Connectivity and Transmission Delay in Large-scale Cognitive Radio Ad Hoc Networks With Unreliable Secondary Links

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Abstract—In this paper, we investigate connectivity and transmission delay of secondary users in large-scale wireless cognitive radio (CR) ad hoc networks from a percolation-based perspective. Using the Random Connection Model, we study the unreliability of wireless secondary links in CR ad hoc networks, which has not been well studied in the existing literature. By introducing two auxiliary random graphs and using continuum percolation theory, we study the impacts of key system parameters on connectivity and transmission delay in a CR network. We first characterize three behavioral regions of connectivity for a secondary network, i.e., disconnectivity, long-term connectivity and instantaneous connectivity regions. We show that the unreliability of secondary links does not affect the disconnectivity region, but affects the long-term connectivity and instantaneous connectivity regions. Using the ergodic theorem, we then study the scaling behavior of transmission delay with respect to the distance between two randomly chosen secondary users in a connected secondary network for two cases. Specifically, when propagation delay is negligible, we show that transmission delay scales linearly and sub-linearly with distance in the long-term connectivity and instantaneous connectivity regions, respectively. When propagation delay is considered, we show that transmission delay scales linearly with respect to distance in both the long-term connectivity and instantaneous connectivity regions.

Index Terms—Connectivity, Transmission Delay, Cognitive Radio Networks, Unreliability

I. INTRODUCTION

TODAY, as wireless communication witnesses explosive growth in the number of customers, the efficiency of spectrum usage has become a major concern. Recent measurements show that in the conventional networks, where spectrum is allocated statically, a large percentage of licensed bands remain unused over 90% of time [1]. To address this spectrum under-utilization problem, the concept of Cognitive Radio (CR) network [2] is introduced. CR network provides higher spectrum efficiency via a heterogeneous wireless structure and dynamic spectrum access techniques. Specifically, in a CR network, a secondary network is overlaid with a primary network. Secondary users detect and reuse temporarily unused frequency bands of primary users without causing unacceptable interference to primary users.

In CR networks, since higher transmission priority is given to primary users, secondary users may suffer from severe connection and delay problems. Therefore, the study of connectivity and transmission delay of secondary users in large-scale CR ad hoc networks has received increasing attention in recent years. Percolation theory, especially continuum percolation theory [3], [4], [5], is a powerful mathematical tool for the analysis of connectivity in large-scale (homogeneous) wireless networks [6], [7], [8], [9]. A fundamental result of continuum percolation reveals a phase transition effect whereby the macroscopic behavior of the system is very different for user density λ below and above the critical density λc [8]. Specifically, for λ < λc (subcritical), there are an infinite number of mutually disconnected finite components almost surely (a.s.), i.e., the network is not percolated. For λ > λc (supercritical), there exists a unique connected component which contains an infinite number of nodes a.s., i.e., the network percolates.

In [10], [11], [12], [13], [14], the analysis of connectivity for large-scale homogeneous wireless networks using percolation theory has been extended to large-scale heterogeneous CR ad hoc networks. Specifically, in modeling primary users in CR ad hoc networks, [10], [12], [13] and [14] differentiate primary transmitters and primary receivers, while [11] does not. Reference [10] considered a static primary network and revealed the tradeoff between the proximity (number of neighbors) and the occurrence of spectrum opportunities. References [11], [12] and [13] considered the i.i.d. random activity of primary users, and characterized connectivity and transmission delay of secondary users. In particular, reference [11] showed that there exist two critical densities λcS and λcP, and characterized the connectivity when secondary user density λS satisfies s(t)λS > λcS and primary user density λP satisfies λP < λcP, where s(t) is the survival function determined by the number of licensed channels and the activity of primary users. References [12] and [13] showed that there exist one critical density λc and three connectivity regions, i.e., disconnectivity, long-term connectivity and instantaneous connectivity regions. Both the long-term connectivity and instantaneous connectivity regions start from λc.

However, [10], [11], [12] and [13] failed to consider the unreliability of secondary links caused by noise, fading and user uncertainty in the real radio environment. Reference [14] considered the unreliability of secondary links and derived the lower and upper bounds on the delay-to-distance ratio. However, [14] failed to consider the interference to primary receivers caused by secondary communications, which significantly affects the boundary between the instantaneous connectivity and long-term connectivity regions. In addition, [14] directly applied the results from references [10] and [12], which actually adopted a reliable model for secondary links. Thus, [14] failed to rigorously prove the connectivity and delay results under the unreliable secondary link model. Therefore,
when fully considering the spectrum sharing mechanisms in large-scale CR ad hoc networks, the impact of unreliable secondary links on connectivity and transmission delay of secondary users is still unknown.

In this paper, we investigate percolation-based connectivity and transmission delay of secondary users in large-scale wireless CR ad hoc networks with unreliable secondary links. Since CR networks are heterogeneous networks with two tiers of nodes, i.e., primary users and secondary users, the analytical results obtained in homogeneous wireless networks [7] - [9] cannot be readily extended to heterogeneous CR networks. In addition, there is a complex coupling between primary users and secondary users induced by the spectrum sharing mechanisms in CR networks. Specifically, secondary users can only exploit spectrum opportunities left by nearby primary transmitters, and should not interfere any primary receivers. As a consequence of this coupling, connectivity and transmission delay in a secondary network depend not only on its own topological patterns, but also on the node density, activity and interference tolerance of primary networks. Finally, the unreliability of secondary links adds extra randomness to the structure of secondary networks. Therefore, the analysis of connectivity and transmission delay in CR networks with unreliable secondary links is very challenging.

In this paper, we address the above mentioned challenges and obtain analytical results to provide insights for the design of CR ad hoc networks. We consider the impact of the unreliability of wireless secondary links, which has not been thoroughly studied in the existing literature. By introducing two auxiliary random graphs corresponding to Boolean Model (BM) and Random Connection Model (RCM) and using continuum percolation theory, we study the impacts of key system parameters, such as primary and secondary node densities, primary node activity, secondary link reliability, transmission ranges and interference ranges on connectivity and delay in CR networks. As a first step, we characterize three behavioral regions of connectivity for secondary networks, i.e., disconnection, long-term connectivity and instantaneous connectivity regions. Specifically, we first show that the boundary between the disconnection and connectivity regions depends solely on the secondary node density and the secondary transmission range, and is not affected by the unreliability of secondary links. Then, we show that the boundary between the long-term and instantaneous connectivity regions is dependent on the unreliability of secondary links. Due to the unreliability of secondary links, extra delay is incurred for successful transmission. Using the ergodic theorem, we then study the scaling behavior of transmission delay with respect to the distance between two randomly chosen secondary users in a connected secondary network for two cases. Specifically, when propagation delay is negligible, we show that transmission delay scales linearly and sub-linearly in the long-term connectivity and instantaneous connectivity regions, respectively. When propagation delay is small but nonnegligible, we show that transmission delay scales linearly in both the long-term connectivity and instantaneous connectivity regions. Finally, using simulation results, we verify our theoretical analysis and intuitions.

The important notations used throughout this paper are summarized in Table I.

II. SYSTEM MODEL

In this section, we elaborate on the primary network model, the secondary network model and the communication model of the secondary network. To make the analysis in Sections III and IV tractable and obtain first-order insights into cognitive radio ad hoc networks, we consider one frequency band of the same communication characteristics over all the frequencies within this band [10] - [14].

A. Primary Network Model

We assume that primary transmitters are distributed according to a two-dimensional Poisson point process with density \( \lambda_P \). The transmission powers of all primary transmitters are the same. Assume that primary transmitters and primary receivers are different devices, and each primary transmitter has a corresponding primary receiver. For each primary transmitter, assume that its primary receiver is uniformly distributed within the transmission range \( R_P \) (dependent on the transmission power) of the primary transmitter. By the displacement theorem [15], we know that primary receivers are also distributed according to a Poisson point process with density \( \lambda_P \).

Assume that time is slotted and indexed by \( t \). Without loss of generality, we assume the slot duration is 1. At each slot, assume that each primary transmitter and its receiver are in the same activity state. We model the activity of each primary transmitter-receiver pair \( i \) at slot \( t \) by a Bernoulli random variable \( W_i(t) \in \{0, 1\} \), where \( W_i(t) = 1 \) indicates that primary transmitter \( i \) is active and \( W_i(t) = 0 \) otherwise. We assume \( \{W_i(t)\} \) are i.i.d. with respect to pair index \( i \) and slot index \( t \). Let \( \eta_i \triangleq \Pr(W_i(t) = 1) \in [0, 1] \) denote the active probability. By the coloring theorem [15], active primary transmitters and active primary receivers form Poisson point processes \( \mathcal{X}_{PT} \) and \( \mathcal{X}_{PR} \), respectively, both with density \( \eta_i \lambda_P \). Note that \( \mathcal{X}_{PT} \) and \( \mathcal{X}_{PR} \) are dependent on each other through the transmission range \( R_P \). Thus, we use \( G_P(\eta_i \lambda_P; R_P) \)

\( \begin{array}{|c|c|}
\hline
\text{Notations} & \text{Description} \\
\hline
\lambda_P, \lambda_S & \text{primary, secondary node density} \\
R_P, R_S & \text{primary, secondary transmission range} \\
r_P, r_S & \text{primary, secondary interference range} \\
\eta_i & \text{active probability of primary transmitters} \\
g(d) & \text{connection function of secondary links} \\
G_P(\eta_i \lambda_P; R_P) & \text{primary network} \\
G_S(\lambda_S g), G_S(\lambda_S h) & \text{standalone secondary network under RCM, BM} \\
G_S(\lambda_S g; G_P; r_P, r_S) & \text{actual secondary network under RCM} \\
T(u, v) & \text{minimum transmission delay between } u \text{ and } v \\
\tau & \text{propagation delay} \\
\hline
\end{array} \)

\(^1\) We shall leave the analysis for multiple heterogeneous frequency channels to the future work.

