the film deposited on the paper. In the Landau-Levich class of problems the thin film deposited is “self-metered”, so that the thickness is determined by the parameters of the problem.

These calculations are generally restricted to small capillary numbers, which depending on how we define our capillary number could exclude a large portion of the pen operating regime. If our capillary number is defined as $Ca = V_0\mu/\sigma$, then using our reference parameters for μ, σ and V_{max} we have $Ca_{max} = 0.5/70 \approx 0.007$, and we are always in the limit of small capillary numbers.

However, in some papers authors define a capillary number taking into account the geometric factor R/h_0 , which is the ratio of the sphere radius to the film thickness. In the following I have generally estimated h_0 to be on the order of $10 \mu\text{m}$, and we know that R is on the order of 1 mm . In this case we have $Ca_{max} \approx 0.7$ and the capillary number is no longer particularly small.

Some experiment is required in order to understand how to choose between these two scalings and find the film thicknesses.

6 Instability of a thin film on a sphere

Given a thin film of dimensionless thickness h_0 , we have shown that the dimensionless linearized diffusion equation for a perturbation \hat{h} in the film thickness is

$$\sin(\theta) \frac{\partial \hat{h}}{\partial t} = -\frac{1}{2} \left(h_0^2 + \frac{4}{3} h_0^3 \right) \left(\frac{\partial}{\partial \theta} \left(\sin(\theta) \frac{\partial P}{\partial \theta} \right) + \frac{\partial}{\partial \phi} \left(\frac{1}{\sin(\theta)} \frac{\partial P}{\partial \phi} \right) \right). \quad (2)$$

We proceed using a perturbation of the form $\hat{h}(\theta, \phi, t) = \epsilon f(\theta, \phi) e^{\sigma t}$, where the function $f(\theta, \phi)$ is a spherical harmonic Y_m^l . Substituting this perturbation into Equation 2 we find that σ is positive for all the spherical harmonics, so that the perturbation is growing and the film unstable.

A reasonable question to ask at this point is how quickly the instability will develop, so we can determine whether or not the instability time scale is short enough to be relevant to the other dynamics of the system. In our paper we find that σ is at least equal to $12A$, where $A = h_0^2 + \frac{4}{3}h_0^3$, and $h_0 = h/R$, which is the film thickness scaled by the sphere's radius.

Substituting in values of h and R , we find that $A \approx 10^{-4}$ so in dimensionless terms τ , the time scale for the development of the instability, is at the most on the order of 10^3 . Dimensionally, we have scaled time by the quantity μ/P_0 , which using our reference value of viscosity and the ambient atmospheric pressure is equal to approximately 10^{-8} s. Thus, the time scale of the instability is at most on the order of 10^{-5} s.

This time scale is incredibly fast, so our analysis seems improbable in this case. This is probably related to the non-Newtonian character of the ink, which would be stabilizing and therefore extend the time scale of the instability.

7 Printer's instability for a roller on a plate

The standard analysis of the printer's instability is in the symmetrical geometry where there are two counterrotating cylinders. However, for the pen the geometry is asymmetric, with a ball above the plane of the paper. As a first step, we can analyze the printer's instability in this asymmetric geometry; in essence studying the printer's instability for a one dimensional pen that transfers ink with a cylinder instead of a ball.

All this neglects that the ball is rolling due to friction with the paper, instead of externally imposed.

7.1 Basic analysis

This problem is analyzed in several papers, but we will follow the calculation of Hakim et al. [2]. The geometry is such that the x - and y -axes are in the plane of the plate, with the y -axis aligned with the axis of the cylinder, and the z -axis pointing upwards. The cylinder of radius R is placed above the plane such that the minimum distance between the cylinder and the plane is b_0 , with the width of the gap being described by $b(x)$.

The cylinder is driven at an externally imposed angular velocity Ω , so the linear velocity of the edge of the cylinder is given by $V_0 = R\Omega$.

Because of the geometry of the system, we assume that we have a local Poiseuille-Couette flow so that the fluid velocity \mathbf{u} is assumed to be

$$\mathbf{u}(x, z) = z \left(\frac{V_0}{b(x)} \hat{\mathbf{x}} + \frac{z - b(x)}{2\mu} \vec{\nabla}_h P \right), \quad (3)$$

where $\vec{\nabla}_h$ denotes the two-dimensional horizontal gradient in x and y and P is the pressure field. The symmetry along the y direction is reflected in the fluid velocity's lack of dependence on y .

