# Implied Stochastic Volatility Model: Matlab and R Code Guide

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**Abstract**

This document explains the use of the attached code for estimating parametric and nonparametric implied stochastic volatility (ISV) models as described in Aït-Sahalia et al. (2020). The Maltab and R code was written by the authors of the paper. If you use it you must refer to the paper as the source.

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# 1 Code organization

This package contains the following folders and files:

| Type | Name | Usage |
|---|---|---|
| Matlab data | `IV_Data_Sample.mat` | Cleaned implied volatility (IV) data sample for S&P 500 index options |
| Folder | `Nonparametric_ISV` | The package for estimating a nonparametric ISV model |
| ↪R script | `NP_Estimation.R` | The main script for nonparametric estimation |
| ↪R function | `data_pre.R` | The R function for preprocessing the IV data |
| ↪R function | `SVestimate.R` | The R function for estimating a nonparametric ISV model |
| Folder | `Parametric_ISV` | The package for estimating a nonparametric ISV model |
| ↪Matlab script | `P_Estimation.m` | The main script for parametric GMM estimation |
| ↪Matlab function | `asy_var.m` | The Matlab function for calculating asymptotic variances |
| ↪Matlab function | `GMM_loss_fun.m` | The loss function in GMM |
| ↪Matlab function | `GMM_loss_fun_cons.m` | The loss function in GMM with the Feller constraint |
| ↪Matlab function | `moment_fun.m` | The moment functions in GMM estimation |
| ↪Matlab function | `moment_derivatives.m` | The gradient of the GMM moment functions |
| ↪Matlab function | `TwoStepGMM.m` | The Matlab function for two-step GMM estimation |
| ↪Matlab function | `sigma_ij.m` | The closed-form formula for the expansion term $\sigma^{(i,j)}$ under the Heston model |

# 2 Data

For illustration purposes, the Matlab data file "`IV_Data_Sample.mat`" contains a data example consisting of a small sample of S&P index options. This is a small subsample of the data covering all the trading days of 2007–2017 used in Aït-Sahalia et al. (2020). For constructing this example subsample, we selected 10% of the trading days (276 trading days) from the full sample and included all the options for the days that were selected in this subsample. This file consists of the following variables: `cp_flag`, `d`, `date`, `IVobs`, `Money`, `Mtr`, `price`, `r`, and `Sobs`. For each variable, each row includes the corresponding data for one day. For example, the first row of `IVobs` includes all the IV data on the first day of the subsample – January 3, 2017.

Here are the variables:

- `cp_flag`: 0 for call options and 1 for put options.
- `d`: Daily dividend yields in percentage.
- `date`: Numeric dates. One can use the Matlab function `datestr` to interpret these dates in traditional form.

- `IVobs`: IV observations.
- `Money`: Log-moneyness.
- `Mtr`: Days-to-expiration.
- `price`: Option prices.
- `r`: Risk-free interest rates in percentage.
- `Sobs`: Daily observations of S&P 500 index.

## 3 Estimating a nonparametric ISV model

The code files are collected in the folder `Nonparametric_ISVM`. The main R script for estimating a nonparametric ISV model is `NP_Estimation.R`, while the other two are required R functions. The main file consists of two parts. Part I applies the data filters described in Section 5 of Aït-Sahalia et al. (2020) and then performs one-trial nonparametric estimation for the model (1)–(2). The nonparametric estimators of coefficient functions $\mu$, $\gamma$, $\eta^2$, $\eta$, and $\rho$ will be shown in a figure without confidence intervals. Part II conducts the bootstrap procedure and calculates the confidence interval of each nonparametric function. The final output is shown in Figure 1, which can be compared to Figure 11 of Aït-Sahalia et al. (2020) for the full sample.

Before running the main script, make sure you set the directory correctly. Comments are provided in each code file.

## 4 Estimating a parametric ISV model

The code files are collected in the folder `Parametric_ISV`. The main Matlab script for estimating a parametric Heston model (30)–(31) is `P_Estimation.m`, while the other files are required Matlab functions. In the main file, we consider two cases – the case of exact identification and the case of overidentification. The former (resp. latter) uses four (resp. five) moment conditions to estimate four model parameters $\kappa$, $\alpha$, $\xi$, and $\rho$, as well as computes the asymptotic standard deviations. The final output is shown in Table 1, which can be compared to Table 3 of Aït-Sahalia et al. (2020) for the full sample.

Before running the main script, make sure you have set your current working directorty as "`..\ISVM_Code\Parametric_ISVM`". You can use the Matlab function `pwd` to check your current working directory. Comments are provided in each code file.