\(^2\) Note that this model has applications in relay-assisted cellular networks and heterogeneous networks.
to represent the random graph composed of active primary transmitters and active primary receivers (following $\mathcal{X}_{PT}$ and $\mathcal{X}_{PR}$, both with density $\eta_1\lambda_P$) and primary links. In the following, we use $\mathcal{G}_P$ to represent $\mathcal{G}_P(\eta_1\lambda_P; R_P)$ for notation simplicity.

B. Secondary Network Model

We assume that secondary users are distributed according to a two-dimensional Poisson point process $\mathcal{X}_S$ with density $\lambda_S$. A secondary user can be a transmitter or a receiver at different time slots in a CR ad hoc network. We assume that the transmission powers of all secondary users are the same. We model the unreliability of secondary links by the Random Connection Model (RCM) [4]. Specifically, we model the unreliability of each link $i$ between two secondary users at slot $t$ by a Bernoulli random variable $Y_i(t) \in \{0, 1\}$, where $Y_i(t) = 1$ indicates that link $i$ is reliable at slot $t$ and $Y_i(t) = 0$ otherwise. We assume that $\{Y_i(t)\}$ are independent across links and i.i.d across slots. For each link $i$ with link length $d$, the reliability is characterized by a connection function $g(d)$, i.e., $g(d) \triangleq \Pr(Y_i(t) = 1)$. In reality, two secondary nodes with a longer distance usually have a smaller chance for maintaining a reliable link, and there exists no link when the distance between them is over the transmission range $R_S$ (dependent on the transmission power of the secondary user). Thus, we assume that $g(d)$ is a monotonically non-increasing function with respect to $d$ (i.e., $0 \leq g(d_2) \leq g(d_1) \leq 1$ when $0 < d_1 < d_2 \leq R_S$), and $g(d) = 0$ when $d > R_S$, as illustrated in Fig. 1 (a). We use $\mathcal{G}_S(\lambda_S; g)$ to represent the random graph composed of secondary users (following $\mathcal{X}_S$ with density $\lambda_S$) and secondary links following RCM with connection function $g(d)$.

Remark 1 (Interpretation of Connection Function): (i) $g(d)$ can be used to model various types of noise, fading and user uncertainty. Thus, our link model is more general than the protocol model. (ii) $g(d)$ can capture the average statistics (averaged over geographic randomness) of the physical interference model to some extent. Thus, our model can be treated as a simplified version of the physical interference model, and is used to get design insights. We would like to leave the analysis using the physical interference model to our future work.

Boolean Model (BM) is a special case of RCM [4] and is introduced as an auxiliary model for the theoretical analysis in this paper. Two secondary users in BM have a reliable link as long as they are within the transmission range $R_S$. Let $h(d)$ denote the connection function of BM, i.e., $h(d) = 1$ when $0 < d \leq R_S$, and $h(d) = 0$ when $d > R_S$, as illustrated in Fig. 1 (b). Given graph $\mathcal{G}_S(\lambda_S; g)$, connecting any two nodes with a distance smaller than $R_S$, we get a new graph, denoted as $\mathcal{G}_S(\lambda_S; h)$. Note that $\mathcal{G}_S(\lambda_S; g) \subset \mathcal{G}_S(\lambda_S; h)$. The new graph $\mathcal{G}_S(\lambda_S; h)$ can be treated as a random graph composed of the same set of secondary users (following the same Poisson point process with density $\lambda_S$ as that in $\mathcal{G}_S(\lambda_S; g)$) and secondary links following BM with connection function $h(d)$.

Note that the random graphs $\mathcal{G}_S(\lambda_S; g)$ and $\mathcal{G}_S(\lambda_S; h)$ represent the standalone secondary networks under RCM and BM, respectively. They are independent of the primary network and will serve as two auxiliary graphs for the purpose of analysis for the actual secondary network, which depends on the primary network.

C. Communication Model of Secondary Network

In CR networks, primary users are guaranteed to have higher priority to access communication channels over secondary users. Hence, we treat secondary and primary users as asymmetric users, which reflects the heterogeneous feature of CR networks. Specifically, secondary users can only exploit the temporarily unused spectrum resources of inactive primary transmitters. Besides, communications in the secondary network should not cause unacceptable interference to active primary receivers. To characterize the interactions between secondary and primary networks, let $r_P$ and $r_S$ denote the interference ranges of primary transmitters and secondary users, respectively. Note that $r_P$ and $r_S$ are named after the sources of the interference. As illustrated in Fig. 2, within the distance $r_P$ of each active primary transmitter, spectrum opportunities are not available for secondary users; within the distance $r_S$ of each active secondary user, there should not exist any active primary receivers.3

Based on the spectrum sharing mechanisms in CR networks discussed above, we introduce the following definition of communication links in the secondary network.

3Note that we only consider the communication of one source-destination pair in the secondary network. At each slot, all the active secondary transmitters are transmitting the same information. Thus, we assume there is no interference among secondary users. This is also the case considered in [10] - [14], [6] · [9].
Definition 1 (Communication Link): For any two secondary nodes $u$ and $v$, there exists a communication link between them if and only if (iff.) the following three conditions hold. (i) There are no active primary receivers within the interference range $r_p$ of both nodes $u$ and $v$. (ii) Both nodes $u$ and $v$ are outside the interference range $r_p$ of any active primary transmitters. (iii) There exists a reliable link $l(u,v)$ between nodes $u$ and $v$, i.e., $l(u,v) \in G_S(\lambda_S; g)$.

Condition (i) ensures that neither node $u$ nor node $v$ causes unacceptable interference to active primary receivers. Condition (ii) ensures that communication between node $u$ and node $v$ is not interfered by active primary transmitters. Condition (iii) guarantees that there exists a reliable secondary link between node $u$ and node $v$. Thus, Conditions (i) and (ii) reflect the dependence of secondary communications on primary activities. Condition (iii) reflects the dependence of secondary communications on the unreliability of secondary links.

A communication link is a reliable secondary link through which secondary communications can actually carry out. Note that our communication model allows multi-hop communication via multiple communication links. Specifically, if there exists a communication link between nodes $u$ and $v$, they can communicate directly through a single hop transmission. Otherwise, two nodes may communicate indirectly via multi-hop transmissions (i.e., multiple communication links).

The primary network $G_P$ impacts the secondary network $G_S(\lambda_S; g)$ through the interference ranges $r_p$ and $r_s$. We use $G_S(\lambda_S; g; G_P; r_p, r_s)$ to model the actual secondary network, which depends on the primary network and is illustrated below. Given a secondary network $G_S(\lambda_S; g)$, we consider the impacts of the primary network $G_P$ by removing all the links in $G_S(\lambda_S; g)$ that are not communication links. Specifically, let each active primary transmitter (receiver) in $G_P$ occupy a circular region with radius $r_p$ ($r_s$). Any link in $G_S(\lambda_S; g)$ which has at least one endpoint within an occupied circular region is removed. Then, the resulting graph is $G_S(\lambda_S; g; G_P; r_p, r_s)$. Thus, as illustrated in Fig. 2, we use $G_S(\lambda_S; g; G_P; r_p, r_s)$ to represent the random graph composed of the same set of secondary users (following the same Poisson point process with density $\lambda_S$) as that in $G_S(\lambda_S; g)$ and the secondary communication links defined in Definition 1. Note that $G_S(\lambda_S; g; G_P; r_p, r_s) \subseteq G_S(\lambda_S; g) \subseteq G_S(\lambda_S; h)$.

D. Preliminaries on Continuum Percolation

As stated in Section II-B and II-C, we investigate the performance of $G_S(\lambda_S; g; G_P; r_p, r_s)$ based on two auxiliary graphs $G_S(\lambda_S; h)$ and $G_S(\lambda_S; g)$. Note that $G_S(\lambda_S; h)$ and $G_S(\lambda_S; g)$ represent the random graphs composed of secondary users and secondary links following BM and RCM, respectively. In this part, we briefly present continuum percolation results on the two graphs. For simplicity, here we call $G_S(\lambda_S; h)$ and $G_S(\lambda_S; g)$ as $G_1$ and $G_2$, respectively. By continuum percolation[4], we know that $G_1$ ($G_2$) exhibits a phase transition effect of connectivity at some critical density $\lambda_{c1}$ ($\lambda_{c2}$). In the subcritical phase (i.e., $\lambda_S < \lambda_{c1}$ ($\lambda_S < \lambda_{c2}$)), graph $G_1$ ($G_2$) is composed of an infinite number of finite small components a.s. In the supercritical phase (i.e., $\lambda_S > \lambda_{c1}$ ($\lambda_S > \lambda_{c2}$)), there exists a unique infinite connected component in $G_1$ ($G_2$) a.s. [4].

Now, we give the mathematical definitions of $\lambda_{c1}$ and $\lambda_{c2}$. Recall that $G_1$ and $G_2$ contain the same set of secondary users, whose positions follow a homogeneous Poisson point process with density $\lambda_S$. Thus, we can choose an arbitrary secondary user as the origins of the two graphs. Denote the connected component containing the origin in $G_1$ as $C_{O_1}$. Define $\theta_1(\lambda_S, h) = \Pr(|C_{O_1}| = \infty)$ as the percolation probability that $C_{O_1}$ contains an infinite number of secondary users. Similarly, we define $C_{O_2}$ and $\theta_2(\lambda_S, g)$ for $G_2$.

Thus, for given $h(\cdot)$ and $g(\cdot)$, the critical densities $\lambda_{c1}$ and $\lambda_{c2}$ of $G_1$ and $G_2$ are defined as $\lambda_{c1} = \inf\{\lambda_S > 0 : \theta_1(\lambda_S, h) > 0\}$ and $\lambda_{c2} = \inf\{\lambda_S > 0 : \theta_2(\lambda_S, g) > 0\}$, respectively. By continuum percolation, we know that $\lambda_{c1}$ is a function of $h(\cdot)$, and $\lambda_{c2}$ is a function of $g(\cdot)$, and $\lambda_{c1} \leq \lambda_{c2}$ [4].