The mean fluid velocity across the thin film is

$$\mathbf{u}(x) = \frac{V_0}{2} \hat{\mathbf{x}} - \frac{b^2(x)}{12\mu} \vec{\nabla}_h P. \quad (4)$$

The equation for the pressure field comes from the incompressibility of the oil

$$\text{div}(b(x)\mathbf{u}(x)) = 0 = \frac{V_0}{2} \frac{db(x)}{dx} - \frac{d}{dx} \left(\frac{b^3(x)}{12\mu} \right) \frac{\partial P}{\partial x} - \frac{b^3(x)}{12\mu} \left(\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} \right). \quad (5)$$

7.2 Dimensional analysis and finding the pressure field

For some reason the paper [2] leaves all the equations in dimensional form, but I will change them to dimensionless form. Our dimensional parameters are the linear speed V_0 , the two lengths R and b_0 from the geometry of the problem and the fluid viscosity, surface tension and density, respectively μ, σ and ρ .

I choose to scale lengths by b_0 because I care more about the gap than imposed geometric curvature, velocities by V_0 , and pressure by the group $V_0\mu/b_0$. Then denoting dimensionless quantities by a $\hat{}$, the velocity field becomes

$$\hat{\mathbf{u}}(\hat{x}, \hat{z}) = \frac{\hat{z}}{\hat{b}(\hat{x})} \hat{\mathbf{x}} + \frac{\hat{z} - \hat{b}(\hat{x})}{2} \hat{\nabla}_h \hat{P} \quad (6)$$

and the mean velocity across the film becomes

$$\hat{\mathbf{u}}(x) = \frac{\hat{\mathbf{x}}}{2} - \frac{\hat{b}^2(\hat{x})}{12} \hat{\nabla}_h \hat{P}. \quad (7)$$

Now, we impose incompressibility and conservation of mass by saying that Q , the flux of oil through the gap is constant, so we have $\hat{Q} = \hat{u}_x(\hat{x})\hat{b}(\hat{x}) = \text{constant}$. By symmetry, the pressure field varies only in the x direction, so we can quickly recover the equation for the pressure field:

$$\frac{d\hat{P}}{d\hat{x}} = \frac{6}{\hat{b}^2} - 12\frac{\hat{Q}}{\hat{b}^3}. \quad (8)$$

We can integrate this equation easily if we approximate $\hat{b}(\hat{x})$ by the osculating³ parabola of the cylinder: $b(x) \approx \frac{x^2}{2R} + b_0$, or in dimensionless terms,

$$\hat{b} = 1 + \beta\hat{x}^2, \quad (9)$$

where we have defined $\beta \equiv b_0/2R$. This means we match the curvature of the boundary at the point of minimum gap width. Furthermore, introduce the angle θ such that $\hat{x} = \beta^{-1/2} \tan \theta$, so that $\hat{b} = 1 + \tan^2 \theta$. Note that the boundary conditions $x = \pm\infty$ correspond to the boundary conditions $\theta = \pm\pi/2$.

Imposing the boundary condition that the pressure is equal to P_0 , the atmospheric pressure, at $\theta = -\pi/2$, the pressure field is given by

$$\hat{P}(\theta, \hat{Q}) - \hat{P}_0 = \frac{3}{\sqrt{\beta}} \left(\theta + \frac{\pi}{2} + \frac{1}{2} \sin 2\theta \right) - \frac{6\hat{Q}}{\sqrt{\beta}} \left(\frac{3}{4} \left(\theta + \frac{\pi}{2} \right) + \frac{1}{2} \sin 2\theta + \frac{1}{16} \sin 4\theta \right). \quad (10)$$

³Meaning the curve touching the boundary with the same tangent. From the Latin *osculari*, ‘to kiss’.

Finally, imposing that the pressure is again equal to the atmospheric pressure at $\theta = \pi/2$ closes the problem and implies that the flux is $\hat{Q} = 2/3$. Substituting this into the expression for the pressure field and simplifying shows that the pressure field is given by

$$\hat{P}(\hat{x}) = -\frac{2}{\sqrt{\beta}} \frac{\tan \theta}{(1 + \tan^2 \theta)} = -2 \frac{\hat{x}}{\hat{b}^2}. \quad (11)$$

It is worth noting that Hakim et al. forget the factor of two in the above expression.

7.3 Analysis of the interface

At the meniscus, we have a Young-Laplace boundary condition for the pressure field:

$$P(x_m) = P_0 - \sigma \left(\frac{2}{b(x)} + \frac{1}{\rho} \right), \quad (12)$$

where $1/\rho$ is the curvature of the interface in the horizontal (xy) plane and $1/b(x_m)$ is the curvature in the vertical (yz) plane. We assume $1/\rho = 0$ because of the symmetry in the y direction, so the dimensionless boundary condition is

$$\hat{P}(\hat{x}_m) = \hat{P}_0 - \frac{2}{Ca} \frac{1}{\hat{b}(\hat{x}_m)}, \quad (13)$$

where $Ca = V_0 \mu / \sigma$ is the capillary number of the system.