## References

Aït-Sahalia, Y., Li, C., Li, C. X., 2020. Implied Stochastic Volatility Models. Review of Financial Studies, forthcoming .

| | Exact identification | | Over identification | |
|---|---|---|---|---|
| Parameter | Estimator | Standard error | Estimator | Standard error |
| $\kappa$ | 10.5 | 1.33 | 9.7 | 1.35 |
| $\alpha$ | 0.040 | 0.0042 | 0.040 | 0.0039 |
| $\xi$ | 1.1 | 0.056 | 0.88 | 0.034 |
| $\rho$ | $-0.617$ | 0.0025 | $-0.606$ | 0.0036 |

Table 1: Parametric ISV model: Estimates based on the data sample "`IV_Data_Sample.mat`"

Note: Except for corresponding to the data sample "`IV_Data_Sample.mat`", all the settings for producing this table are the same as those for producing Table 3 in Aït-Sahalia et al. (2020).

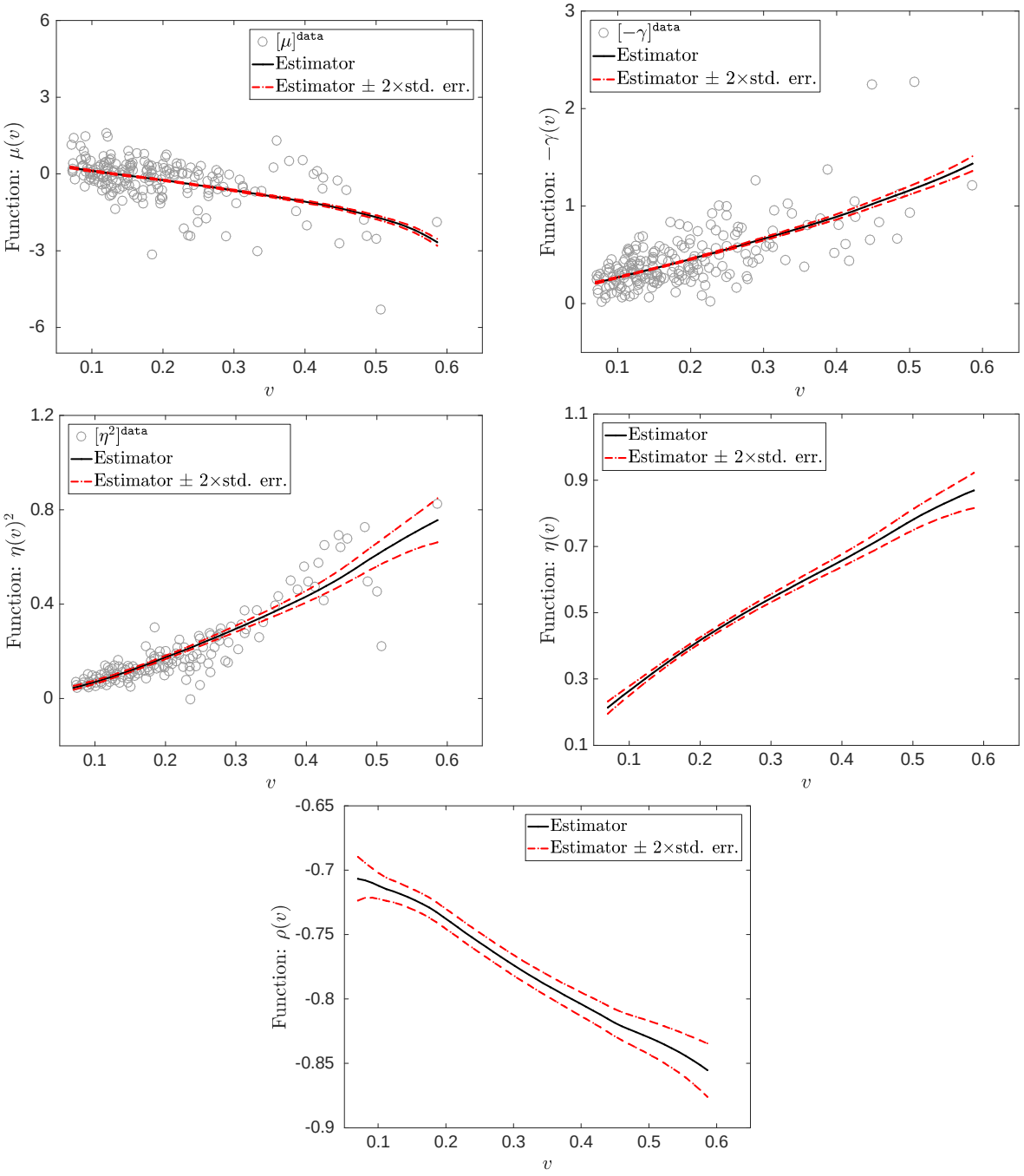


Figure 1: Nonparametric ISV model based on the data sample "`IV_Data_Sample.mat`"

Except for corresponding to the data sample "`IV_Data_Sample.mat`", all the settings for producing these five panels are the same as those for producing Figure 11 in Aït-Sahalia et al. (2020).