III. CONNECTIVITY

In this section, we characterize three behavioral regions of connectivity for a secondary network, i.e., disconnectivity region, long-term connectivity region and instantaneous connectivity region, in terms of the node density pair ($\lambda_S, \lambda_P$).

A. Definitions on Connectivity

As stated in Section II-C, the performance of a secondary network can be characterized by the primary and secondary node densities ($\lambda_S$, $\lambda_P$), primary transmitter activity ($\eta_1$), secondary link reliability ($g(\cdot)$), transmission ranges ($R_P$, $R_S$) and interference ranges ($r_p$, $r_s$) of the primary and secondary networks. In the analysis for connectivity of a secondary network, we assume that $\eta_1, g(\cdot), R_P, R_S, r_p, r_s$ are fixed, and only consider the impacts of the two key parameters $\lambda_S$ and $\lambda_P$ on connectivity and transmission delay in the secondary network $G_S(\lambda_S; g; G_P; r_p, r_s)$, where $G_P$ is short for $G_P(\eta_1; \lambda_P; R_P)$.

In this paper, we assume that transmission delay in a secondary network consists of waiting delay and propagation delay. Waiting delay includes the waiting time for spectrum opportunities from primary users and reliable secondary links. Propagation delay is the time for packets to travel over communication links. Let $T(u,v)$ denote the minimum transmission delay between two secondary users $u$ and $v$. Based on $T(u,v)$, we give the definitions on disconnectivity and connectivity of a secondary network.

Definition 2 (Disconnectivity of Secondary Network): A secondary network $G_S(\lambda_S; g; G_P; r_p, r_s)$ is disconnected iff. $T(u,v) = \infty$ a.s. (i.e., $T(u,v) < \infty$ with probability zero) for two randomly chosen secondary nodes $u$ and $v$.

Definition 3 (Connectivity of Secondary Network): A secondary network $G_S(\lambda_S; g; G_P; r_p, r_s)$ is connected iff. $T(u,v) < \infty$ with a positive probability (w.p.p.) for two randomly chosen secondary nodes $u$ and $v$. 

Note that the reliability of a secondary link in $G_S(\lambda_S; g)$ between any two nodes is i.i.d. across slots. For simplicity, we omit the subscript $t$ in the notations, such as $C_{O_2}$ and $\theta_2(\lambda_S, g)$. Secondary links in $G_S(\lambda_S; g)$ do not change over slots.
By Definition 2 and Definition 3, a secondary network is either disconnected or connected. Next, we further classify connectivity into two categories, i.e., instantaneous connectivity and long-term connectivity. Denote the connected component containing the origin in $G_S(\lambda_S; g; G_P; r_P, r_S)$ at each time slot by $C_{O3}$.

First, we define $\theta_3(\lambda_S, \lambda_P)$ as the probability that $C_{O3}$ has an infinite number of nodes, i.e.,

$$\theta_3(\lambda_S, \lambda_P) = \Pr(|C_{O3}| = \infty). \quad (1)$$

Based on $\theta_3(\lambda_S, \lambda_P)$, we give the definitions of instantaneous connectivity and long-term connectivity of a secondary network, respectively.

**Definition 4 (Instantaneous Connectivity of Secondary Network):** A connected secondary network $G_S(\lambda_S; g; G_P; r_P, r_S)$ has instantaneous connectivity iff. $\theta_3(\lambda_S, \lambda_P) > 0$.

**Remark 2 (Interpretation of Definition 4):** We interpret the instantaneous connectivity for the case in which propagation delay is ignored as below. Since $\theta_3(\lambda_S, \lambda_P) > 0$, a unique infinite connected component formed by communication links exists a.s., as illustrated in Section II-D. Suppose one secondary node inside the infinite connected component transmits a message. All the nodes within the infinite connected component can receive it immediately. In other words, for two randomly chosen secondary nodes $u$ and $v$, $T(u, v) = 0$ w.p.p. Thus, we say that a secondary network has instantaneous connectivity when $\theta_3(\lambda_S, \lambda_P) > 0$.

**Definition 5 (Long-term Connectivity of Secondary Network):** A connected secondary network $G_S(\lambda_S; g; G_P; r_P, r_S)$ has long-term connectivity iff. $\theta_3(\lambda_S, \lambda_P) = 0$.

**Remark 3 (Interpretation of Definition 5):** We interpret the long-term connectivity for the case in which propagation delay is ignored as below. As $\theta_3(\lambda_S, \lambda_P) = 0$, graph $G_S(\lambda_S; g; G_P; r_P, r_S)$ contains an infinite number of finite connected components, as illustrated in Section II-D. In this case, any two secondary nodes residing in different connected components at one slot have to wait for spectrum opportunities and reliable secondary links to communicate with each other. In other words, for two randomly chosen secondary nodes $u$ and $v$, $T(u, v) > 0$ a.s. Thus, we say that a secondary network has long-term connectivity when $\theta_3(\lambda_S, \lambda_P) = 0$.

Based on Definitions 2, 3, 4 and 5, we partition the $(\lambda_S, \lambda_P)$ plane into three different regions, as illustrated in Fig. 3.

In the remainder of this section, we characterize the boundaries between different regions in terms of $(\lambda_S, \lambda_P)$.

### B. Disconnectivity Region and Connectivity Region

Define the disconnectivity region (connectivity region) as the set of $(\lambda_S, \lambda_P)$ for which the secondary network $G_S(\lambda_S; g; G_P; r_P, r_S)$ is disconnected (connected). The following theorem characterizes the boundary between the disconnectivity region and the connectivity region.

**Theorem 1:** A secondary network $G_S(\lambda_S; g; G_P; r_P, r_S)$ is disconnected when $\lambda_S < \lambda_{c1}$ and is connected when $\lambda_S > \lambda_{c1}$, where $\lambda_{c1}$ is the critical density of $G_S(\lambda_S; h)$.

Proof: Please refer to Appendix A.

We interpret Theorem 1 as below. First, Theorem 1 can be easily understood for the special case in which $\eta_1 = 0$ (primary users are inactive a.s.) and $g(d) = h(d)$ (secondary links follow BM). In this case, graph $G_S(\lambda_S; g; G_P; r_P, r_S)$ reduces to graph $G_S(\lambda_S; h)$. For ease of illustration, denote $T(u, v)$ in graph $G_S(\lambda_S; h)$ as $T_h(u, v)$. As illustrated in Section II-D, when $\lambda_S < \lambda_{c1}$, graph $G_S(\lambda_S; h)$ is composed of an infinite number of finite connected components a.s. Two randomly chosen secondary nodes $u$ and $v$ reside in different connected components (for all the time slots) a.s. Thus, when $\lambda_S < \lambda_{c1}$, there exists a unique infinite connected component and two randomly chosen secondary nodes $u$ and $v$ reside in the infinite component w.p.p. Any two nodes $u$ and $v$ within the infinite connected component satisfy $T_h(u, v) < \infty$. Thus, when $\lambda_S > \lambda_{c1}$, $T_h(u, v) < \infty$ w.p.p.

Next, we interpret the general case with $\eta_1 \in [0, 1)$ and a general connection function $g(d)$ by comparing $T(u, v)$ and $T_h(u, v)$ discussed above. Due to the activity of primary users and the unreliability of secondary links, it is obvious that $T(u, v) \geq T_h(u, v)$ a.s. When $\lambda_S < \lambda_{c1}$, for two randomly chosen secondary nodes $u$ and $v$, $T_h(u, v) = \infty$ a.s. implies $T(u, v) = \infty$ a.s. Thus, by Definition 2, the secondary network is disconnected. When $\lambda_S > \lambda_{c1}$, for two randomly chosen secondary nodes $u$ and $v$, $T_h(u, v) < \infty$ w.p.p. By properties of the first passage time, we can show that $T(u, v) - T_h(u, v) < \infty$ a.s. Thus, for two randomly chosen secondary nodes $u$ and $v$, we have $T(u, v) < \infty$ w.p.p.

Now, we discuss the impacts of key system parameters on connectivity of secondary networks. By Theorem 1, we know that the boundary between the disconnectivity and connectivity regions on the $(\lambda_S, \lambda_P)$ plane is $\lambda_S = \lambda_{c1}$. As illustrated in Section II-D, we know that $\lambda_{c1}$ is a function of $h(\cdot)$ (i.e., $R_S$) only. In addition, as long as $\eta_1 < 1$ and $g(d) > 0$ for all $d \leq R_S$, the waiting time for spectrum opportunities left by primary users and reliable secondary links between any two nodes within the transmission range $R_S$ is finite a.s. Therefore, the connectivity (disconnectivity) of the secondary network $G_S(\lambda_S; g; G_P; r_P, r_S)$ only depends on secondary node density $\lambda_S$ and secondary transmission range $R_S$, and is independent of primary node density $\lambda_P$, primary transmission range $R_P$, primary node active probability $\eta_1$, connection function $g(\cdot)$ and primary and secondary interference ranges $r_P$ and $r_S$. Note that, since $\lambda_{c1}$ is not affected by the unreliability of
secondary links, the boundary between the disconnectivity and connectivity regions with unreliable secondary links is the same as that under the reliable link model (i.e., BM).

C. Instantaneous Connectivity Region and Long-term Connectivity Region

Define the instantaneous (long-term) connectivity region as the set of \((\lambda_S, \lambda_P)\) for which the secondary network \(G_S(\lambda_S; g; G_P; r_P, r_S)\) has instantaneous (long-term) connectivity. In the following, we characterize the boundary between the instantaneous connectivity region and the long-term connectivity region.

First, we define \(\Delta_P(\lambda_S) \equiv \inf \{\lambda_P : \theta_S(\lambda_S, \lambda_P) = 0, \lambda_S > \lambda_{c1}\}\). We characterize three basic properties of \(\Delta_P(\lambda_S)\) in the following theorem.