Expressing conservation of fluid at the interface takes the form

$$\hat{V}_n \hat{b}(\hat{x}_m) (1 - F(Ca)) = (\mathbf{n} \cdot \hat{\mathbf{u}}(\hat{\mathbf{r}}_m) - F(Ca)) \hat{b}(\hat{x}_m), \quad (14)$$

where V_n is the normal velocity of the interface at the point \mathbf{r}_m , \mathbf{n} is the outward normal of the meniscus, and the function $F(Ca)$ is the constant of proportionality in the relationship $Q = \alpha V_0 b(x_m)$ which Landau and Levich [3] calculated through matching asymptotics.

For the form of F , we use the result that Hakim et al. use, which is

$$F(Ca) = 0.12(1 - \exp(-8.6Ca^{2/3})). \quad (15)$$

7.4 Stability

Now, we do a linear stability analysis by introducing perturbations. The location $\xi(y)$ of the interface is assumed to be

$$x_m^{(1)} = \xi(y) = x_m + \varepsilon(t) \sin(ky) \quad (16)$$

and the pressure field

$$P(\mathbf{r}) = P^{(0)}(x) + \eta(t) q_k(x) \sin(ky), \quad (17)$$

where $P^{(0)}$ is the solution for the pressure field in front of a planar interface as determined above.

The perturbations non-dimensionallize in the obvious way. Substituting these perturbations into the dimensionless versions of equations 4 and 5 which determine the pressure field, we find that $\hat{q}_k(\hat{x})$ satisfies

$$\frac{d^2 \hat{q}_k}{d\hat{x}^2} + \frac{3}{\hat{b}} \frac{d\hat{b}}{d\hat{x}} \frac{d\hat{q}_k}{d\hat{x}} - \hat{k}^2 \hat{q}_k = 0, \quad (18)$$

which goes to zero as $\hat{x} \rightarrow -\infty$.

The boundary condition for the pressure on the interface becomes

$$\frac{b_0}{\sigma} \eta(t) q_k(x_m) = \hat{\varepsilon}(t) \left(-Ca \left[\frac{d\hat{P}^{(0)}}{d\hat{x}} \right]_{\hat{x}=\hat{x}_m} + \frac{2}{\hat{b}^2} \left[\frac{d\hat{b}}{d\hat{x}} \right]_{\hat{x}=\hat{x}_m} + \hat{k}^2 \right). \quad (19)$$

The conservation equation at the interface (14) gives the time dependence of ε and η :

$$\frac{d\hat{\varepsilon}}{dt} (1 - F(Ca)) = -\hat{\varepsilon}(\hat{t}) \frac{\hat{Q}}{\hat{b}^2(\hat{x}_m)} \left[\frac{d\hat{b}}{d\hat{x}} \right]_{\hat{x}=\hat{x}_m} - \frac{\hat{b}^2(\hat{x}_m)}{12} \hat{q}'_k(\hat{x}_m) \hat{\eta}(\hat{t}), \quad (20)$$

where $\hat{q}'_k = d\hat{q}_k/d\hat{x}$. Then $\hat{\varepsilon}$ has exponential time dependence $\hat{\varepsilon} \sim \exp(\hat{\tau}\hat{t})$ with $\hat{\tau}$ given by

$$\hat{\tau} = \frac{1}{1 - F(Ca)} \left(-\frac{\hat{Q}}{\hat{b}^2(\hat{x}_m)} \left[\frac{d\hat{b}}{d\hat{x}} \right]_{\hat{x}_m} + \frac{\hat{b}^2(\hat{x}_m)}{12} \frac{\hat{q}'_k(\hat{x}_m)}{\hat{q}_k(\hat{x}_m)} \left[\left[\frac{d\hat{P}^{(0)}}{d\hat{x}} \right]_{\hat{x}_m} - \frac{2}{Ca} \frac{1}{\hat{b}^2} \left[\frac{d\hat{b}}{d\hat{x}} \right]_{\hat{x}_m} - \frac{1}{Ca} \hat{k}^2 \right] \right). \quad (21)$$

The film will be unstable if $\hat{\tau} > 0$, which is the case if the term above involving $\hat{q}'_k(\hat{x}_m)$ dominates the term involving the volume flux \hat{Q} . To carry out this computation one

should solve the equation 18 for $\hat{q}_{\hat{k}}$. In order to simplify the calculation we assume that $(1/\hat{b})(d\hat{b}/d\hat{x})$ is constant, which is to say that we assume that \hat{b} is an exponential function of \hat{x} . Then, defining

$$\frac{1}{h} = \frac{3}{2\hat{b}} \left(\frac{d\hat{b}}{d\hat{x}} \right)_{\hat{x}_m}, \quad (22)$$

we have the approximate solution

$$\hat{q}_k \sim e^{m_k \hat{x}}, \text{ with } m_k = \frac{1}{h} \left(\sqrt{1 + h^2 \hat{k}^2} - 1 \right). \quad (23)$$