**Theorem 2:** For a connected \((\lambda_S > \lambda_{c1})\) secondary network \(G_S(\lambda_S; g; G_P; r_P, r_S)\), \(\Delta_P(\lambda_S)\) has the following three properties.

(i) \(\Delta_P(\lambda_S)\) is non-decreasing with respect to \(\lambda_S\); (ii) \(\Delta_P(\lambda_S)\) satisfies \(0 \leq \Delta_P(\lambda_S) \leq \frac{\lambda_{c1}}{\eta_1(R_P - h_S)}\), where \(\lambda_{c1}\) is the critical density of \(G_S(\lambda_S; h)\) with \(R_S = 1\); (iii) \(\Delta_P(\lambda_S) = 0\) when \(\lambda_S < \lambda_{c2}\) and there exists \(\lambda_{c1} \geq \lambda_{c2}\) such that \(\Delta_P(\lambda_S) > 0\) when \(\lambda_S > \lambda_{c3}\), where \(\lambda_{c2}\) is the critical density of \(G_S(\lambda_S; g)\).

**Proof:** Please refer to Appendix B.

In the following theorem, we show that \(\Delta_P(\lambda_S)\) serves as the boundary between the instantaneous connectivity region and the long-term connectivity region.

**Theorem 3:** A connected \((\lambda_S > \lambda_{c1})\) secondary network \(G_S(\lambda_S; g; G_P; r_P, r_S)\) has instantaneous connectivity when \(\lambda_P < \Delta_P(\lambda_S)\) and long-term connectivity when \(\lambda_P > \Delta_P(\lambda_S)\).

**Proof:** Please refer to Appendix C.

Now, we discuss the impacts of key system parameters on instantaneous connectivity and long-term connectivity of connected \((\lambda_S > \lambda_{c1})\) secondary networks. By Theorem 3, we know that the boundary between the instantaneous connectivity and long-term connectivity regions on the \((\lambda_S, \lambda_P)\) plane is \(\lambda_P = \Delta_P(\lambda_S)\), which depends on all system parameters, i.e., \(\lambda_P, \lambda_S, R_P, R_S, \eta_1, g(\cdot), r_P\) and \(r_S\) (cf. proof of Theorem B). In contrast, for a secondary network with reliable links (BM), it has been proved that \(\lambda_{c1} = \lambda_{c2} = \lambda_{c3}\) \[16\]. Therefore, we know that due to the unreliability of secondary links, the instantaneous connectivity region is reduced.

IV. TRANSMISSION DELAY

In this section, we characterize the scaling behavior of minimum transmission delay \(T(u, v)\) with respect to distance \(d(u, v)\) between two secondary users \(u\) and \(v\) in a connected secondary network for the cases without propagation delay and with propagation delay, respectively.

A. Transmission Delay

When minimum transmission delay \(T(u, v)\) between two nodes \(u\) and \(v\) is infinite, we cannot transmit information efficiently between them. Thus, we only focus on the case in which \(T(u, v)\) is finite. In a disconnected secondary network, for two randomly chosen nodes \(u\) and \(v\), \(T(u, v) = \infty\) a.s. Thus, we do not consider the disconnected case. For a connected secondary network, \(T(u, v) < \infty\) w.p.p. Specifically, when two randomly chosen nodes in a connected secondary network are within the infinite connected component of \(G_S(\lambda_S; h)\), denoted by \(C(G_S(\lambda_S; h))\), we have \(T(u, v) < \infty\) a.s. Note that \(C(G_S(\lambda_S; h))\) does not necessarily contain the origin\(^7\) and does not change over time slots. Therefore, in the following, we randomly choose two nodes within \(C(G_S(\lambda_S; h))\) in a connected secondary network and study the scaling behavior of transmission delay with respect to distance.

As illustrated in Section IV, we assume that transmission delay in the secondary network consists of waiting delay and propagation delay. In practical systems, waiting delay is in the order of seconds, minutes or even larger in wireless CR ad hoc networks, while propagation delay is usually in the order of milliseconds \[9\]. Thus, propagation delay is much smaller than waiting delay. Therefore, we first ignore propagation delay to capture the first-order insights into the scaling behavior of transmission delay in Section IV-B. Then, we consider small propagation delay and study the scaling behavior of transmission delay in Section IV-C. In the following, we use \(\tau\) to denote the propagation delay over each link. We assume \(\tau \in [0, 1)\) to ensure the success of propagating over a link when a reliable spectrum opportunity lasts.\(^8\)

B. Without Propagation Delay

When propagation delay is negligible (\(\tau = 0\)), we present the scaling behavior of transmission delay with respect to distance in the instantaneous connectivity and long-term connectivity regions in the following theorem.

**Theorem 4:** Assume \(\tau = 0\). In a connected \((\lambda_S > \lambda_{c1})\) secondary network \(G_S(\lambda_S; g; G_P; r_P, r_S)\), for two randomly chosen secondary users \(u\) and \(v\) within the infinite connected component \(C(G_S(\lambda_S; h))\), the following results hold.

(i) If the secondary network has instantaneous connectivity, we have

\[
\Pr\left(\lim_{d(u,v) \to \infty} \frac{T(u,v)}{d(u,v)} = 0\right) = 1.
\]

(ii) If the secondary network has long-term connectivity, there exists \(\gamma \in (0, \infty)\) such that

\[
\Pr\left(\lim_{d(u,v) \to \infty} \frac{T(u,v)}{d(u,v)} = \gamma\right) = 1.
\]

**Proof:** Please refer to Appendix D.

**Remark 4 (Interpretation of Theorem 4):** Theorem 4 shows that without propagation delay, transmission delay scales sub-linearly and linearly with distance when \(G_S(\lambda_S; g; G_P; r_P, r_S)\)

\(^7\) According to continuum percolation theory \[4\], a connected secondary network contains an infinite connected component a.s., but that the connected component containing origin is infinite is of positive probability. Our focus is to study the transmission delay between two randomly chosen nodes within the infinite connected component in the network, which exists with probability 1. Thus, we do not require \(C(G_S(\lambda_S; h))\) to contain the origin.

\(^8\) Note that we assume the time slot duration is 1. In addition, we assume that the activity of primary users and unreliability of secondary links are i.i.d. across time slots. Hence, the minimum duration for a reliable spectrum opportunity is 1.
has instantaneous connectivity and long-term connectivity, respectively. Theorem 4 can be interpreted as below. Without propagation delay, a message can be immediately disseminated among nodes residing in the same connected component formed by communication links. Thus, when instantaneous connectivity is achieved, a message can travel over an infinite distance in the infinite connected component in a single time slot. This results in the sub-linear scaling behavior of transmission delay with respect to distance in the instantaneous connectivity region. When long-term connectivity is achieved, there exist an infinite number of small finite connected components in the secondary network a.s. Let $D_i$ be the longest distance between any two nodes in a connected component $C_i$. Then, a message can only travel between two nodes in $C_i$ with the maximum distance $D_i$ in a single time slot. This results in the linear scaling behavior of transmission delay with respect to distance in the long-term connectivity region.

Next, we compare our results at $\tau = 0$ under the unreliable link model (RCM) with those obtained under the reliable link model (BM) [16]. As illustrated in Section III-C, the unreliability of secondary links affects the boundary between the long-term connectivity and instantaneous connectivity regions. In the instantaneous connectivity region, at each time slot, a message can travel an infinite distance under both RCM and BM. Therefore, for both RCM and BM, transmission delay scales sub-linearly in the instantaneous connectivity region. In the long-term connectivity region, the distributions of $D_i$ in RCM and BM are different. However, for both RCM and BM, transmission delay scales linearly in the instantaneous connectivity region.

### C. With Propagation Delay

When propagation delay is small but nonnegligible ($0 < \tau < 1$), we present the scaling behavior of transmission delay with respect to distance in the instantaneous connectivity and long-term connectivity regions in the following theorem.

**Theorem 5:** Assume $0 < \tau < 1$ and $\frac{1}{\tau}$ is an integer.\(^{10}\) In a connected $(\lambda_S > \lambda_P)$ secondary network $\mathcal{G}_S(\lambda_S; g, g_P, r_P, r_S)$, for two randomly chosen secondary users $u$ and $v$ within the infinite connected component $C(\mathcal{G}_S(\lambda_S; h))$, there exist $\gamma_1(\tau)$ and $\gamma_2(\tau)$ satisfying $\frac{1}{\tau} \leq \gamma_1(\tau) < \gamma_2(\tau) < \infty$ such that the following results hold.

(i) If the secondary network has instantaneous connectivity, we have

$$\Pr \left( \lim_{d(u,v) \to \infty} \frac{T(u,v)}{d(u,v)} = \gamma_1(\tau) \right) = 1. \quad (4)$$

(ii) If the secondary network has long-term connectivity, we have

$$\Pr \left( \lim_{d(u,v) \to \infty} \frac{T(u,v)}{d(u,v)} = \gamma_2(\tau) \right) = 1. \quad (5)$$

\(^{9}\) Note that delay-to-distance ratio $\gamma$ under RCM is greater than that under BM.\(^{10}\) For technical tractability, in Theorem 5, we only consider the case in which $\frac{1}{\tau}$ is an integer. We can simply use $\left\lfloor \frac{1}{\tau} \right\rfloor$ and $\left\lceil \frac{1}{\tau} \right\rceil$ and get two limiting ratios if $\frac{1}{\tau}$ is not an integer. These two limiting ratios can serve as upper and lower bounds of the delay-to-distance ratio. Hence, our results on the scaling behavior of transmission delay still hold for the general case in which $\frac{1}{\tau}$ is not an integer.

Moreover, $\gamma_1(\tau)$ and $\gamma_2(\tau)$ are non-decreasing with respect to $\tau$. As $\tau \to 0$, we have $\gamma_1(\tau) \to 0$ and $\gamma_2(\tau) \to \gamma$, where $\gamma$ is given in Theorem 4.

**Proof:** Please refer to Appendix E.

**Remark 5 (Interpretation of Theorem 5):** Theorem 5 shows that with propagation delay $\tau \in (0, 1)$, transmission delay scales linearly with distance in both instantaneous connectivity and long-term connectivity regions. We interpret Theorem 5 as below. When propagation delay is considered, at each time slot, a message can go through at most $\frac{1}{\tau}$ communication links in a connected component, i.e., at most distance $\frac{R_S}{\tau}$. In the instantaneous connectivity region, at each time slot, although an infinite number of nodes in the secondary network are connected by communication links, a message can only be disseminated at most over distance $\min\{\frac{R_S}{\tau}, \infty\} = \frac{R_S}{\tau}$. This results in the linear scaling behavior of transmission delay with respect to distance in the instantaneous connectivity region. In the long-term connectivity region, there exist an infinite number of small finite connected components with limited sizes a.s. Let $D_i$ be the longest distance between any two nodes in a connected component $C_i$. Then, a message can travel between two nodes in $C_i$ with the maximum distance $\min\{\frac{R_S}{\tau}, D_i\}$ in a single time slot. Therefore, transmission delay also scales linearly in the long-term connectivity region.

Next, we compare our results at $\tau \in (0, 1)$ under the unreliable link model (RCM) with those obtained under the reliable link model (BM) [16]. As illustrated in Section III-C, the unreliability of secondary links affects the boundary between the long-term connectivity and instantaneous connectivity regions. In the instantaneous connectivity region, at each time slot, a message can travel at most distance $\frac{R_S}{\tau}$ under both RCM and BM. Therefore, for both RCM and BM, transmission delay scales linearly in the instantaneous connectivity region. In the long-term connectivity region, the distributions of $D_i$ in RCM and BM are different. However, for both RCM and BM, transmission delay scales linearly in the instantaneous connectivity region.\(^{11}\)

### V. Simulations

In this section, we present simulation results to verify the theoretical analysis of connectivity and transmission delay in large-scale CR ad hoc networks with unreliable secondary links. In the simulations, we use Visual Studio C++ as the simulation tool. We consider three Poisson point processes $\mathcal{X}_{P_P}, \mathcal{X}_{P_R}$ and $\mathcal{X}_S$ with densities $\lambda_P, \lambda_P$ and $\lambda_S$, respectively, in a square region $[-20,20] \times [-20,20]$. We choose the primary transmission range $R_P = 2$, secondary transmission range $R_S = 1.4$, primary interference range $r_P = 2.37$, and secondary interference range $r_S = 1.5$. Note that $R_P > R_S$ and $r_P > r_S$, as the transmission power and interference of primary transmitters are usually larger than those of secondary users in reality. Also note that $r_P > R_P$ and $r_S > R_S$, as for each user, its interference range is generally larger than its transmission range. We choose the active probability of

\(^{11}\) Note that delay-to-distance ratio $\gamma_2(\tau)$ under RCM is greater than that under BM.
primary transmitter-receiver pairs $\eta_1 = 0.3$. We consider the following connection function

$$g(d) = \begin{cases} 
(1 - \frac{d}{2R_S})^2, & d \leq R_S \\
0, & d > R_S
\end{cases}.$$ 

Note that $g(d)$ satisfies our assumption in Section II-B and is also used in [7].

A. Critical Densities

In this part, we identify the critical densities $\lambda_{c1}$ and $\lambda_{c2}$ by simulations based on the existence of an infinite connected component under BM and RCM, respectively. Denote the largest connected components in the BM graph $G_S(\lambda; h)$ and the RCM graph $G_S(\lambda; g)$ by $C_{max1}$ and $C_{max2}$, respectively. By continuum percolation theory [4], there is a unique infinite connected component a.s. for a connected network. Fig. 4 illustrates the probabilities that the largest connected component is infinite for BM and RCM, i.e., $\Pr(|C_{max1}| = \infty)$ and $\Pr(|C_{max2}| = \infty)$, versus the secondary node density $\lambda_S$, respectively. From Fig. 4, we can see that for BM, $\Pr(|C_{max1}| = \infty) < 1$ when $\lambda_S < 1.08$ and $\Pr(|C_{max1}| = \infty) = 1$ when $\lambda_S > 1.08$. In addition, for RCM, $\Pr(|C_{max2}| = \infty) < 1$ when $\lambda_S < 1.83$ and $\Pr(|C_{max2}| = \infty) = 1$ when $\lambda_S > 1.83$. Therefore, we identify the critical densities $\lambda_{c1} \approx 1.08$ and $\lambda_{c2} \approx 1.83$. In the remainder of this section, the obtained critical densities will be used to verify the analytical results.

B. Connectivity

In this part, we verify the three behavioral regions of connectivity for secondary networks. First, we verify the boundary between the disconnection region and the connectivity region. According to Definitions 2 and 3, for a connected secondary network, the transmission delay between any two secondary nodes is finite w.p.p.. In our simulations, we observe that for one successful transmission between two randomly chosen nodes, the transmission delay generally takes a value less than 200. Thus, we consider $T(u, v) = 2000$ as infinity to indicate a transmission failure. Fig. 5 illustrates the probability $\Pr(T(u, v) < 2000)$ versus the secondary node density $\lambda_S$ and the primary node density $\lambda_P$. The red dashed line represents the boundary between $\Pr(T(u, v) < 2000) = 0$ and $\Pr(T(u, v) < 2000) > 0$. From Fig. 5, we can see that $\Pr(T(u, v) < 2000) = 0$ when $\lambda_S < 1.08$ and $\Pr(T(u, v) < 2000) > 0$ when $\lambda_S > 1.08$. Note that by Fig. 4, we know $\lambda_{c1} = 1.08$. Therefore, Fig. 5 shows that a secondary network is disconnected when $\lambda_S < \lambda_{c1}$ and is connected when $\lambda_S > \lambda_{c1}$. This verifies Theorem 1.

Next, for a connected secondary network, we identify the boundary between the long-term connectivity region and the instantaneous connectivity region. Denote the largest connected component in the graph $G_S(\lambda_S; g; G_P; r_P, r_S)$ at each time slot by $C_{max3}$. By Definition 4, Definition 5 and the stationarity of the CR network model, for an instantaneously connected secondary network, we know that $|C_{max3}| = \infty$ a.s. Fig. 6 illustrates the probability $\Pr(|C_{max3}| = \infty)$ versus the secondary node density $\lambda_S$ and the primary node density $\lambda_P$. The green dashed curve indicates the boundary between $\Pr(|C_{max3}| = \infty) < 1$ and $\Pr(|C_{max3}| = \infty) = 1$. The blue dashed curve is the projection of the green curve on the $(\lambda_S, \lambda_P)$ plane, indicating $\lambda_P = \Lambda_P(\lambda_S)$. From Fig. 6, we can see that $\Pr(|C_{max3}| = \infty) < 1$ when $\lambda_P < \Lambda_P(\lambda_S)$ and $\Pr(|C_{max3}| = \infty) = 1$ when $\lambda_P > \Lambda_P(\lambda_S)$. Therefore, Fig. 6 shows that a connected secondary network has instantaneous connectivity when $\lambda_P < \Lambda_P(\lambda_S)$ and long-term connectivity when $\lambda_P > \Lambda_P(\lambda_S)$. This verifies Theorem 3. Furthermore, from Fig. 6, we can also see that the boundary $\lambda_P = \Lambda_P(\lambda_S)$ is non-decreasing with respect to the secondary density $\lambda_S$ and is upper bounded by 1. This verifies Theorem 2 (i) and (ii). Besides, $\Lambda_P(\lambda_S)$ equals zero when $\lambda_S < 1.83$, and intersects the $\lambda_S$-axis at some point $\lambda_S = \lambda_{c3} \approx 1.88$. Recall that by Fig. 4, we know $\lambda_{c2} = 1.83$. This verifies Theorem 2 (iii).
C. Transmission Delay

In this part, we verify the scaling behavior of minimum transmission delay $T(u, v)$ with respect to distance $d(u, v)$ between two secondary users $u$ and $v$ in a connected secondary network. First, we consider the case in which propagation delay is negligible, i.e., $\tau = 0$. By Fig. 6, we choose $\lambda_S = 2.4$ and $\lambda_P = 0.01$, which corresponds to a secondary network with instantaneous connectivity. Fig. 7a illustrates the delay-to-distance ratio $\frac{T(u,v)}{d(u,v)}$ versus the source-destination distance $d(u,v)$ at $\tau = 0$, $\lambda_S = 2.4$ and $\lambda_P = 0.01$. From Fig. 7a, it can be seen that $\frac{T(u,v)}{d(u,v)}$ converges to zero as $d(u,v)$ increases. In other words, transmission delay scales sub-linearly with respect to distance if a connected secondary network has instantaneous connectivity. This verifies Theorem 4 (i). On the other hand, by Fig. 6, we choose $\lambda_S = 2.4$ and $\lambda_P = 0.3$, which corresponds to a secondary network with long-term connectivity. Fig. 7b illustrates the delay-to-distance ratio $\frac{T(u,v)}{d(u,v)}$ versus the source-destination distance $d(u,v)$ at $\tau = 0$, $\lambda_S = 2.4$ and $\lambda_P = 0.3$. From Fig. 7b, we can see that $\frac{T(u,v)}{d(u,v)}$ approaches a constant ($\approx 1.05$) as $d(u,v)$ increases. In other words, transmission delay scales linearly with respect to distance when the secondary network has long-term connectivity. This verifies Theorem 4 (ii) with $\gamma = 1.05$.