Under this assumption, $\hat{q}'_k(\hat{x}_m)/\hat{q}_k(\hat{x}_m) = m_k$ and so

$$\hat{\tau} = \frac{1}{1 - F(Ca)} \left(-\frac{\hat{Q}}{\hat{b}^2(\hat{x}_m)} \left[\frac{d\hat{b}}{d\hat{x}} \right]_{\hat{x}_m} - \frac{\hat{b}^2(\hat{x}_m)}{12} m_k \left[\left[\frac{d\hat{P}^{(0)}}{d\hat{x}} \right]_{\hat{x}_m} + \frac{2}{Ca} \frac{1}{\hat{b}^2} \left[\frac{d\hat{b}}{d\hat{x}} \right]_{\hat{x}_m} + \frac{1}{Ca} \hat{k}^2 \right] \right). \quad (24)$$

Given equation 24, we can calculate the capillary number at onset of instability for a given geometry and set of material properties. In our case, the order of magnitude for the critical velocity is approximately 100 mm/s.

8 Extension to the problem in three dimensions

As far as I can tell, the printer's instability in fully three dimensional geometry has never been treated in the literature. I have found one paper considering thin films between a sphere and a planar surface. Bico et al. consider the motion of a sphere down an inclined plane coated with a thin liquid film in [1]. This problem is much closer in spirit to the problem we are considering, both because of its fully three-dimensional nature and the fact that the speed of the ball is not externally imposed but determined by the physics of the system.

Unfortunately, the problem is difficult, and as they say, “the flow is a three-dimensional free-surface problem, which introduces many complexities and even thinking about two-dimensional variants . . . appears nontrivial.”

However, among their experimental results is one particularly interesting finding: that there are several flow regimes that develop depending on the value of the capillary number. Neglecting what they call the “overhang” case as less relevant to our current problem, they find a transition from the “circular” to the “cusp” regime when $Ca \approx 1$.

At low capillary numbers the system is in the circular flow regime, so named because the meniscus around the sphere is nearly circular. However, as the angle and therefore the capillary number is increased beyond the critical value ≈ 1 , a cusp appears at the rear (i.e. downstream) end of the meniscus and the sphere’s rolling and sliding velocities jump. Clearly, in this higher capillary flow regime the drag forces on the sphere are decreased.

In our particular system we consider the viscosity and surface tension to be constant, so the capillary number, $Ca = \mu V R / \sigma b_0$ is determined by the velocity of the pen tip. One experiment related to this instability would be to see if there were a threshold velocity above which the pen were much easier to drag across the paper; simple experiments done at home suggest that if such a velocity does exist, it is either far too low to be noticeable or far too high to be attained during normal use.

A simple calculation suggests that the latter is the case: using the reference values of μ and σ , the velocity required to produce a capillary number 1 is on the order of 0.7 m/s, which is on the order of magnitude of the highest speed likely to be attained by a person dragging a pen across paper.

However, this may be where the non-Newtonian characteristics of the ink become important.

9 Conclusion

In this paper, we have analyzed the behavior of a ballpoint pen. To do this we first had to break the system down into its constituent parts, then find modes of failure and pose relevant questions, and finally attempt to solve them. In particular we were concerned with skipping or pooling behavior of the ink, both of which would stem from instabilities

in the ink being transferred to the writing surface.

To begin with, we estimated the pressures on the ball at the tip of the pen, and concluded that for some surfaces, the pen must fail to write because there is insufficient friction between the ball and the surface to bring ink from the reservoir.

Then we analyzed the instability of a thin film on a sphere, in this case the ball. The relevant time scales for the instabilities were far too fast to play much of a role in the modes of failure of the pen. However, in this case we saw why the non-Newtonian rheology of the ink may be desirable.

Afterwards, we spent the bulk of the paper analyzing the printer's instability in a geometry close to that of the pen-paper system. As in the majority of the literature, we focused on the two-dimensional case, where essentially we considered a pen whose writing tip was a long roller. In this case, there is an instability, but it only develops at relatively high velocities that are unlikely to be seen in everyday use of the pen.

Finally, we gave some indications on how the three-dimensional aspects of the problem might change the results of our two-dimensional calculation. There is very little theory available for this problem, but experimental results suggest that there is still an instability and that it also only develops at relatively high velocities.

In summary, ballpoint pens are devices that fundamentally use surface tension and do theoretically exhibit instabilities. However, these instabilities are sufficiently far outside of their normal operating range that they are actually very robust in everyday use.

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