Next, we consider the case in which propagation delay is small but nonnegligible, i.e., $0 < \tau < 1$. In our simulations, we consider two different values of propagation delay, i.e., $\tau = 0.2$ and $\tau = 0.5$. We choose $\lambda_S = 2.4$ and $\lambda_P = 0.01$ for a secondary network with instantaneous connectivity. Fig. 7c illustrates the delay-to-distance ratio $\frac{T(u,v)}{d(u,v)}$ versus the source-destination distance $d(u,v)$ at $\tau = 0.2$ ($\tau = 0.5$), $\lambda_S = 2.4$ and $\lambda_P = 0.01$. From Fig. 7c, we can see that as $d(u,v)$ increases, $\frac{T(u,v)}{d(u,v)}$ approaches 0.25 and 0.59 for $\tau = 0.2$ and $\tau = 0.5$, respectively. In other words, transmission delay scales linearly with respect to distance if a connected secondary network has instantaneous connectivity. This verifies Theorem 5 (i) with $\gamma_1(0.2) = 0.25$ and $\gamma_1(0.5) = 0.59$. On the other hand, we choose $\lambda_S = 2.4$ and $\lambda_P = 0.3$ for the secondary network with long-term connectivity. Fig. 7d illustrates the delay-to-distance ratio $\frac{T(u,v)}{d(u,v)}$ versus the source-destination distance $d(u,v)$ at $\tau = 0.2$ ($\tau = 0.5$), $\lambda_S = 2.4$ and $\lambda_P = 0.3$. Fig. 7d shows that $\frac{T(u,v)}{d(u,v)}$ approaches 1.62 for $\tau = 0.2$ and 2.23 for $\tau = 0.5$ as $d(u,v)$ increases. In other words, transmission delay scales linearly with respect to distance when the secondary network has long-term connectivity. This verifies Theorem 5 (ii) with $\gamma_2(0.2) = 1.62$ and $\gamma_2(0.5) = 2.23$.

Note that from Fig. 7c and Fig. 7d, we can observe that for $\tau = 0.2$, $\frac{\tau}{\lambda_P} = 0.14 \leq \gamma_1(\tau) = 1.62$; for $\tau = 0.5$, $\frac{\tau}{\lambda_P} = 0.36 > \gamma_1(\tau) = 2.23$ and $\gamma_1(\tau) < \gamma_2(\tau)$. This verifies $\frac{\tau}{\lambda_P} \leq \gamma_1(\tau) < \gamma_2(\tau) < \infty$ in Theorem 5. In addition, combining Fig. 7c and Fig. 7d, we can also see that both $\gamma_1(\tau)$ and $\gamma_2(\tau)$ are non-decreasing with respect to $\tau$. This also verifies Theorem 5.

VI. CONCLUSION

In this paper, we study percolation-based connectivity and transmission delay of secondary users in large-scale CR ad hoc networks with unreliable secondary links. By introducing two auxiliary random graphs and using continuum percolation, we first study the impacts of key system parameters on connectivity of secondary networks. Using the ergodic theorem, we then study the scaling behavior of transmission delay with respect to the distance between two randomly chosen users in a connected secondary network for two cases, i.e., with propagation delay and without propagation delay. Using numerical simulations, we verify the theoretical analysis of connectivity and transmission delay.

APPENDIX A

PROOF OF THEOREM 1

Proof: When $\lambda_S < \lambda_{c1}$, there are an infinite number of finite connected components in $\mathcal{G}_S(\lambda_S; h)$ a.s. Thus, for two randomly chosen secondary nodes $u$ and $v$, there exists no path formed by links in $\mathcal{G}_S(\lambda_S; h)$ a.s. Since $\mathcal{G}_S(\lambda_S; g; \mathcal{G}_P; r_p, r_s)$ is a subgraph of $\mathcal{G}_S(\lambda_S; h)$, there is no path formed by communication links between two randomly chosen secondary nodes $u$ and $v$ a.s., i.e., $T(u,v) = \infty$ a.s.

When $\lambda_S > \lambda_{c1}$, there exists a unique infinite connected component in $\mathcal{G}_S(\lambda_S; h)$ a.s., which contains a strictly positive portion of all the secondary nodes. For any two nodes $u$ and $v$ residing in the infinite connected component, there is at least one path $\pi$ between $u$ and $v$ formed by finite number of links $\{l_i(u_i, u_{i+1})\}$ in $\mathcal{G}_S(\lambda_S; h)$. Then the minimum transmission delay $T(u,v)$ among all paths connecting $u$ and $v$ is upper bounded by the transmission delay $T(\pi)$ along the path $\pi$. Next, we only need to establish the finiteness of the transmission delay in one single link $l_i(u_i, u_{i+1})$.

Lemma 1: For a secondary network $\mathcal{G}_S(\lambda_S; g; \mathcal{G}_P; r_p, r_s)$, the transmission delay $T(d_i)$ for any two nodes $u_i$ and $u_{i+1}$ satisfying $d_i = d(u_i, u_{i+1}) < R_S$ is finite a.s.

Proof of Lemma 1: To prove the a.s. finiteness of $T(d_i)$ for a single link $l_i$ with length $d_i$ in $\mathcal{G}_S(\lambda_S; h)$, we show that the waiting delay for $l_i$ to become a communication link is finite a.s. We denote the probability that $l_i$ is a communication link by $\xi(d_i)$. Let $\mathbb{E}(u, r, r_x/t_x)$ denote the event that there exists no active primary receivers/transmitters within the circle of radius $r$ centered at the secondary user $u$, and $\mathbb{E}(d_i)$ denotes the event that there exists a reliable link for two secondary users with a distance $d_i$ in $\mathcal{G}_S(\lambda_S; g)$. According to Definition 1, we have $\xi(d_i) = \Pr(\mathbb{A}(u_i, r_s, r_x) \cap \mathbb{A}(u_{i+1}, r_s, r_x) \cap \mathbb{A}(u_i, r_t, t_x) \cap \mathbb{A}(u_{i+1}, r_t, t_x) \cap \mathbb{E}(d_i)) = \Pr(\mathbb{A}(u_i, r_t, t_x) \cap \mathbb{A}(u_{i+1}, r_t, t_x) \cap \mathbb{E}(d_i))$. Since active primary receivers form a Poisson point process $\mathcal{X}_{PR}$ with density $\eta_1 \lambda_P$, we have

$$\Pr(\mathbb{A}(u_i, r_t, t_x) \cap \mathbb{A}(u_{i+1}, r_t, t_x)) = \exp(-\eta_1 \lambda_P (2\pi r_x^2 - |A_1(d_i, r_S)|))$$

where $|A_1(d_i, r_S)|$ is the Lebesgue measure of intersection $A_1$ of two circles with radii $r_S$ and centered $d_i$ apart (Fig. 8 (a)). Active primary users also form a Poisson point process $\mathcal{X}_{PR}$ with density $\eta_1 \lambda_P$. We remove the active primary users whose corresponding receivers are within the interference.
positive, variable with parameter passage time, according to the connection function $r x$

$$P_r(\text{centered at } u)$$

Since waiting delay $T(u, v)$ versus the source-destination distance.

Combining (6) (7) and (8), we get the probability $\tau^2$.

Next, we prove that $\theta_3(\lambda_S, \lambda_P)$ is non-increasing with respect to $\lambda_P$. Consider two secondary networks $G_S(\lambda_S; g; G_P; r, r_S)$ and $G_S(\lambda_S; g; G_P; r, r_S)$ with $\lambda_S < \lambda_S$. By superposition theorem[15], the Poisson point process $\mathbb{X}_S$ with density $\lambda_S$ is statistically equivalent to the supposition of $\mathbb{X}_S$. Then any realisation of $G(\lambda_S; g; G_P; r, r_S)$ can be generated by adding more secondary nodes and thus more communication links to a realisation of $G_S(\lambda_S; g; G_P; r, r_S)$. If follows that if an infinite connected component exists in $G_S(\lambda_S; g; G_P; r, r_S)$, so will $G_S(\lambda_S; g; G_P; r, r_S)$. Thus, $\theta_3(\lambda_S, \lambda_P) < \theta_3(\lambda_S, \lambda_P)$ for $\lambda_S < \lambda_S$.

Next, we prove that $\theta_3(\lambda_S, \lambda_P)$ is non-increasing with respect to $\lambda_P$. Consider two secondary networks $G_S(\lambda_S; g; G_P; r, r_S)$ and $G_S(\lambda_S; g; G_P; r, r_S)$ with the same parameters except $\lambda_P < \lambda_P$. Similarly, any realisation of $G_S(\lambda_S; g; G_P; r, r_S)$ can be generated by independently removing primary transmitters and corresponding receivers in a realisation of $G_S(\lambda_S; g; G_P; r, r_S)$ with probability $1 - \frac{\lambda_S}{\lambda_P}$. Thus, $G_S(\lambda_S; g; G_P; r, r_S)$ have the same secondary nodes but more communication links than $G_S(\lambda_S; g; G_P; r, r_S)$. If follows that $\theta_3(\lambda_S, \lambda_P) < \theta_3(\lambda_S, \lambda_P)$ for $\lambda_P < \lambda_P$.
Proof of Lemma 2 (ii): To prove Lemma 2 (ii), it suffices to show that \( \theta(\lambda_S, \lambda_P) = 0 \) for the case in which we only consider the impacts of active primary transmitters. By Definition 1, within the distance \( r_P \) of each active primary transmitter, secondary nodes cannot see spectrum opportunities. For each communication link \( l_i \) with length \( d_i \), the minimum distance between the communication link and active primary transmitters is denoted by \( d_0(d_i) \). Since in reality, the transmission power of primary transmitters is usually several times larger than that of secondary users, we only focus on the case with \( r_P > R_S \). Then, we get the lower bound of \( d_0(d_i) \) by \( d_0(d_i) = (r_P^2 - d_i^2/4)^{\frac{1}{2}} \geq (r_P^2 - R_S^2/4)^{\frac{1}{2}} \). Therefore, each active primary transmitter has to keep at least \( d_0(R_S) = (r_P^2 - R_S^2/4)^{\frac{1}{2}} \) away from any communication links. Thus, we establish a Boolean Model \( B(X_{PT}, d_0(R_S), \eta \lambda_P) \), where nodes (active primary transmitters) are distributed following a Poisson point process \( X_{PT} \) with density \( \eta \lambda_P \) and each node occupies a disk with radius \( d_0(R_S) \). Then, we simplify the impacts of primary network on secondary users by only considering \( B(X_{PT}, d_0(R_S), \eta \lambda_P) \). By [4], there is a critical density \( \frac{\lambda_{c1}}{4d_0(R_S)^2} \) such that a unique infinite occupied component exists a.s. for \( \eta \lambda_P > \frac{\lambda_{c1}}{4d_0(R_S)^2} \), where \( \lambda_{c1} \) is the critical density for \( B(X_{PT}, \frac{1}{2}, \eta \lambda_P) \) with disk radius \( \frac{1}{2} \) (i.e., unit transmission range). Next, we prove that for \( \eta \lambda_P > \frac{\lambda_{c1}}{4d_0(R_S)^2} \), probability \( \theta_3(\lambda_S, \lambda_P) = 0 \).

We place a square lattice \( L \) with edge length \( d_e \) on \( \mathbb{R}^2 \). Consider a sequence of annuli \( \{G_i\}, i \geq 1 \) around the origin. Each annulus \( G_i \) consists of four rectangles, as illustrated in Fig. 9.

![Annulus Diagram](image)

Fig. 9. A sequence of annuli \( \{G_i\}, i \geq 1 \) around the origin.

\[
\begin{align*}
A_i^+ &= \left[ -\frac{d_e}{2}, \frac{d_e}{2} \right] \times \left[ -\frac{d_e}{2}, \frac{d_e}{2} \right], \\
A_i^- &= \left[ -\frac{d_e}{2}, \frac{d_e}{2} \right] \times \left[ -\frac{d_e}{2}, -\frac{d_e}{2} \right], \\
B_i^+ &= \left[ -\frac{d_e}{2}, \frac{d_e}{2} \right] \times \left[ -\frac{d_e}{2}, \frac{d_e}{2} \right], \\
B_i^- &= \left[ -\frac{d_e}{2}, \frac{d_e}{2} \right] \times \left[ -\frac{d_e}{2}, -\frac{d_e}{2} \right].
\end{align*}
\]

Let \( \tilde{A}_i^+ (\tilde{A}_i^-) \) be the event that \( A_i^+ (A_i^-) \) is crossed from left to right by a connected component in \( B(X_{PT}, d_0(R_S), \eta \lambda_P) \). Similarly, let \( \tilde{B}_i^+ (\tilde{B}_i^-) \) be the event that \( B_i^+ (B_i^-) \) is crossed from top to bottom by a connected component in \( B(X_{PT}, d_0(R_S), \eta \lambda_P) \). Let \( \tilde{G}_i \) be the event that \( A_i^+, A_i^-, B_i^+, B_i^- \) occur simultaneously. [17] shows that when \( \eta \lambda_P > \frac{\lambda_{c1}}{4d_0(R_S)^2} \), there exists \( j < \infty \) such that \( \tilde{G}_j \) occurs with the probability 1. This indicates that there must exist an annulus containing a continuous occupied region of primary transmitters that surrounds the origin. In this case, the connected component containing the origin \( C_{O_A} \) is limited to a finite region. Thus, when \( \eta \lambda_P > \frac{\lambda_{c1}}{4d_0(R_S)^2} \), we have \( \theta_3(\lambda_S, \lambda_P) = \text{Pr}(|C_{O_A}| = \infty) = 0 \).

Proof of Lemma 2 (iii): Since \( \lambda_{c2} \) is the critical density of \( G_S(\lambda_S; g) \), \( \theta_2(\lambda_S, g) = 0 \) when \( \lambda_S < \lambda_{c2} \). Note that \( G_S(\lambda_S; g; G_P; r_P, R_S) \) is a subgraph of \( G_S(\lambda_S; g) \). Thus, we have \( \theta_2(\lambda_S, \lambda_P) \leq \theta_2(\lambda_S, g) = 0 \) when \( \lambda_S < \lambda_{c2} \). Thus, Lemma 2 (iii) is clear.

Proof of Lemma 2 (iv): Let \( \epsilon = \min\{g(d), 0 < d \leq R_S\} \). We construct a new secondary network \( G_S(\lambda_S; g'; G_P; r_P, R_S) \) with the connection function \( g'(d) \) defined as
\[
g'(d) = \begin{cases} 
\epsilon, & 0 < d \leq R_S \\
0, & d > R_S 
\end{cases}
\]

Thus, each secondary link in the new secondary network \( G_S(\lambda_S; g'; G_P; r_P, R_S) \) is reliable with probability \( \epsilon \). Since \( g'(d) \leq g(d), \forall 0 < d \leq R_S \), by the coupling argument, the existence of an infinite connected component formed by communication links in \( G_S(\lambda_S; g'; G_P; r_P, R_S) \) implies the existence of such an infinite connected component in \( G_S(\lambda_S; g; G_P; r_P, R_S) \). Now consider a site percolation process in \( G_S(\lambda_S; h; G_P; r_P, R_S) \), where each secondary node is independently open with probability \( \epsilon \). Secondary links following BM are open between open nodes. For closed nodes, all the associated secondary links are closed. By the Thinning Theorem [15], the resulting graph is an infinite connected component satisfying the condition \( \epsilon \lambda_S > \lambda_{c1} \), where \( \lambda_{c1} \) is the critical density for \( G_S(\epsilon \lambda_S; h) \). It is known that the occurrence of site percolation implies the occurrence of bond percolation[18]. [19]. Then when \( G_S(\epsilon \lambda_S; h; G_P; r_P, R_S) \) has an infinite connected component, \( G_S(\lambda_S; g'; G_P; r_P, R_S) \) also has an infinite one. Since \( G_S(\lambda_S; g'; G_P; r_P, R_S) \) can be treated as a subgraph of \( G_S(\lambda_S; g; G_P; r_P, R_S) \), there exists \( \lambda_P > 0 \) such that \( \theta_3(\lambda_S, \lambda_P) > 0 \) when \( \lambda_S > \lambda_{c2} \) and \( \lambda_P < \lambda_P \).

By Lemma 2 (i), we establish the monotonocity of \( \theta_3(\lambda_S, \lambda_P) \) with respect to \( \lambda_S \) and \( \lambda_P \). Suppose for some \( \lambda_S \) and \( \lambda_P, \theta_3(\lambda_S, \lambda_P) = 0 \). Then for any \( \lambda_S < \lambda_S \), we also have \( \theta_3(\lambda_S, \lambda_P) = 0 \). Since \( \lambda_P(\lambda_S) \) is defined as the infimum \( \lambda_P \) satisfying \( \theta_3(\lambda_S, \lambda_P) = 0 \), we know \( \lambda_P(\lambda_S) \leq \Lambda(\lambda_S) \) when \( \lambda_S < \lambda_S \). Thus, Theorem 2 (ii) is proved.

Theorem 2 (ii) and (iii) follow directly from Lemma 2 (ii) and (iii), which state the upper bound of the boundary \( \Lambda(\lambda_S) \) and that \( \Lambda(\lambda_S) = 0 \) when \( \lambda_S < \lambda_{c2} \), respectively. Combining Lemma 2 (ii), (iii), (iii) and (iv), we complete the proof of Theorem 2 (iii).

APPENDIX C

PROOF OF THEOREM 3

Proof: The proof of Theorem 3 is very straightforward. Since \( \Lambda(\lambda_S) \) is defined as the infimum \( \lambda_P \) satisfying
$\theta_3(\lambda_s, \lambda_P) = 0$, we have $\theta_3(\lambda_s, \lambda_P t) > 0$ when $\lambda_P < \Lambda_P(\lambda_s)$. Thus, by Definition 4, the secondary network has instantaneous connectivity when $\lambda_P < \Lambda_P(\lambda_s)$. According to Lemma 2, $\theta_3(\lambda_s, \lambda_P)$ is non-increasing with respect to $\lambda_P$. Thus, for any $\lambda_P > \Lambda_P(\lambda_s)$, $\theta_3(\lambda_s, \lambda_P t) = 0$. Therefore, by Definition 5, the secondary network has long-term connectivity when $\lambda_P > \Lambda_P(\lambda_s)$.

**APPENDIX D**

**PROOF OF THEOREM 4**

Proof: Consider any two secondary nodes $u, v \in C(\mathcal{G}_s(\lambda_s; h))$. The proof of Theorem 4 (i) is based on the Subadditive Ergodic Theorem [20]. First, using the scaling argument [4, Chapter 2.2], we can scale the whole CR networks to get a unit transmission range ($R_s = 1$) without loss of generality. Suppose the secondary network is in the long-term connectivity region. Then $u, v$ cannot lie in the same connected component formed by communication links as $d(u, v) \to \infty$. We set $u$ as the origin, and the line connecting $u$ and $v$ as the x-axis. For every $(i, 0)$ on the x-axis, we denote the node in $C(\mathcal{G}_s(\lambda_s; h))$ with the smallest Euclidean distance to $(i, 0)$ by $w_i \triangleq \arg \min_{w \in C(\mathcal{G}_s(\lambda_s; h))} d(w, (i, 0))$. Then $u = w_0$. Let $(n, 0)$ be the closest integer coordinate to $v$. Since $n - 1 < d(u, v) < n + 1$, we have

$$
\frac{T(w_0, w_n) - T(w_n, v)}{n + 1} \leq \frac{T(u, v) - T(w_0, w_n) + T(w_n, v)}{n - 1}.
$$

If $w_n = v$, then $T(w_n, v) = 0$. If $w_n \neq v$, since $d(w_n, (n, 0)) < d(v, (n, 0)) \leq \frac{1}{2}$, we have $d(w_n, v) < d(w_n, (n, 0)) + d(v, (n, 0)) \leq 1 = R_s$, and by Lemma 1, $T(w_n, v) < \infty$.

Let $T_m, n \triangleq T(w_m, w_n)$, $0 \leq m \leq n$. Then, we only need to prove $0 < \lim_{n \to \infty} \frac{T_m, n}{n} = \gamma < \infty$ by the following two lemmas.

**Lemma 3:** Let $\gamma \triangleq \lim_{n \to \infty} \max_{T_m, n} = 1$, then $\gamma = \inf_{n \geq 1} \frac{E[T_m, n]}{n}$ and $\lim_{n \to \infty} \frac{T_m, n}{n} = \gamma$ a.s.

**Lemma 4:** $0 < \gamma = \lim_{n \to \infty} \frac{T_m, n}{n} < \infty$.

**Proof of Lemma 3:** We prove Lemma 3 by using the Subadditive Ergodic Theorem [20].

**Lemma 5:** [20, Theorem 1.10] Let $\{S_{m, n}\}$ be a collection of random variables indexed by integers $0 \leq m < n$. Suppose $\{S_{m, n}\}$ has the following properties: (i) $S_{m, n} \leq S_{m, n+k} + S_{m+k, n}$; (ii) $\{S_{m, n+k, n}\}$ is a stationary process for each $k$; (iii) $S_{m, n+k, k \geq 0} = S_{m+1, n+k+1, k \geq 0}$ in distribution for each $m$; (iv) $\frac{E[S_{m, n}]}{n} \leq \gamma$ for each $n$.

Then (a) $\gamma \triangleq \lim_{n \to \infty} \frac{E[S_{m, n}]}{n} = \inf_{n \geq 1} \frac{E[S_{m, n}]}{n}$; $\gamma \triangleq \lim_{n \to \infty} \frac{S_{m, n}}{n}$ exists a.s. and $E[S] = \gamma$. Furthermore, if (v) the stationary process in (ii) is ergodic, then (b) $S = \gamma$ a.s.

To prove Lemma 3, it suffices to show that $T_m, n \leq m < n$ satisfies the conditions (i)-(v). Since $T_m, n$ represents the minimum transmission delay along all paths connecting $w_m$ and $w_n$, it is obvious that condition (i) holds. By the stationarity of the CR network model, conditions (ii) and (iii) also hold. Now, we prove that conditions (iv) and (v) are satisfied. First, by giving the following lemmas from [17], we prove $E[T_{m, n}] < \infty$ for each $n$.

**Lemma 6:** [17] For each $n < \infty$, $d(w_0, w_n) < \infty$.

**Lemma 7:** [17] For any two nodes $u$ and $v$ in the infinite connected component $\mathcal{C}(\mathcal{G}_s(\lambda_s; h))$, let $Q(u, v)$ denote the minimum number of links among paths in $\mathcal{G}_s(\lambda_s; h)$ connecting node $u$ to node $v$. If $d(u, v) < \infty$, then $Q(u, v) < \infty$.

Since $w_0, w_n \in C(\mathcal{G}_s(\lambda_s; h))$, there exists a path $\pi(w_0, w_n)$ connecting $w_0$ to $w_n$ with the minimum number of links in $\mathcal{G}_s(\lambda_s; h)$. By Lemma 6 and Lemma 7, we get the minimum number of links $Q(w_0, w_n) < \infty$. Let $T_{m, n}$ denote the maximum expected single-link delay for all link lengths. By Appendix A, the probability that a communication link exists is strictly positive for all $d \in (0, R_s]$. We can easily get that $T_{m, n} < \infty$. Thus, we have $E[T_{m, n}] \leq E[T(\pi(w_0, w_n))] \leq T_{m, n}Q(u, v) < \infty$. Then condition (iv) holds.

Next we show condition (v) holds by proving that $T_{(n-1)k, nk, n \geq 1}$ is mixing. Consider $T_{(n-1)k, nk}$ and $T_{(n-1)k, (n+1)k}$. We construct two sets of square annuli, $\{G_j\}$ and $\{G_j'\}$, centered at $(\frac{2n-1}{2}, k, 0)$ and $(\frac{2n-1}{2}, k, 0)$, respectively. By the proof of Lemma 2 in Appendix B, we know that there exists a path from $w_{(n-1)k} (w_{(n-1)k+1})$ to $w_{nk} (w_{nk+m+1})$ that is surrounded by some annulus $G_j (G_j')$ with finite $j$ ($j'$). As $m \to \infty$, the two annuli $G_j$ and $G_j'$ are so separated that they neither share the secondary links nor are affected by the same active primary users. Thus, $T_{(n-1)k, nk}$ and $T_{(n-1)k, (n+1)k}$ get independent of each other as $m \to \infty$. Then we have $\lim_{m \to \infty} |Pr(T_{(n-1)k, nk} < t \cap T_{(n-1)k, (n+1)k} < t') - Pr(T_{(n-1)k, nk} < t)Pr(T_{(n-1)k, (n+1)k} < t')| = 0$. Then condition (v) holds and we conclude Lemma 3.

**Proof of Lemma 4:** By Lemma 3, we know that $\gamma = \lim_{n \to \infty} \frac{E[T_{m, n}]}{n} = \inf_{n \geq 1} \frac{E[T_{m, n}]}{n} \leq E[T(0, 1)] < \infty$. Thus the finiteness of $\gamma$ is proved. Lemma 4 [16] shows that $\gamma > 0$ for the special case in which $g(d) = h(d)$. Denote $\gamma$ in this case as $\gamma_h$. Considering the unreliability of the secondary links, it is obvious that $\gamma \geq \gamma_h$.

To prove Theorem 4 (ii), we consider a secondary network with instantaneous connectivity. Then there exists a unique infinite connected connected component formed by communication links in $\mathcal{G}_s(\lambda_s; g; G; r_P, r_s)$ at time $t$, denoted by $\mathcal{C}_t$, a.s. Since $Pr(u \in \mathcal{C}_t) > 0$ and independent of $t$, there exists $t_0 < \infty$ such that $u \in \mathcal{C}_t_0$. At $t = t_0$, if the other node $v \in \mathcal{C}_t_0$, $u$ can communicate with $v$ instantaneously and $T(u, v) = t_0 < \infty$. If the other node $v \in C(\mathcal{G}_s(\lambda_s; h)))$ but $\notin \mathcal{C}_t_0$, we can find a node $w$ in $\mathcal{C}_t_0$ which has the smallest Euclidean distance to $v$, i.e., $w \triangleq \arg \min_{w \in \mathcal{C}_t_0} d(w, v)$. Then, we can construct a path $\pi(w, u)$ formed by communication links in $\mathcal{C}_t_0$, and another path $\pi(w, v)$ with the minimum number of links in $C(\mathcal{G}_s(\lambda_s; h))$. Transmission delay $T(u, v)$ from $u$ and $v$ is upper bounded by $t_0 + T(\pi(u, w)) + T(\pi(w, v)) = t_0 + T(\pi(u, v))$.

Next, to prove Theorem 4 (ii), it suffices to show the finiteness of $T(\pi(u, v))$. First, we show that $d(w, v)$ is finite

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14 A measure preserving transformation $H$ on $(\Omega, F, P)$ is called strong mixing if for all measurable sets $A$ and $B$, $\lim_{n \to \infty} |Pr(A \cap H^{-n} B) - Pr(A) Pr(B)| = 0$. A sequence $\{X_n, n \geq 0\}$ is called strong mixing if the shift on sequence space is strong mixing [21].
Lemma 8: When the secondary network has instantaneous connectivity, denote the infinite connected component formed by communication links at time slot $t$ by $C_t$. For a node $v \in C_t$, node $w \in C_t$ is the node in $C_t$ with the smallest Euclidean distance to $v$. Then, we have $d(w, v) < \infty$ a.s.

Proof of Lemma 8: The proof is similar to [16] and is omitted due to space limitation.

According to Lemma 7 and Lemma 8, the number of links $Q(\pi(w, v))$ between $u$ and $v$ in graph $G_S(\lambda_S; h)$ is finite, which results in $E[T(\pi(w, v))] < \infty$. Consequently, $T(\pi(w, v)) < \infty$ a.s. Thus, we have $T(u, v) < \infty$ a.s. when the secondary network has instantaneous connectivity. This completes the proof of Theorem 4 (ii).

APPENDIX E

PROOF OF THEOREM 5

Proof: Denote the infinite connected component formed by communication links at time slot $t$ by $C_t$. By the Subadditive Ergodic Theorem, Theorem 5 (i) for the case in which for some $t_0$, $u$ and $v$ are both in $C_{t_0}$ and Theorem 5 (ii) can be proved using similar techniques in the proof of Theorem 4 (ii). Thus, when the secondary network has instantaneous connectivity, for two nodes $u$ and $v$ in $C_{t_0}$, we have

$$\lim_{d(u, v) \to \infty} \frac{T(u, v)}{d(u, v)} = \gamma_1(\tau).$$

To prove Theorem 5 (i) for the case in which $u \in C_{t_0}$ and $v \in C(G_S(\lambda_S; h))$ but $v \notin C_{t_0}$, we consider $w \in C_{t_0}$ with the smallest Euclidean distance to $v$. Then, we have

$$\lim_{d(u, v) \to \infty} \frac{T(u, v)}{d(u, v)} = \gamma_1(\tau) + \lim_{d(u, v) \to \infty} \frac{T(w, v)}{d(u, v)} = \gamma_1(\tau) + \gamma_2(\tau) = \gamma_1(\tau) + \gamma_2(\tau) = \gamma_1(\tau),$$

which is strictly positive. Thus, the proof of Theorem 5 (i) is proved.

Using the coupling argument, $\gamma_1(\tau)$ and $\gamma_2(\tau)$ are decreasing with respect to $\tau$. Since $\gamma_1(\tau)$ and $\gamma_2(\tau)$ are strictly positive for $\tau \in (0, 1)$, $\lim_{\tau \to 0} \gamma_1(\tau)$ and $\lim_{\tau \to \infty} \gamma_2(\tau)$ exist and is equal to 0. (ii). In the instantaneous connectivity and long-term connectivity regions, at each time slot, messages can travel at most $1/R_S$ links, i.e., at most distance $R_S$. Thus, we get the lower bound of delay-to-distance ratios $\gamma_1(\tau)$ and $\gamma_2(\tau)$ as

$$\gamma_1/\tau = R_S.$$

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